

# Demand Estimation

## *MIXTAPE SESSION*

Jeff Gortmaker and Ariel Pakes



## Last Class

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subject to  $s_{jt} = \sum_{i \in \mathcal{I}_t} w_{it} \cdot \frac{\exp[\delta_{jt} + \mu_{ijt}(\theta)]}{1 + \sum_{k \in \mathcal{J}_t} \exp[\delta_{kt} + \mu_{ikt}(\theta)]}$

- On day 2, adding preference heterogeneity  $\mu_{ijt}$  gave more realistic substitution patterns.

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  - Most common form is  $\mu_{ijt} = x'_{jt}(\Sigma\nu_{it} + \Pi y_{it})$  for  $\nu_{it} \sim N(0, I)$  and  $y_{it}$  from census data.
  - Implements random coefficients  $\beta_{it} \sim N(\beta + \Pi y_{it}, \Sigma\Sigma')$  on characteristics  $x_{jt}$  in utility.

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- This required adding consumer type  $i$  data to supplement our product  $j$  data from day 1.
- Let's go over your second coding exercise.

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  - Same cereals in each market, so no choice set variation along mushy dimension.
  - Results in unrealistically limited substitution between similar cereals.
- Also can't estimate a parameter in  $\Pi$  on log income alone.
  - Market fixed effects are collinear with market-level income means.
  - Unrealistic that overall cereal preference doesn't vary with income.

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- “Micro data” has information about individual choices, not just market-level quantities.
- Typical example is consumer survey data.
  - Internal surveys conducted by firms.
  - Ad-hoc surveys conducted by academics.
  - Marketing research datasets (e.g. NielsenIQ’s Consumer Panel).
  - Regulatory agencies like the UK’s antitrust authority (Reynolds and Walters, 2008).

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  - Within-mushy substitution is precisely what we hope this parameter will increase!
- Let’s incorporate answers to these questions into estimation.
  - We’ll set up a general framework and come back to these when we have notation to do so.



# Roadmap

Micro BLP Estimation

Choosing Micro Moments

Using More Information

Coding Exercise 3

## Micro BLP Estimator

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  - Ratios (e.g. mean income given mushy), correlations (e.g. between income and price), etc.
- The resulting “micro BLP” estimator is used a lot in industrial organization.

# Micro BLP Popularity

- First popularized by  
Petrin (2002) and BLP (2004).

Paper	Demand Estimation		
	Industry	Country	Years
Petrin (2002)	Automobiles	United States	1981–1993
Berry, Levinsohn, and Pakes (2004)	Automobiles	United States	1993
Thomadsen (2005)	Fast Food	United States	1999
Goeree (2008)	Personal Computers	United States	1996–1998
Ciliberto and Kuminoff (2010)	Cigarettes	United States	1993–2002
Nakamura and Zerom (2010)	Coffee	United States	2000–2004
Beresteanu and Li (2011)	Automobiles	United States	1999–2006
Li (2012)	Automobiles	United States	1999–2006
Copeland (2014)	Automobiles	United States	1999–2008
Starc (2014)	Health Insurance	United States	2004–2008
Ching, Hayashi, and Wang (2015)	Nursing Homes	United States	1999
Li, Xiao, and Liu (2015)	Automobiles	China	2004–2009
Nurski and Verboven (2016)	Automobiles	Belgium	2010–2011
Barwick, Cao, and Li (2017)	Automobiles	China	2009–2011
Murry (2017)	Automobiles	United States	2007–2011
Wollmann (2018)	Commercial Vehicles	United States	1986–2012
Li (2018)	Automobiles	China	2008–2012
Li, Gordon, and Netzer (2018)	Digital Cameras	United States	2007–2010
Backus, Conlon, and Sinkinson (2021)	Cereal	United States	2007–2016
Grieco, Murry, and Yurukoglu (2021)	Automobiles	United States	1980–2018
Neilson (2021)	Primary Schools	Chile	2005–2016
Armitage and Pinter (2022)	Automobiles	United States	2009–2017
Döpper, MacKay, Miller, and Stiebale (2022)	Retail	United States	2006–2019
Durrmeyer (2022)	Automobiles	France	2003–2008
Weber (2022)	Trucks	United States	2010–2018
Bodéré (2023)	Preschools	United States	2010–2018
Montag (2023)	Laundry Machines	United States	2005–2015
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- Used a lot. But each paper has different notation.
- We'll use the standardized framework for PyBLP from [Conlon and Gortmaker \(2025\)](#).

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- There are two new components.
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  2. Their model analogues  $f(v(\theta)) = [f_1(v(\theta)), \dots, f_M(v(\theta))]'$ .
- Statistically, we need  $f(\bar{v}) \rightarrow f(v(\theta_0))$  as the micro dataset expands.
  - This gives what we'll call  $m = 1, \dots, M$  different "micro moments."
  - A bit different from our "aggregate moments"  $\mathbb{E}[\xi_{jt} \cdot z_{jt}] = 0$ .

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- Different weights  $w_{dijt}$ , values  $v_{pijt}$ , and functions  $f_m(\cdot)$  support most summary stats.

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- Second, you need to define your **micro part**  $p$ .
  - Micro values  $v_{pijt} = \text{income}_{it}$  implies  $\bar{v}_p$  is mean surveyed income.
  - In practice, you specify a second function to compute a matrix of values for each market  $t$ .

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- To match  $\bar{v}_1$ , the mean income of cereal purchasers, PyBLP needs some info.
- First, you need to define your **micro dataset**  $d$ .
  - Sampling weights  $w_{dijt} = 1\{j \neq 0\}$  imply only cereal purchasers were surveyed.
  - In practice, you specify a function to compute a matrix of weights for each market  $t$ .
  - You also need to specify the number of survey observations  $N_d = |\mathcal{N}_d|$ .
- Second, you need to define your **micro part**  $p$ .
  - Micro values  $v_{pijt} = \text{income}_{it}$  implies  $\bar{v}_p$  is mean surveyed income.
  - In practice, you specify a second function to compute a matrix of values for each market  $t$ .
- Lastly, you need to define your **micro moment**  $m$ .
  - The identity function  $f_m(\bar{v}_p) = \bar{v}_p$  just matches the mean surveyed income.
  - You also need to specify the actual value of the micro statistic  $\bar{v}_1$ .



# Model Analogues

$$f_m(\bar{v}_p) \rightarrow f_m(v_p(\theta_0))$$

- For each guess of  $\theta$ , PyBLP will compute the model analogue  $f_m(v_p(\theta))$ .

# Model Analogues

$$f_m\left(\frac{1}{N_d} \sum_{n \in \mathcal{N}_d} v_{pi_n j_n t_n}\right) \rightarrow f_m\left(\frac{\cdots v_{pijt}}{\cdots}\right)$$

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  4. Selected to be in the survey with known probability  $w_{di_n j_n t_n}$ .

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  - Start by choosing a single micro moment that “targets” the parameter.
- What if you could estimate a parameter with either aggregate or micro variation?
  - Could just choose the variation that seems more “credible.” Often the micro moment.
  - Can use both. Micro moments can reduce large SEs from limited aggregate variation.

# Targeting Micro Moments

- Simplest case: 1 characteristic  $x_{jt}$  (e.g. price) and 1 demographic  $y_{it}$  (e.g. income).

$$u_{ijt} = \beta_1 + \pi_1 y_{it} + (\beta_x + \pi_x y_{it}) x_{jt} + \xi_{jt} + \varepsilon_{ijt}$$

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  - Relationship between income and price targets how income shifts price sensitivity.
  - Other common examples include “ $\mathbb{E}[y_{it} \mid x_{jt} < \bar{x}]$ ” and “ $\mathbb{E}[x_{jt} \mid y_{it} < \bar{y}]$ .”

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  - Other common examples include “ $\mathbb{E}[y_{it} \mid x_{jt} < \bar{x}]$ ” and “ $\mathbb{E}[x_{jt} \mid y_{it} < \bar{y}]$ .”
- Micro data is not directly informative about “linear parameters”  $\beta_1$  or  $\beta_x$ .
  - Mean utility  $\delta_{jt} = \beta_1 + \beta_x x_{jt} + \xi_{jt}$  is already pinned down by market shares  $s_{jt}$ .

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- The literature has had success with incorporating second choice data (e.g. [BLP, 2004](#)).
  - Survey asks consumers which  $k_n \neq j_n$  they'd choose if their first choice weren't available.
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  - Micro weights and values now just have an extra index:  $w_{dijkt}$  and  $v_{pijkt}$ .
- Direct measures of substitution are very informative about  $\Sigma$ .
  - Recall the red bus/blue bus example that motivated adding preference heterogeneity.
  - Each second choice is like observing a new market with the first choice removed.

## Second Choice Moments

- Extend the simple example from before with unobserved preference heterogeneity:

$$u_{ijt} = \beta_1 + \sigma_1 \nu_{1it} + \pi_1 y_{it} + (\beta_x + \sigma_x \nu_{2it} + \pi_x y_{it}) x_{jt} + \xi_{jt} + \varepsilon_{ijt}$$

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- To target  $\sigma_x$ , we want a measure of how much people substitute within  $x_{jt}$ .
  - In your exercise, you'll match the share " $\mathbb{P}(\text{mushy}_{jt} \text{ and } \text{mushy}_{kt} \mid j \neq 0)$ ."
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- **Diversion ratios** are straightforward to interpret and collect.

# Outside Substitution and Market Size

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  - How many consumers will stop purchasing soda if all sodas are taxed?
  - How much larger will a market grow if innovation reduces production costs?

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- On day 1, we discussed how choosing a market size is neither easy nor innocuous.
  - Assuming a small market size  $M_t$  means assuming a small outside share  $s_{0t}$ .
  - Typically, this results in relatively high inside qualities  $\xi_{jt}$ .
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  - Typically, this results in relatively high inside qualities  $\xi_{jt}$ .
  - Implies little substitution to the outside good in counterfactuals.
- Directly matching an outside diversion ratio will help discipline outside substitution.
  - If  $\hat{\sigma}_1$  is large, many people will dislike all inside goods and usually choose  $j = 0$ .
  - This reduces the *effective* market size, helping to compensate for a too-large  $M_t$ .
  - See [Zhang \(2024\)](#) for more on how  $\sigma_1$  can help and other solutions.

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  - **Compatibility**: Aggregate and micro data may come from different sampling schemes.
  - **Clarity**: Matching a single statistic makes it clear where identification comes from.
- But adding more info can greatly increase the precision of our estimates.
  - Ideally we'd observe a complete micro dataset  $\{t_n, j_n, k_n, y_{i_n t_n}\}_{n \in \mathcal{N}_d}$ .

# Maximum Likelihood

- If we only had micro data, we may want to just work with its log likelihood:  
(technically, this likelihood is conditional on the aggregate data)

$$\log \mathcal{L}(\theta, \delta) = \sum_{n \in \mathcal{N}_d} \log \mathbb{P}(t_n, j_n, k_n, y_{i_n t_n} \mid n \in \mathcal{N}_d; \theta, \delta)$$

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  2. Run an IV regression of  $\hat{\delta}_{jt}$  on  $x_{jt}$  to recover linear parameters  $\hat{\beta}$  like in day 1.
- For a modern take on this “MLE” approach, see [Grieco, Murry, Pinkse and Sagl \(2025\)](#).
  - Combine 1, 2, and the likelihood for aggregate market shares into a single objective.
  - Their Julia package [Grumps.jl](#) efficiently handles the high-dimensional  $\delta = \{\delta_{jt}\}_{j,t}$ .

# Optimal Micro Moments

- In micro BLP, **optimal micro moments** match the first-order conditions in MLE:

$$f^*(\bar{v}) = \frac{1}{N_d} \sum_{n \in \mathcal{N}_d} \frac{\partial \mathbb{P}(t_n, j_n, k_n, y_{i_n t_n} \mid n \in \mathcal{N}_d; \theta)}{\partial \theta}$$

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- These use all the information in a micro dataset (**Conlon and Gortmaker, 2025**).
  - Intuition for statistical efficiency here is just that MLE is efficient.
- Can be a bit tricky to compute, but only a few lines of code with PyBLP.
  - Like optimal IVs, can update along with the weighting matrix for a second GMM step.

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  - Estimating a nesting parameter.
- I'll post the remaining solutions today after questions.

Good luck with estimating your own demand systems!

[j.gortmaker@nyu.edu](mailto:j.gortmaker@nyu.edu)

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