Machine Learning and Causal Inference

Mixtape Track

MIXTAPE SESSIONS

Allow me to introduce myself

- Economics professor at Brigham Young University in Utah

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- 4 kids, most of whom can now run and mountain bike faster than me

Allow me to introduce myself

- Economics professor at Brigham Young University in Utah
- 4 kids, most of whom can now run and mountain bike faster than me
- A big fan of causal inference in observational settings:
  - Quasi-experimental evaluations of the effects of unions (Frandsen 2016, 2017, 2021; Chen, Frandsen, Grabowski, Town, Sojourner 2015)
  - Distributional effects (Frandsen and Lefgren 2018, 2021; Frandsen, Froelich, Melly 2012)
- And of exploring machine learning in applied economics:
  - Teach Machine Learning for Economists at BYU
  - Research on the power of ML in empirical strategies (Angrist and Frandsen 2022)

Welcome to the Machine: Where we're going

Machine Learning + Causal Inference I

Day 1 (today)

- Prediction vs. Causality
- Conceptual and practical (python!) intro to supervised machine learning methods
  - Lasso
  - Ridge
  - Elastic nets
  - Random Forests
- Day 2: How modern prediction methods can be deployed in the service of causal inference
  - Post double selection lasso (PDS lasso)
  - Double/de-biased machine learning (DML)

Machine Learning + Causal Inference II (starts May 15)

- Predicting heterogeneous treatment effects
- Random Causal Forests

Prediction vs. Causality

Imagine you are a life insurance underwriter. You receive an application for life insurance from someone with the following characteristics:

- male

Prediction vs. Causality

Imagine you are a life insurance underwriter. You receive an application for life insurance from someone with the following characteristics:

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- age 67

Prediction vs. Causality

Imagine you are a life insurance underwriter. You receive an application for life insurance from someone with the following characteristics:

- male
- age 67
- high blood pressure

Prediction vs. Causality

Imagine you are a life insurance underwriter. You receive an application for life insurance from someone with the following characteristics:

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- high cholesterol

Prediction vs. Causality

Imagine you are a life insurance underwriter. You receive an application for life insurance from someone with the following characteristics:

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- high cholesterol
- family history of heart disease

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- and . . .

Prediction vs. Causality

Imagine you are a life insurance underwriter. You receive an application for life insurance from someone with the following characteristics:

- male
- age 67
- high blood pressure
- high cholesterol
- family history of heart disease
- and . . .
- was admitted to the hospital yesterday

Prediction vs. Causality

Now imagine you are a loved one of someone with the following characteristics:

- male

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Now imagine you are a loved one of someone with the following characteristics:

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- and . . .
- is having chest pains.

Prediction vs. Causality

Now imagine you are a loved one of someone with the following characteristics:

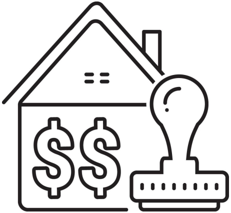
- male
- age 67
- high blood pressure
- high cholesterol
- family history of heart disease
- and . . .
- is having chest pains.
- Should you take him to the hospital?

Prediction vs. Causality: Purpose

Prediction vs. Causality: Purpose

Prepare

- A loan officer wants to know the likelihood of an individual repaying a loan based on income, employment, and other characteristics.



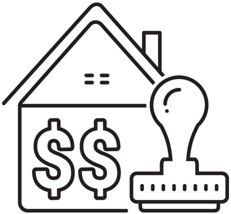
Prediction vs. Causality: Purpose

Prepare

- A loan officer wants to know the likelihood of an individual repaying a loan based on income, employment, and other characteristics.

Influence

- A mortgage lender wants to know if direct debit will increase loan repayments



Prediction vs. Causality: Purpose

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Influence

- In order to decide whether to invest in a start-up, an investor needs to know how likely the start-up is to succeed, given the entrepreneur's experience and the characteristics of the industry.

Prediction vs. Causality: Purpose

Prepare

- In order to decide whether to invest in a start-up, an investor needs to know how likely the start-up is to succeed, given the entrepreneur's experience and the characteristics of the industry.

Influence

- An entrepreneur needs to know what the effect of receiving funding from a private equity investor (rather than getting a loan) is on the ultimate success of an enterprise.

Prediction vs. Causality: Purpose

Prepare

Influence

- A bail hearing judge needs to know how likely a defendant is to flee before trial, given his or her charges, criminal history, and other characteristics

Prediction vs. Causality: Purpose

Prepare

- A bail hearing judge needs to know how likely a defendant is to flee before trial, given his or her charges, criminal history, and other characteristics

Influence

- A policy maker needs to know the effect of being released on bail (rather than detained) prior to trial on ultimate conviction

Prediction vs. Causality: Purpose

Prepare

Influence

- A home seller wants to know what price homes with the characteristics of his or her home typically sell for

Prediction vs. Causality: Purpose

Prepare

- A home seller wants to know what price homes with the characteristics of his or her home typically sell for

Influence

- A home seller wants to know by how much installing new windows will raise the value of his or her home

Prediction vs. Causality: Purpose

Prepare

Influence

- A Harvard admissions officer wants to know how likely an applicant with given credentials is to graduate in 4 years

Prediction vs. Causality: Purpose

Prepare

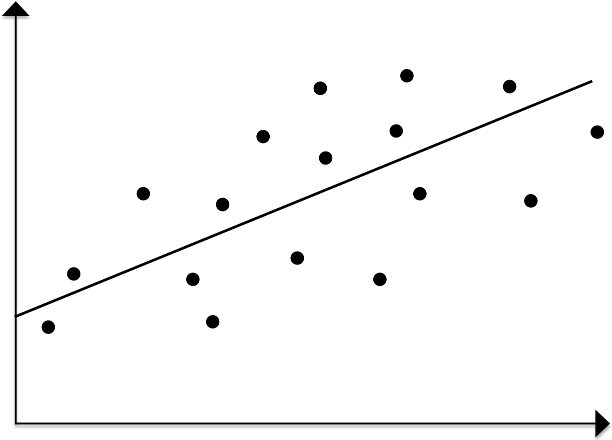
- A Harvard admissions officer wants to know how likely an applicant with given credentials is to graduate in 4 years

Influence

- A labor economist wants to know whether individuals of a certain ethnic background are less likely to get into Harvard than applicants with similar academic credentials

Prediction vs. Causality: Target

$$y_i = \alpha + \beta x_i + \varepsilon_i$$



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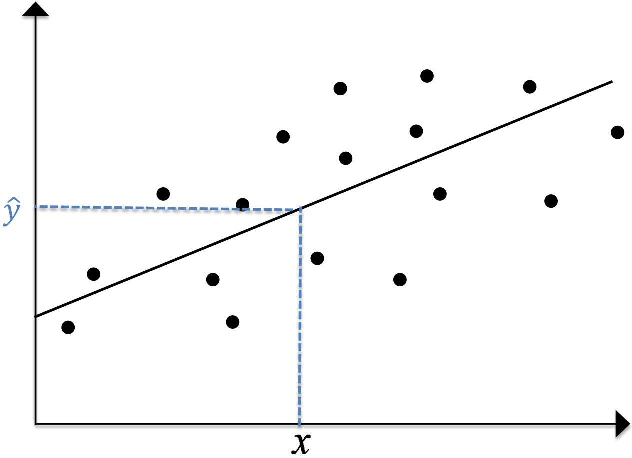

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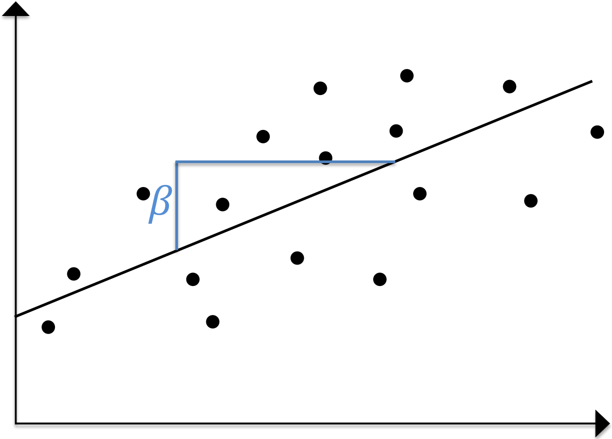

Prediction vs. Causality: Methods

Causality

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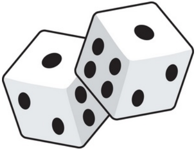
- Gold standard: RCT



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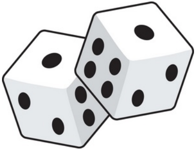

- Aluminum standard: Regression or IV strategies that approximate controlled experiments

Prediction vs. Causality: Methods

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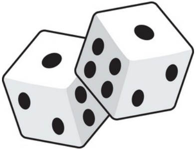

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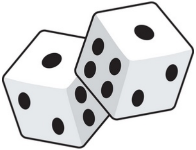

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Prediction

- Supervised machine learning algorithms

Prediction vs. Causality: Where shall the twain meet?

We've seen that prediction and causality

- answer different questions

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Different strokes for different folks, or complementary tools in an applied economist's toolkit?

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Different strokes for different folks, or **complementary tools in an applied economist's toolkit**?

- Illustrate using the Oregon Health Insurance Experiment (go to python)

Where ML fits into causal inference

Traditional regression strategy:

1. Regress Y_i on X_i and compute the residuals,

$$\begin{aligned}\tilde{Y}_i &= Y_i - \hat{Y}_i^{OLS}, \\ \hat{Y}_i^{OLS} &= X_i' \left(X'X\right)^{-1} X'Y\end{aligned}$$

2. Regress D_i on X_i and compute the residuals,

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3. Regress $\tilde{Y}_i$ on $\tilde{D}_i$.

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When OLS might not be the right tool for the job:

- there are many variables in X_i
- the relationship between X_i and Y_i or D_i may not be linear

Where ML fits into causal inference

ML-augmented regression strategy:

1. Predict Y_i using X_i with ML and compute the residuals,

$$\begin{aligned} \tilde{Y}_i &= Y_i - \hat{Y}_i^{ML}, \\ \hat{Y}_i^{ML} &= \text{prediction generated by ML} \end{aligned}$$

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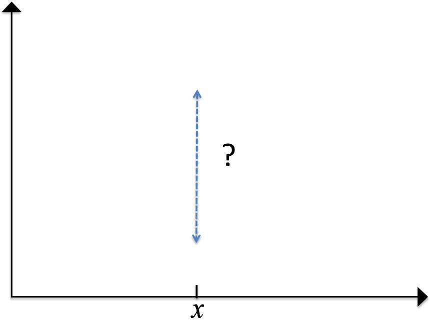
Most common ML methods in applied economics:

- Lasso
- Ridge
- Elastic net
- Random forest

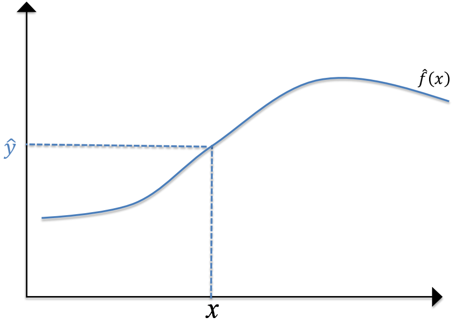
Getting serious about prediction

- **Goal:** Predict an out-of-sample outcome Y



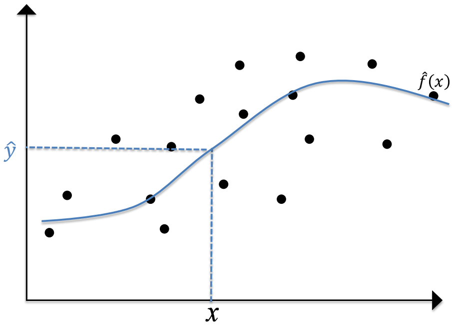
Getting serious about prediction

- **Goal:** Predict an out-of-sample outcome Y
- as a function, $\hat{f}(X)$, of **features** $X = (1, X_1, X_2, \ldots, X_K)'$.



Getting serious about prediction

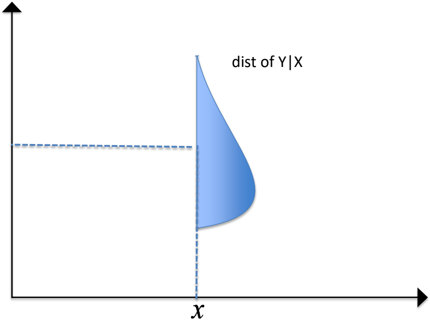
- **Goal:** Predict an out-of-sample outcome Y
- as a function, $\hat{f}(X)$, of **features** $X = (1, X_1, X_2, \ldots, X_K)'$.
- Estimate the function $\hat{f}$ (aka "train the model") based on **training sample** $\{(Y_i, X_i); i = 1, \ldots, N\}$



Cutting our losses

- Want our prediction to be "close," i.e. minimize the expected **loss function**:

$$\min_{f(x)} E\left[L\left(f\left(x\right) - Y\right) | X = x\right]$$



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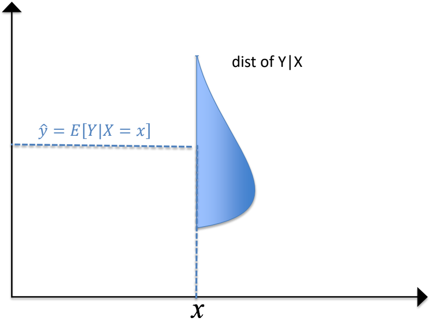

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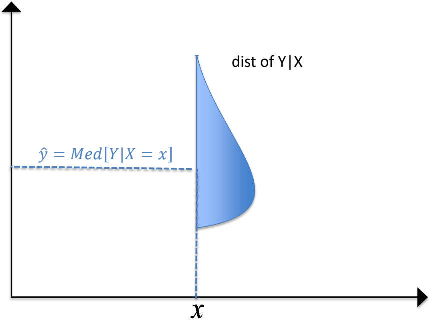

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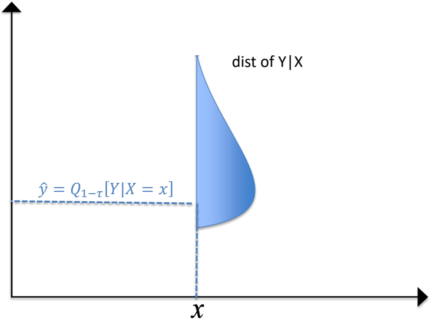
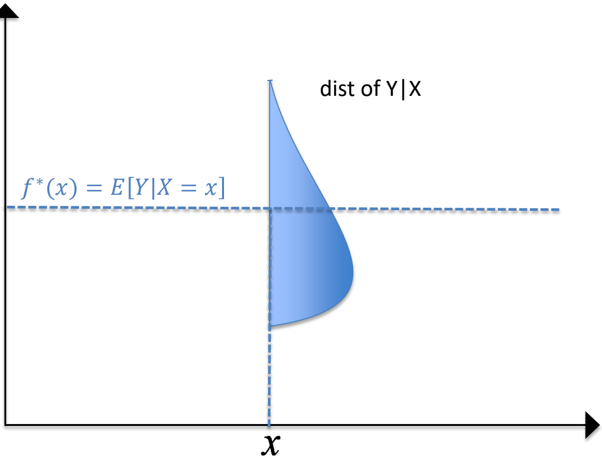
- **Absolute loss**: $L(d) = |d| \implies f^*(x) = Med\left[Y|X = x\right]$
- **Asymmetric loss**:
  $L_\tau\left(d\right) = d\left(\tau - 1\left(d < 0\right)\right) \implies f^*(x) = Q_{1-\tau}\left[Y|X = x\right]$



Navigating the Bias-Variance Tradeoff

- Prediction problem solved if we knew $f^*(x) = E[Y|X = x]$



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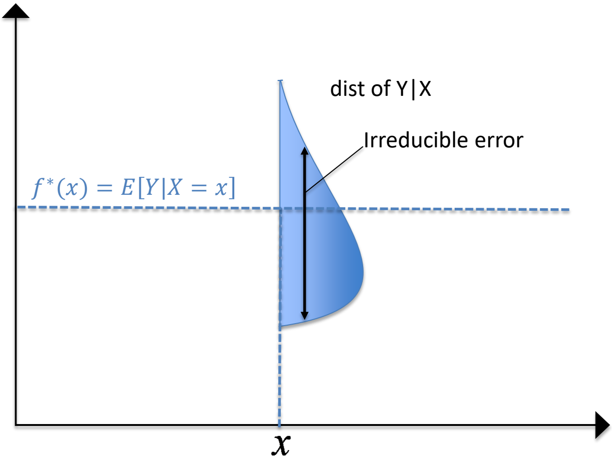

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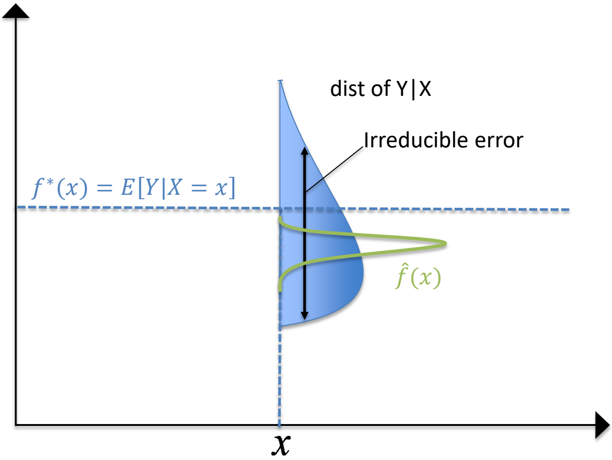

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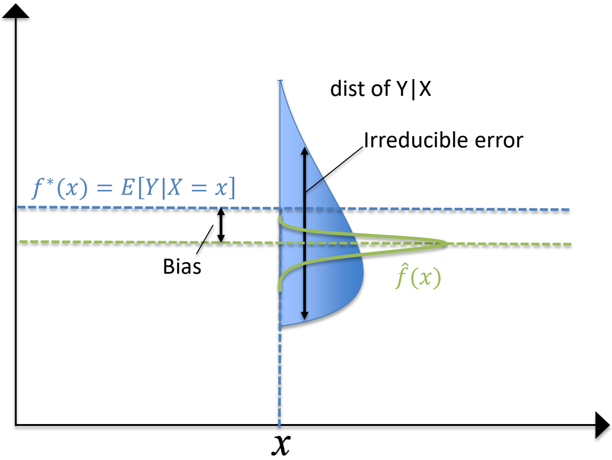

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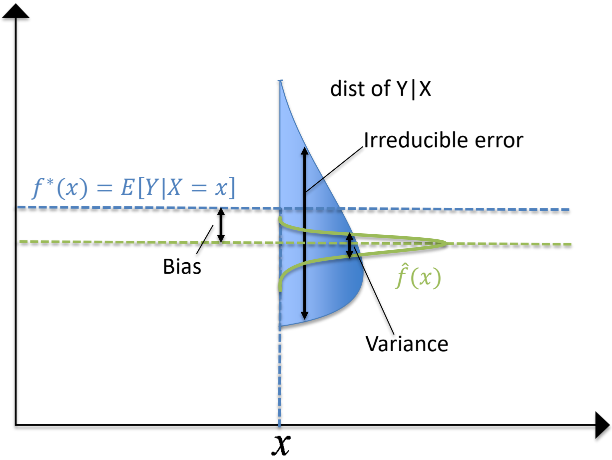

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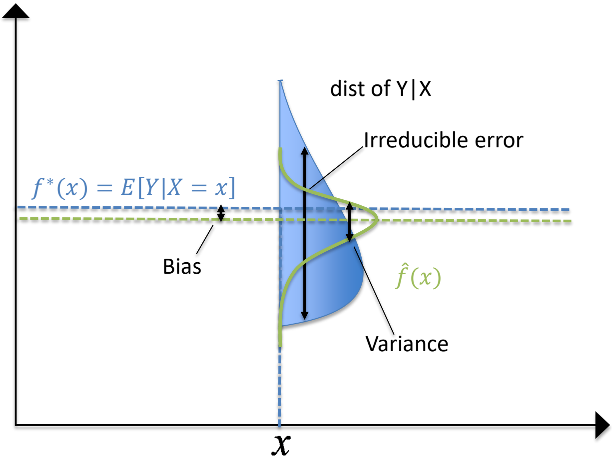

Navigating the Bias-Variance Tradeoff

- Prediction problem solved if we knew $f^*(x) = E\left[Y|X = x\right]$
- But we have to settle for an estimate: $\hat{f}\left(x\right)$;

$E\left[\left(Y - \hat{f}\left(x\right)\right)^2 \middle| X = x\right]$ becomes:

$$
\begin{aligned}
&\left(E\left[\hat{f}\left(x\right) - f^*\left(x\right)\right]\right)^2 && \text{prediction bias squared} \\
&+ E\left[\left(\hat{f}\left(x\right) - E\left[\hat{f}\left(x\right)\right]\right)^2\right] && \text{prediction variance} \\
&+ E[(Y - f^*\left(x\right))^2|X = x] && \text{irreducible error.}
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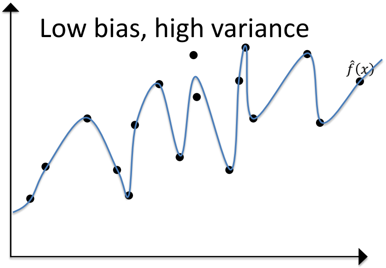

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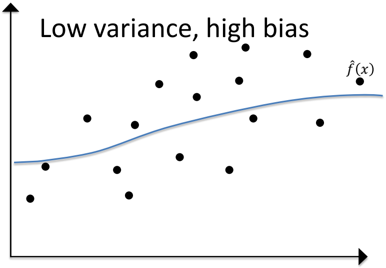


Python example: predicting earnings in the NLSY

Penalized Regression: Lasso

- When is it the right tool for the job:
  - When you have a large number of potential regressors (including powers or other transformations), maybe even more than the sample size!
  - Out of these, only a relatively few (but you don't know which) really matter (what do we mean by "matter?"). We call this **approximate sparsity**
- Theoretical definition:

$$\arg\min_b \sum_{i=1}^{n} \left(y_i - x_i' b\right)^2 + \lambda \sum_{j=1}^{k} |b_j|$$

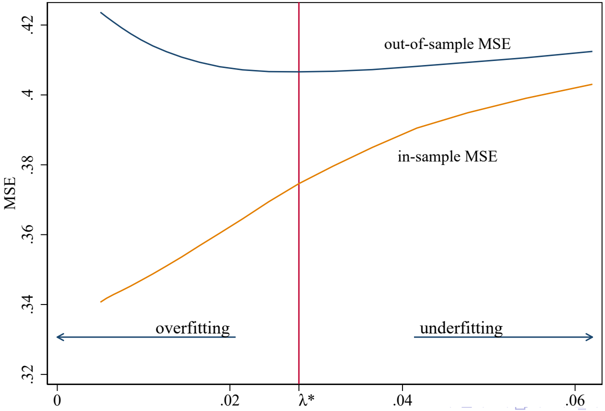
  What does λ do and how do we choose it?
- Caveats and considerations:
  - Important to standardize regressors pre-lasso
  - Can give unexpected results with dummy variables
  - Resist the temptation to interpret coefficients or the included variables as the "true model!"
- Let's give it a go in python!

Choosing Tuning Parameters: Cross-Validation

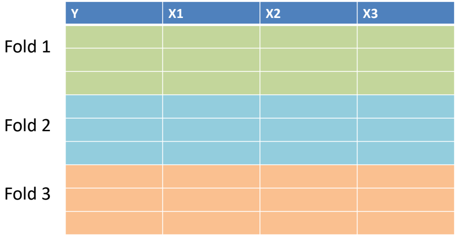
All supervised ML methods have tuning parameters:

- Lasso: λ
- Ridge: α
- Random forests: tree depth, etc.

Tuning parameters are the rudder by which we navigate the bias-variance tradeoff.



Choosing Tuning Parameters: Cross-Validation



Cross-validation procedure: Divide sample in K folds

- Choose some value of the tuning parameter, λ
- For each fold $k = 1, \ldots, K$
    1. Train model leaving out fold k
    2. Generate predictions in fold k
    3. Compute MSE for fold k: $MSE_k = \frac{1}{n_k} \sum_{i \in k} \left(Y_i - \hat{Y}_i \right)^2$
- Compute overall MSE correponding to the current choice of λ: $MSE(\lambda) = \frac{1}{K} \sum_{k=1}^{K} MSE_k$

Repeat the above for many values of λ, and choose the value λ^* with the lowest cross-validated MSE—time for python!

Penalized Regression: Ridge

- When is it the right tool for the job:
  - When you have a large number of regressors including highly collinear ones
- Theoretical definition:

$$
\begin{aligned}
& \arg\min_b \sum_{i=1}^{n} \left(y_i - x_i' b\right)^2 + \alpha \sum_{j=1}^{k} b_j^2 \\
= \quad & \left(X'X + \alpha I\right)^{-1} X'Y
\end{aligned}
$$

- Caveats and considerations:
  - Important to standardize regressors pre-ridge
  - Shrinks (biases) coefficients towards zero, but not all the way (unlike lasso)
- Let's give it a go in python!

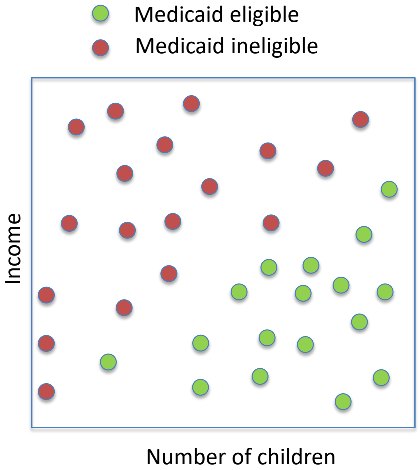
Penalized Regression: Elastic Net

- Combines lasso and ridge approaches
- Theoretical definition:

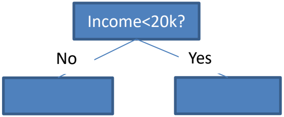
$$\arg\min_b \sum_{i=1}^{n} \left(y_i - x_i' b\right)^2 + \alpha\gamma \sum_{j=1}^{k} |b_j| + .5\alpha\,(1-\gamma) \sum_{j=1}^{k} b_j^2$$

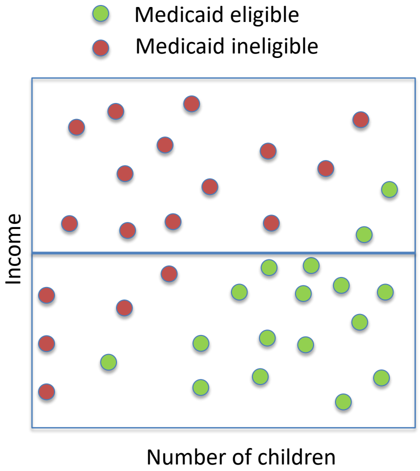
- Caveats and considerations:
  - Two tuning parameters: α and γ
  - Important to standardize regressors pre-ridge
  - Zeros out many regressors, shrinks (biases) remaining coefficients towards zero
- Let's give it a go in python!

Decision Trees

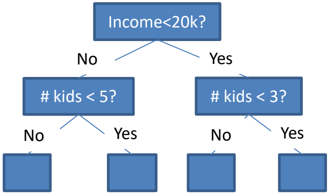


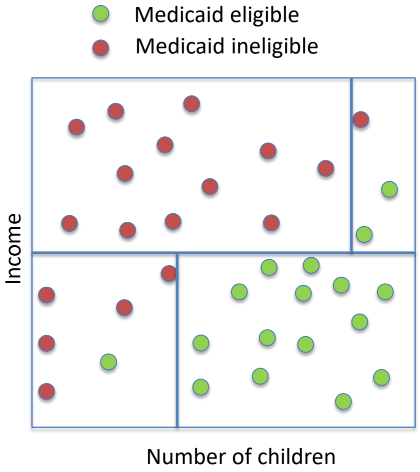
Decision Trees



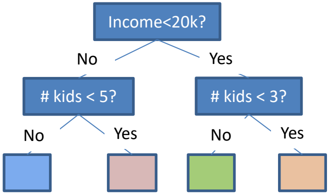


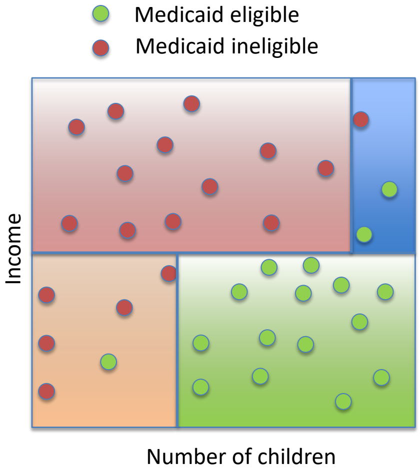
Decision Trees



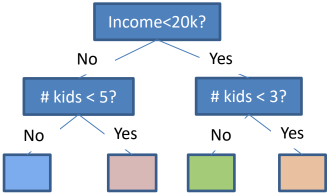


Decision Trees



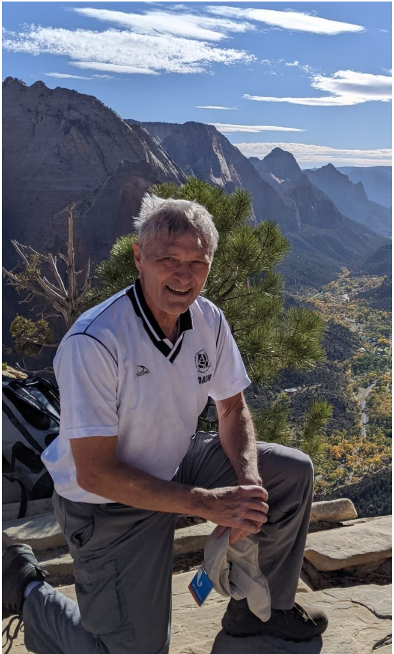


Decision Trees

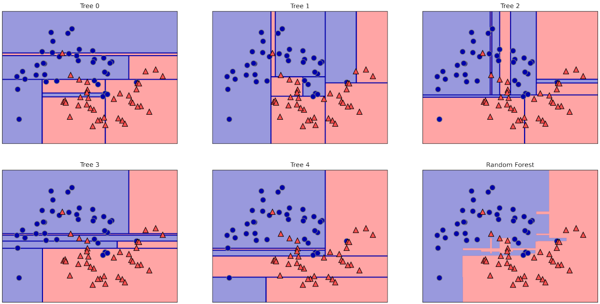


- Where to split:
  Choose the feature from $\{x_1, \ldots, x_p\}$ and the value of that feature to minimize MSE in the resulting child nodes
- Tuning parameters
  - Max depth
  - Min training obs per leaf
  - Min improvement in fit in order to go ahead with a split
- Let's try it in python!

Wisdom of the crowd: predict my father's age!



Forest for the Trees



- Value proposition: reduce variance by averaging together multiple predictions
- The catch: individual trees need to be **de-correlated**
- Algorithm:
  - Grow B trees, each on a different bootstrapped sample
  - At each split, consider only a random subset of features
  - Average together the individual predictions
- Let's grow some trees in python!