# 2023-2024 Paper

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1. (a) 对 $\theta \in (0,\infty)$

The interval $\Rightarrow (\theta, 2\theta)$

The MME: $\frac{3\theta}{2} = \bar{X}$, $\hat{\theta}_{MME} = \frac{2\bar{X}}{3}$

The MLE:

$$f = \frac{1}{2\theta - \theta} = \frac{1}{\theta}$$

$L(x;\theta) = (\frac{1}{\theta})^n I(x)$ where $I(x) = \begin{cases} 0, & x \notin (\theta, 2\theta) \\ 1, & x \in (\theta, 2\theta) \end{cases}$

when $\hat{\theta} = X_{(1)}$, the Likelyhood function $\rightarrow$ maximum

where $X_{(1)} < X_{(2)} < \dots < X_{(n)}$

(b) 对 $\theta \in (-\infty, 0)$

The interval $\Rightarrow (\theta, 0)$

The MME: $\frac{\theta}{2} = \bar{X}$, $\hat{\theta}_{MME} = 2\bar{X}$

The MLE:

$$f = \frac{1}{0-\theta} = -\frac{1}{\theta}$$

$L(x;\theta) = (-\frac{1}{\theta})^n I(x)$ where $I(x) = \begin{cases} 0, & x \notin (\theta, 0) \\ 1, & x \in (\theta, 0) \end{cases}$

when $\hat{\theta} = -X_{(n)}$ the Likelyhood function $\rightarrow$ maximum

where $X_{(1)} < \dots < X_{(n)}$

(c) 对 $\theta \in R$

The interval $\Rightarrow (\theta, \theta + |\theta|)$

The MME $\Rightarrow \frac{2\theta + |\theta|}{2} = \bar{X}$, $2\theta + |\theta| = 2\bar{X} \Rightarrow$ 对 $\theta > 0$, $\hat{\theta} = \frac{2}{3}\bar{X}$; $\theta < 0$, $\hat{\theta} = 2\bar{X}$

The MLE

$$f = \frac{1}{|\theta|}$$

$L = (\frac{1}{|\theta|})^n I(x)$ where $I(x) = \begin{cases} 0; & x \notin (\theta, |\theta| + \theta) \\ 1; & x \in (\theta, |\theta| + \theta) \end{cases}$

when $\hat{\theta} = |X_{(?)}|$

where $|X_{(?)}| < |X_{(others)}|$

2.

(a) $P(X_i = k) = \theta(1-\theta)^k = e^{\log\theta} e^{\log(1-\theta)^k} = e^{\log\theta + k\log(1-\theta)}$

let $k(x) = x$ $p(\theta) = \log(1-\theta)$. $S(x) = 0$ $q(\theta) = \log\theta$.

Then $\sum k(x_i) = \sum x_i$ is the sufficient and complete statistic due to properties of the exp function class.

(b)

$$\sum x_i = \theta^n (1-\theta)^{\sum x_i}, \quad x_i = 0, 1, 2, \dots$$

(c)

$$E(\sum X_i) = E(X_1) + E(X_2) + \dots + E(X_n) = \frac{n}{p}$$

$$E(\frac{1}{\sum X_i}) = \frac{p}{n}. \quad E(n \cdot \frac{1}{\sum X_i}) = p$$

So the UMVUE is $\frac{n}{\sum X_i}$

3.

So the UMVUE is $\frac{1}{5}\sum X_i$

3.

(a) Hypothesis: An asserction about the distribution of one or more variable.

Type I Error: If $H_0$ is true but reject $H_0$

Type II Error: If $H_0$ is False but accept $H_0$

(b)

Let $\theta_1 = \frac{1}{2}$ and $\theta_2 < \frac{1}{2}$, then we have

$$\frac{\theta_1^{\sum x_i}(1-\theta_1)^{n-\sum x_i}}{\theta_2^{\sum x_i}(1-\theta_2)^{n-\sum x_i}} \le k \qquad \frac{\theta_1^{\sum x_i}(1-\theta_2)^{\sum x_i}}{\theta_2^{\sum x_i}(1-\theta_1)^{\sum x_i}} \le k\frac{(1-\theta_2)^n}{(1-\theta_1)^n}$$

$\Rightarrow \sum X_i \le C$ . It is the best critical region

We set critical region $C = \{x_1 \dots x_5 \mid \sum_{i=1}^{5} x_i \le C\}$

which is a UMP test

(c)

$$\gamma_C = P_{\theta=\theta_2}\{x_1 \dots x_5 \mid \sum_{i=1}^{5} x_i \le C\}$$

(d)

$$\alpha = \max P_{\theta=\theta_1=\frac{1}{2}}\{x_1 \dots x_5 \mid \sum_{i=1}^{5} x_i \le C\}$$

(e)

All parameter space $\Omega := \{\theta \le \frac{1}{2}\}$ . the subset of $H_0$ $\omega := \{\theta = \frac{1}{2}\}$

Let $L(\hat{\omega}) = \max_{\theta\in\omega} L(x_1 \dots x_5; \theta = \frac{1}{2})$

$L(\hat{\Omega}) = \max_{\theta\in\Omega} L(x_1 \dots x_5; \theta \le \frac{1}{2})$

$$\Lambda(x_1 \dots x_n) = L(\hat{\omega}) / L(\hat{\Omega}) = \max L(x_1 \dots x_5; \theta = \tfrac{1}{2}) / \max L(x_1 \dots x_5; \theta \le \tfrac{1}{2})$$

$$= L(x_1 \dots x_5; \theta = \tfrac{1}{2}) / L(x_1 \dots x_5; \theta \le \tfrac{1}{2})$$

Note that $L(x_1 \dots x_5; \theta = \frac{1}{2}) / L(x_1 \dots x_5; \theta \le \frac{1}{2})$

$$= (\tfrac{1}{2})^{\sum x_i}(\tfrac{1}{2})^{n-\sum x_i} / \theta^{\sum x_i}(1-\theta)^{n-\sum x_i} \le 1$$

Define $\lambda \in (0,1) \Rightarrow \Lambda(x_1 \dots x_n) \le \lambda$, which means it is the same as the UMP Test

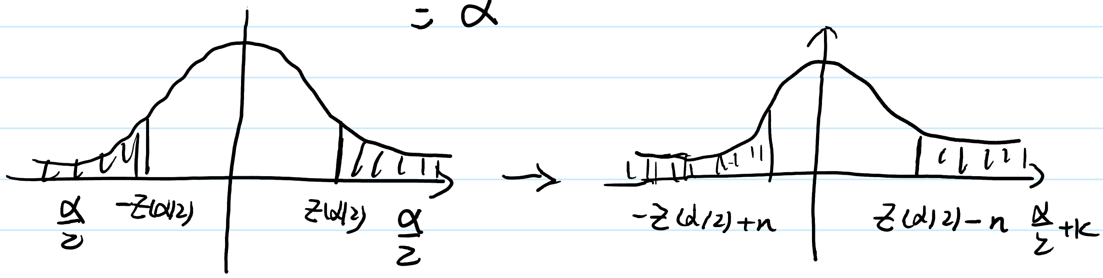
4.

$\bar{X} = \frac{1}{n}\sum X_i \sim N(0, \frac{1}{\sqrt{n}})$ with $H_0$. $\Rightarrow \sqrt{n}\bar{X} \sim N(0,1)$ with $H_0$

$\sqrt{n}|\bar{x}| > z(\alpha/2) \Rightarrow \sqrt{n}\bar{x} > z(\alpha/2)$ or $\sqrt{n}\bar{x} < -z(\alpha/2)$

So $P(\sqrt{n}|\bar{x}| > z(\alpha/2)) = P(\sqrt{n}\bar{x} > z(\alpha/2)) + P(\sqrt{n}\bar{x} < -z(\alpha/2))$

$= \alpha$




which means the Original power of test is $\alpha$

Consider power function

$\bar{X} = \frac{1}{n}\sum X_i \sim N(\frac{\mu}{n}, \frac{1}{\sqrt{n}})$ with $H_0 \Rightarrow \sqrt{n}\bar{X} \sim N(\frac{\mu}{\dots}, 1) \rightarrow \sqrt{n}\bar{X} - \frac{\mu}{\dots} \xrightarrow{D} N(0,1)$

[illegible]

Consider power function

$\bar{X} = \frac{1}{n} \sum x_i \sim N(\frac{\mu}{n}, \frac{1}{\sqrt{n}})$ with Ha $\Rightarrow \sqrt{n}\bar{X} \sim N(\frac{\mu}{\sqrt{n}}, 1) \Rightarrow \sqrt{n}\bar{X} - \frac{\mu}{\sqrt{n}} \xrightarrow{D} N(0,1)$

$\sqrt{n}|\bar{x}| > z(\alpha/2) \Rightarrow \sqrt{n}x > z(\alpha/2)$ or $\sqrt{n}\bar{x} < -z(\alpha/2)$

$\gamma_c = 2P(\sqrt{n}\bar{x} - \frac{\mu}{\sqrt{n}} > z(\alpha/2) - \frac{\mu}{\sqrt{n}}) \geq \alpha$, which means there may exist a $\mu$ such that $\gamma_c \geq$ Original power of test. Hence, it is not a UMP Test.

5.

(a) Need Help.