Finding Communities in Weighted Networks Through Synchronization

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Abstract:

Community detection in weighted networks is an important challenge. In this paper, we introduce a local weight ratio scheme for identifying the community structures of weighted networks within the context of the Kuramoto model by taking into account weights of links. The proposed scheme takes full advantage of the information of the link density among vertices and the closeness of relations between each vertex and its neighbors. Using this method, we explore the connection between community structures and dynamic time scales of synchronization. Moreover, we can also unravel the hierarchical structures of weighted networks with a well-defined connectivity pattern by the synchronization process. The performance of the proposed method is evaluated on both computer-generated and real-world networks.

<u>Lead Paragraph</u>: **Community structure, the gathering of vertices into groups such that there is a higher density of edges within groups than between them, is one important feature of networks. Identifying the community structure of complex networks is an important tool for understanding the network structure, analyzing the functional properties of complex networks, as well as revealing a hierarchy of nodes and connections within a complex structure. Different from early community detection approaches such as the modularity-maximum method [1], the clique percolation method [2] and the Potts method [5], Arenas et al. [7, 9] proposed a new method in the framework of synchronization of phase oscillators. The fundamental mechanism behind this method is based on the scenario that highly densely interconnected sets of oscillators are synchronizing more easily than those with sparse connections. Subsequently, some algorithms have been developed for detecting communities through synchronization [11, 13, 14]. However, these algorithms are designed in the context of binary networks. As is widely recognized, many real-life networks are intrinsically weighted. Therefore, characterizing and detecting such community structures in weighted networks has very important practical significance [17]. This paper aims at finding community structure for weighted networks**

via a local weight ratio scheme. This scheme takes full advantage of the information of the link density among vertices and the closeness of relations between each vertex and its neighbors. Simulations on artificial networks and real-world networks are carried out to evaluate the performance of the proposed method based on the maximum measurement of weighted modularity.

1 Introduction

There has been a surge of interest in the detection of community structure of a network in recent years, due to its important characterization to help understanding the network structure and analyzing the functional properties of complex networks. In order to identify communities of networks, many different algorithms have been recently developed, such as various modularity optimization methods building on the work by Newman and Girvan [1], the clique percolation method [2], and methods based on statistical inference [3, 4]. Among them, the authors proposed a method to find modules based on the statistical properties of a system of spins (namely, q-state Potts spins) associated to the nodes of the graphs in [5]. In [6], a simulated annealing strategy is used to maximize the modularity and identify the community structure in a complex network based on a k-means iterative procedure. However, most of these methods are detecting communities only based on the topological structure of networks and possess a high computational complexity.

Complementary to such methods, Arenas et al. [7, 9] proposed a new method in the framework of synchronization of phase oscillators. For more details on synchronization of phase oscillators, we refer the reader to [10] and references therein. The fundamental mechanism behind this method is based on the scenario that highly densely interconnected sets of oscillators are synchronizing more easily than those with sparse connections [7, 11]. Through the dynamical process to synchronization, a hierarchical organization of communities in complex networks can be extracted. In [12], Boccaletti et al. presented a so-called dynamic clustering approach in the framework of desynchronization for community detection. A modified Kuromoto model with the reference to the so-called opinion changing ration (OCR) model is introduced to depict the dynamics of each vertex of the network. This method has the advantage of using different types of information but is sensitive to the initial intrinsic frequency of each vertex and needs the tuning of a parameter related to the coupling strength in order to detect modules.

Based on the above two groundbreaking works, a number of algorithms have been developed for detecting communities through a synchronization [7, 11, 13, 14] or desynchronization [12, 15, 16] process. These algorithms, however, are designed in the context of binary networks, i.e., the edges between vertices are either present or not, which limits their applications. As is widely recognized, many real networks are intrinsically weighted. Therefore characterizing and detecting such community structures in weighted networks has very important practical significance [17].

In this paper, we propose another community detecting method for weighted networks from the viewpoint of a dynamic synchronization process. By modifying the coupling between coupled phase oscillators of the Kuromoto model with the information of local weight ratio together with normalized weighted coupling, the proposed local weighted ratio scheme shows the relationship between the emergence of the synchronization patterns and the network topology. Sequentially, the hierarchical organization of communities in weighted networks is revealed through the time scales of the synchronization process. Finally, numerical experiments on a variety of simulated networks and real-world networks are carried out to evaluate the performance of the proposed method based on the maximum measurement of weighted modularity.

2 Kuramoto oscillators

In order to reveal the synchronization patterns of networks, we introduce the following weighted Kuromoto model of N coupled phase oscillators instead of the unweighted Kuromoto model [7]:

$$\dot{\theta}_i(t) = \omega_i + \sigma \sum_{j=1}^{N} W_{ij} \sin(\theta_j(t) - \theta_i(t)),\ i \in \mathbb{Z}_N = \{1, 2, \cdots, N\}. \tag{1}$$

with $\theta_i(t)$ the phase of the ith oscillator having natural frequency ω_i, σ the coupling strength between any two oscillators, and W_{ij} is the ijth entry of the symmetric weighted coupling matrix which represents the weight of the link between node i and node j. W_{ij} $(i \neq j)$ is endowed with some nonzero real constant if there is a connection between node i and node j and $W_{ij} = 0$ otherwise, $W_{ii} = 0$ for all $i \in \mathbb{Z}_N$. The natural frequencies ω_i $(i \in \mathbb{Z}_N)$ are distributed with probability density $g(\omega)$, where $g(\omega)$ is assumed to be symmetric and unimodal about the mean frequency Ω, i.e., $g(\Omega + \omega) = g(\Omega - \omega)$. When θ_i is shifted to $\theta_i + \Omega t$ by choosing a suitable rotating frame, where Ω is the first moment (mean) of $g(\omega)$. The model (1) can be transformed to an equivalent system of phase oscillators whose natural frequencies have a zero mean [8]. Therefore, it can be concluded that $g(\omega) = g(-\omega)$ for all ω. To measure the phase coherence, the order parameter $r(t)$ has been widely used

$$r(t) = \Big| \frac{1}{N} \sum_{i=1}^{N} e^{\kappa \theta_i(t)} \Big|, \tag{2}$$

where $\kappa^2 = -1$, and $0 \leq r(t) \leq 1$. Obviously, when all the oscillators have the same phase the quantity equals one ($r(t) = 1$), which corresponds to full synchronization. The degree of synchronization is equal to zero ($r(t) = 0$) when all the oscillators are independent and are having different phases. In this paper, we shall use a different order parameter (6) to characterize the phase coherence.

3 The proposed method

In this section, the weighted modularity Q_w proposed by Newman will be recalled firstly, and then our local weight ratio scheme for identifying communities and dynamic synchronization affinity matrix for evaluating synchronization will be presented.

3.1 Weighted modularity

To evaluate the partition of networks quantitatively, Newman and Girvan have defined a modularity Q in binary networks [18]. Since in weighted networks community structure is not only related to the link density among nodes but also to the closeness of their relations, Newman has generalized the modularity Q to weighted modularity Q_w [19]:

$$Q_w = \frac{1}{2W_s} \sum_{ij} \Big[W_{ij} - \frac{W_i W_j}{2W_s} \Big] \delta_{c_i, c_j}, \tag{3}$$

where $i, j \in \mathbb{Z}_N$, N represents the number of nodes, W_{ij} is the link weight between node i and j, $W_i = \sum\limits_j W_{ij}$ is strength of node i, and the total strength is $2W_s = \sum\limits_{i,j} W_{ij}$. c_i shows that vertex i belongs to community c_i. The Kronecker delta function δ_{c_i,c_j} takes the value 1 if node i and node j belong to the same community, 0 otherwise. In fact, Q_w is the total fraction of link weights with both links ending in the same community subtracted from the same value if the links were placed randomly. Basically, when the value of the modularity Q_w is larger, the partition is better.

3.2 The local weight ratio scheme

Here a local weight ratio method will be presented for community detection of weighted networks, which is inspired by the work in [13] where a proper weighting approach combining the edge betweenness centrality and common neighborhood ratio was presented for binary networks. The proposed method attempts to take full advantage of the information of the link density among vertices and the closeness of relations between each vertex and its neighbors.

First, normalizing the weighted coupling matrix W_{ij} $(i, j \in \mathbb{Z}_N)$ yields $\bar{W}_{ij} = W_{ij} L_s / W_s$, where $W_s = \frac{1}{2} \sum_{ij} W_{ij}$ and L_s is the total number of links in the network. We then define the local weight ratio

$$C_{ij} = \frac{2\Big[W_{ij}\delta_{ij} + \sum\limits_{k=1}^{N} \delta_{ik}\delta_{jk}(W_{ik} + W_{jk})/2\Big]}{\sum\limits_{k=1}^{N} W_{ik}\delta_{ik} + \sum\limits_{k=1}^{N} W_{jk}\delta_{jk}},\ i, j \in \mathbb{Z}_N, \tag{4}$$

with $\delta_{ik} = 1$ if node i and node k are connected, $\delta_{ik} = 0$ otherwise.

Intuitively, the measurement C_{ij} contains not only the information of link weights between i and j, but also the information of their common neighbors (because only the link weights between vertices i, j and their common neighbors will be activated in view of the term $\delta_{ik}\delta_{jk}$ in the numerator, while $\delta_{ik}\delta_{jk} = 0$ in other cases). Thus it reflects how close the relation between node i and node j is, and is supposed to be higher for vertices inside a community since these vertices are strongly connected, while lower for those vertices that are not in the same community. Now let us combine the normalized matrix $\bar{W}_{ij}$ and the local weight ratio C_{ij}, and define the modified weighted coupling matrix $\tilde{W}_{ij} = \bar{W}_{ij}C_{ij},\ i, j \in \mathbb{Z}_N$. Thereby, the modified dynamical system of weighted Kuromoto model (1) is given by

$$\dot{\theta}_i(t) = \omega_i + \sigma \sum_{j=1}^{N} G_{ij} \sin(\theta_j(t) - \theta_i(t)),\ i \in \mathbb{Z}_N, \tag{5}$$

where $G_{ij} = \tilde{W}_{ij} \Big/ \sum_j \tilde{W}_{ij}, i, j \in \mathbb{Z}_N$.

Figure 1 shows a small synthetic network with two communities. One can readily get the weighted coupling matrix of the network:

$$W = \begin{bmatrix} 0 & 2 & 2 & 0 & 1 & 0 & 0 & 0 & 0 \\ 2 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 \\ 2 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 1 \\ 0 & 2 & 2 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 2 & 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 2 \\ 0 & 0 & 1 & 0 & 2 & 0 & 0 & 2 & 0 \end{bmatrix}.$$

Through the above local weight ratio scheme, we obtain the normalized modified weighted coupling matrix as follows:

$$G = \begin{bmatrix} 0 & 0.4868 & 0.4463 & 0 & 0.0669 & 0 & 0 & 0 & 0 \\ 0.3059 & 0 & 0.3882 & 0.3059 & 0 & 0 & 0 & 0 & 0 \\ 0.2849 & 0.3945 & 0 & 0.2849 & 0 & 0 & 0 & 0 & 0.0356 \\ 0 & 0.4868 & 0.4463 & 0 & 0 & 0 & 0.0669 & 0 & 0 \\ 0.1158 & 0 & 0 & 0 & 0 & 0.4211 & 0 & 0 & 0.4632 \\ 0 & 0 & 0 & 0 & 0.2069 & 0 & 0.4138 & 0.3793 & 0 \\ 0 & 0 & 0 & 0.0643 & 0 & 0.4678 & 0 & 0.4678 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.3793 & 0.4138 & 0 & 0.2069 \\ 0 & 0 & 0.0984 & 0 & 0.4723 & 0 & 0 & 0.4293 & 0 \end{bmatrix}.$$

It is observed that this new weighted coupling matrix can easily distinguish the weights between the inner and outer community. Note that $G = (G_{ij})_{n\times n}$ is non-symmetric, which reflects the connection strength between each vertex and its neighbors. In fact, modifying the weighted coupling matrix in such a manner will increase the interaction of the vertices of the same community by strengthening their intra-community edges and decrease the rest. Therefore, the dynamical process of Kuromoto oscillators will be influenced based on the new coupling matrix. Consequently, it is possible to reveal the community structure in weighted networks through the dynamic time scales of synchronization through this scheme. The main idea that we propose here to detect communities of the networks with the weighted coupling matrix W, is to assimilate all nodes with identical oscillators following the modified dynamical system (5). Owing to the synthesized information in G, the dynamical process towards synchronization depends on on the edge weights and on the topology.

3.3 Synchronization measurement

In order to identify the local dynamic effects of coupled oscillators towards synchronization, we use the following local order parameter [7]

$$\rho_{ij}(t) = \langle \cos[\theta_i - \theta_j] \rangle, \ i, j \in \mathbb{Z}_N \tag{6}$$

to measure the coherence of the population of N oscillators, where $\langle \cdot \rangle$ is an average over many realizations with different random initial phases. In [7], the authors introduced a dynamic connectivity matrix to determine the borders between different local clusters. Here we utilize the time depending matrix introduced in [14] to reveal the topology scales of weighted networks:

$$D_t(T)_{ij} = \begin{cases} t_{ij} & \text{if } \rho_{ij}(t) > T, \\ 0 & \text{if } \rho_{ij}(t) < T, \end{cases} \tag{7}$$

where $i, j \in \mathbb{Z}_N$, t_{ij} denotes the time needed for the nodes i and j to reach synchronization in the sense of $\rho_{ij}(t) > T$. T is a given threshold for characterizing the degree of synchronization between pairs of oscillators. At a time t, define $D_{t_m}(T) = \max_{i,j}\{D_t(T)_{ij}\}$ and let $D_t(T)_{ij} = D_{t_m}(T)$ when $D_t(T)_{ij} = 0$ $(i \neq j)$ which implies that node i and node j still haven't synchronized until this time. Therefore, different structural patterns observed along the synchronization process can be exhibited by the matrix $D_t(T)_{ij}$ at different times. To visually show the structural patterns, let us normalize $D_t(T)_{ij}$ as $S_t(T)_{ij} = D_t(T)_{ij} \big/ D_{t_m}(T)$, which will be called as normalized time-depending affinity matrix (or sync-affinity matrix, for short).

4 Results

In this section, we present several tests of our method on artificial weighted networks and on real-world weighted networks for which the community structures are already known. For all cases, we perform numerical simulations over 100 realizations with synchronization threshold $T = 0.99$ and coupling strength $\sigma = 15$. As described in [9], the value of the synchronization threshold T is prescribed to 0.99. Other choices only modify the relative time scales without altering the results. Normally, a large coupling strength σ increases the synchronizability of phase oscillators. Here, we choose the coupling strength σ as 15 in order to make the dendrogram of community structure (such as Figure 5 and Figure 6(b)) more readable through the synchronization process.

4.1 Artificial weighted networks

4.1.1 Computer-generated network

First, to verify the performance of our method, we have applied it to a weighted network with fixed community structure [19]. This network is constructed with $n = 128$ vertices, divided into four communities of 32 vertices each. Edges among vertices are randomly chosen such that the average degree is fixed at 16. In the sequel, we refer to the average number of inter-community edges per vertex and the average number of intra-community edges per vertex as z_{out} and z_{in}, respectively. Let W_{out} and W_{in} denote the weights among inter-community edges and intra-community edges, respectively. We assign $W_{\text{in}} > 1$ to the intra-community edges and keep the fixed weight 1 for inter-community edges. For the sake of clarity, we show a random version of the network in Figure 2(a). Through the synchronization process of Kuromoto model (5) using the local weight ratio scheme, Figure 2(b) shows the normalized sync-affinity matrix $S_t(T)$ for describing the time needed for each pair of nodes to synchronize at the peak in the weighted modularity which is depicted in Figure 3(a). Figure 3 shows the dendrogram together with the weighted modularity Q_w, plotted as a function of position in the dendrogram. As we can see, the modularity has a single clear peak at the point where the network breaks into four communities, as we would expect. The peak value equals 0.4375 which is typical.

4.1.2 Weighted hierarchical network

The second synthetic example is shown in Figure 4(a) with multi-scale hierarchical organization. It is worth mentioning that there is no significant difference among the degrees of vertices. However, we assign different weights (numbers adjacent to the edges in Figure 4(a)) to the inter-community edges and the intra-community edges. In this way, a hierarchical clustering network is generated. By employing the criterion of modularity in conjunction with the hierarchical tree mentioned above, we can accurately find community structures in networks. We can obtain the maximal modularity $Q = 0.4947$, corresponding to a 'best' partition with 5 communities. The visualization of $S_t(T)$ reveals community structures at different hierarchical levels in Figure 4(b). It is noted that the sync-affinity matrix shows a set of dissimilar time scales corresponding to the different hierarchical levels. Figure 5 shows the weighted modularity Q_w on the left and the dendrogram of hierarchical structure on the right of the figure found via the proposed scheme. This example suggests that our method is effective at extracting hierarchical community structure from weighted networks.

4.2 Real-world weighted networks

In this section, we perform our method on several real-world networks for which the community structure is already known from other sources.

4.2.1 Zachary's karate club network

The first real-world network is derived from the well known karate club study of Zachary [20]. Nodes in the network stand for club members and the links reflect the social relations between them. The network has a total 34 vertices representing members of the karate club and 78 edges representing the friendship between members of the club which was observed over a period of two years. The edge weights ranging from 1 to 11 of the network represent the strength of ties between members. We substitute the weighted coupling matrix of the network into Kuromoto model (5) and attempt to identify communities involved in the split of club using our scheme. As a result, we obtain a suboptimal partition represented in Figure 6(a) and giving four communities: $\mathcal{C}_1 = \{1, 2, 3, 4, 8, 12, 13, 14, 18, 20, 22\}$, $\mathcal{C}_2 = \{5, 6, 7, 11, 17\}$, $\mathcal{C}_3 = \{9, 10, 15, 16, 19, 21, 23, 24, 27, 28, 30, 31, 33, 34\}$, $\mathcal{C}_4 = \{25, 26, 29, 32\}$ with the weighted modularity of $Q_w = 0.4439$. Compared with the greedy algorithm [21] with $Q_w = 0.4345$, our method gives a better result, though the weighted modularity is $Q_w = 0.4449$ for the optimal community structure where vertices $24, 28$ are included in community $\mathcal{C}_4$ instead of community $\mathcal{C}_3$. Clearly, The bipartition consisting of the two following communities: $\mathcal{C}_1 + \mathcal{C}_2$ and $\mathcal{C}_3 + \mathcal{C}_4$, corresponds exactly to the split of the karate club, as observed by Zachary. Figure 6(b) show the hierarchical clustering tree of communities produced by our method during the dynamical process towards synchronization.

4.2.2 Victor Hugo's Les Misérables dataset

For our next application to real-world networks, we look at the Victor Hugo's Les Misérables dataset [22, 23]. The dataset describes the relationships between characters in Victor Hugo's Les Misérables dataset. A graph was built with 77 vertices associated to characters which interact and 257 edges associated with coappearance of characters in one or more scenes. The edge weights represent the number of scenes where the corresponding characters are jointly appearing. This network was studied by Newman and Girvan [1] with their betweenness-based divisive hierarchical algorithm, leading to a partition into 11 communities with a modularity $Q = 0.54$. Making use of the edge weights in the matrix G and applying our method to the network, the partition obtained with our method has a strong modularity $Q_w = 0.5464$ and gives a 6-communities partition shown in Figure 7. The obtained results can be compared with the results on this network by edge ratio algorithm (11-communities with $Q_w = 0.4933$, [24]) and the simulated annealing algorithm (5-communities with $Q_w = 0.5460$, [25]).

4.2.3 Network of American college football teams

Since an unweighted network with an adjacency matrix instead of a weighted coupling matrix is a special case of weighted networks, our scheme is also applicable to community detection in unweighted networks. As an application of our algorithm, we turn to the network of American college football team. The network represents the game schedule of the 2000 season of Division I of the American college football league [18]. The nodes in the network represent 115 teams and edges represent regular season games between the two teams that they connect. The teams are divided into conferences containing around 8 to 12 each. Games are more frequent between members of the same conference than between members of different conferences. The optimal community structure split of the network obtained using our algorithm has a strong modularity of $Q_w = 0.6044$ and gives 9 communities as shown in Figure 8, outperforming some existing methods [18, 6, 26].

5 Conclusions

In this paper, we have introduced a new method to identify the community structure in weighted networks based on the phase synchronization of Kuromoto oscillators. Our method works by using information about local weight ratio to detect communities and unravels the dependence of the dynamics of coupled oscillators and the network topology. Using the criterion of weighted modularity, the method can accurately find community structures in weighted networks. Moreover, according to the normalized dynamic affinity matrix and the dendrogram of vertices during the dynamical process towards synchronization, the hierarchical organization of communities in weighted networks can be uncovered by taking snapshots of the evolution at discrete times. In addition, we have demonstrated the efficacy and utility of our method with two artificially generated weighted networks and three real-world networks with known community structure.

We hope that the ideas and methods presented described here will prove useful in studying community structure of many other types of networks, and offer insight into the complex networks with a multi-scale description.

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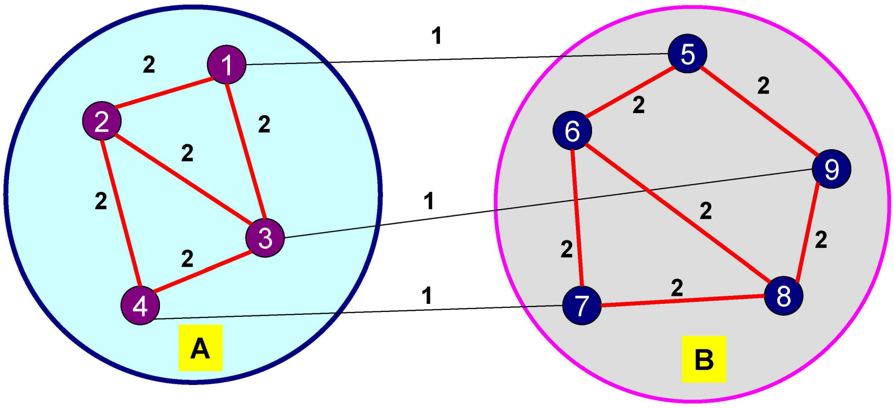

Figure 1: A small synthetic network example with 9 labeled nodes. The numbers between two different nodes represent their coupling weights. The first four nodes are in the same community and highlighted in the same circles. The left five nodes are marked in another community.

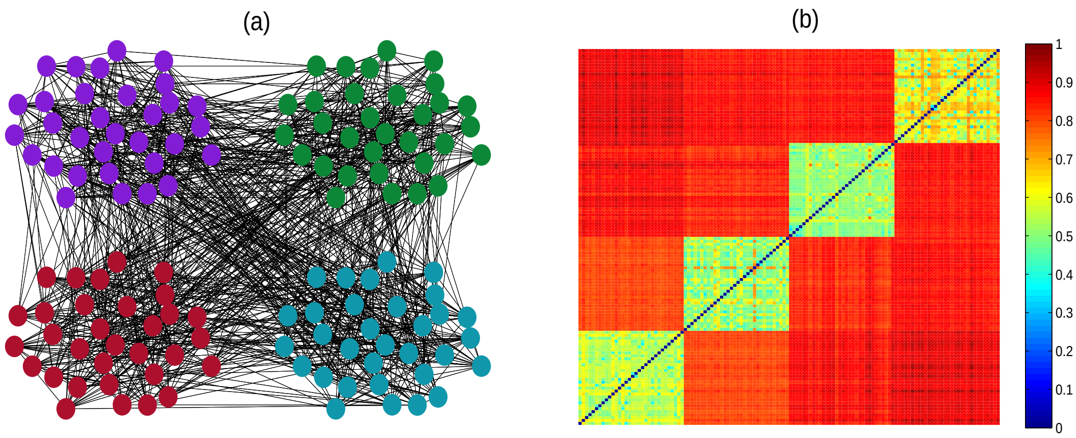

Figure 2: (a) Computer-generated weighted network of 128 nodes has 4 modules comprising 32 nodes each. Here, $z_{\text{out}} = 8, z_{\text{in}} = 8, W_{\text{out}} = 1, W_{\text{in}} = 2.2$. (b) Normalized sync-affinity matrix $S_t(T)$ obtained by our method for the computer-generated network. The color code denotes the normalized synchronization times. The dark bluer colors in the scale represent groups of nodes that are more synchronized.

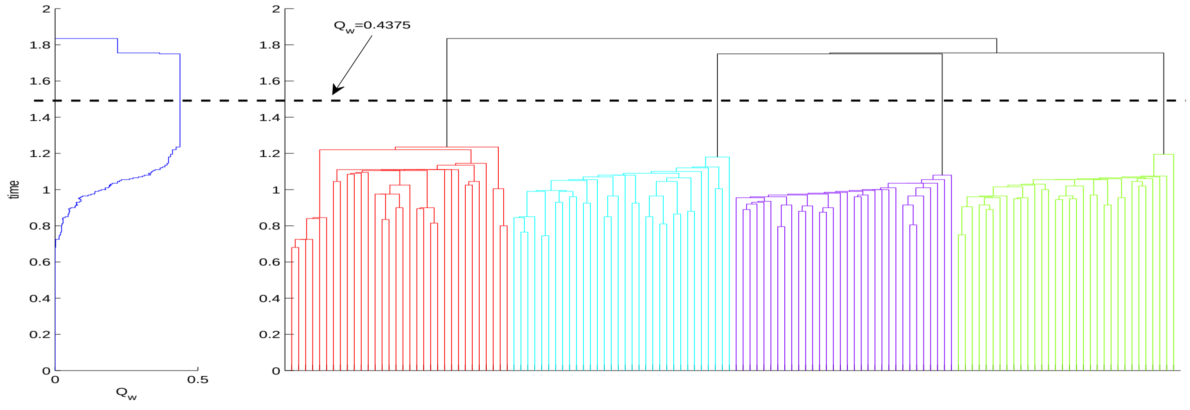

Figure 3: The dendrogram and weighted modularity Q_w of community structure in the weighted network found by our method.

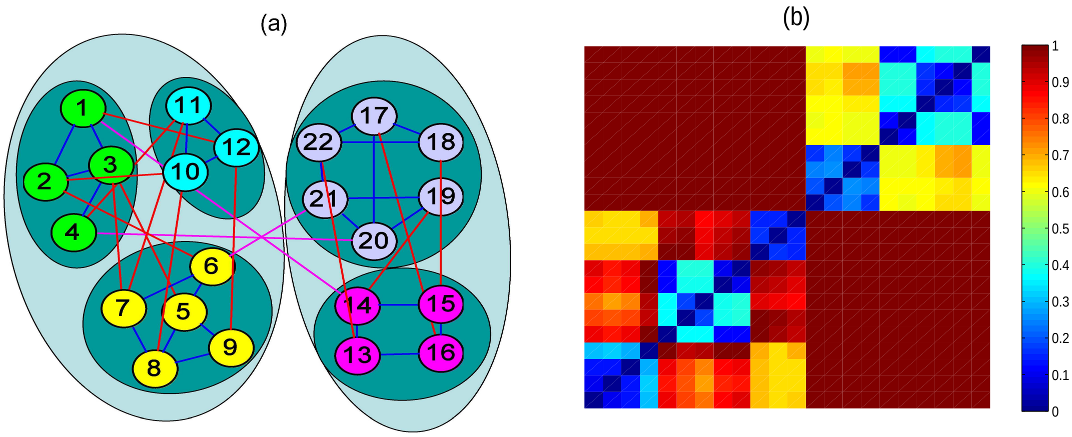

Figure 4: (a) A weighted hierarchical network of 22 nodes with 5 communities. Each community is shaded with a different color. Blue, red, and pink links represent the edge weights with values $1.4, 1.2$, and 1.0, respectively. (b) Normalized dynamic affinity matrix $S_t(T)$ in the weighted hierarchical network.

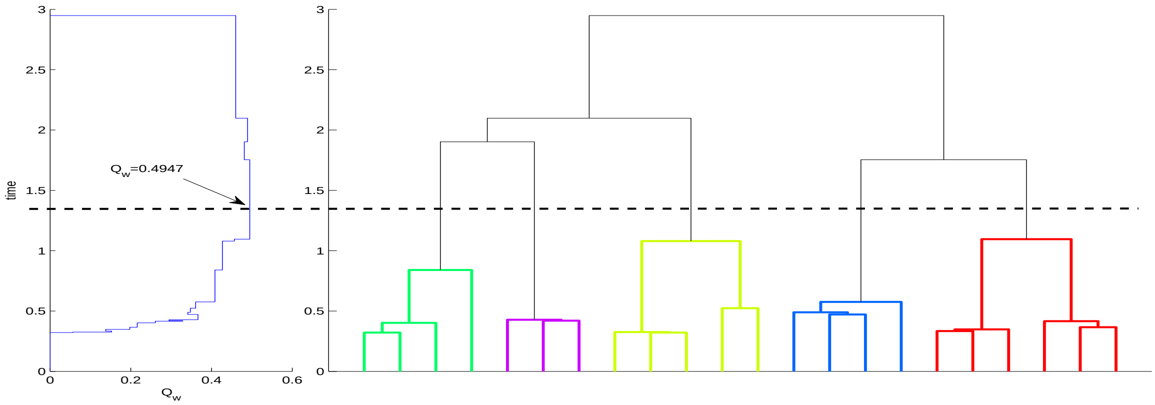

Figure 5: The dendrogram of community structure in the weighted hierarchical network found by our method. The dashed line corresponds to the optimal partition with maximal $Q_w = 0.4947$.

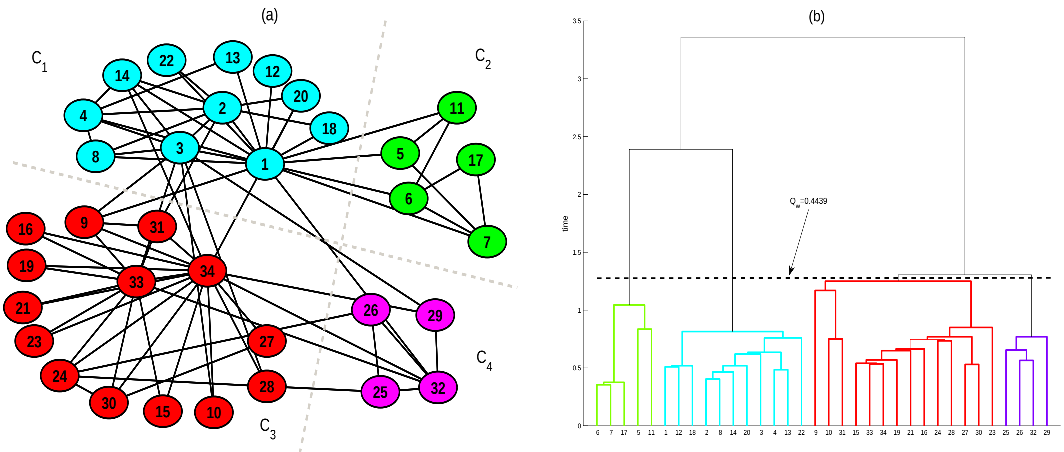

Figure 6: (a) The community structure of Zachary's Karate Club network detected by the proposed method. (b) The dendrogram of community structure found by our method.

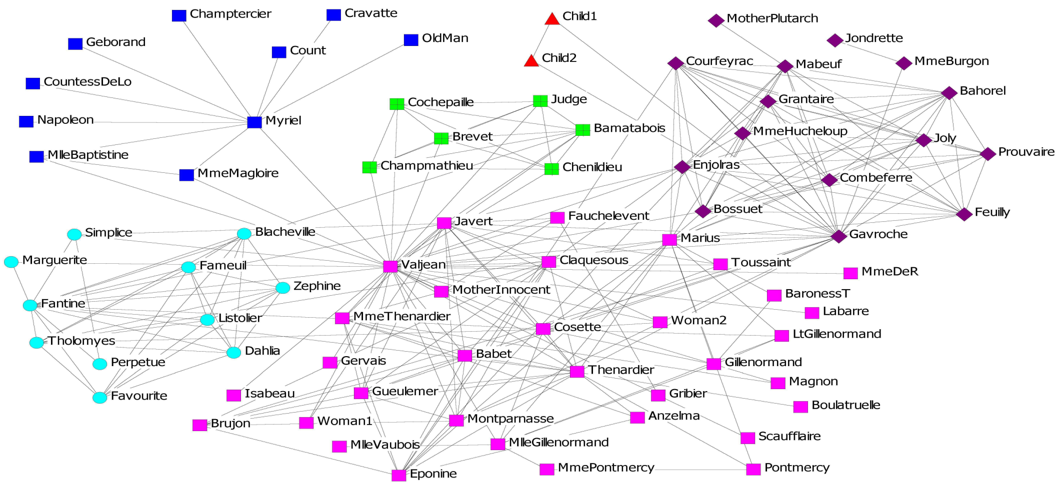

Figure 7: The community structure of the network of interactions between major characters in the novel Les Misérables by Victor Hugo. The largest modularity achieved by using the locally weighted ratio scheme is $Q_w = 0.5464$ and corresponds to a 6-communities partition. Each community is shaded with a different color and a different shape.

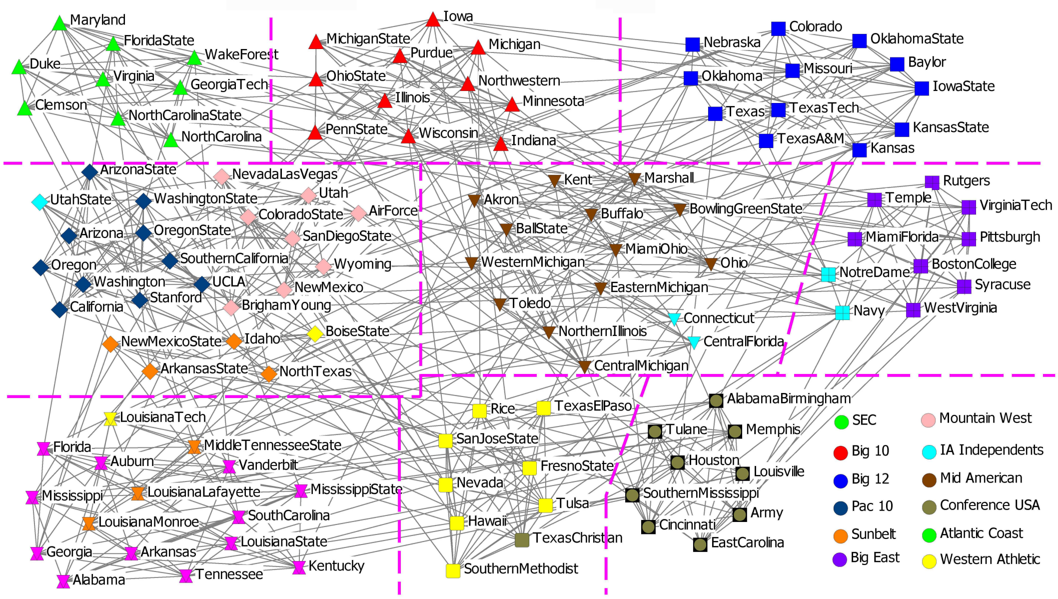

Figure 8: The conference structure (a 9-communities partition) of the American football team network obtained by our method. Different communities are separated by dashed lines and each community is shaded with a different shape. Colors represent the real 12 conferences.