Centrality

CS224W: Social and Information Network Analysis

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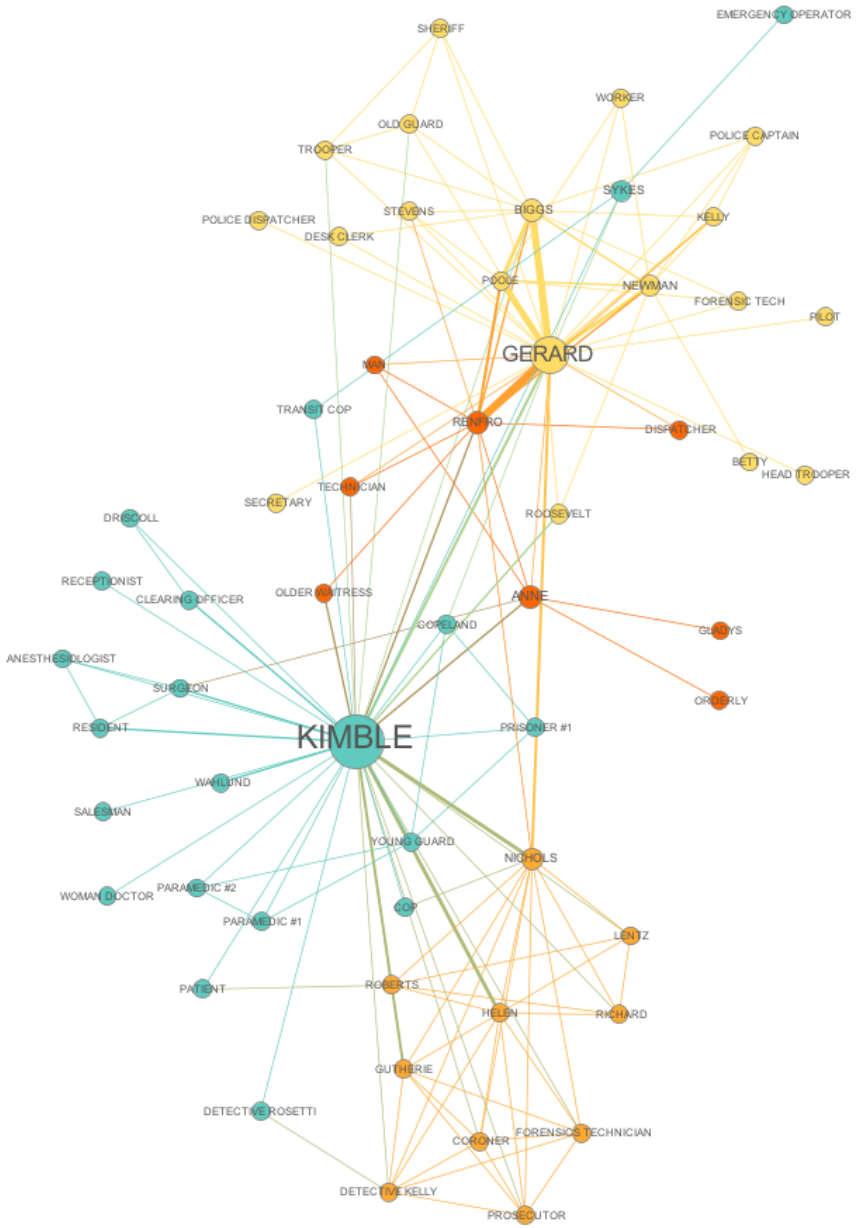
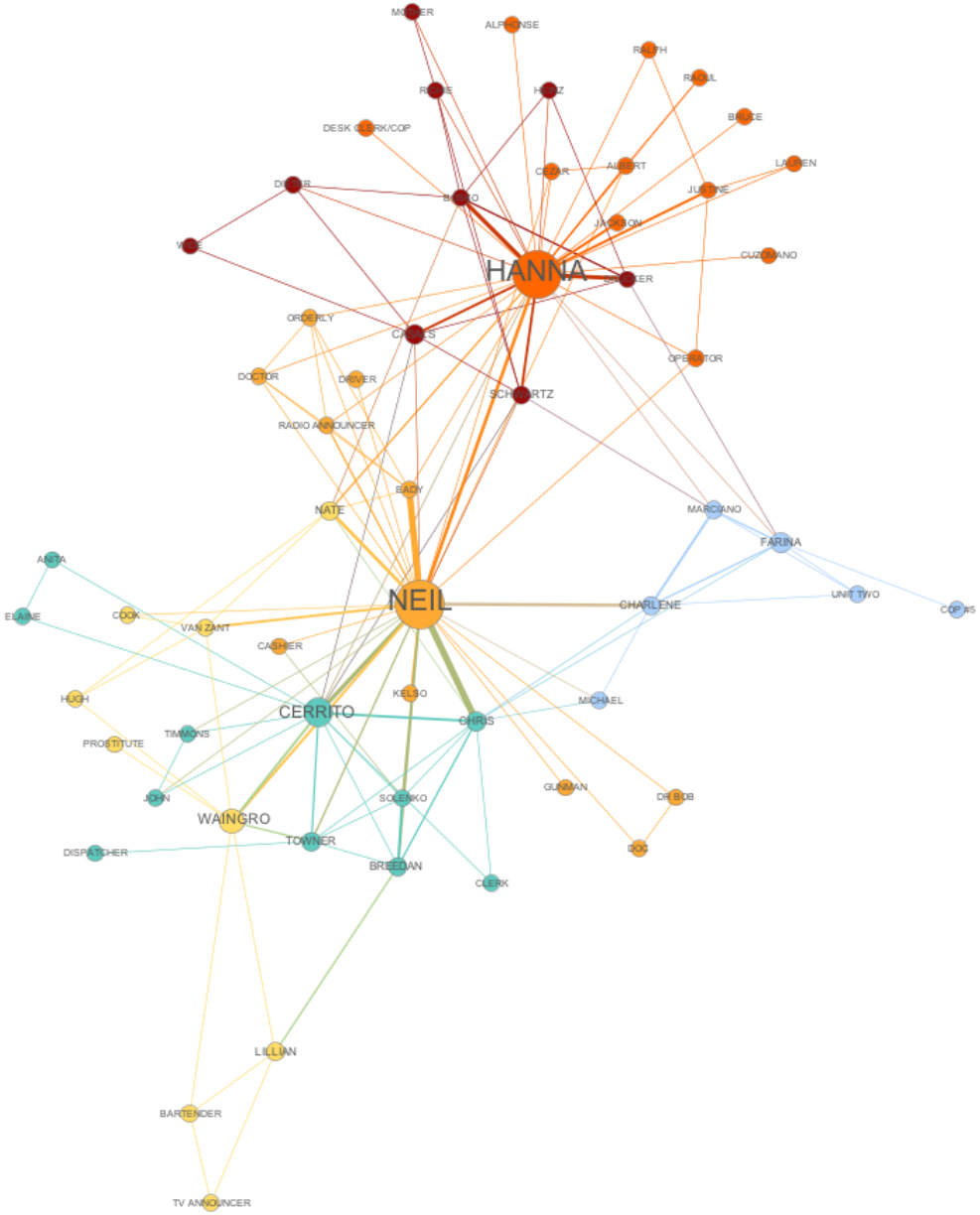
http://cs224w.stanford.edu

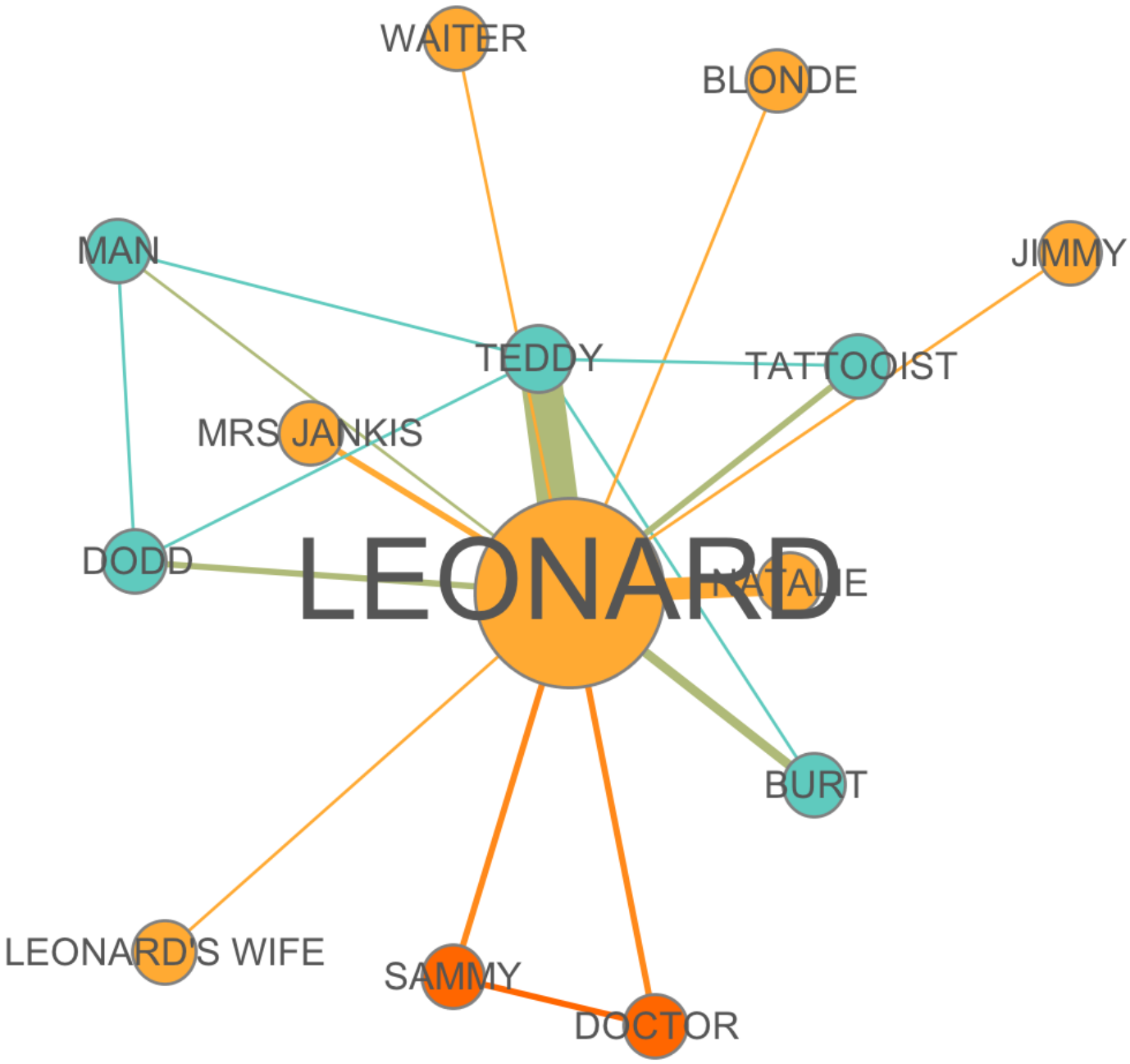
The Fugitive (1993)



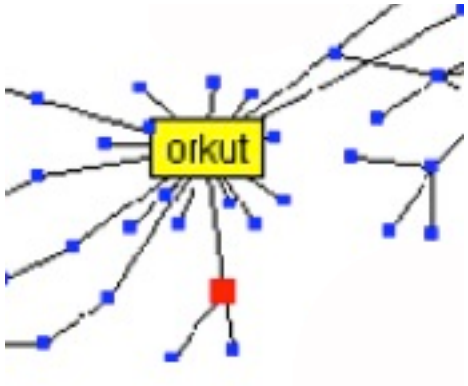
Heat (1995)



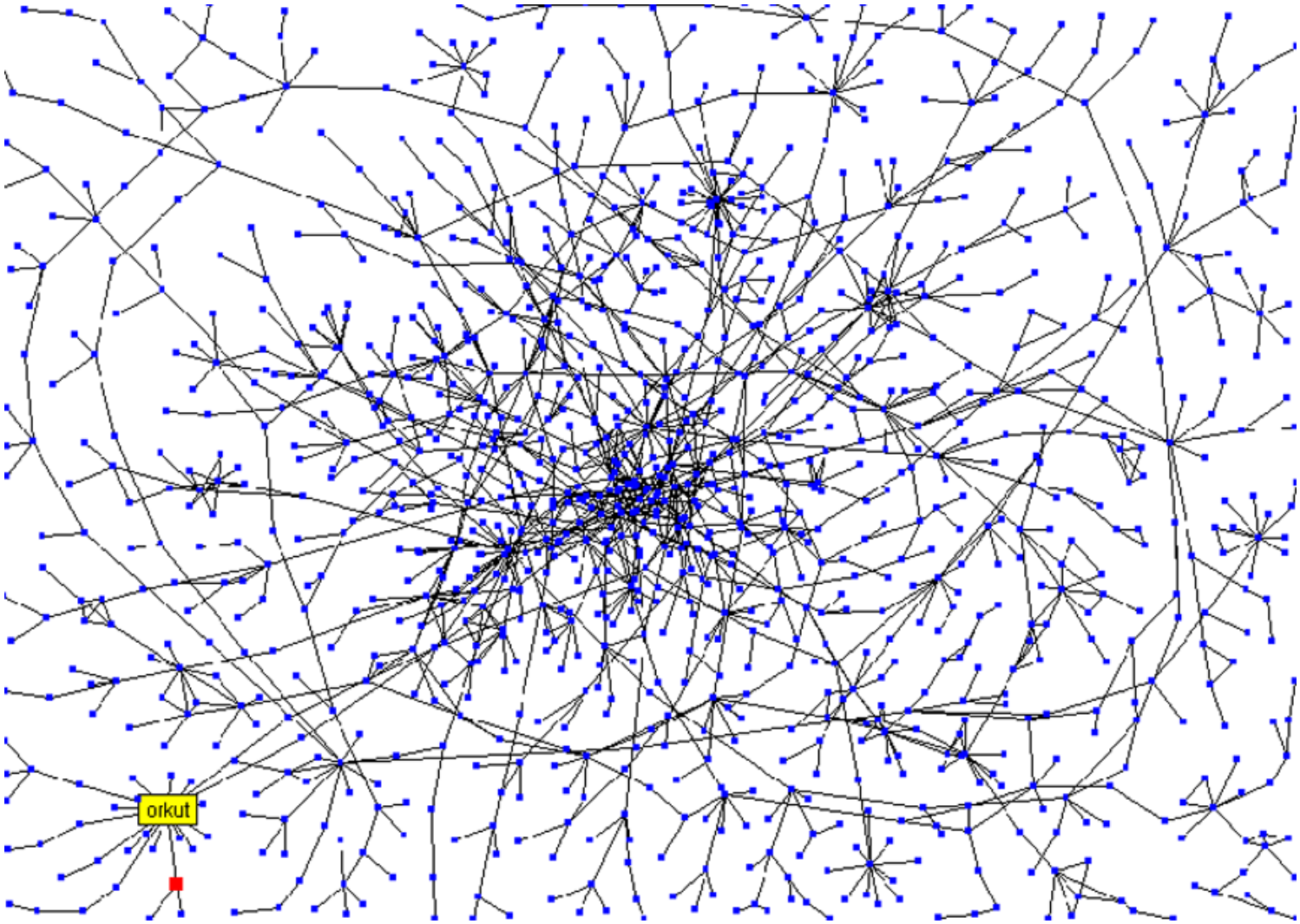
Memento (2000)



Is counting the edges enough?



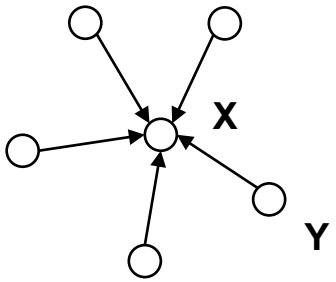
Stanford Social Web (ca. 1999)



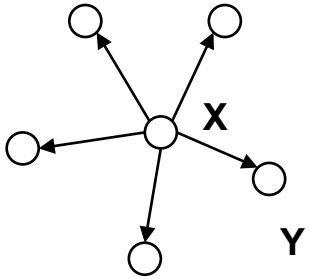
network of personal homepages at Stanford

different notions of centrality

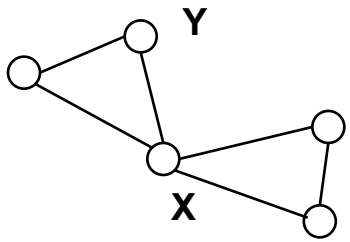
In each of the following networks, X has higher centrality than Y according to a particular measure



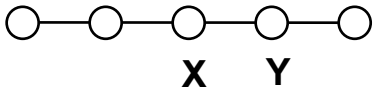
indegree



outdegree

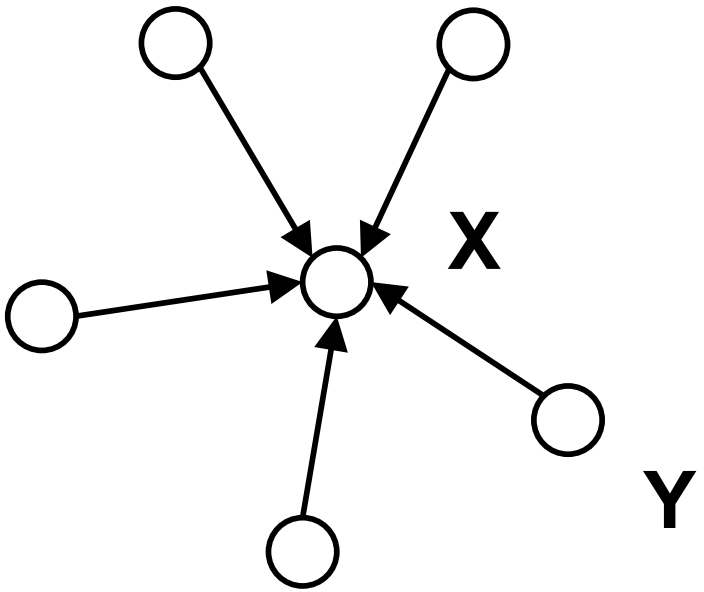


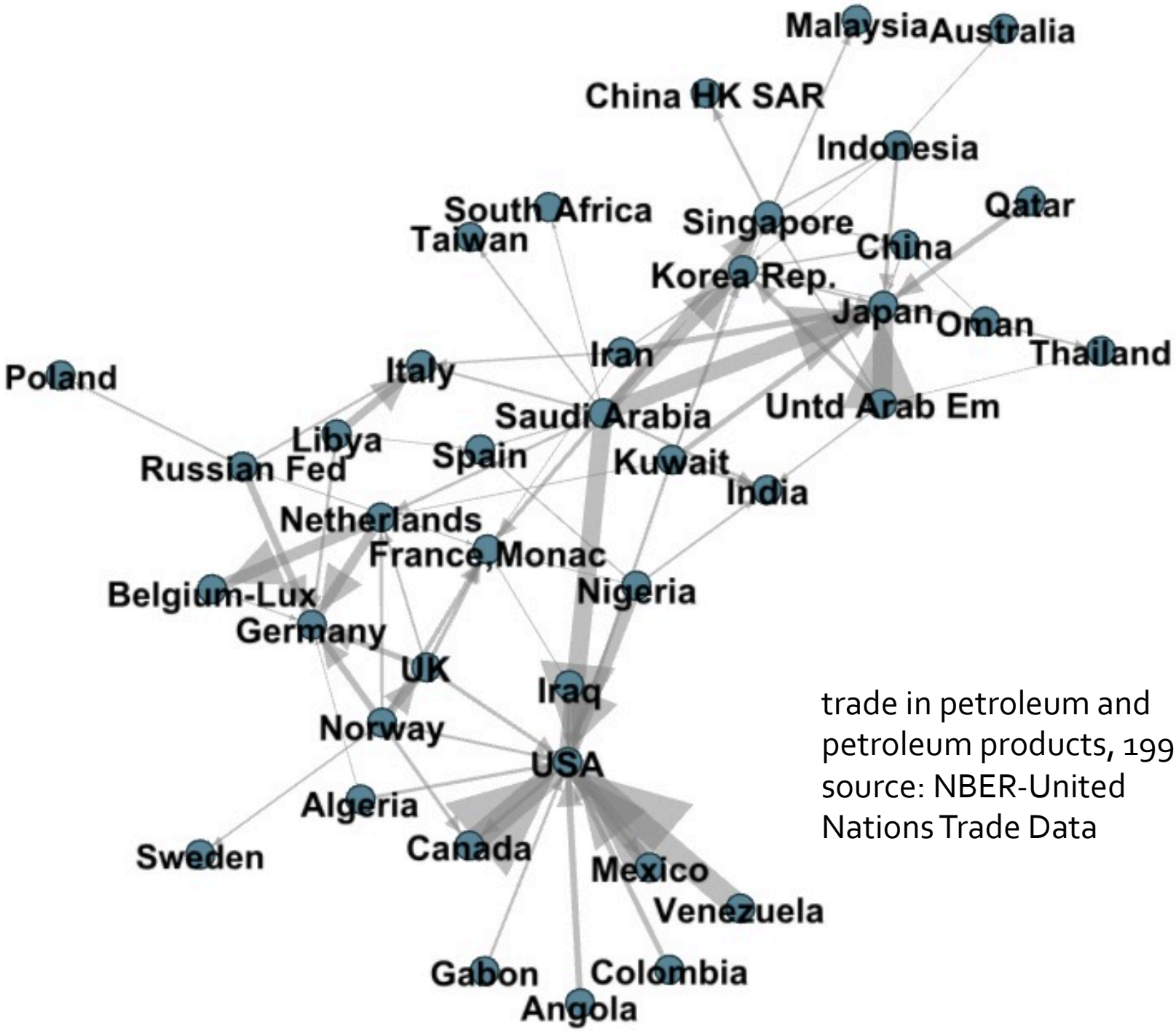
betweenness



closeness

review: indegree

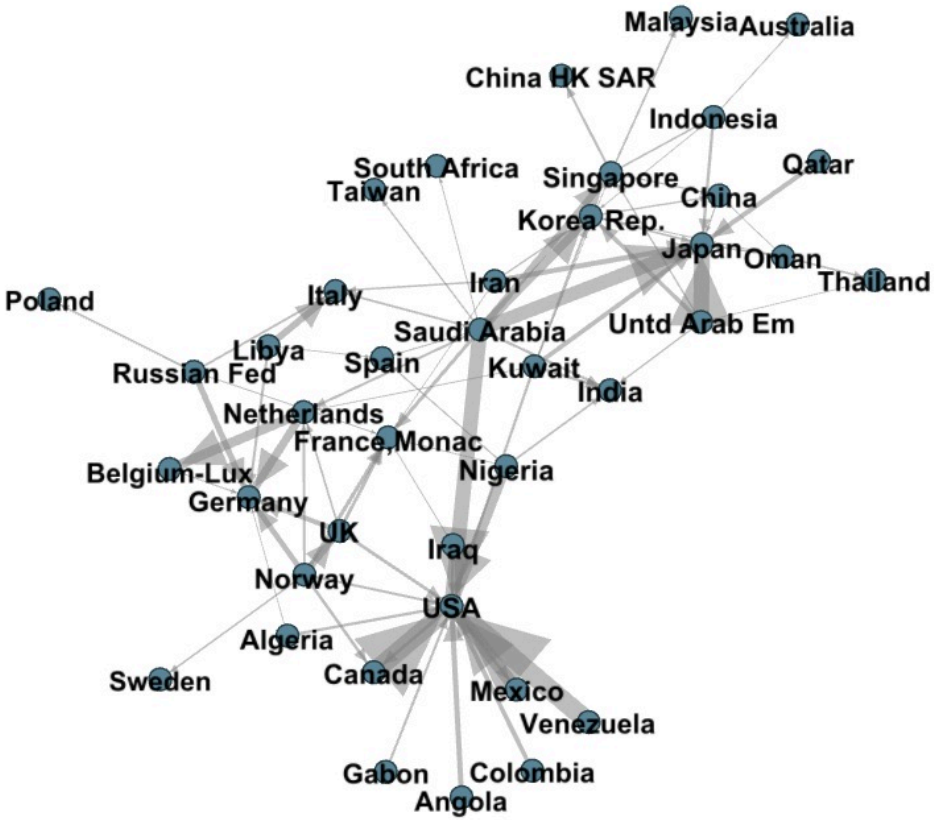




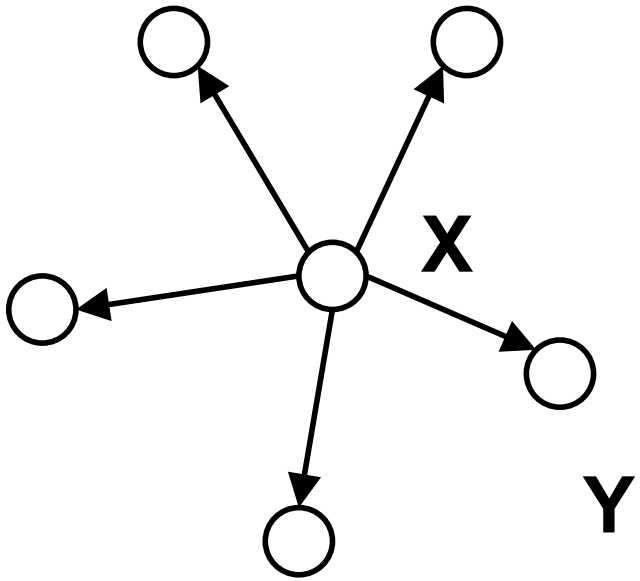
trade in petroleum and petroleum products, 1998, source: NBER-United Nations Trade Data

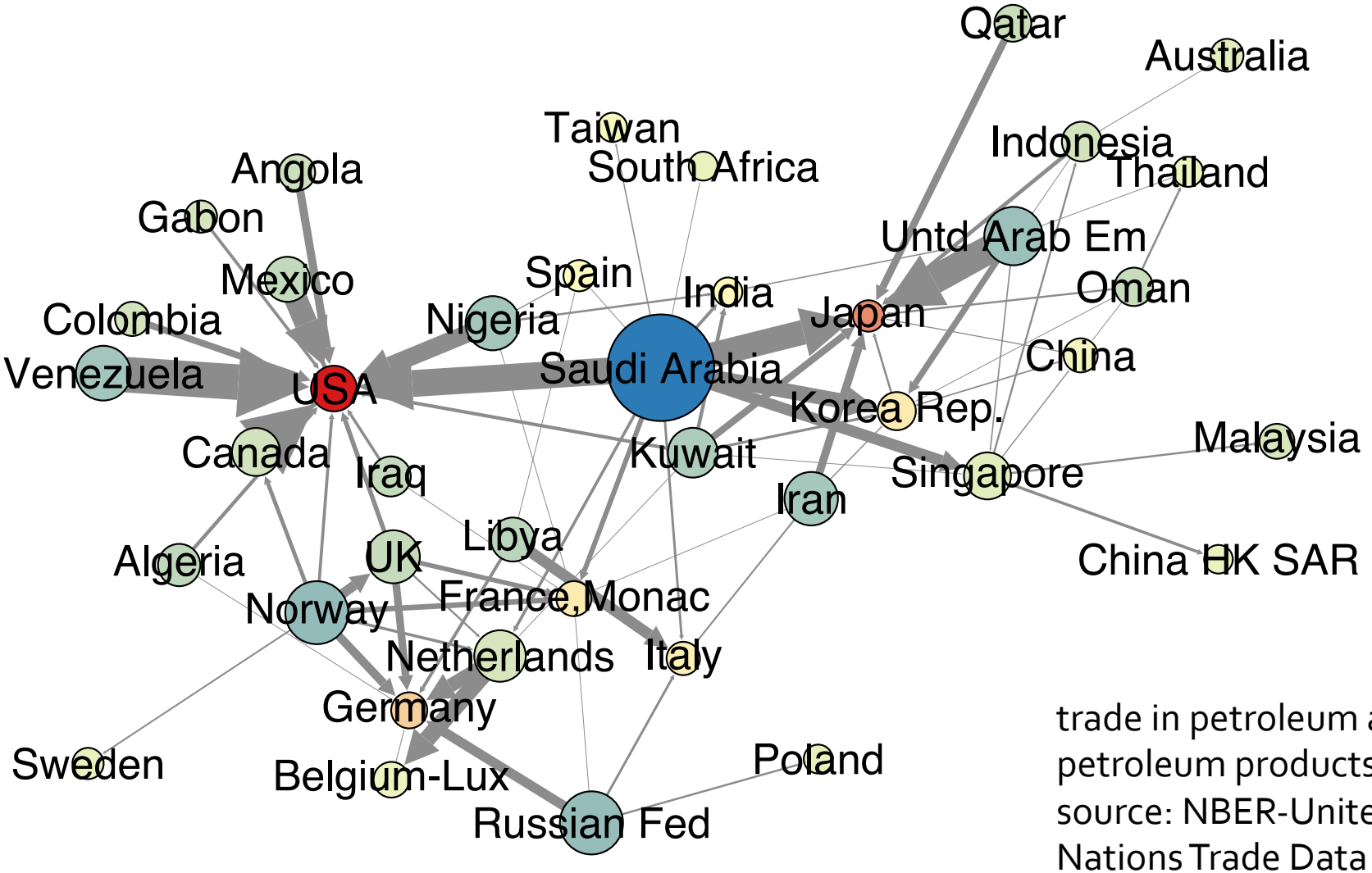
Q: high indegree

- Which countries have high indegree (import petroleum and petroleum products from many others)
  - Saudi Arabia
  - Japan
  - Iraq
  - USA
  - Venezuela



review: outdegree

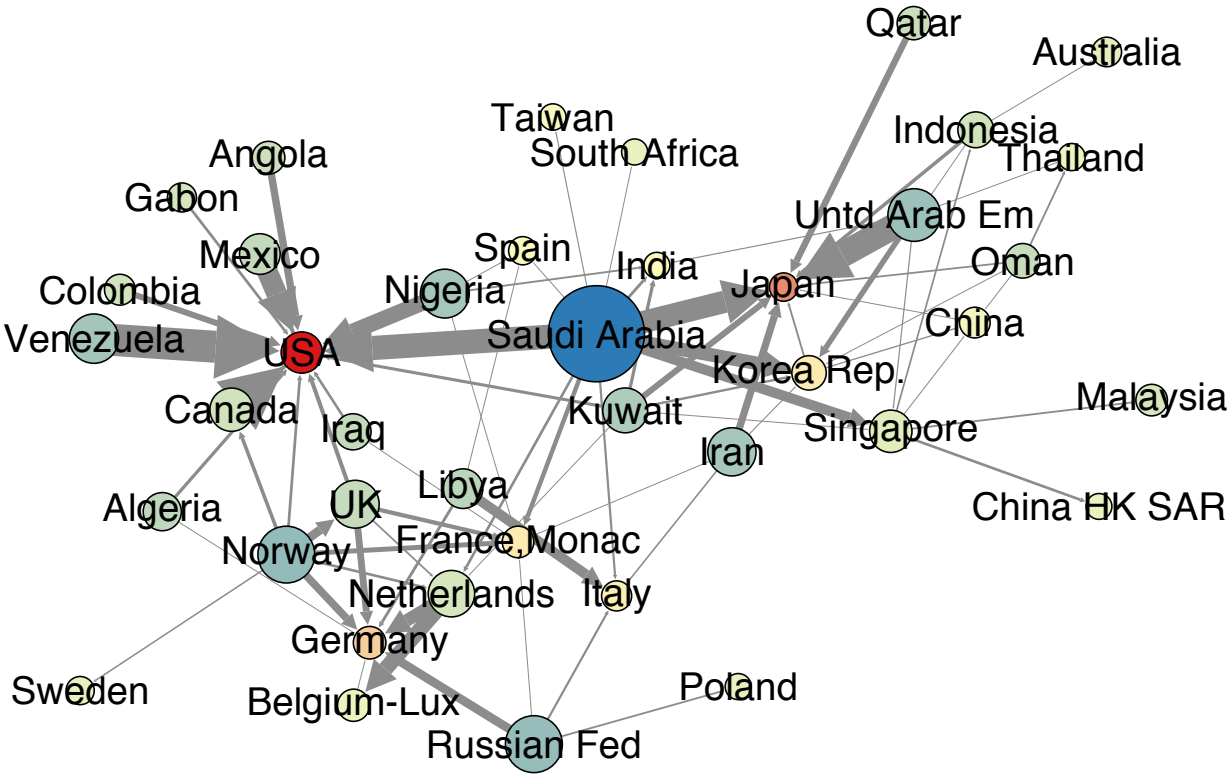


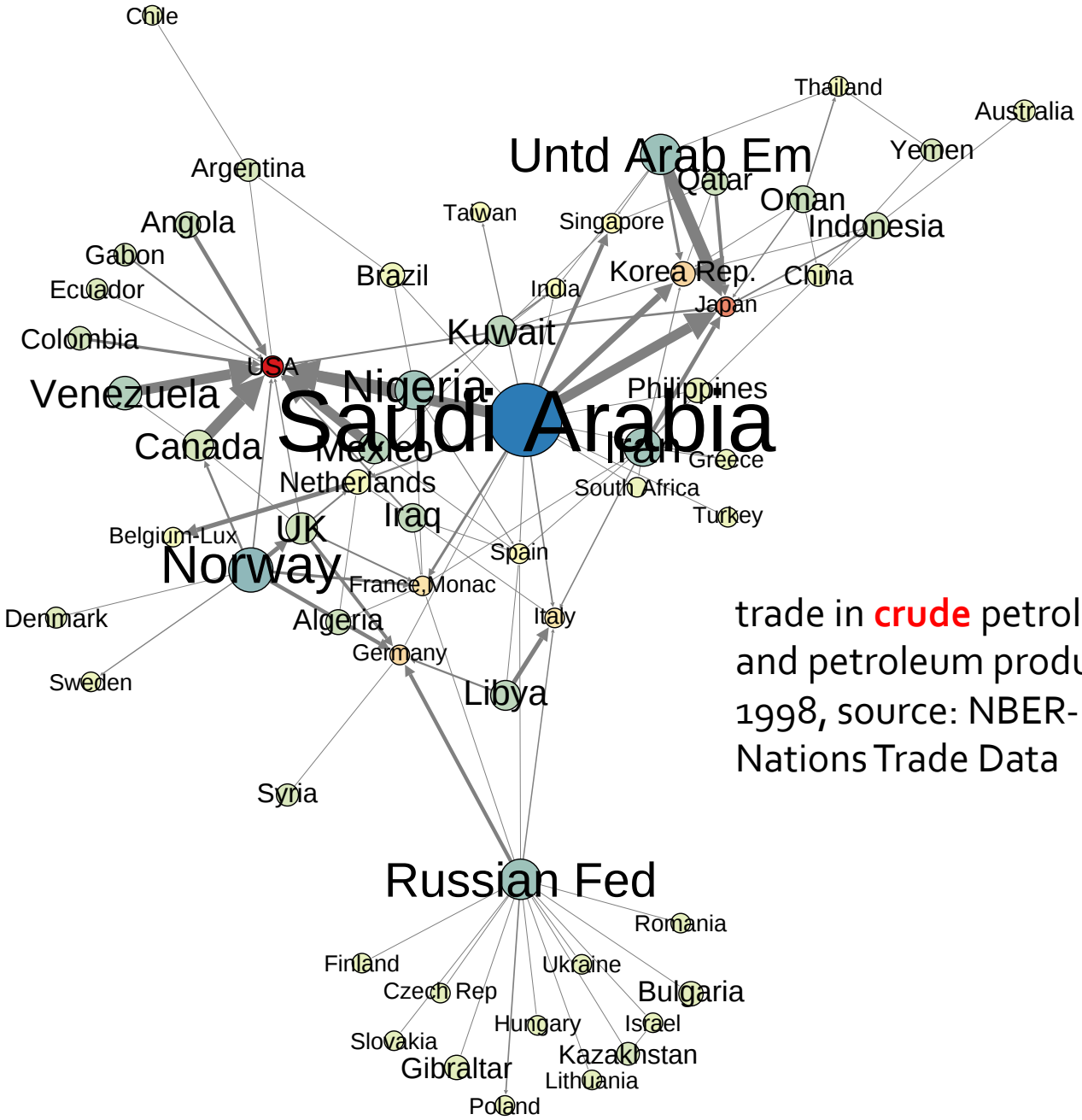


trade in petroleum and petroleum products, 1998
source: NBER-United Nations Trade Data

Q: low outdegree

- Which country has low outdegree but exports a significant quantity (thickness of the edges represents $$ value of export) of petroleum products
  - Saudi Arabia
  - Japan
  - Iraq
  - USA
  - Venezuela

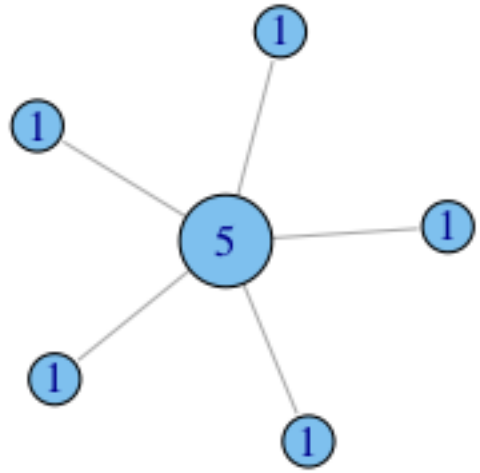




trade in **crude** petroleum and petroleum products, 1998, source: NBER-United Nations Trade Data

putting numbers to it

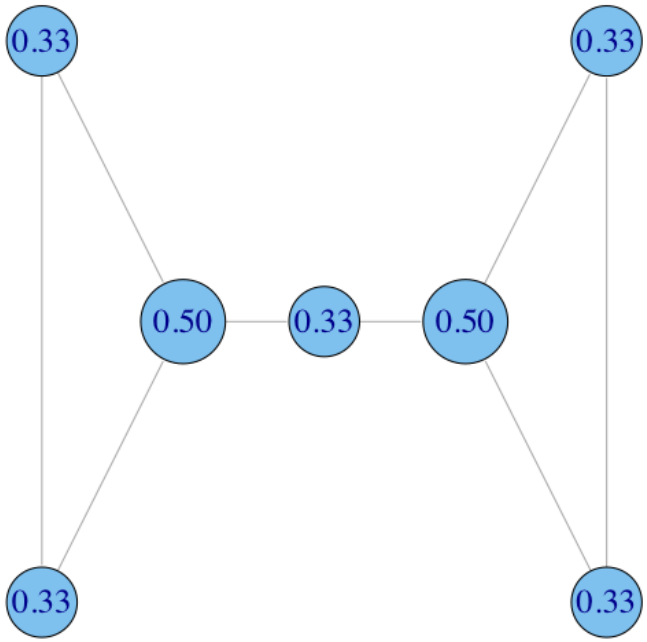
Undirected degree, e.g. nodes with more friends are more central.

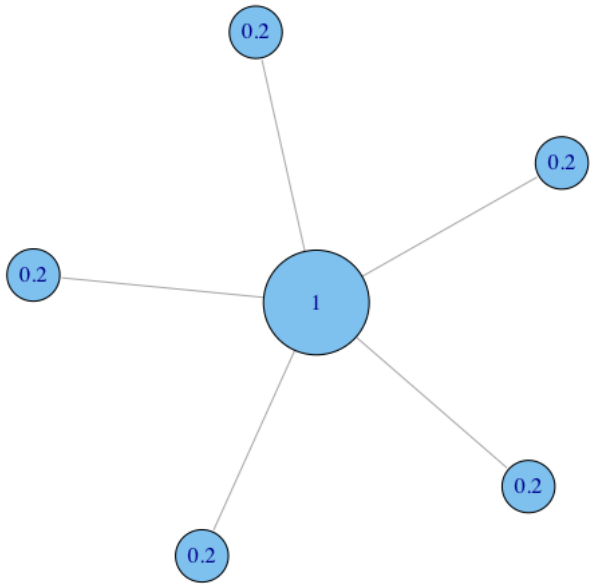


Assumption: the connections that your friend has don't matter, it is what they can do directly that does (e.g. go have a beer with you, help you build a deck...)

normalization

divide degree by the max. possible, i.e. (N-1)





centralization: skew in distribution

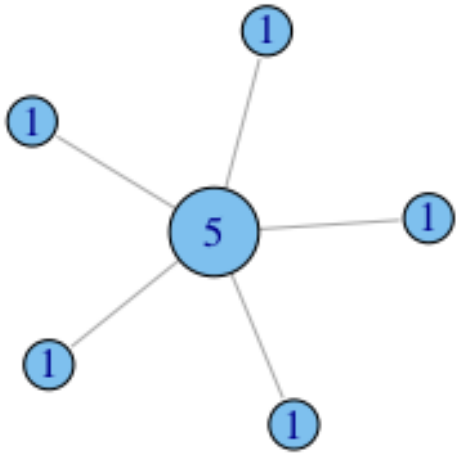
How much variation is there in the centrality scores among the nodes?

Freeman's general formula for centralization (can use other metrics, e.g. gini coefficient or standard deviation):

maximum value in the network

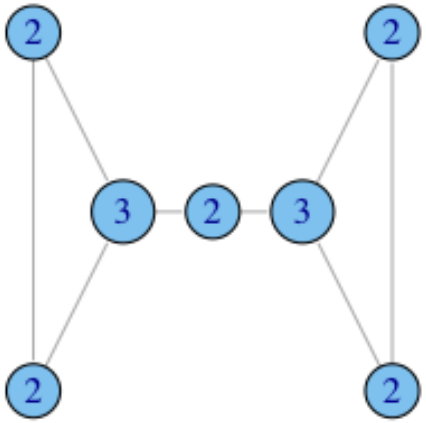
$$C_D = \frac{\sum_{i=1}^{g} \left[C_D(n^*) - C_D(i) \right]}{[(N-1)(N-2)]}$$

degree centralization examples



$C_D = 1.0$

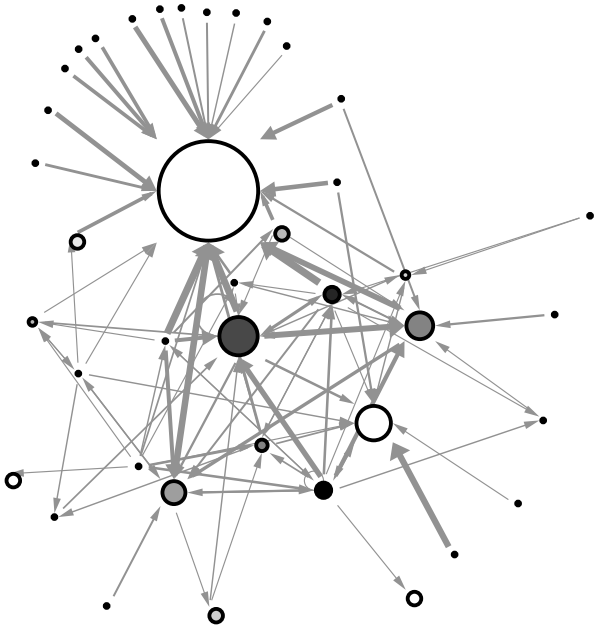
$C_D = 0.167$



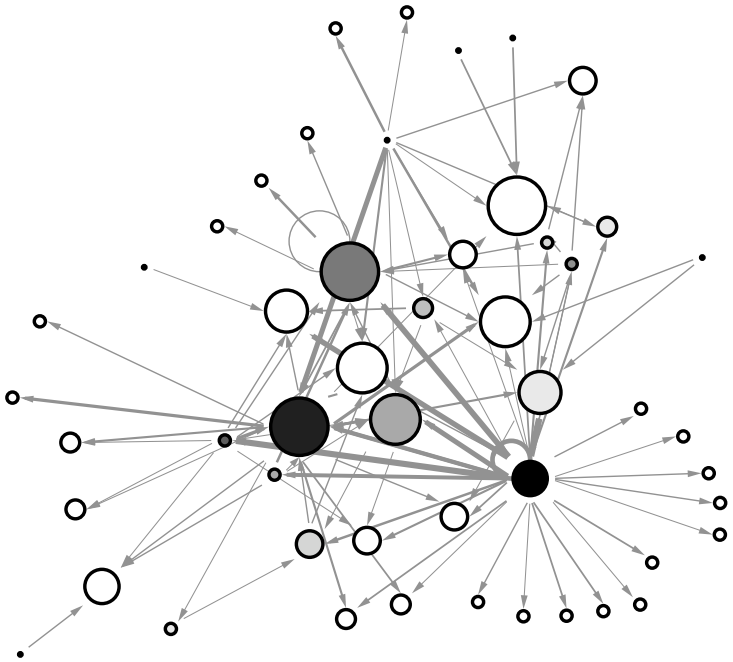
$C_D = 0.167$

real-world examples

example financial trading networks



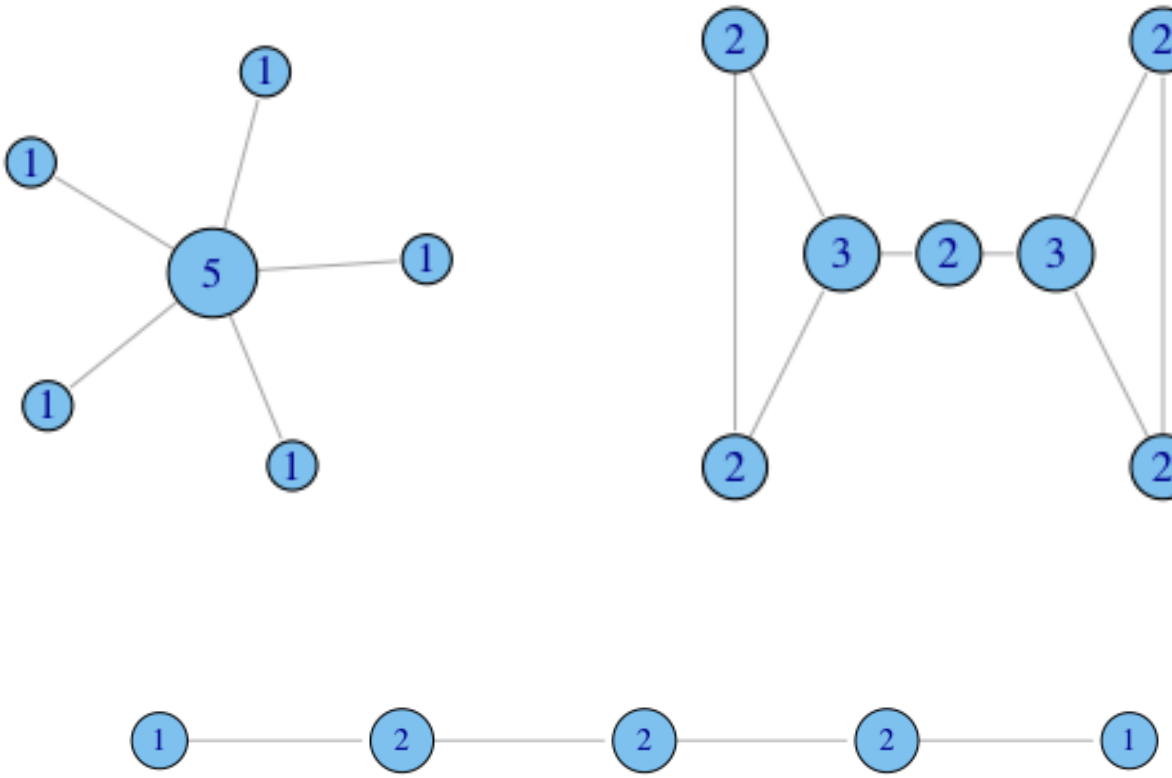
high in-centralization:
one node buying from
many others



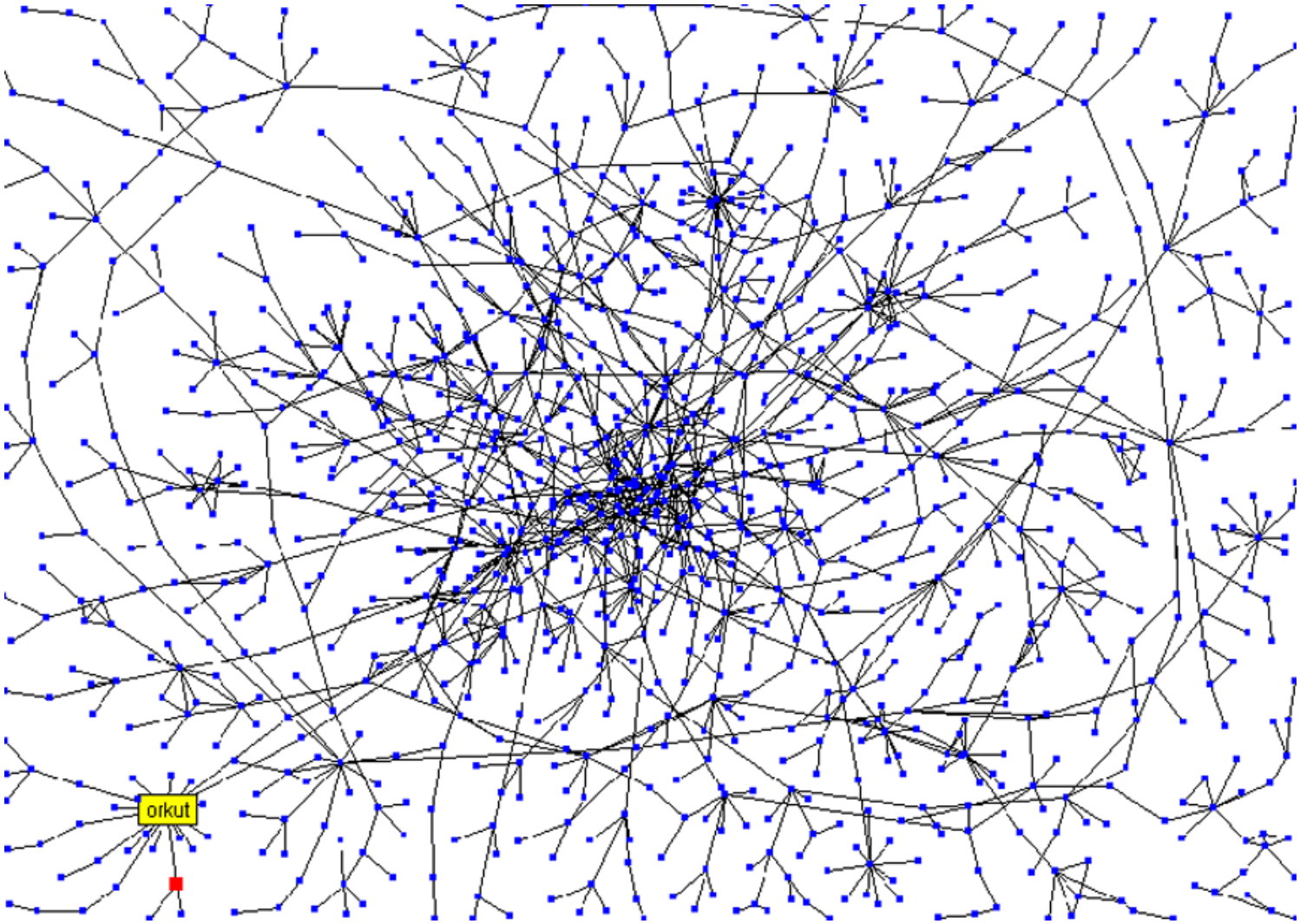
low in-centralization:
buying is more evenly
distributed

what does degree not capture?

In what ways does degree fail to capture centrality in the following graphs?

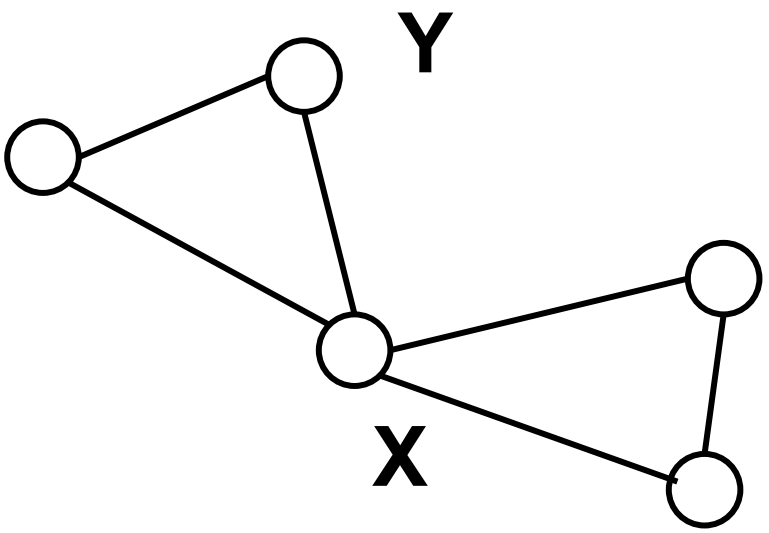


Stanford Social Web (ca. 1999)

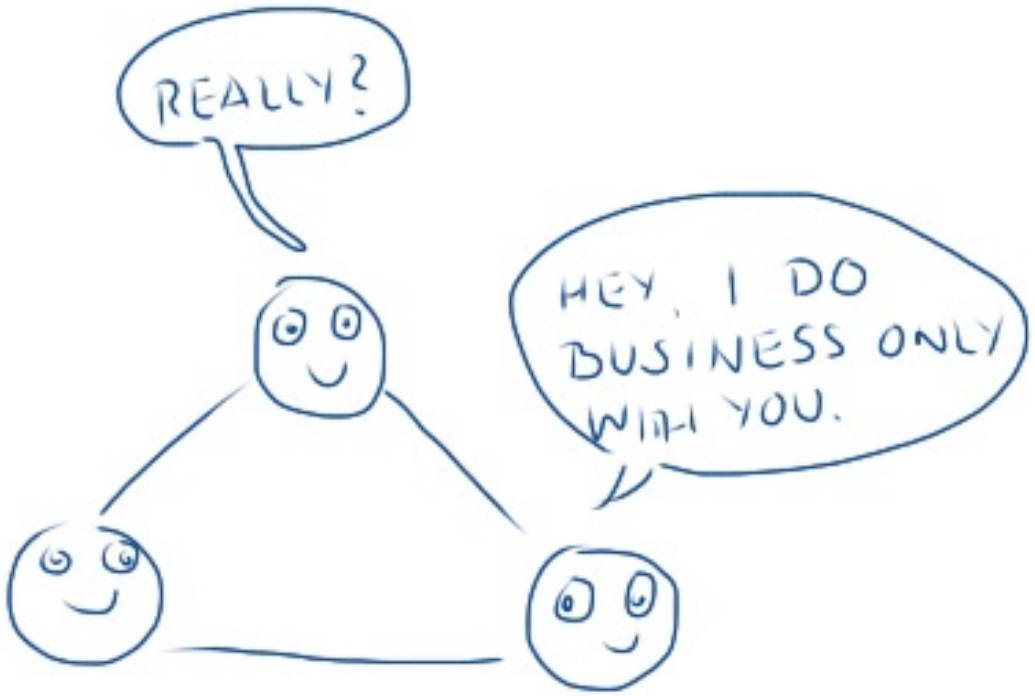


network of personal homepages at Stanford

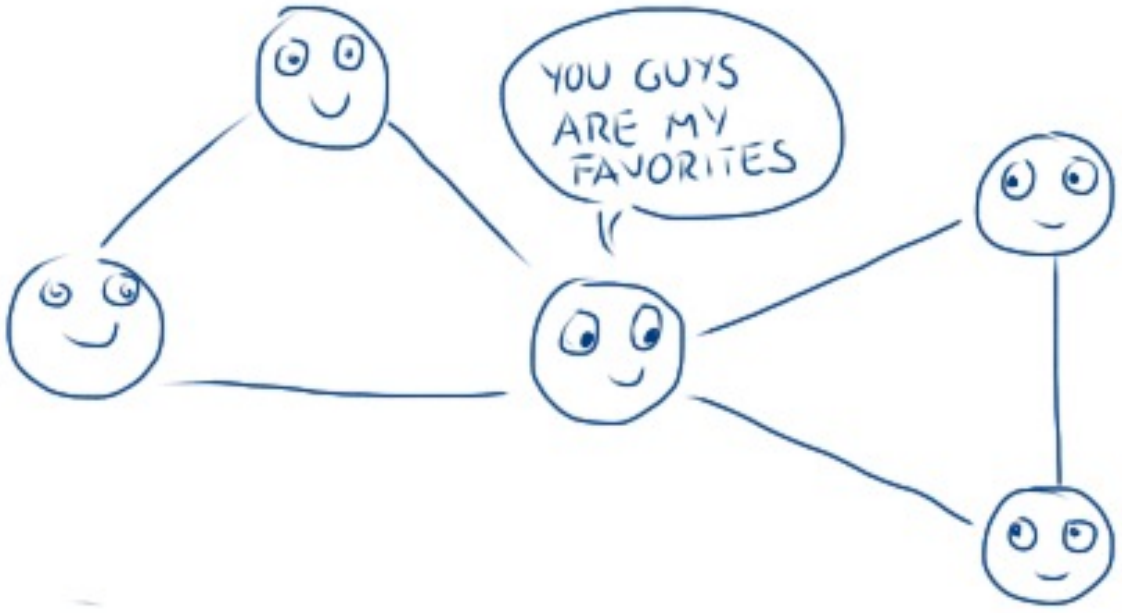
Brokerage not captured by degree



Constraint

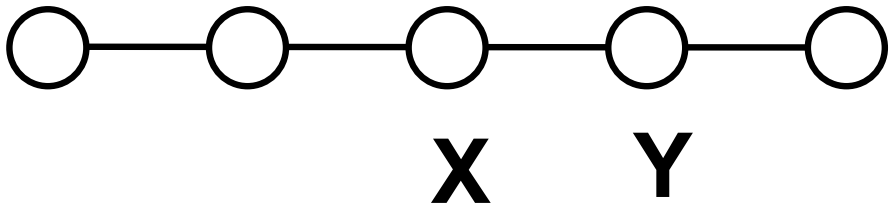


constraint



Betweenness: capturing brokerage

- intuition: how many pairs of individuals would have to go through you in order to reach one another in the minimum number of hops?



Betweenness: definition

$$C_B(i) = \sum_{j<k} g_{jk}(i) / g_{jk}$$

Where g_{jk} = the number of shortest paths connecting *jk*
$g_{jk}(i)$ = the number that actor *i* is on.

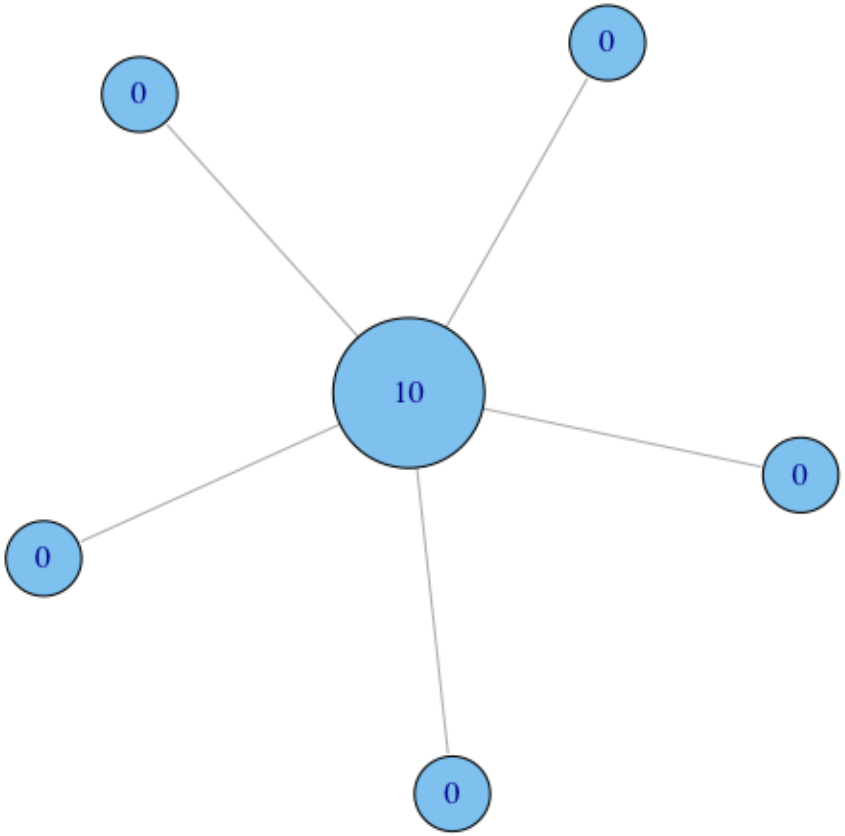
Usually normalized by:

$$C'_B(i) = C_B(i\ ) / [(n-1)(n-2)/2]$$

number of pairs of vertices excluding the vertex itself

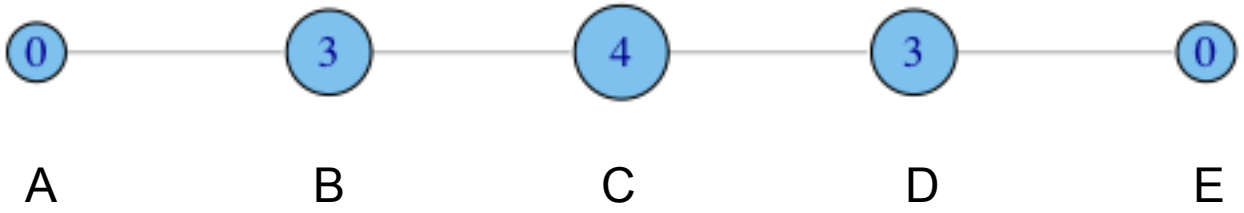
Betweenness on toy networks

- non-normalized version:



Betweenness on toy networks

- non-normalized version:

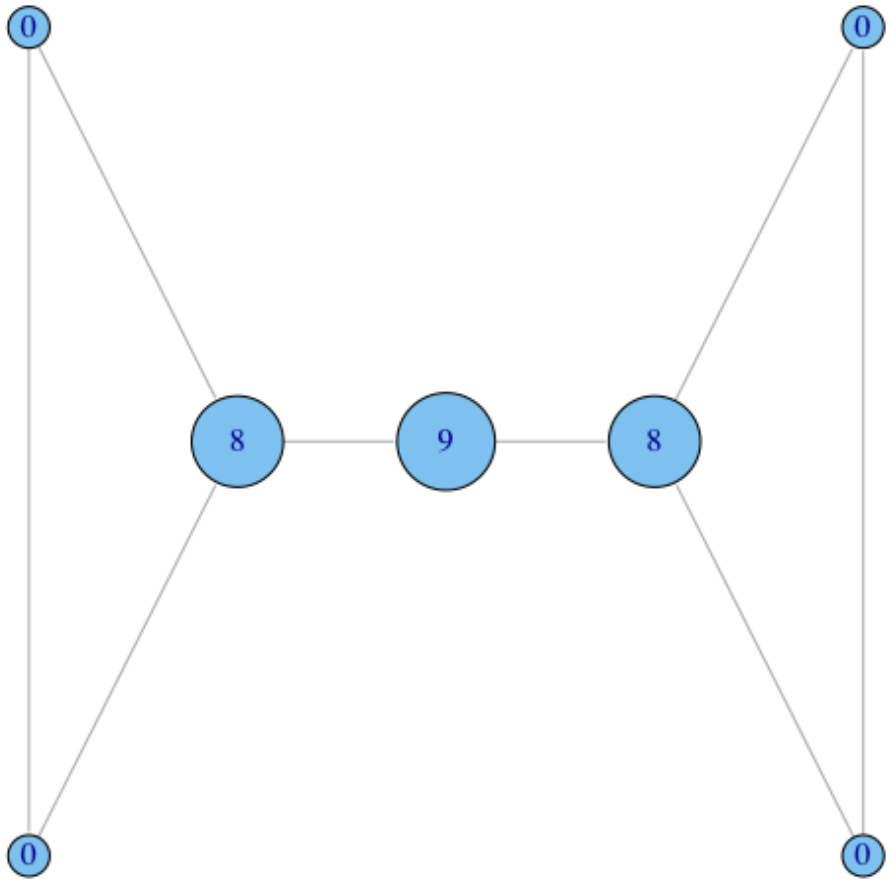


- A lies between no two other vertices
- B lies between A and 3 other vertices: C, D, and E
- C lies between 4 pairs of vertices (A,D),(A,E),(B,D),(B,E)

- note that there are no alternate paths for these pairs to take, so C gets full credit

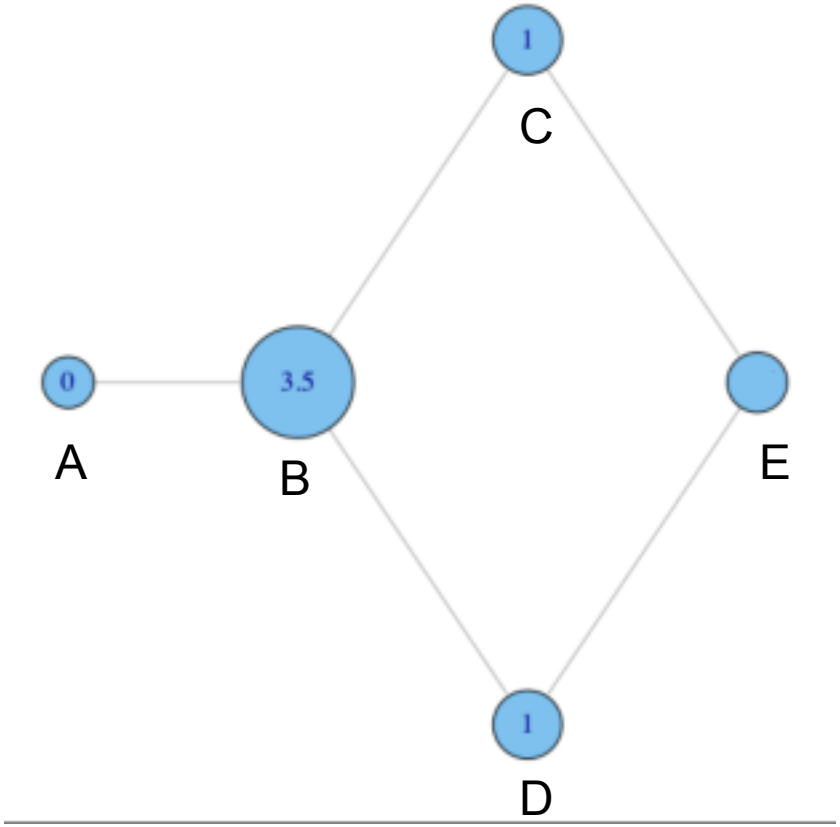
Betweenness on toy networks

- non-normalized version:



Betweenness on toy networks

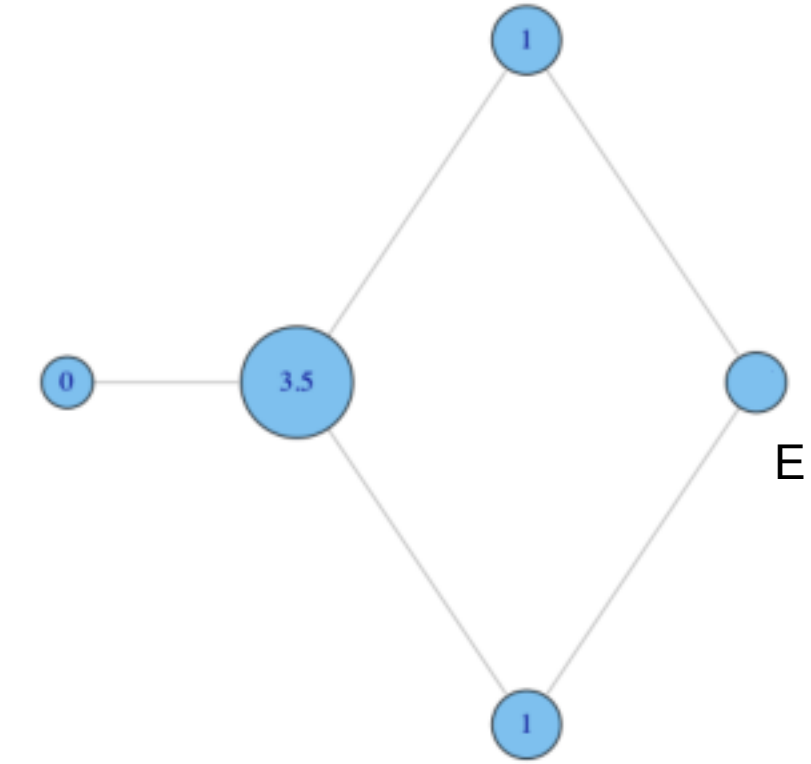
- non-normalized version:



- why do C and D each have betweenness 1?
- They are both on shortest paths for pairs (A,E), and (B,E), and so must share credit:
  - ½+½ = 1

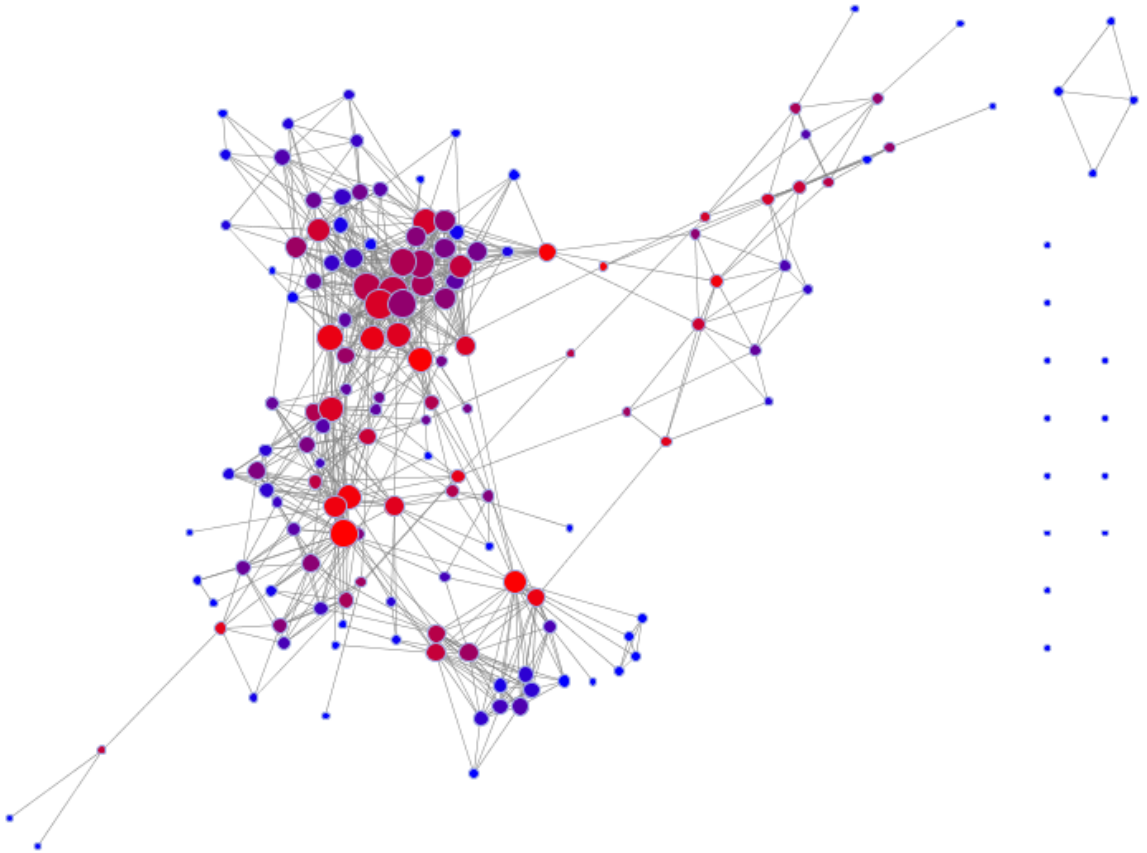
Q: betweenness

- What is the betweenness of node E?



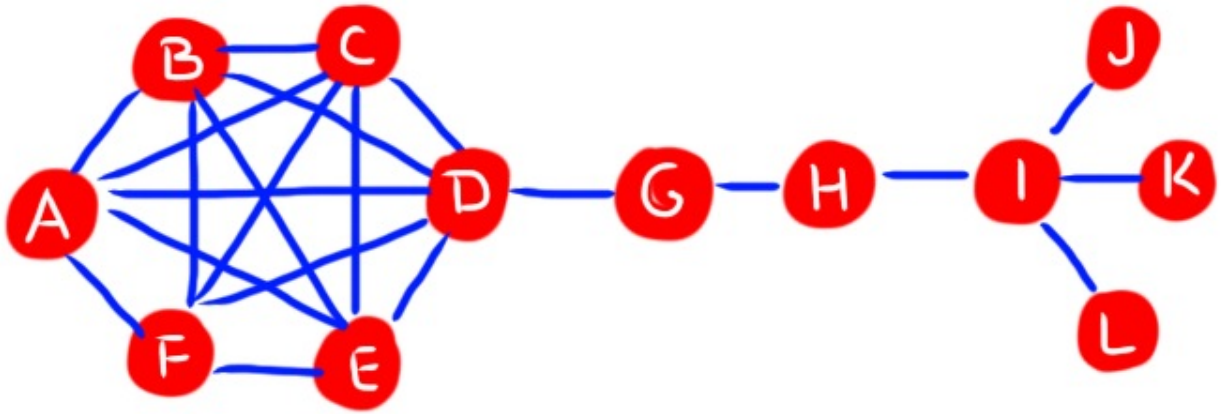
betweenness: example

Lada's old Facebook network: nodes are sized by degree, and colored by betweenness.



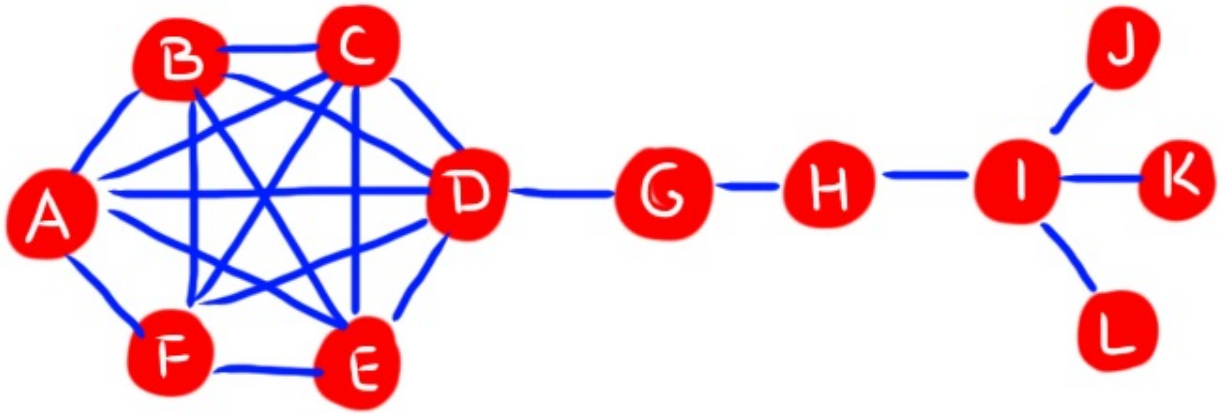
Q: high betweenness, low degree

- Find a node that has high betweenness but low degree



Q: low betweenness, high degree

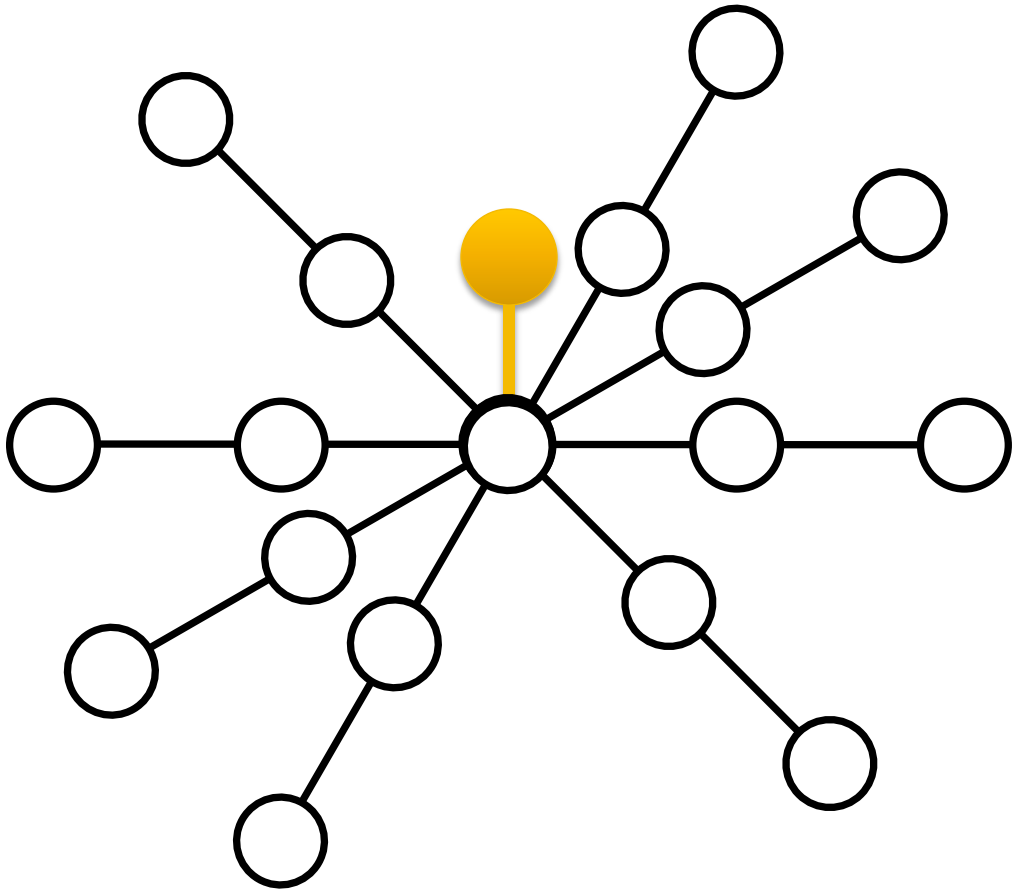
- Find a node that has low betweenness but high degree



closeness

- What if it’s not so important to have many direct friends?
- Or be “between” others
- But one still wants to be in the “middle” of things, not too far from the center

need not be in a brokerage position



closeness: definition

Closeness is based on the length of the average shortest path between a node and all other nodes in the network

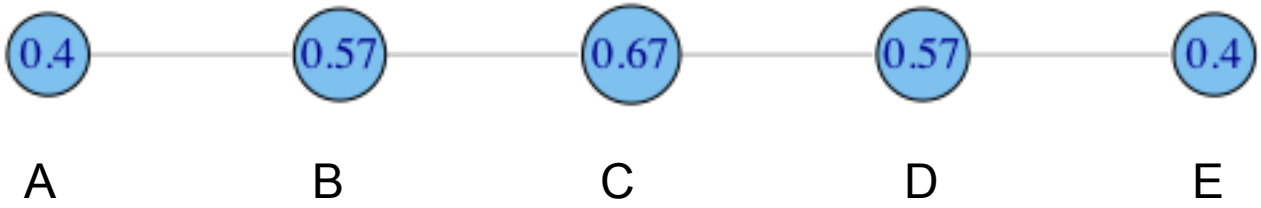
Closeness Centrality:

$$C_c(i) = \left[\sum_{j=1}^{N} d(i,j) \right]^{-1}$$

Normalized Closeness Centrality

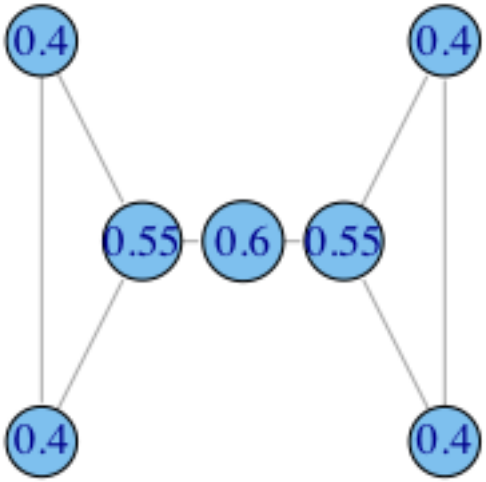
$$C_C'(i) = (C_C(i))/(N-1)$$

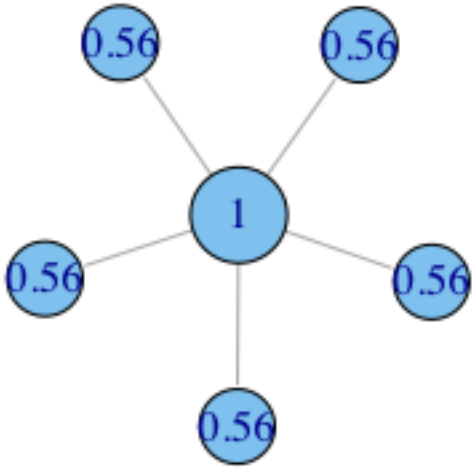
Closeness: toy example



$$C_c^{'}(A) = \left[\frac{\sum_{j=1}^{N} d(A,j)}{N-1} \right]^{-1} = \left[\frac{1+2+3+4}{4} \right]^{-1} = \left[\frac{10}{4} \right]^{-1} = 0.4$$

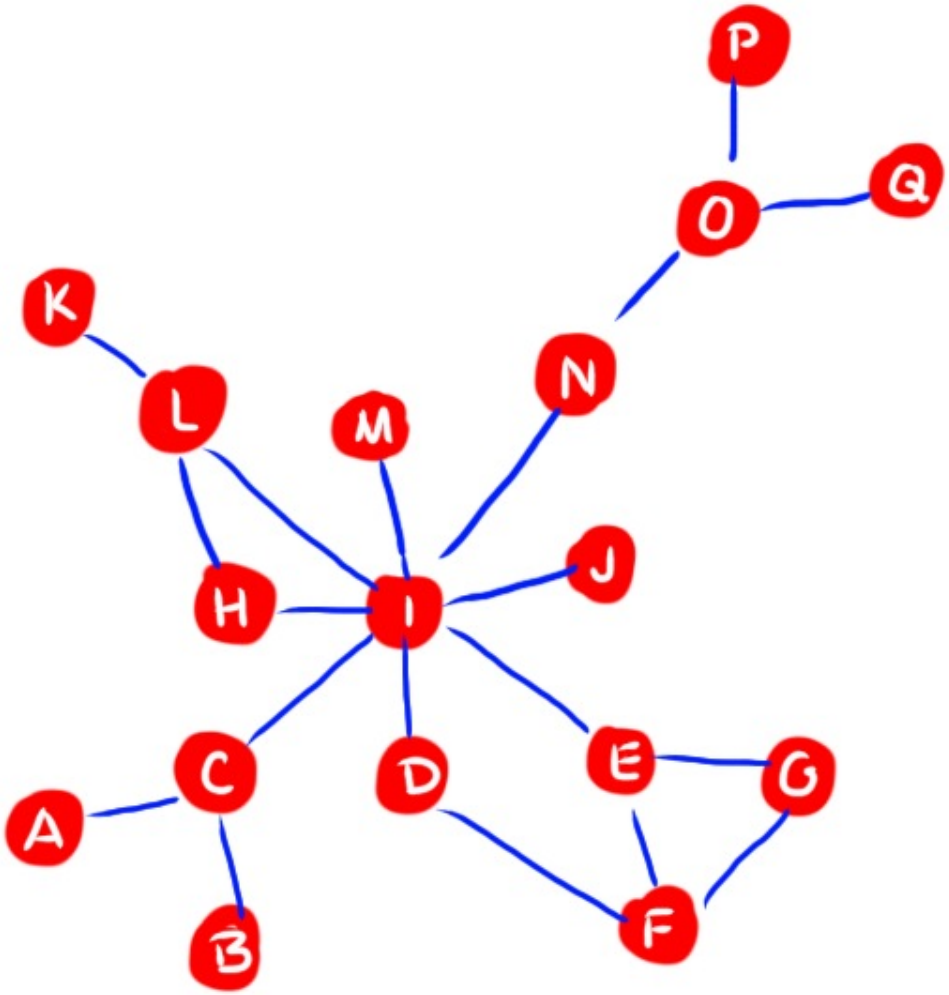
Closeness: more toy examples





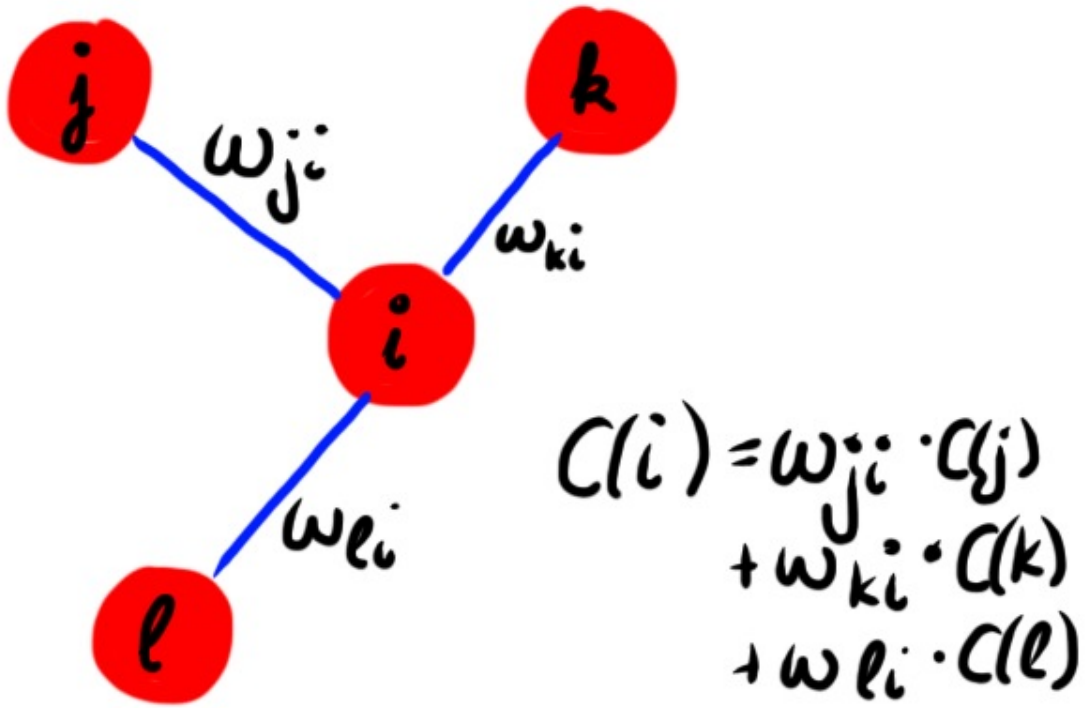
Q:high degree, low closeness

Which node has relatively high degree but low closeness?



Eigenvector centrality

- How central you are depends on how central your neighbors are



Bonacich eigenvector centrality

$$c_i(\beta) = \sum_j (\alpha + \beta c_j) A_{ji}$$

$$c(\beta) = \alpha(I - \beta A)^{-1} A1$$

- α is a normalization constant
- β determines how important the centrality of your neighbors is
- $\mathbf{A}$ is the adjacency matrix (can be weighted)
- $\mathbf{I}$ is the identity matrix (1s down the diagonal, 0 off-diagonal)
- $\mathbf{1}$ is a matrix of all ones.

Bonacich Power Centrality: attenuation factor β

small β ➔ high attenuation

only your immediate friends matter, and their importance is factored in only a bit

high β ➔ low attenuation

global network structure matters (your friends, your friends' of friends etc.)

β = 0 yields simple degree centrality

$$c_i(\beta) = \sum_j (\alpha \quad) A_{ji}$$

Bonacich Power Centrality: attenuation factor β

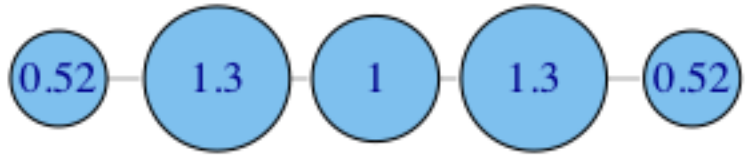
If $\beta > 0$, nodes have higher centrality when they have edges to other central nodes.

If $\beta < 0$, nodes have higher centrality when they have edges to less central nodes.

Bonacich Power Centrality: examples

β=.25

β=-.25



Why does the middle node have lower centrality than its neighbors when β is negative?

Centrality in directed networks

- WWW
- food webs
- population dynamics
- influence
- hereditary
- citation
- transcription regulation networks
- neural networks

Betweenness centrality in directed networks

- We now consider the fraction of all directed paths between any two vertices that pass through a node

betweenness of vertex i

paths between j and k that pass through i

$$C_B(i) = \sum_{j,k} g_{jk}(i) / g_{jk}$$

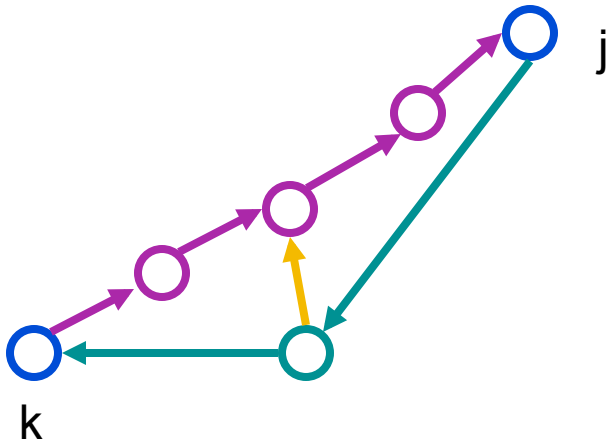
all paths between j and k

- Only modification: when normalizing, we have (N-1)*(N-2) instead of (N-1)*(N-2)/2, because we have twice as many ordered pairs as unordered pairs

$$C_B'(i) = C_B(i) / [(N-1)(N-2)]$$

Directed geodesics

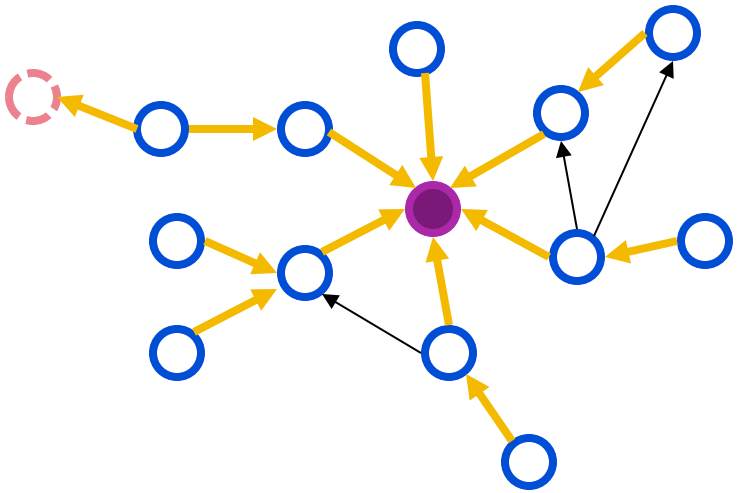
- A node does not necessarily lie on a geodesic (shortest path) from *j* to *k* if it lies on a geodesic from *k* to *j*



Directed closeness centrality

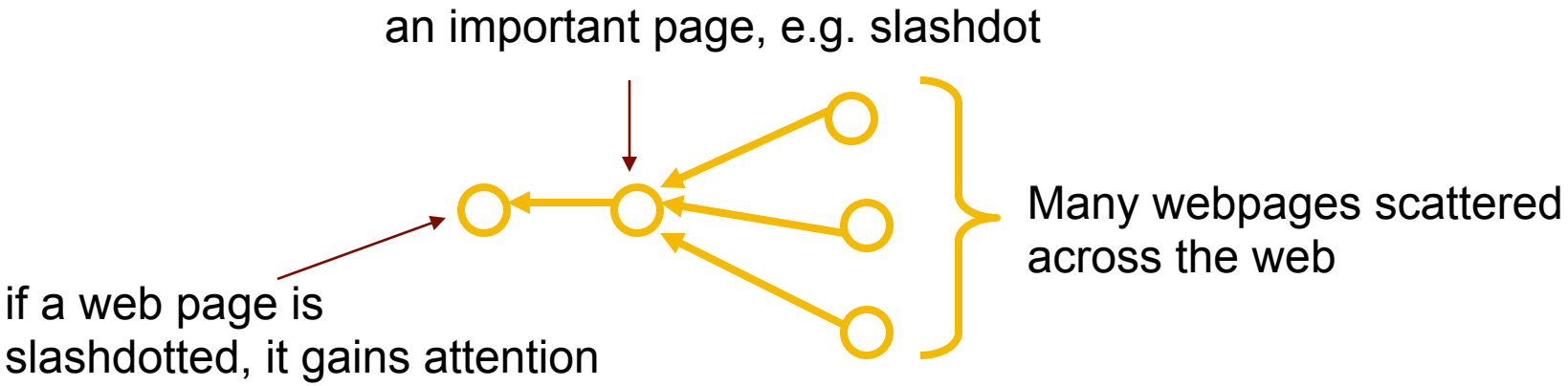
- choose a direction
  - in-closeness (e.g. prestige in citation networks)
  - out-closeness

- usually consider only vertices from which the node *i* in question can be reached



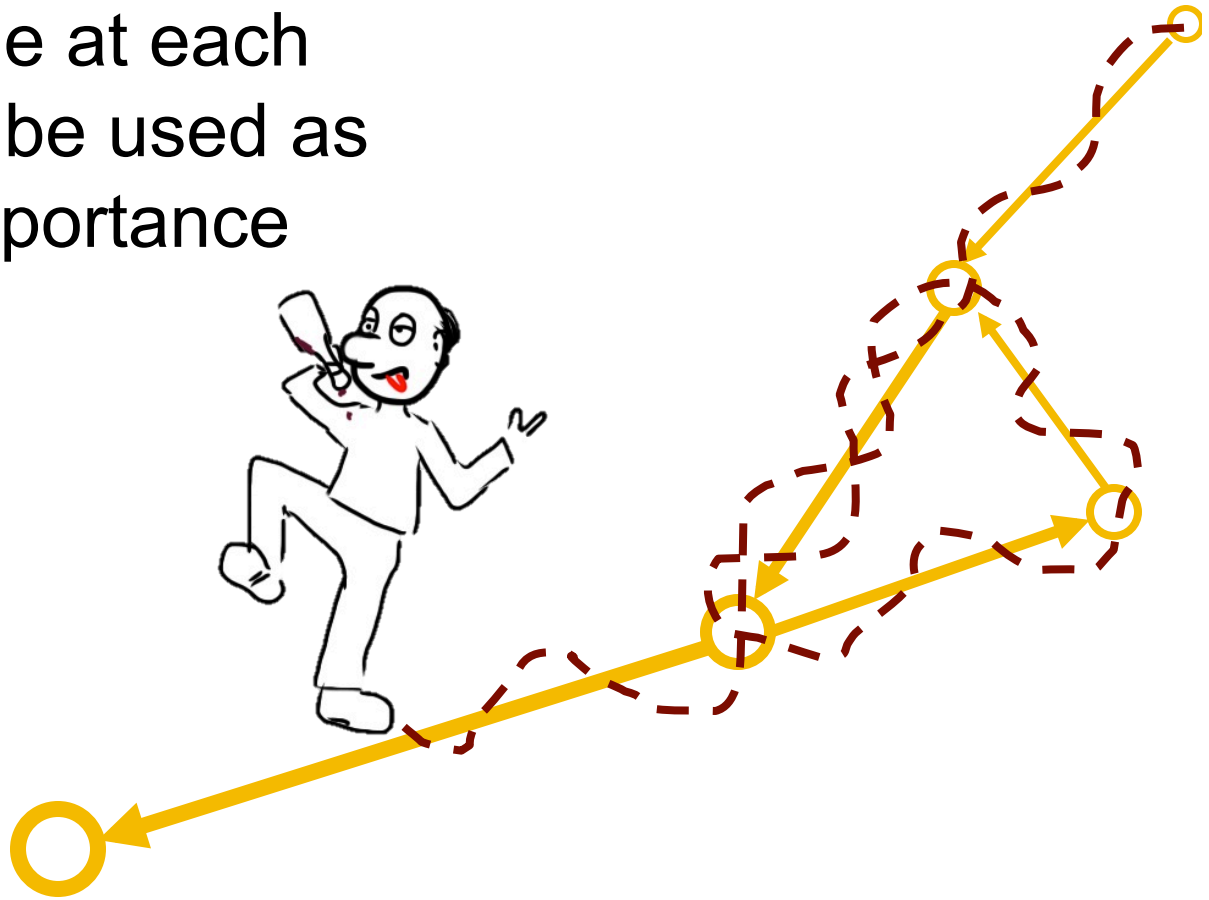
Eigenvector centrality in directed networks

- PageRank (centrality) brings order to the Web:
  - it's not just the pages that point to you, but how many pages point to those pages, etc.
  - more difficult to artificially inflate centrality with a recursive definition



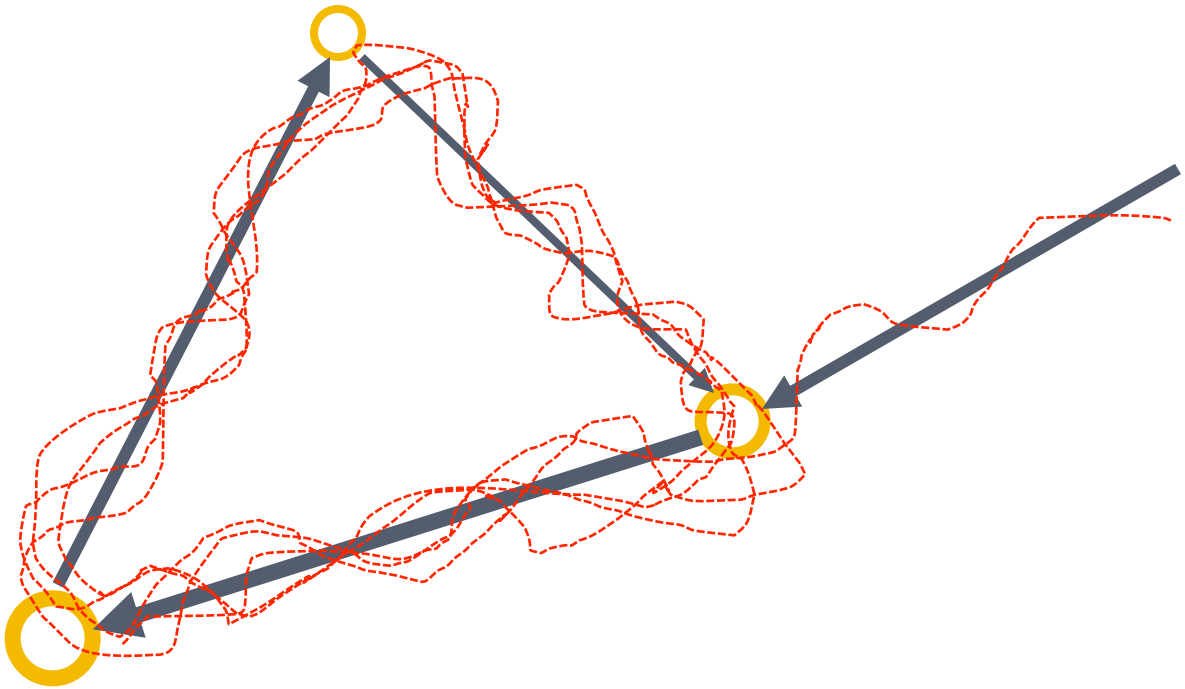
Ranking pages by tracking a drunk

- A random walker following edges in a network for a very long time will spend a proportion of time at each node which can be used as a measure of importance



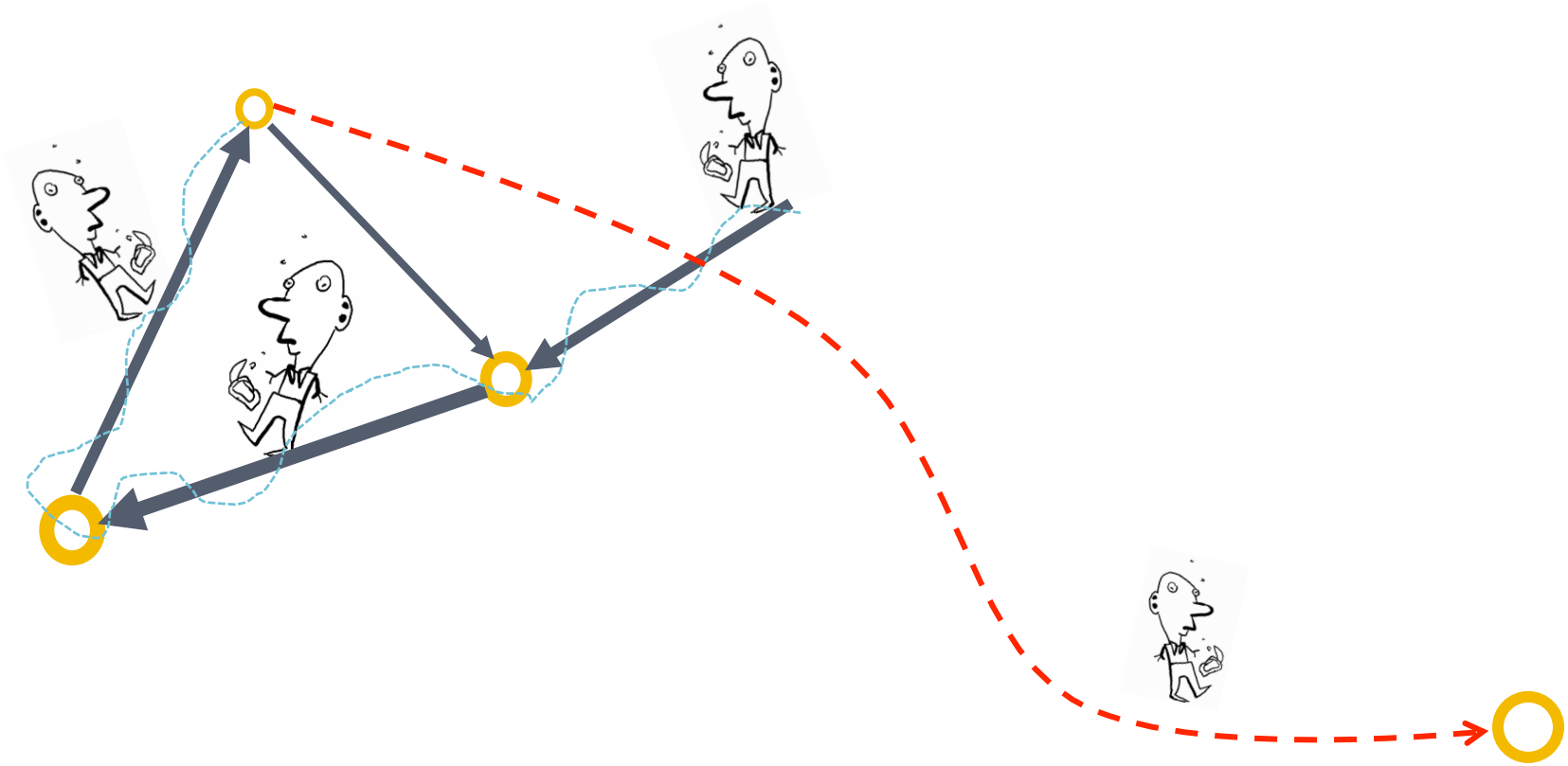
Trapping a drunk

- Problem with pure random walk metric:
  - Drunk can be “trapped” and end up going in circles



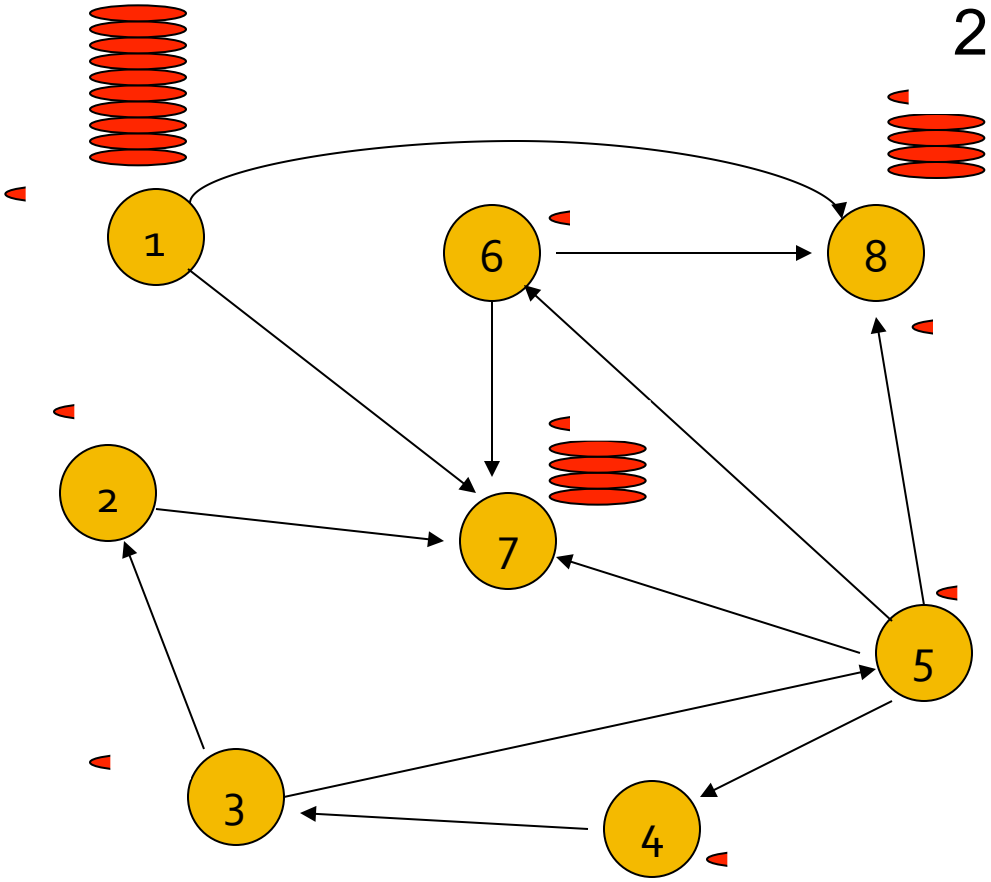
Ingenuity of the PageRank algorithm

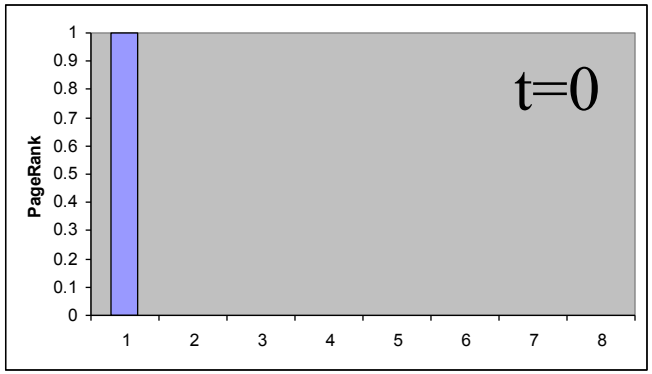
- Allow drunk to teleport with some probability
  - e.g. random websurfer follows links for a while, but with some probability teleports to a “random” page (bookmarked page or uses a search engine to start anew)

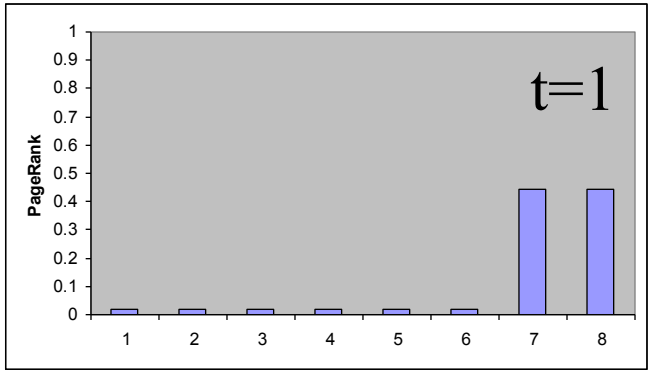


Example: probable location of random walker after 1 step

20% teleportation probability

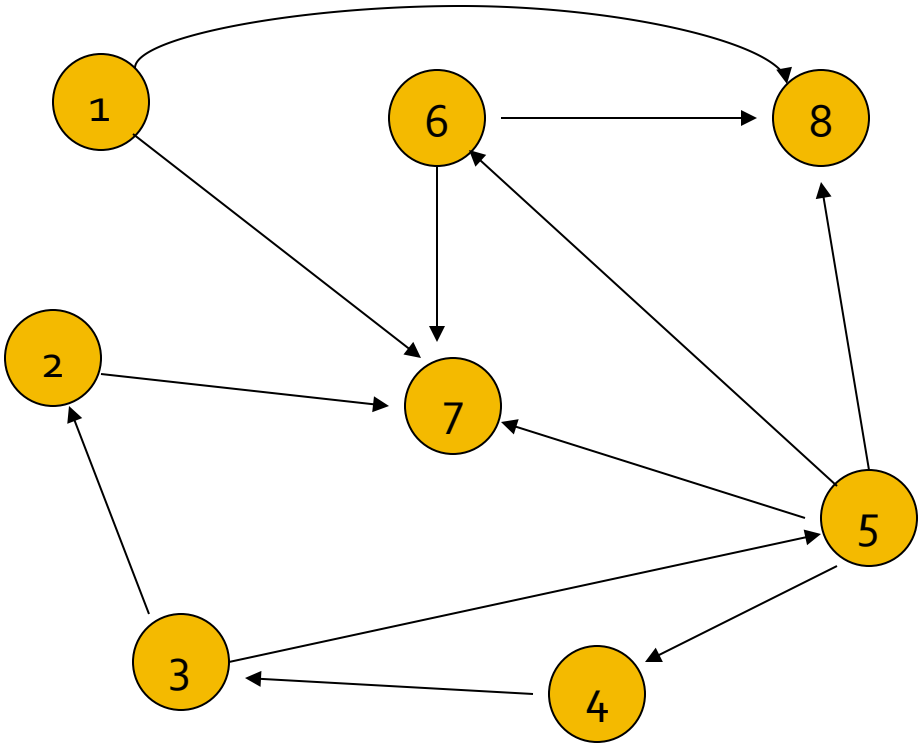


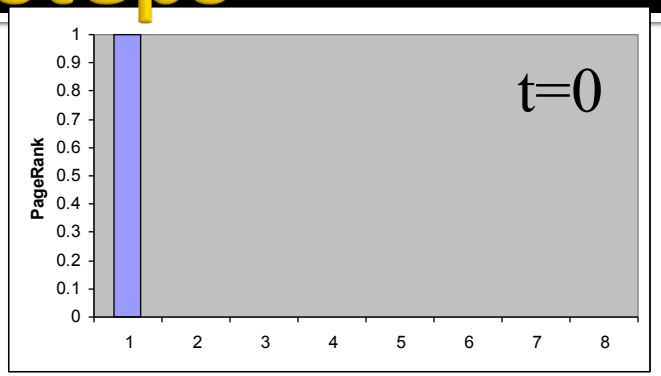


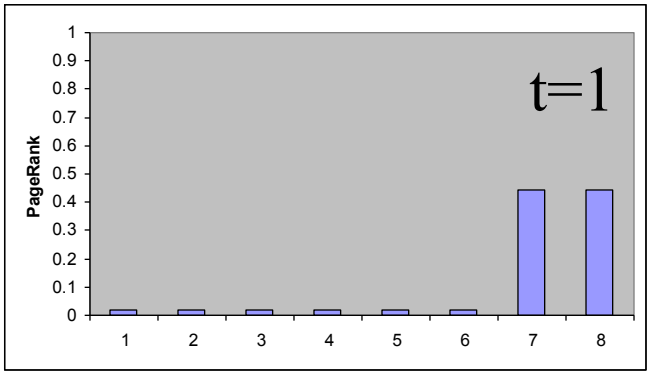


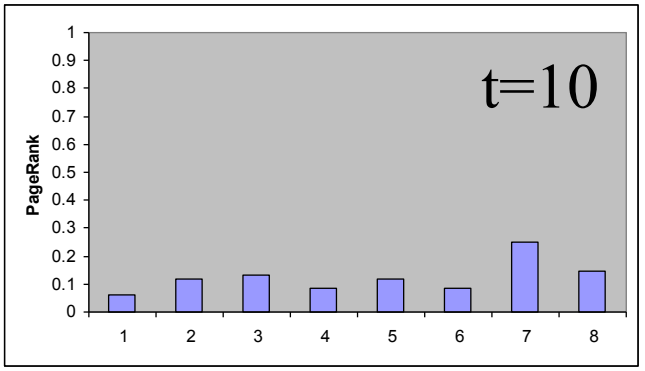
slide adapted from: Dragomir Radev

Example: probable location of random walker after 10 steps



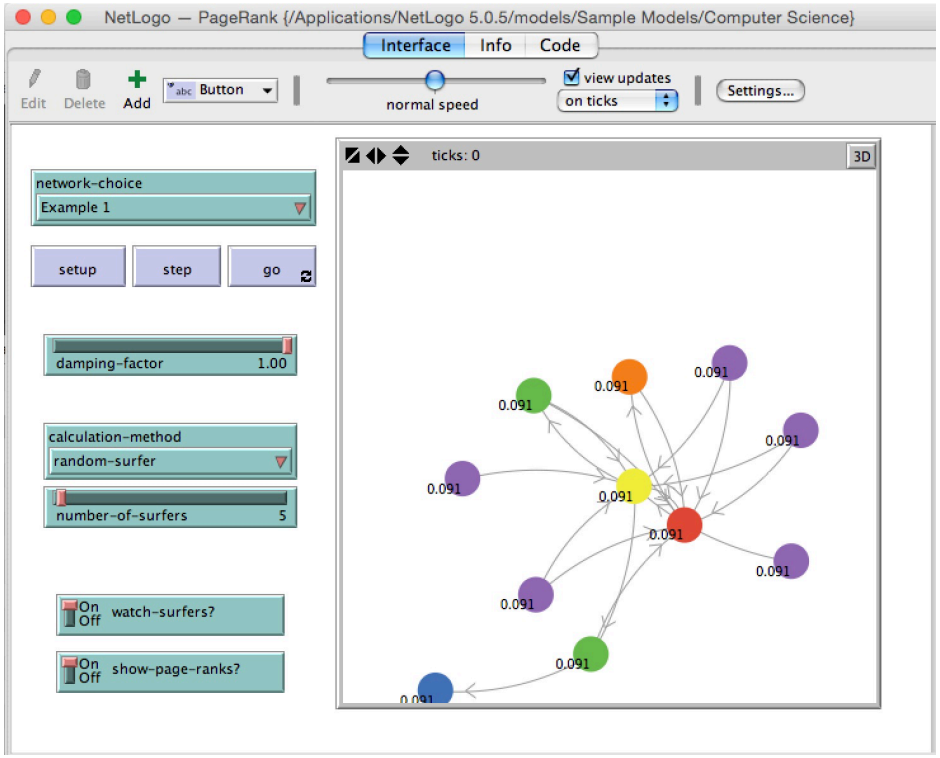






slide from: Dragomir Radev

Q: PageRank's damping factor



- What happens to the relative PageRank scores of the nodes as you increase the teleportation probability (decrease the damping factor)?
  - they equalize
  - they diverge
  - they are unchanged

PageRank.nlogo part of the built-in suite of network models for NetLogo

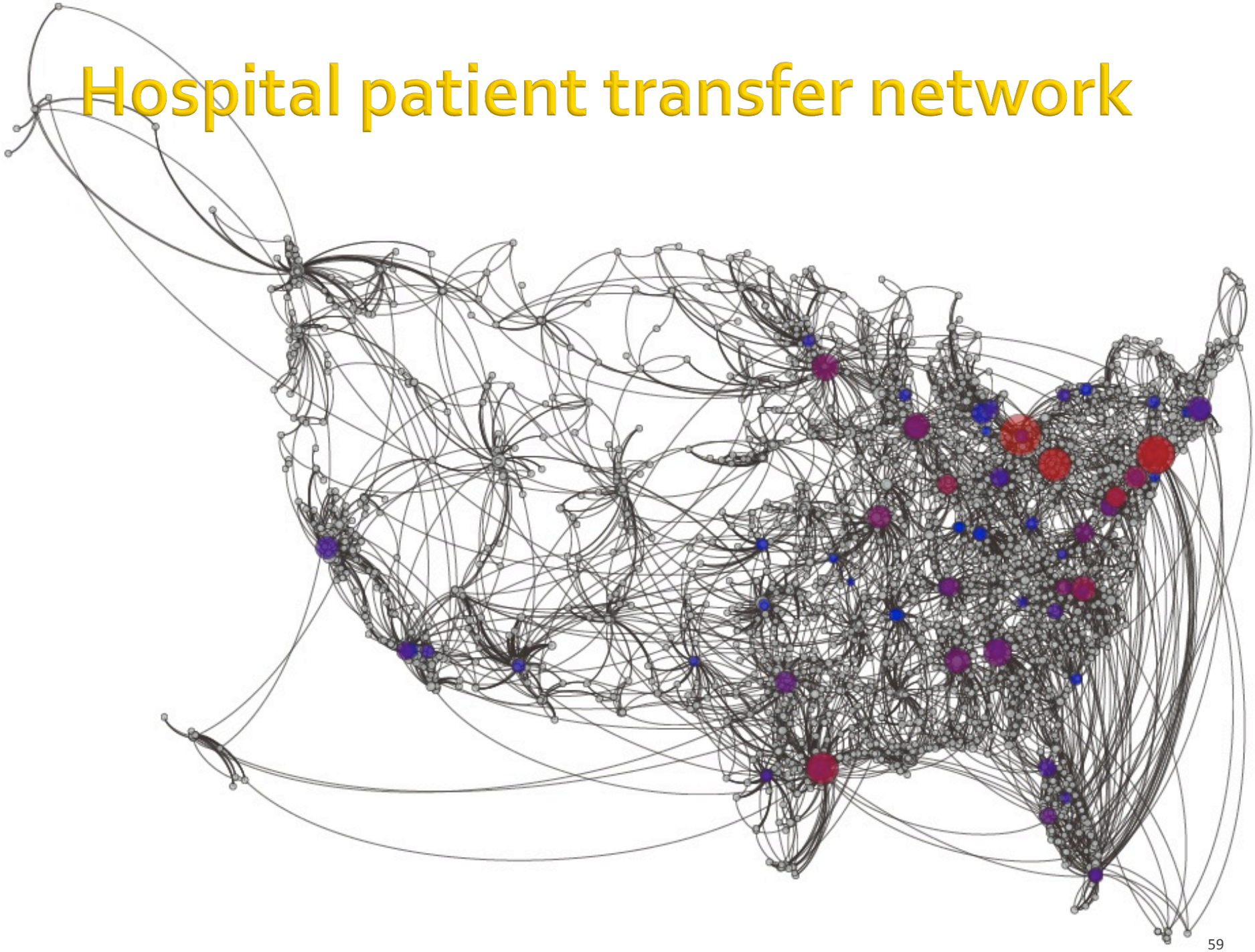
Summary

- Centrality
  - many measures: degree, betweenness, closeness, eigenvector
  - may be unevenly distributed
    - measure via distributions and centralization
  - in directed networks
    - indegree, outdegree, PageRank
  - consequences:
    - benefits & risks (Baker & Faulkner)
    - information flow & productivity (Aral & Van Alstyne)

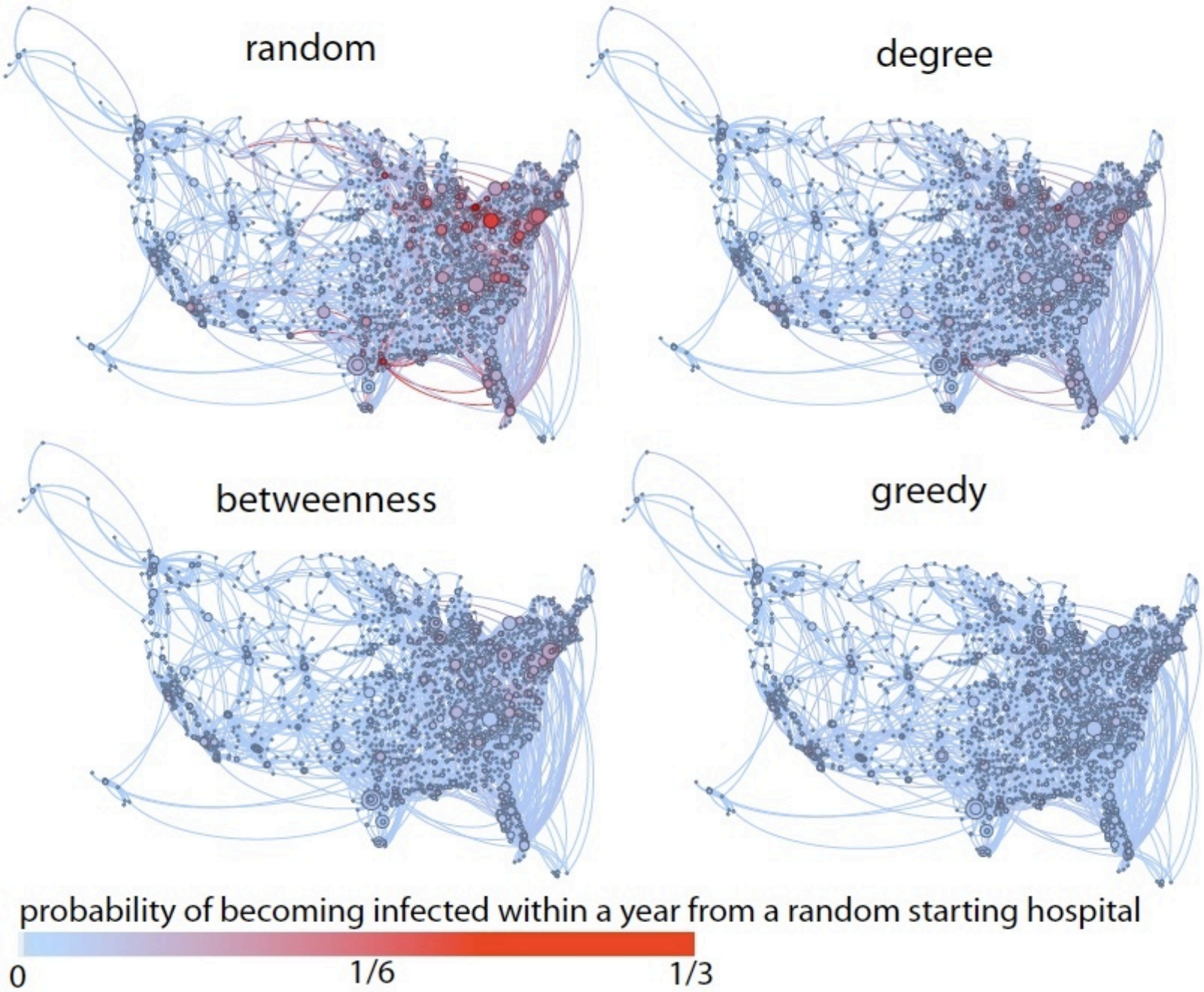
Some applications

(time permitting)

Hospital patient transfer network

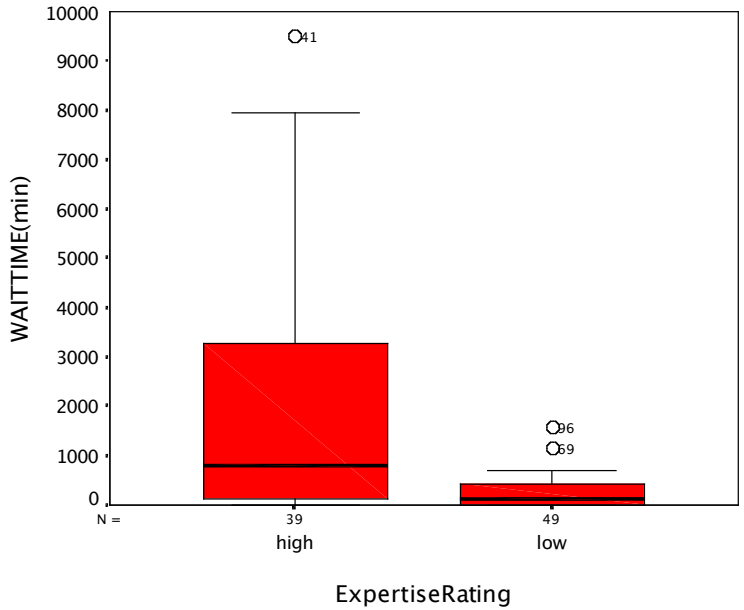


Infection prevention strategies in a hospital patient transfer network



Identifying expertise

- The Response Time Gap



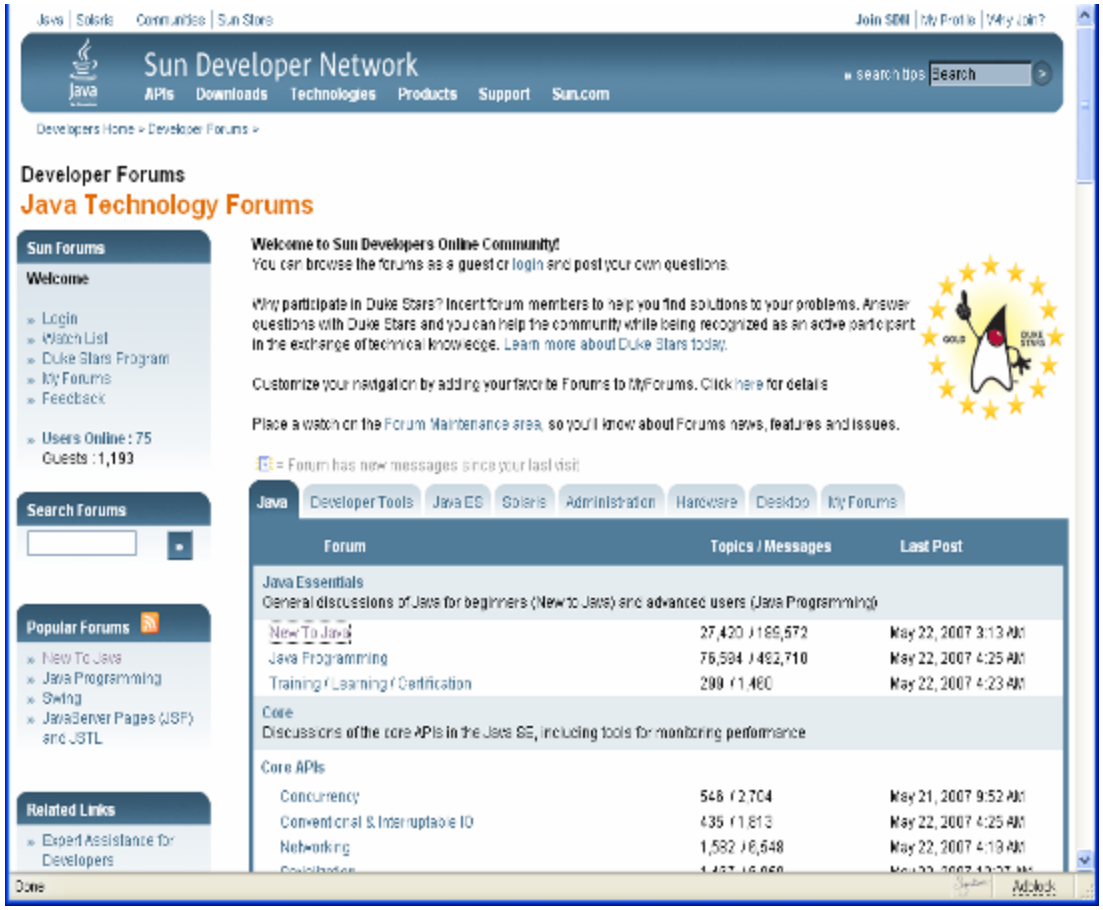
- The Expertise Gap
- Difficult to infer reliability of answers

Automatically ranking expertise may be helpful.

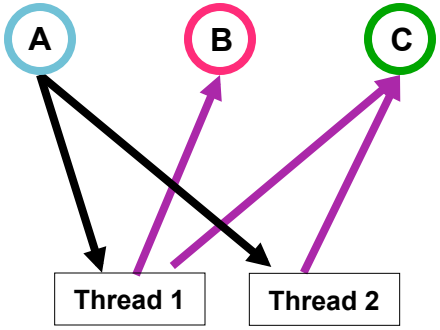
Zhang, Ackerman, Adamic, WWW'07

Java Forum

- 87 sub-forums
- 1,438,053 messages
- community expertise network constructed:
  - 196,191 users
  - 796,270 edges

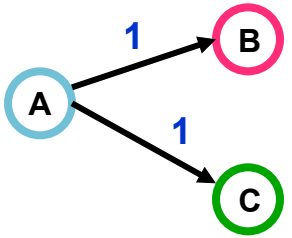


Constructing an expertise network

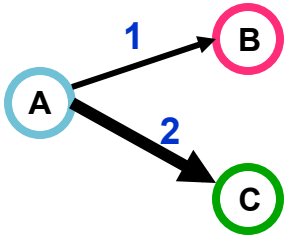


Thread 1: Large Data, binary search or hashtable? *user A*
Re: Large… *user B*
Re: Large… *user C*
Thread 2: Binary file with ASCII data *user A*
Re: File with… *user C*

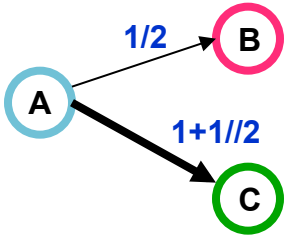
unweighted



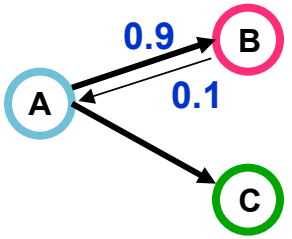
weighted by # threads



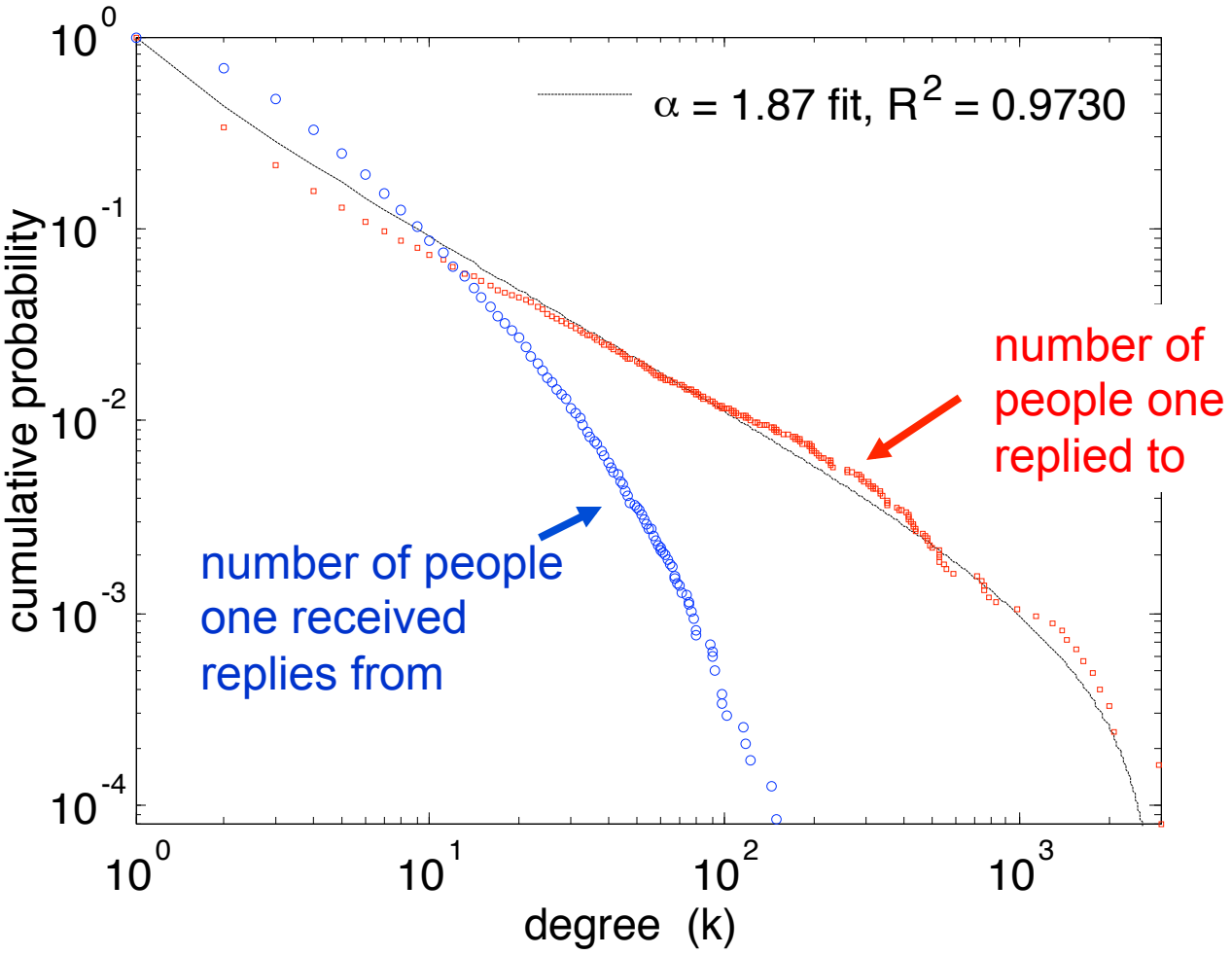
weighted by shared credit



weighted with backflow

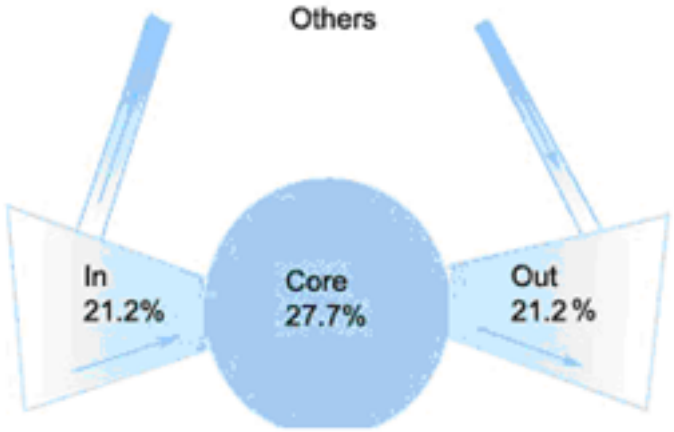


Uneven participation

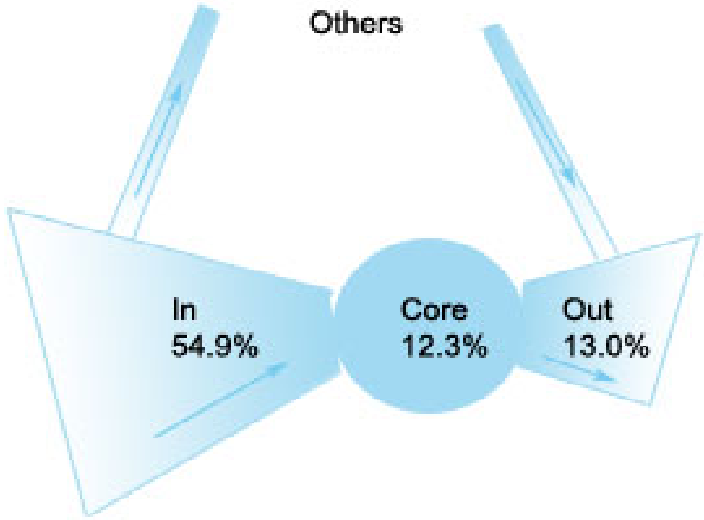


- 'answer people' may reply to thousands of others
- 'question people' are also uneven in the number of repliers to their posts, but to a lesser extent

Not Everyone Asks/Replies



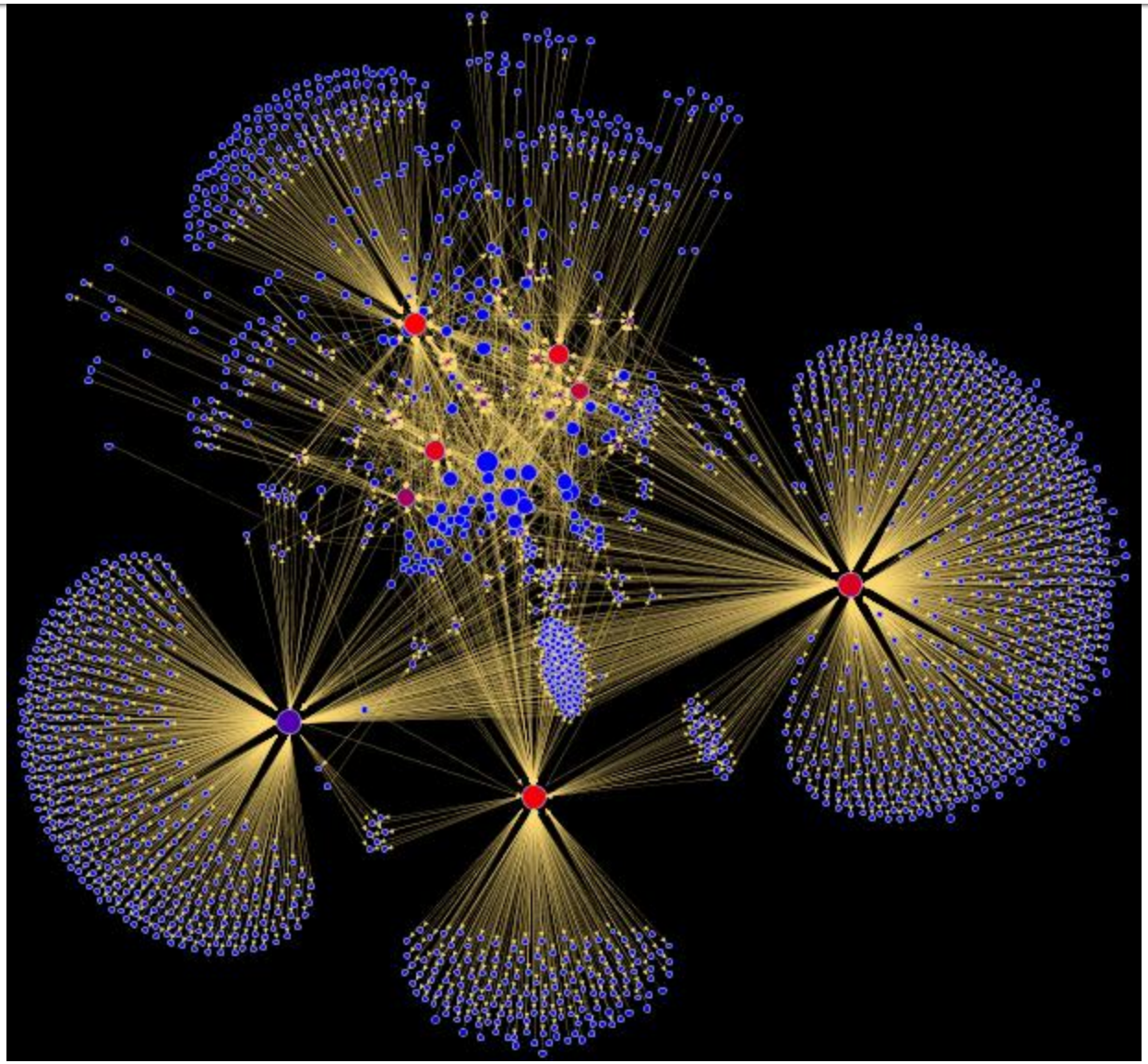
The Web is a bow tie



The Java Forum network is an uneven bow tie

- Core: A strongly connected component, in which everyone asks and answers
- IN: Mostly askers.
- OUT: Mostly Helpers

fragment of the Java Forum

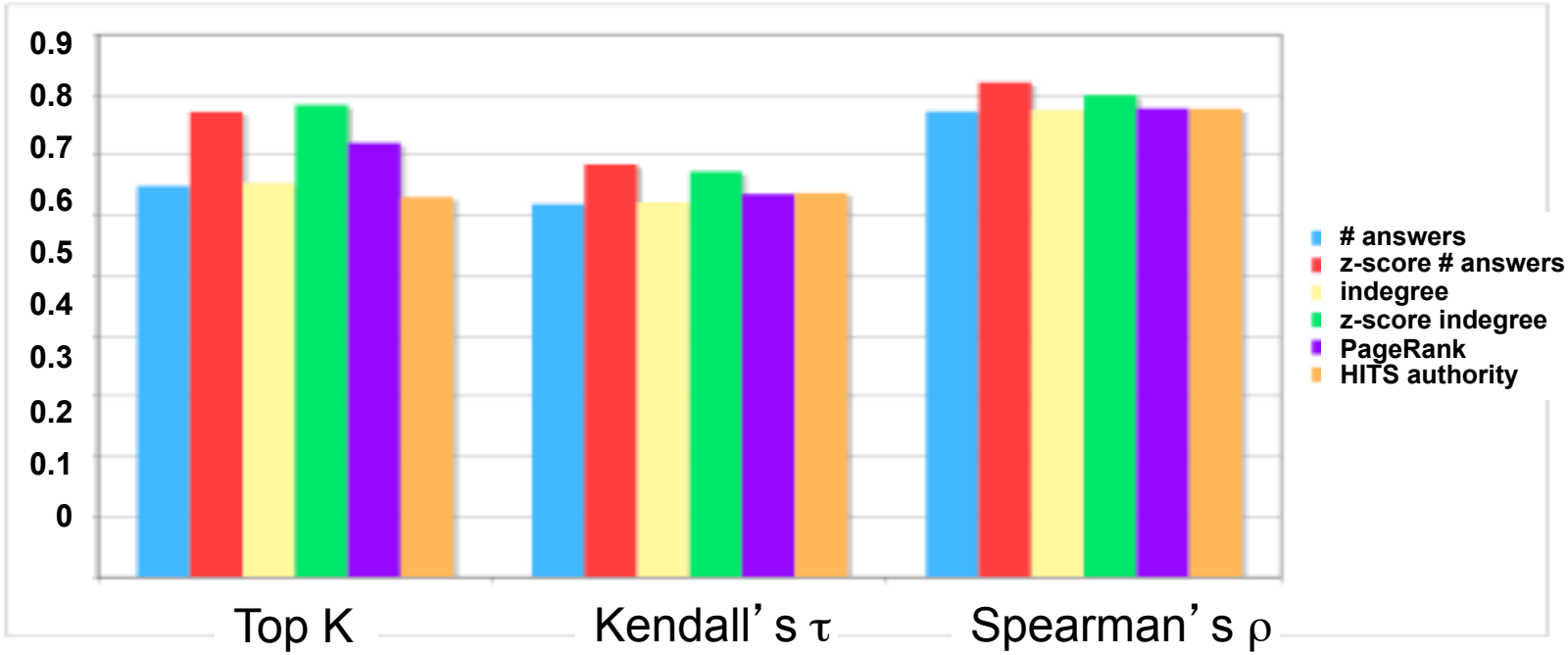


Relating network structure to expertise

- Human-rated expertise levels
  - 2 raters
  - 135 JavaForum users with >= 10 posts
  - inter-rater agreement (τ = 0.74, ρ = 0.83)
  - for evaluation of algorithms, omit users where raters disagreed by more than 1 level (τ = 0.80, ρ = 0.83)

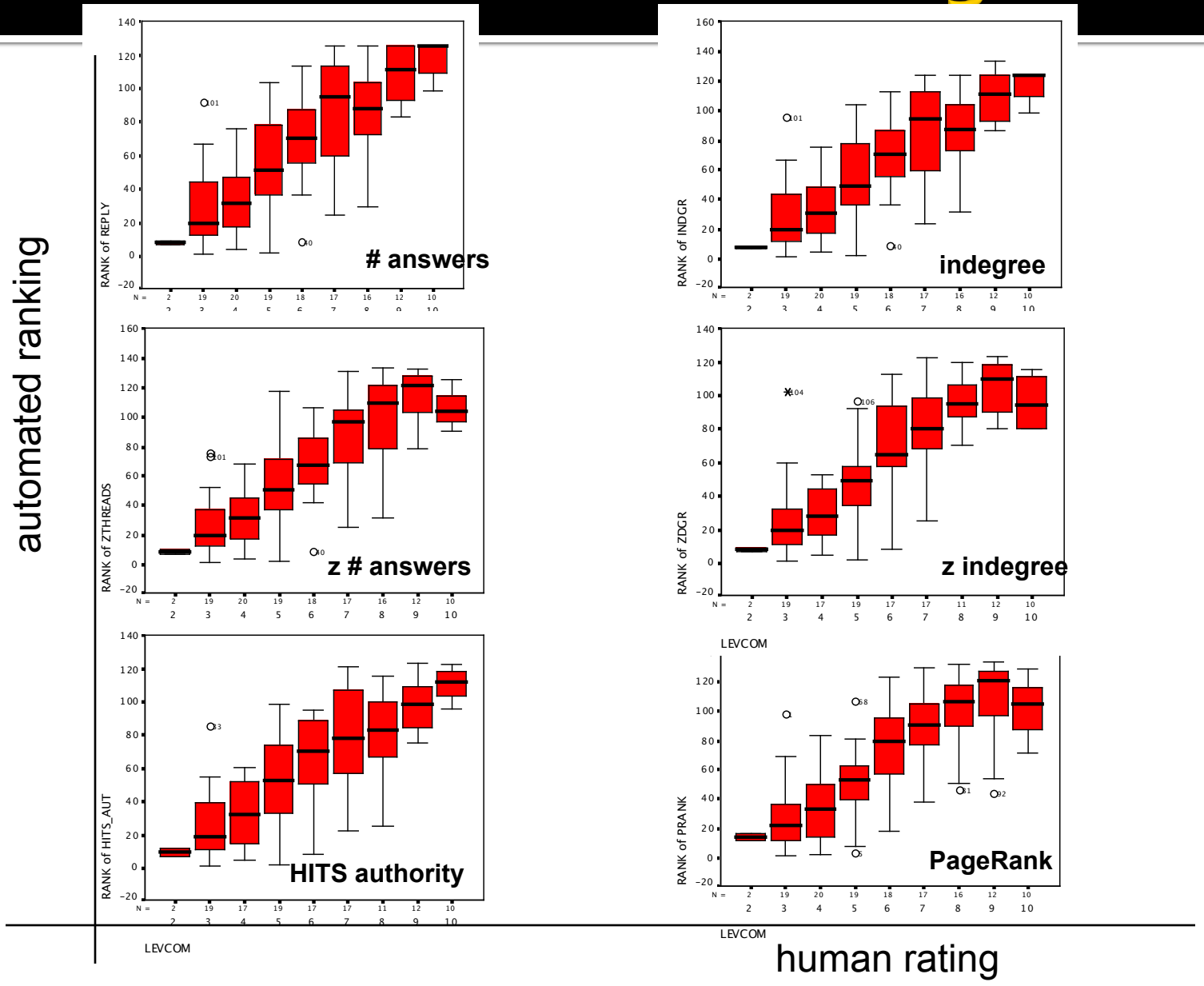
L	Category	Description
5	Top Java expert	Knows the core Java theory and related advanced topics deeply.
4	Java professional	Can answer all or most of Java concept questions. Also knows one or some sub topics very well,
3	Java user	Knows advanced Java concepts. Can program relatively well.
2	Java learner	Knows basic concepts and can program, but is not good at advanced topics of Java.
1	Newbie	Just starting to learn java.

Algorithm Rankings vs. Human Ratings

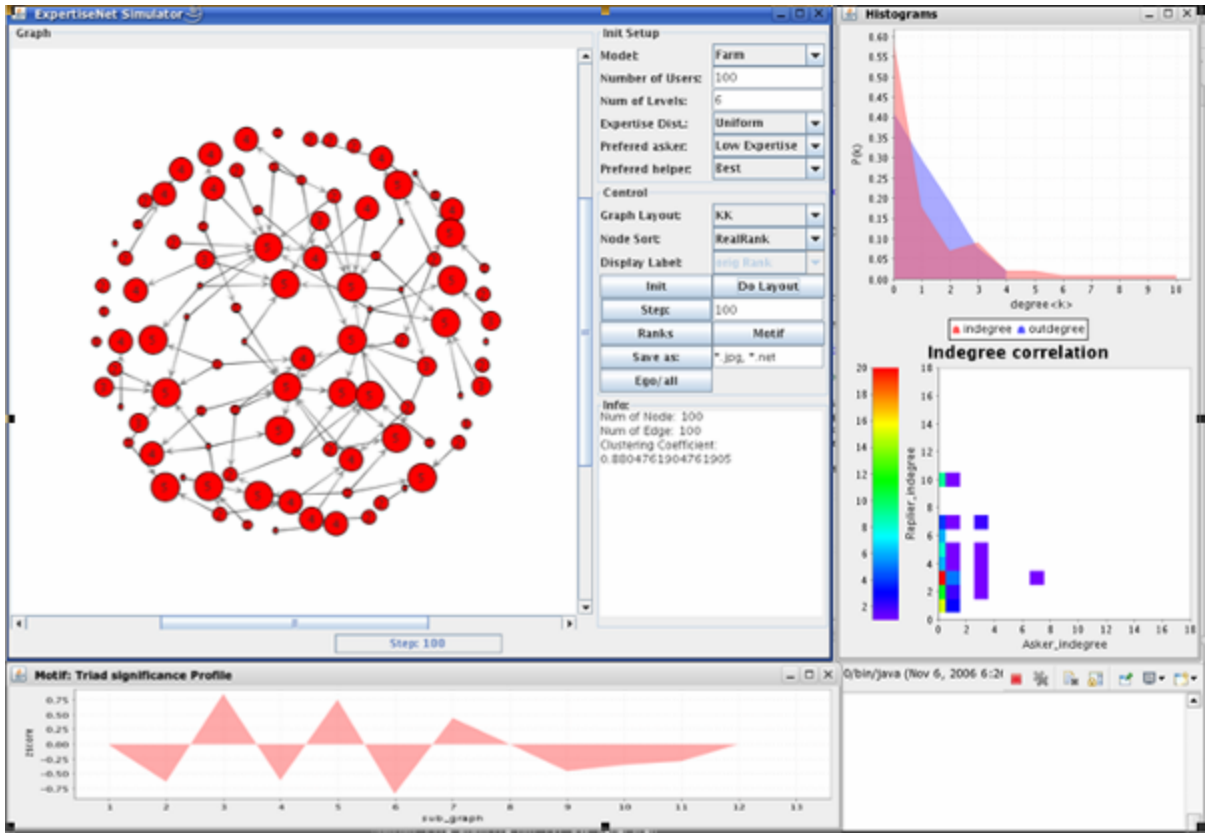


simple local measures do as well (and better) than measures incorporating the wider network topology

automated vs. human ratings



Modeling expertise network formation



ExpertiseNet Simulator

Control Parameters:

- Distribution of expertise
- Who asks questions most often?
- Who answers questions most often?
  - best expert most likely
  - someone a bit more expert

Simulating probability of expertise pairing

suppose:

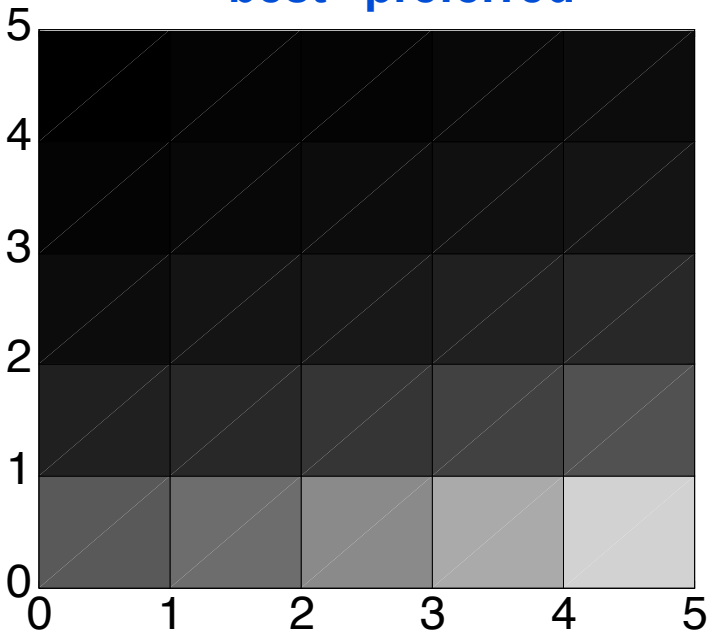
expertise is uniformly distributed

probability of posing a question is inversely proportional to expertise

p_{ij} = probability a user with expertise *j* replies to a user with expertise *i*

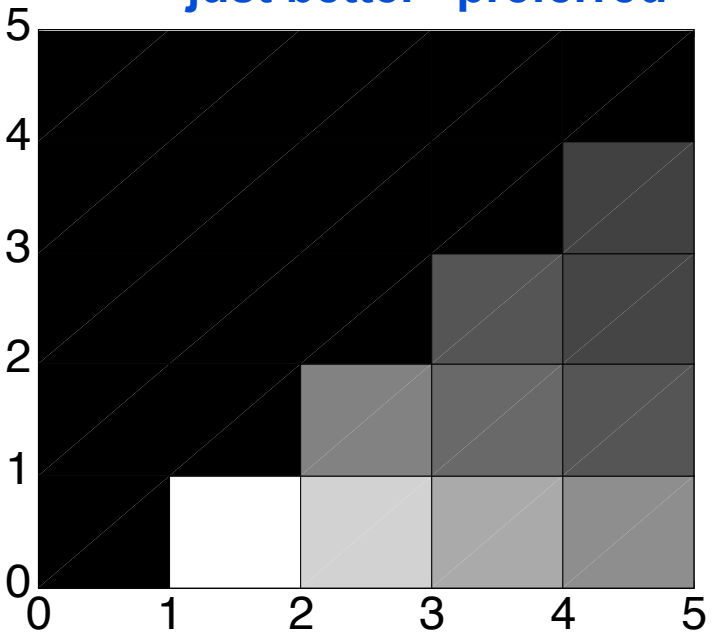
2 models:

'best' preferred



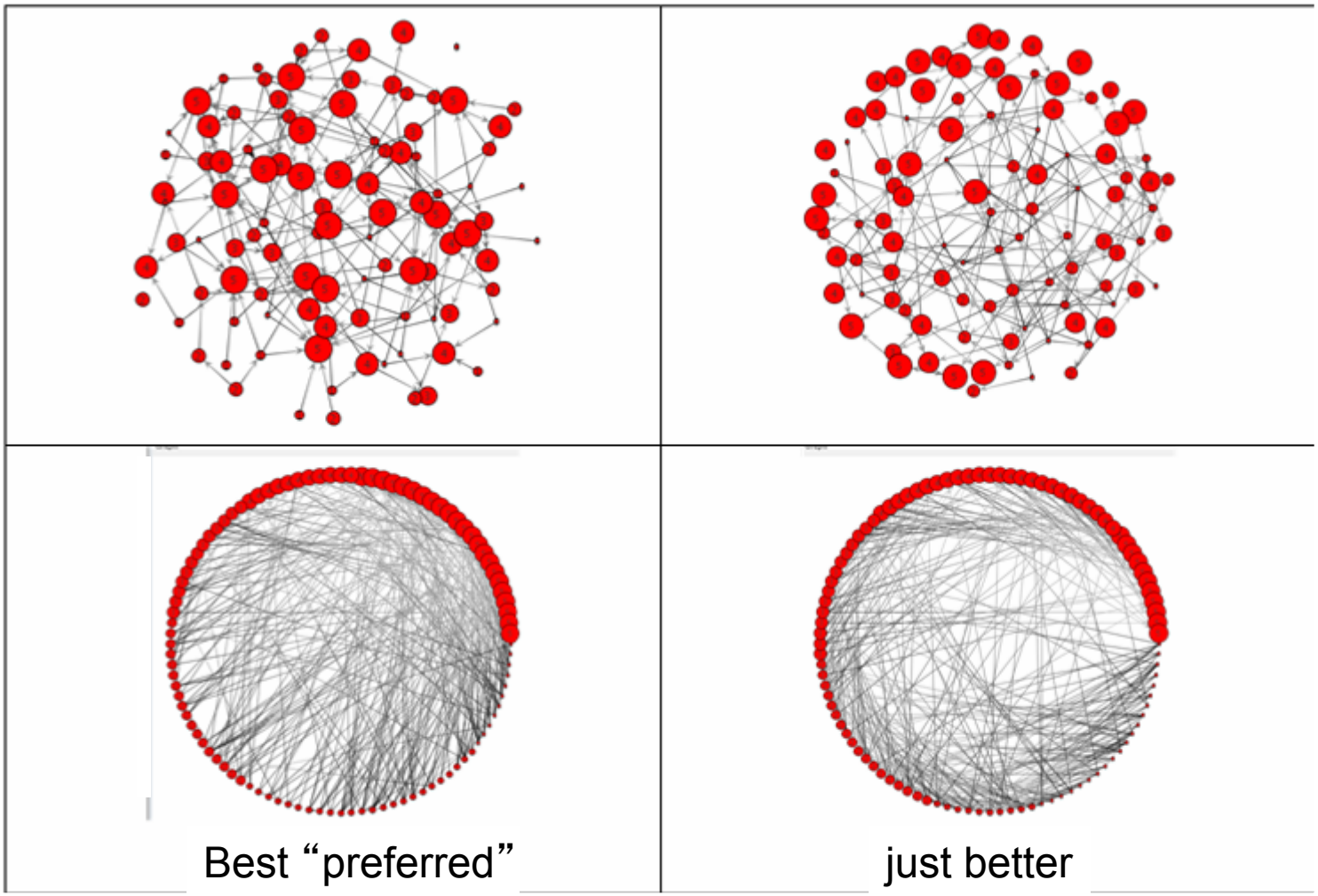
$$p_{ij} \sim e^{\beta(j-i)} / i$$

'just better' preferred

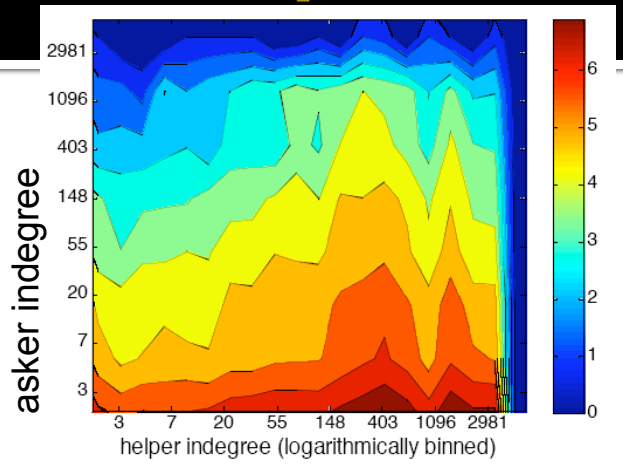


$$p_{ij} \sim e^{\gamma(i-j)} / i \qquad j>i$$

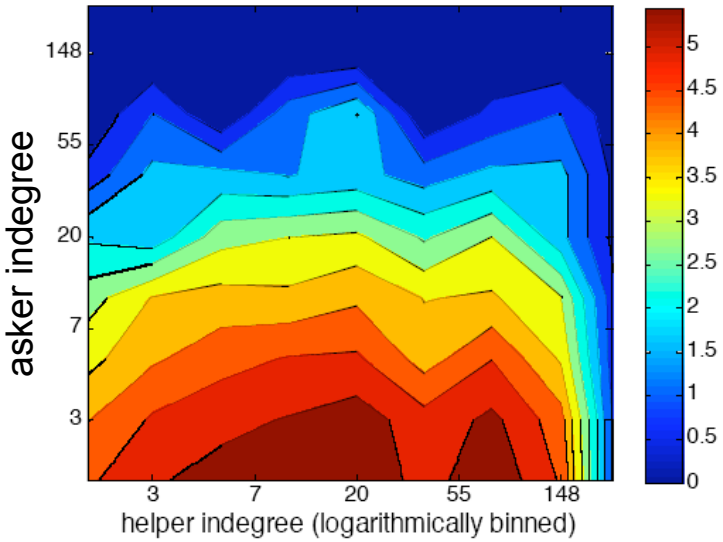
Visualization



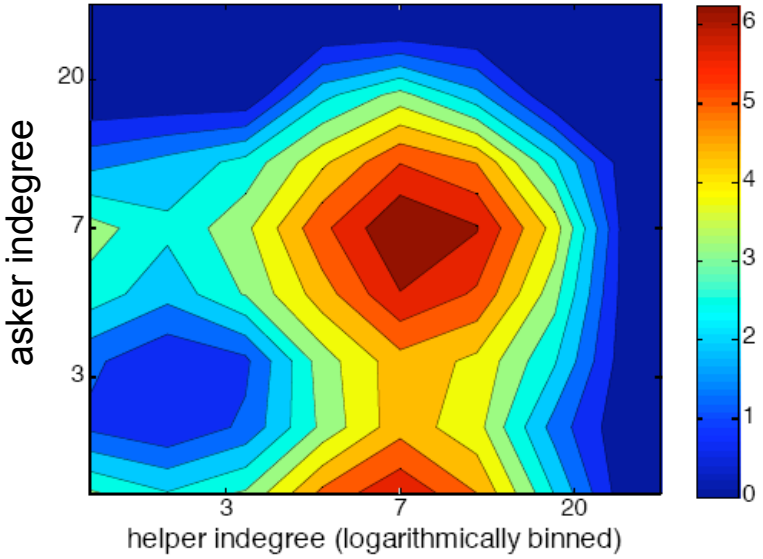
Degree correlation profiles



Java Forum Network

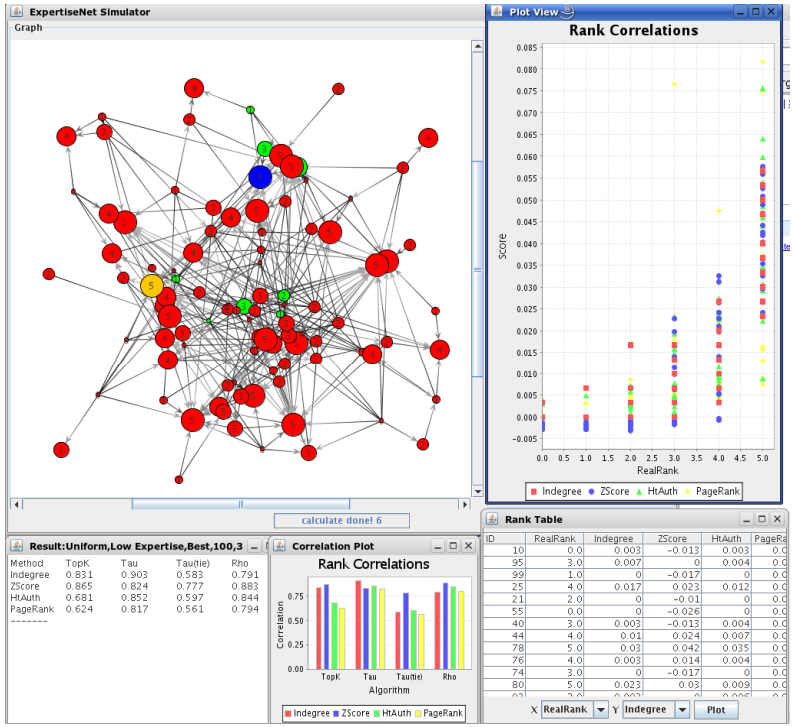


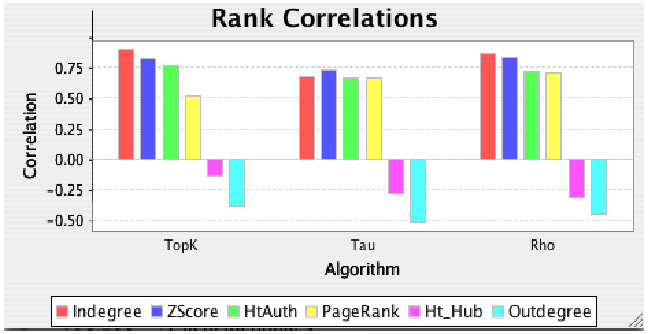
best preferred (simulation)



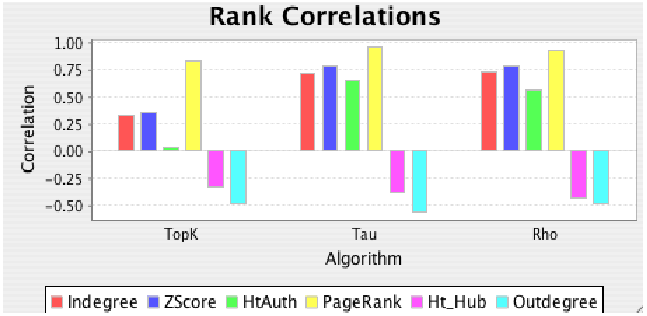
just better (simulation)

Algorithm selection



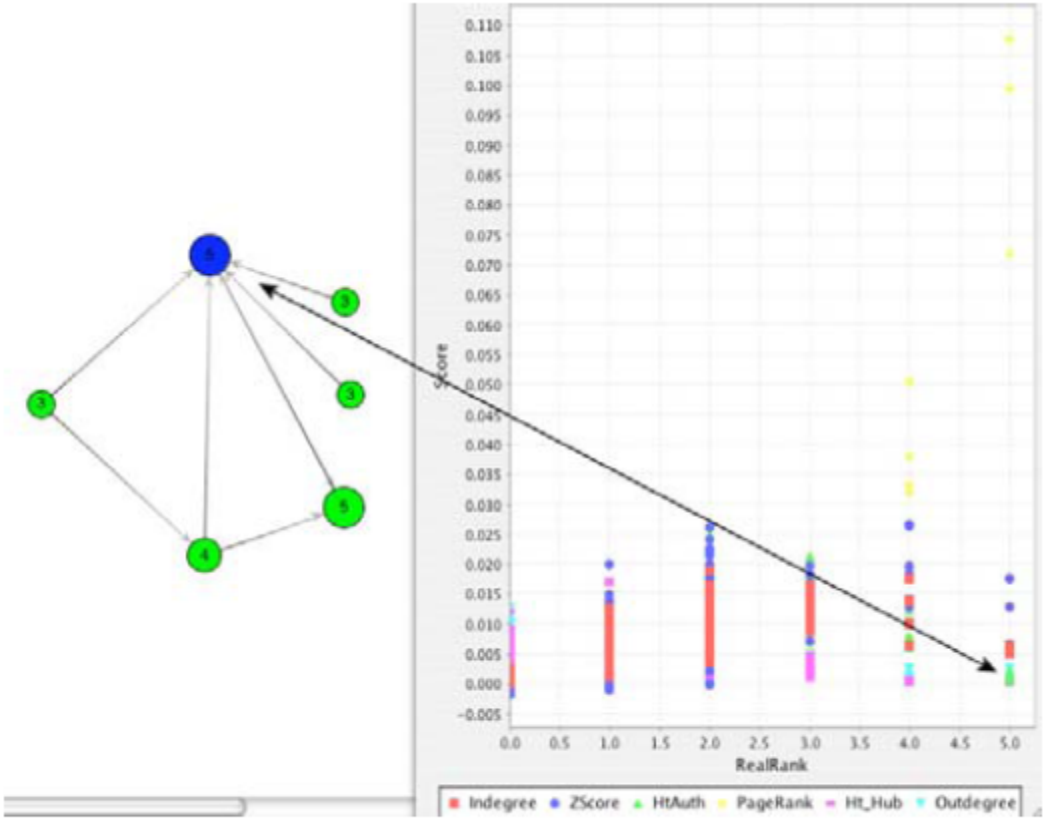


Preferred Helper: ‘best available’



Preferred Helper: ‘just better’

Algorithm evaluation



In the ‘just better’ model, a node is correctly ranked by PageRank but not by HITS

Conclusion

- Node centrality can reveal the relative importance of nodes within the network
- Choose a measure appropriate to the question you are asking