

THESIS

**A Study of Baseline Compensation  
System for Stable Operation of  
Gravitational-wave Telescope**

**Koseki Miyo**

*Department of Physics  
University of Tokyo*

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# Chapter 1

## Background

### 1.1 Gravitational-wave

#### 1.1.1 ...

### 1.2 Sources of Gravitational-wave

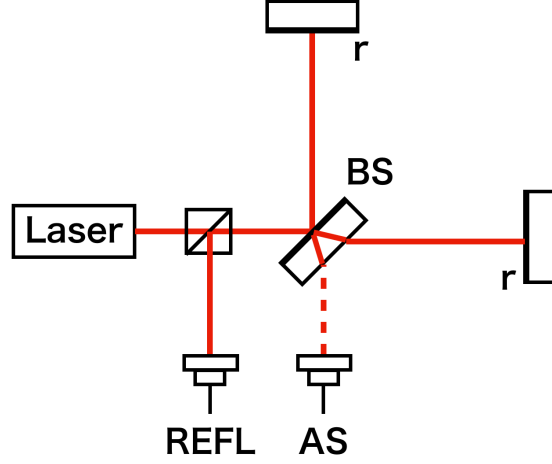
#### 1.2.1 ...

### 1.3 Interferometric Gravitational-wave detection

#### 1.3.1 Detection Principle

地上の大型重力波検出器の基本要素は Michelson 型レーザー干渉計である。プラスモードの重力波が Fig.4.4 に示すような Michelson 型レーザー干渉計を垂直に通過する場合を考える。

### 1.3.2 Michelson Interferometer



**Figure 1.1:** Michelson Interferometer.

Michelson interferometer is a converter from the optical phase difference of two lights to the amplitude modulation of a single light. Consider about the interferometer shown in Fig. 1.1. Incident light can be written as,

$$E_{\text{in}} = E_0 e^{i\omega t}, \quad (1.1)$$

where  $E_0$  is the amplitude and  $\omega_0$  is the angular frequency of the laser field. Two lights split by the Beam Splitter (BS) interfere at the Anti-symmetric (AS) port and Reflection (REFL) port. The output field at the AS port is represented as,

$$E_{\text{AS}} = -\frac{1}{2}rE_0e^{i(\omega_0t-\phi_x)} + \frac{1}{2}rE_0e^{i(\omega_0t-\phi_y)}, \quad (1.2)$$

where  $r$  denotes the amplitude reflectivity of the end mirrors, and  $\phi_x$  and  $\phi_y$  are the phase delay due to the light traveling in the  $x$  and  $y$  arms. This output signal can be represented as a single field as,

$$E_{\text{AS}} = irE_0e^{i(\omega_0t-(\phi_x+\phi_y)/2)} \sin\left(\frac{\phi_x-\phi_y}{2}\right). \quad (1.3)$$

We find that the amplitude of the output light is a function of the difference between two phases;  $\phi_x - \phi_y$ . Furthermore, the power of output light at the AS port is obtained by squaring the Eq. 1.3,

$$P_{\text{AS}} = (rE_0)^2 [1 - \cos(\phi_x - \phi_y)] \quad (1.4)$$



Similarly, power of the output light as REFL port is written as,

$$P_{\text{REFL}} = (rE_0)^2 [1 + \cos(\phi_x - \phi_y)]. \quad (1.5)$$

Therefore, we can measure the optical phase difference as the amplitude changes using a Photo Detector (PD) and detect GWs.

### 1.3.3 Null Measurement

#### Shot Noise

Shot noise is the optical readout noise associated with the discrete nature of photons and electric charges. 光のパワーは

#### Control Noise

## 1.4 Summary of the Chapter



## Chapter 2

# KAGRA

### 2.1 Overview

#### 2.1.1 ...

#### 2.1.2 ...

### 2.2 KAGRA Tunnel

#### 2.2.1 Tunnel Design

KAGRA tunnel is excavated in the Kamioka mine in Hida, Gifu, Japan [1]. The tunnel is consisted of two floors. 干渉計を構成するほとんどの鏡は 1 階に設置された防振装置で懸架されているが、腕共振器を構成する 4 つの鏡は 1 階から 14m の高さにある 2 階から懸架されている。

The tunnel is locate under 200 m from ground surface to decrease the seismic noise effectively.

#### 2.2.2 Geological features

Hida region to which Kamioka belongs is a ancient region in Japan island [2].

The main bedrock is the geniss.

## 2.3 Main Interferometer

### 2.3.1 Overview

KAGRA is a cryogenic intergerometric gravitational-wave detector constructed at the underground site of Kamioka mine [3].

### 2.3.2 Main Interferometer

#### Design

The design of KAGRA interferometer is dual recycled Fabry-Perot Michelson interferometer [4][5].

## 2.4 Vibration Isolation System

### 2.4.1 Overview

KAGRA has 4 types vibration isolation system.

### 2.4.2 Type-A Suspension System

Type-A suspensions are developed [6].

## 2.5 Summary of the Chapter

## Chapter 3

# Underground Seismic Noise

### 3.1 Introduction

Seismic noise cause two main problems to the terrestrial gravitational-wave detectors.

First one is the limitation of the sensitivity. DARM which receive the gravitational-wave signal is disturbed by the seismic noises in these frequencies. Amplitude spectrum density of this, is empirically known as

$$\sim \frac{10^{-8}}{f^2} \text{ m}/\sqrt{\text{Hz}}, \quad (3.1)$$

where  $f$  is a frequency of the spectrum. The noise limits the sensitivity of the detectors in lower frequency typically below 10 Hz even after the attenuation by the vibration isolation systems. Therefore, low-level seismic noise is an important in this frequency region.

Second one is the decrease of the duty cycle. 1Hz 以下の地面振動、とくに 200mHz 付近にピークをもつ脈動は、レーザー干渉計の安定的な稼働に大きな影響を与える。この帯域では地面振動の RMS の値はおおよそ数  $\mu\text{m}$  である一方で、腕共振器が共振して重力波に対する感度を保つためには、鏡の RMS 振幅を数 nm 以下に抑えなければならない。しかしながら、現実的に動作可能な防振装置の固有周波数は数 100Hz 程度であり、200Hz 付近にピークをもつ脈動は十分に防振することができない。このような防振できない地面振動は、腕共振器のロックを不安定にし、検出器の duty cycle を下げてしまう。

Underground can resolve these problems. Underground is more quiet than the surface of the ground [7]. Especially, the underground seismic noise above

1 Hz is effectively reduced than the noise on surface of the ground [8], because the noises in this band are correlated to human activities such as industrial machinery, cars and trains, or human footsteps [9] and then are able to be eliminated in underground. Moreover, a laser interferometer gravitational wave antenna with a baseline length of 20 m (LISM) constructed underground have demonstrated the stable performance of the detector by resulting the high duty cycle of 99.8 % [10].

しかし、KAGRA のような 3km の長期線のレーザー干渉計では、LISM のように安定して可動させることは難しいとされている。なぜならば、地面振動による基線長変動は、基線長が長いほどその影響は大きいためである。後述する §3.4 によれば、0.2Hz の脈動による基線長変動への影響は、KAGRA は LISM の 150 倍ある一方で、表 3.1 に示すように、線幅はおよそ 17 倍しかない。つまり KAGRA は LISM と比べて、線幅に対して地面振動による基線長伸縮はおよそ 1 桁大きいことを意味する。このような長期線化による問題は、KAGRA などの第二世代の検出器だけの問題ではなく、ET などの数 10km の基線長を計画する第 3 世代の検出器にとって同様の問題となる。

**Table 3.1:** Comparison of the line width of the arm cavity

|       | Finess | Laser Wavelength [ $\mu\text{m}$ ] | Line width [ $\mu\text{m}$ ] |
|-------|--------|------------------------------------|------------------------------|
| LISM  | 25000  | 1064                               | 0.021                        |
| KAGRA | 1500   | 1064                               | 0.35                         |

本章では、重力波検出器にとって問題となる地面振動の主な励起源とその時間依存性や、それらが基線長変動に与える影響を述べる。まず §3.2 で主な励起源を述べる。これら励起源は周波数帯域で大きく分類することができ、3.2 にまとめた。次に ?? では、その時間依存性をのべる。とくに脈動について、1 年間の地震計データをもとに、その時間依存性を調べた。そして最後に §3.4 と ?? では、地面振動が基線長伸縮に与える影響をのべ、実際に 2 台の地震計をつかった測定と比較した。

**Table 3.2:** Baseline changes caused by several seismic sources

| Sources                   | Frequency Band [Hz] | RMS Amplitude [ $\mu\text{m}$ ] | Detail |
|---------------------------|---------------------|---------------------------------|--------|
| Human,traffic             | $> 1$               | $< 1$                           | §3.2.1 |
| Ocean waves (Microseisms) | 0.1–0.3             | 0.1-10                          | §3.2.2 |
| Large earthquakes         | $> 0.05$            | $> 100$                         | §3.2.3 |
| Moon, Sun (Earth tides)   | $< 1^{-5}$          | $> 100$                         | §3.2.4 |

## 3.2 Seismic Noise

### 3.2.1 Human activity

(あとで書く) Virgo の地面振動の昼と夜の比較についてのべる。[11] ...

...

...

### 3.2.2 Microseisms

Microseisms which power spectrum has peaks in 50–200 mHz are excited by oceanic waves. These seismic waves can be categorized by the generating mechanism of these [12].

The primary ocean microseisms are generated only in shallow waters in coastal regions. In this regions, the water wave energy can be converted directly into seismic energy either through vertical water pressure variations, or by the impacts of surf on the shores. There are correlation between this microseismic peak and the swell at the beaches was known starting from the data sets studied by [13].

The secondary ocean microseisms could be explained by the superposition of ocean waves of equal period traveling in opposite directions. Therefore, generating standing gravity waves of half the period [14].

The RMS amplitude spectral of both type of the microseisms are strongly depends on the low pressure on the ocean [15].

### 3.2.3 Large Earthquakes

Large amplitude earthquakes around the world would interrupt the operation of the gravitational wave detector and reduce their duty cycle. 実際、観測中に地震でロックが落ちたあと、復帰するまでに数時間かかる場合がある [16]。規模の大きな地震ほど、振幅がおおきいことはもちろん、長周期で減衰までにかかる時間が長い。長周期地震に対して重力波検出器が無力なのは、その防振装置が、せいぜい 100 mHz の地面振動揺れに対して最適化しているためである [ ]。そのためこのような制御方式を使う以上、制御ノイズが増大して感度が落ちることを犠牲にしてでも、地震が来る前に、地震でロックロスしないような制御フィルターの切り替えを必要とする。この切替のために *Seismon* とよばれる早期地震アラートシステム [17] を用いた制御フィルターの切り替えが試みられている [18]。

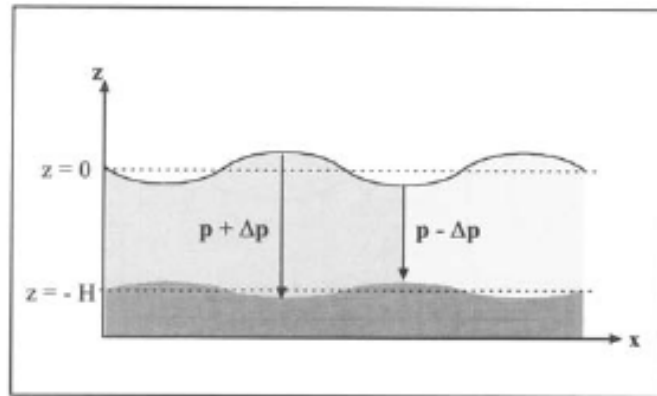
### 3.2.4 Earth Tides

(あとで書く) (書くかどうか迷っている。) ...

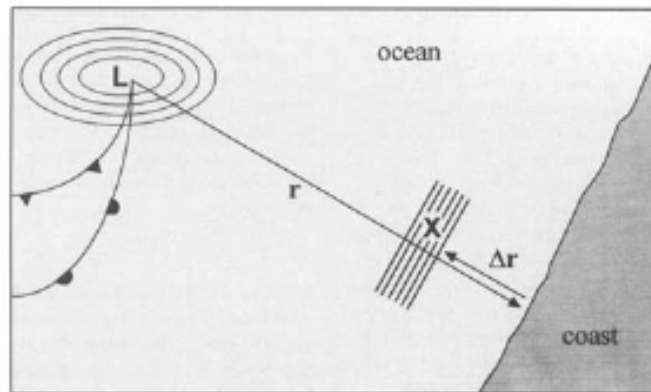
...

...





(a) Generating mechanism of the primary microseisms.  
引用先の図をコピーしているので、自分で描いたのに差し替える。



(b) Generating mechanism of the secondary microseisms. 引用先の図をコピーしているので、自分で描いたのに差し替える

**Figure 3.1:** Generating mechanism of the microseisms. (a) describes the mechanism of the primary microseisms. (b) describes the mechanism of the secondary microseisms.

### 3.3 Long-term Study of the seismic environment at KAGRA

#### 3.3.1 Overview

本節では、地面振動の大きさが時間帯や季節の違いでどのように変化するか調べた。基本的に地震などの突発現象を除けば、地面振動は定常的な振る舞いを示すが、その RMS 振幅は時間依存である。例えば、人間由来の地面振動は夜間になると静かで、波浪由来の脈動は天気が悪化するとうるさくなる。また日本の場合、初秋は台風によって、冬は日本海低気圧によって脈動がうるさくなることが知られている [ ]。とくに後者の脈動がうるさいと、干渉計の稼働が妨げられ、DutyCycle を低下してしまう。このような理由から、地面振動の時間依存性を知ることは、干渉計の稼働の安定性を議論するうえで重要である。

地面振動の振幅スペクトルの分布は、およそ 1 年間の地震計の時系列データからもとめた。このデータには、地震計のメンテナンスによるデータの欠損や、地震などの突発的な地面振動のデータが含まれている。そのため、スペクトルを計算するためにこれらデータを取り除き、およそ 1 時間 (4096 秒) 区切りのデータ・セットを用意した。これらデータ・セットごとに振幅スペクトルを計算し、分布をもとめた。

#### 3.3.2 Experimental Arrangement

Seismic motion is measured by a seismometer installed on the second floor of the X-end area. This area is placed 200 m underground from the surface of the mountain. Comparison to corner area, human activity in the end area is less because the corner area has parking lots. Comparison to the Y-end area, there is no entrance connected to other mines. Therefore, the X-end area is relatively quiet in the KAGRA mine, regarding the seismic noise induced by human activity.

In this study, Trillium 120-QA which is known as three-component, very broadband, and low-noise seismometer, was used. These three outputs are proportional to the ground velocity of two horizontal and one vertical, respectively. The feature of the low-noise can resolve Peterson's new low-noise model (NLNM) and new high-noise model (NHNM) [19].

As shown in fig 3.2, the seismometer is housed in the black thermal insulation cover according to the installation manual [20]. Thermal insulation protects two broad categories of thermal couplings that can cause unwanted noise [20]. First

### 3.3. LONG-TERM STUDY OF THE SEISMIC ENVIRONMENT AT KAGRA19



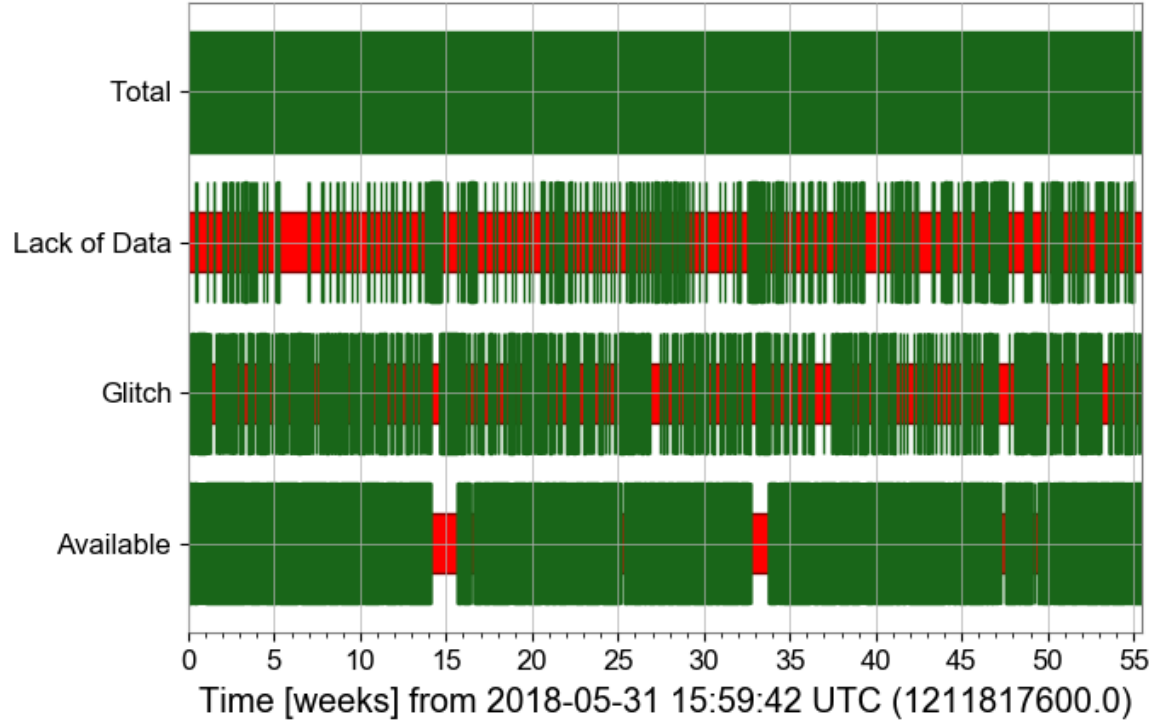
**Figure 3.2:** Trillium 120-QA installed on the second floor at X-end area, which is covered by black thermal insulation cover

is the direct coupling to the sensitivity. This coupling typically increases the noise of the vertical channel as a periodic diurnal variation caused by the day-to-night temperature cycle, because the springs that suspended the inertial masses are temperature sensitive. The second is the coupling to tilt from the thermal fluctuation. Tilt converts the vertical acceleration of gravity into horizontal acceleration. This thermally induced tilt noise on the horizontal will be larger than the direct thermal coupling on the vertical channel. To be low sensitivity to both tilt and temperature, this model has a function to center the inertial mass after the initial installation.

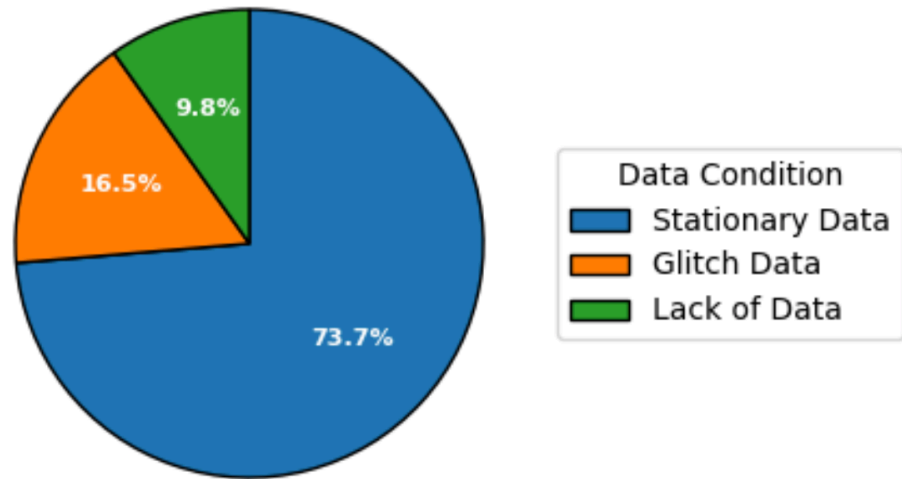
The signals of the seismometer is recorded through the data acquisition system developed by LIGO [21]. The analog signal is converted to digital signal by the 16 bit analog-to-digital converters (ADC) with 16384 Hz sampling. This analog signal is amplified with 30 db so that the ADC noise does not mask this signal.

#### 3.3.3 Data Selection

およそ1年分の地震計のデータをつかった。スペクトルの計算のために、一年分のデータを4096秒のセグメントに分割し、その中からデータの欠損などの異常値を取り除いた。解析に使ったセグメントをFig3.3に示す。14-15週と33-34週に生じた大きなデータの欠損期間を除いて、一年を通してデータが取得できている。セグメントはXX個あり、全体のXX%を占める。



(a) 各状態の時間分布。



(b) 各状態の割合。(a) を円グラフにしたもの。

**Figure 3.3:** Available data from June 01 2018 to 2019-06-24 09:40:14JST. Total:全体のセグメント。Lack of Data:データの欠損が合ったセグメント。Glitch:Glitchがあったセグメント。Available:欠損も Glitch もなかったセグメント

### 3.3. LONG-TERM STUDY OF THE SEISMIC ENVIRONMENT AT KAGRA21

#### 3.3.4 Data Processing

振幅スペクトルの推定は 50%オーバーラップした 32 個のセグメントの平均で得た。それぞれのセグメントの FFT の計算は、まず dtrend をして線形成分を取り除き、Hanning 窓にかけてから行った。32 回の平均をおこなったスペクトルは自由度 32 のカイ二乗分布に従う。自由度  $\nu$  のときの  $100(1 - \alpha)\%$  の信頼区間は、周波数  $f$  でのスペクトルの推定量を  $\hat{G}(f)$  とすると、

$$\frac{\nu \hat{G}(f)}{\chi^2(\nu, 1 - \frac{\alpha}{2})} \leq G(f) \leq \frac{\nu \hat{G}(f)}{\chi^2(\nu, \frac{\alpha}{2})} \quad (3.2)$$

で与えられる。したがって、95%の信頼区間は

$$\nu / \chi^2(\nu, 1 - \frac{\alpha}{2}) \leq G(f) / \hat{G}(f) \leq \nu / \chi^2(\nu, \frac{\alpha}{2}) \quad (3.3)$$

となり、自由度 32 の場合、推定量の 0.65 から 1.75 の範囲になる。

#### 3.3.5 Results

すべてのセグメントから求めた振幅スペクトル密度をを振幅に換算したものを Fig. ??に示す。赤の実線は垂直成分の 50 パーセンタイルで、下と上に 10 と 90 パーセンタイルを示す。青の実線は X 軸と Y 軸の二乗和から求めた並進成分であり、同様に 10,50,90 パーセンタイルを示す。緑点線は Trillium120 のデータシートから引用した Selfnoise である。黒の点線は Peterson の NLNM と NHNM である。この Peterson のスペクトルは、世界の 75 箇所の基地にある地震計の数年分のデータから得たものである。これらデータは地表と地下両方のデータを含み、並進成分と垂直成分を含む。NHNM は、内陸の沖積層で地面が揺れやすい場所や、沿岸地域で脈動や人間の活動の影響を受けやすい場所の地面振動を反映している。一方で NHNM は、とくに 0.1Hz 以下では、広範囲の複数の地震計の垂直成分で得られた地面振動のノイズフロアである。

測定で得られた地面振動は、40mHz 以上では並進成分も垂直成分も同じ振幅スペクトル密度をもつ。40mHz 以下で並進成分が垂直成分よりも大きい、これは付録で後述しているとおり、無相関なノイズである。おそらく温度ゆらぎから生じる傾斜カップリングだと考えられる。

Peterson のスペクトルと測定で得た 10 パーセンタイルを比較すると、0.1 から 2Hz をのぞいて、NLNM と同じである。0.1Hz 以下では垂直成分は地面振動のノイズレベルと同等であり、2Hz 以上は、並進も垂直成分も、地下環境のおかげで静かである。

対照的に 0.1 から 2Hz の帯域では、並進成分も垂直成分も NLNM より数倍大きい。これは KAGRA が富山湾から 40km の距離にあり、比較的脈動の影響を受

けやすいためと考えられる。

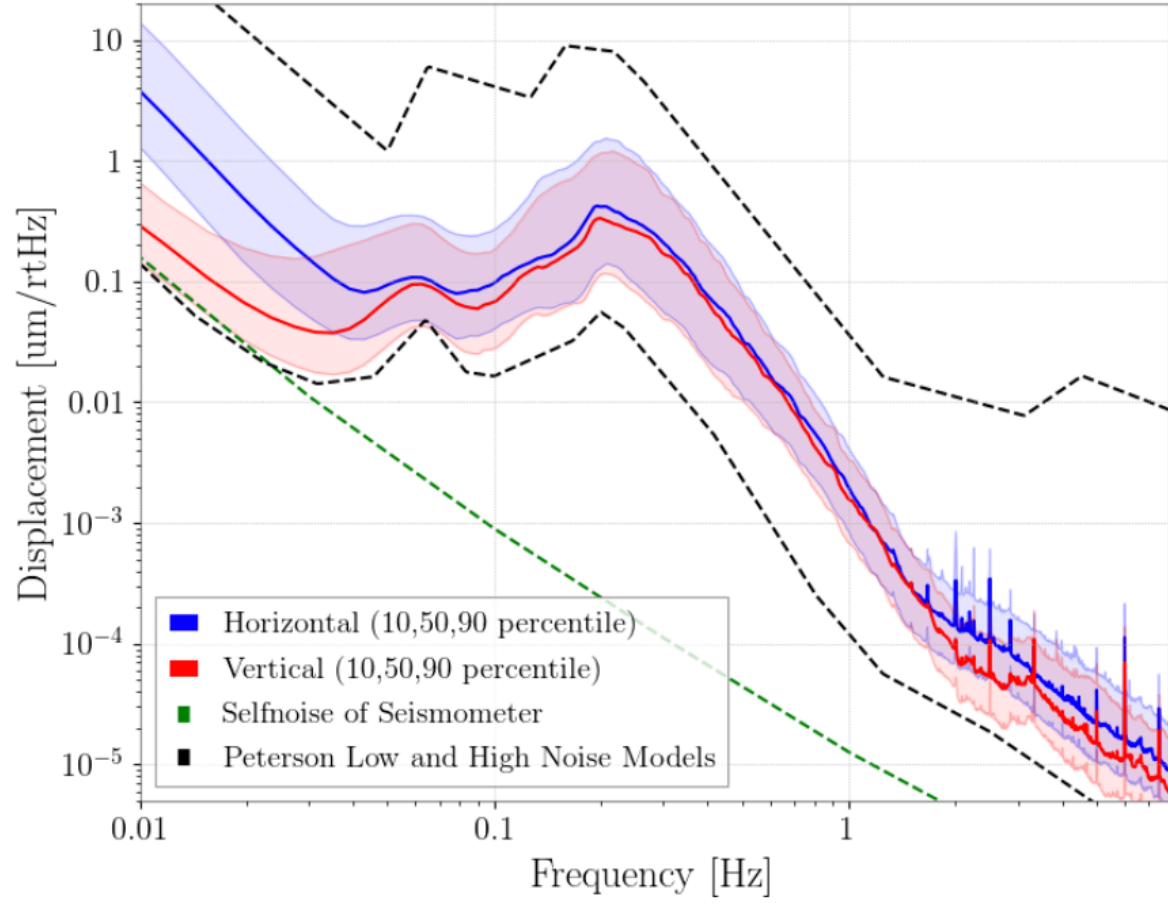


Figure 3.4

## 3.4 Differential Motion Reduction

### 3.4.1 Introduction

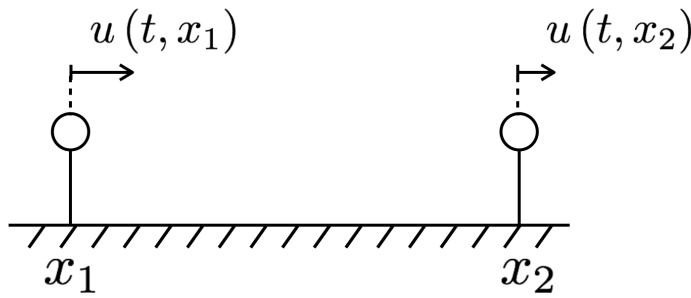
The motion of two mirrors in the cavity have two modes. One is differential motion, which is the length change of that. Another one is common motion, which is the motion of the center of the cavity. In terms of the length control, it is important that the RMS amplitude of differential motion is as small as possible. Actually, the amplitude of these two motions are the same each other when the mirrors moves with no coherence. However, when a coherence exists, the common motion tends to be larger than the differential one.

As discussed in this section, the coherence depends on both, the arm length and the wavelength of seismic waves. For example, if the arm length is much more smaller than the wavelength, the mirrors move together. This means that the common motion is greater than the differential motion.

The ratio of the amplitudes of the differential motion over common motion is newly defined as Common and Differential Motion Ratio (CDMR). It is usefull to know how the ground redacts the differential motion or increase the common motion.

### 3.4.2 Differential Motion Reduction

#### Differential Motion and Common Motion



**Figure 3.5:** The displacements of the two points which are sparated L in X axis.

Motions of the two points can be represented as the differential motion and the common motion. Displacement of both differential motion and common

motion of the two points shown in Figure(??) are defined as

$$u_{\text{diff}} \equiv \frac{u_1 - u_2}{\sqrt{2}}, \quad u_{\text{comm}} \equiv \frac{u_1 + u_2}{\sqrt{2}} \quad (3.4)$$

where  $u_1(x, t)$  and  $u_2(x, t)$  are the displacement of each points. These two motions defined in Eq.(3.4) are normalized by  $\sqrt{2}$  due to conserve the total power.

### Common and Differential Motion Ratio (CDMR)

CDMR is defined as the powers of common motion over the differential motion as bellow,

$$\text{CDMR} \equiv \sqrt{\frac{\text{Common Motion}}{\text{Differential Motion}}} = \sqrt{\frac{P_{\text{comm}}(\omega)}{P_{\text{diff}}(\omega)}} \quad (3.5)$$

where  $P_{\text{comm}}, P_{\text{diff}}$  are the power spectral densities (PSDs) of the differential motion and common motion, respectively. Each PSDs are converted from the autocorrelation function of these by the Wiener-Khinchin theorem.

First, autocorrelation function  $C_{\text{diff}}$  of the differential motion is given by its definition in Eq.(3.4)

$$C_{\text{diff}}(\tau) = \frac{1}{2} \left\langle \left[ x_1(t) - x_2(t) \right] \left[ x_1(t + \tau) - x_2(t + \tau) \right] \right\rangle \quad (3.6)$$

$$= \frac{1}{2} \left[ C_{11}(\tau) - C_{12}(\tau) - C_{21}(\tau) + C_{22}(\tau) \right], \quad (3.7)$$

,where  $C_{ij}$  are the autocorrelation functions of each point and defined as  $C_{ij} \equiv \langle x_i(t) x_j(t + \tau) \rangle$ , ( $i = 1, 2, j = 1, 2$ ). Therefore, the power spectrum density of differential motion  $P_{\text{diff}}(\omega)$  can be computed as

$$P_{\text{diff}}(\omega) = \frac{1}{2} \left[ P_1(\omega) + P_2(\omega) - P_{12}(\omega) - P_{12}^*(\omega) \right] \quad (3.8)$$

$$= \frac{1}{2} \left[ P_1 + P_2 - \text{Re}[\gamma] \times 2\sqrt{P_1 P_2} \right] \quad (3.9)$$

where  $P_1(\omega), P_2(\omega)$  are the power spectrum densities of each points, and  $P_{12}(\omega)$  are the cross spectrum between two point. The parameter  $\gamma$  is the complex coherence between them defined below,

$$\gamma \equiv \frac{P_{12}}{\sqrt{P_1 P_2}}. \quad (3.10)$$

Here, assuming that seismic wave propagating each points does not decay, which means  $P_1 = P_2 \equiv P$ , one can compute the  $P_{\text{diff}}(\omega)$  as

$$P_{\text{diff}}(\omega) = P(1 - \text{Re}[\gamma]). \quad (3.11)$$



Therefore, the PSDs of the common motion can be calculated as

$$P_{\text{comm}}(\omega) = P(1 + \text{Re}[\gamma]). \quad (3.12)$$

Finally, CDMR defined Eq.(3.5) in case the seismic wave does not decay is represented as

$$\text{CDMR} = \sqrt{\frac{1 + \text{Re}[\gamma]}{1 - \text{Re}[\gamma]}}. \quad (3.13)$$

Eq.(3.13) indicate that CDMR can be expressed by only the coherence  $\gamma$  between of two points. For example, CDMR tends to be larger when  $\gamma$  close to 1. This means that the differential motion is more less than the common motion because the two points move together in the same direction.

## 3.5 Measurement of Differential Motion Reduction

### 3.5.1 Overview

Fig.3.6 に示すように



**Figure 3.6:** Seismometers for measurement of the differential motion reduction

### 3.5.2 Reduction in X-arm Scale



NO IMAGE

Figure 3.7: ...

### 3.5.3 Reduction in Other Short Scale



Figure 3.8: ...

## 3.6 Summary of the Chapter

## Chapter 4

# Geophysics Interferometer (GIF)

### 4.1 Overview

#### 4.1.1 Laser Strainmeter for Geophysics

#### 4.1.2 Motivation in GW detectors

### 4.2 Working Principle

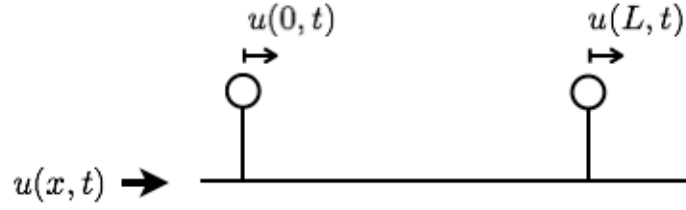
#### 4.2.1 Asymmetric Michelson Interferometer

$$\phi = 2\pi \frac{2(l_x - l_y)}{\lambda} \sim 4\pi \frac{l_x}{\lambda} \quad (4.1)$$

$$|d\phi| = 4\pi \frac{l_x}{\lambda} \left( \left| \frac{d\lambda}{\lambda} \right| + \left| \frac{dl_x}{l_x} \right| \right) \quad (4.2)$$

#### 4.2.2 Response to the seismic strain

The response of the strainmeter to seismic waves have characteristics of the low pass filter. To calculate this response, it is assumed that the plane seismic waves which displacement  $u(x, t)$  is represented as  $u(x, t) = u_0 e^{i(\omega t - kx)}$  with angular frequency of  $\omega$  and wave number of  $k$ , propagate along with the direction of the base-line of the strainmeter. The length fluctuation between two mirrors



**Figure 4.1:** The displacements of the two points which are separated  $L$  in  $X$  axis.

separated with  $L$  can be expressed as

$$\Delta L(t) \equiv u(0, t) - u(L, t) \quad (4.3)$$

$$= u(0, t) - u(0, t - \tau), \quad (4.4)$$

where  $\tau = L/v$  is the time delay. The transfer function from the displacement to the length fluctuation is

$$H_{\text{disp}}(s) \equiv \frac{\Delta L(s)}{u(s)} = 1 - \exp(-\tau s) \quad (4.5)$$

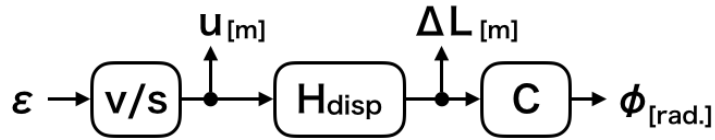
Because the strain amplitude  $\epsilon(x, t)$  is defined as  $\epsilon(x, t) \equiv \frac{du}{dx}$ , the strain

$$\epsilon(x, t) \equiv \frac{du}{dx} = \frac{du}{dt} \frac{dt}{dx} \quad (4.6)$$

$$= u(x, t)' \frac{1}{v} \quad (4.7)$$

Therefore, the response of the strainmeter to the seismic strain is given

$$H_{\text{strain}}(s) \equiv \frac{\Delta L(s)}{\epsilon(s)} = \frac{\Delta L(s)}{\frac{s}{v} u(s)} = (1 - \exp(-\tau s)) \frac{v}{s} \quad (4.8)$$



**Figure 4.2**

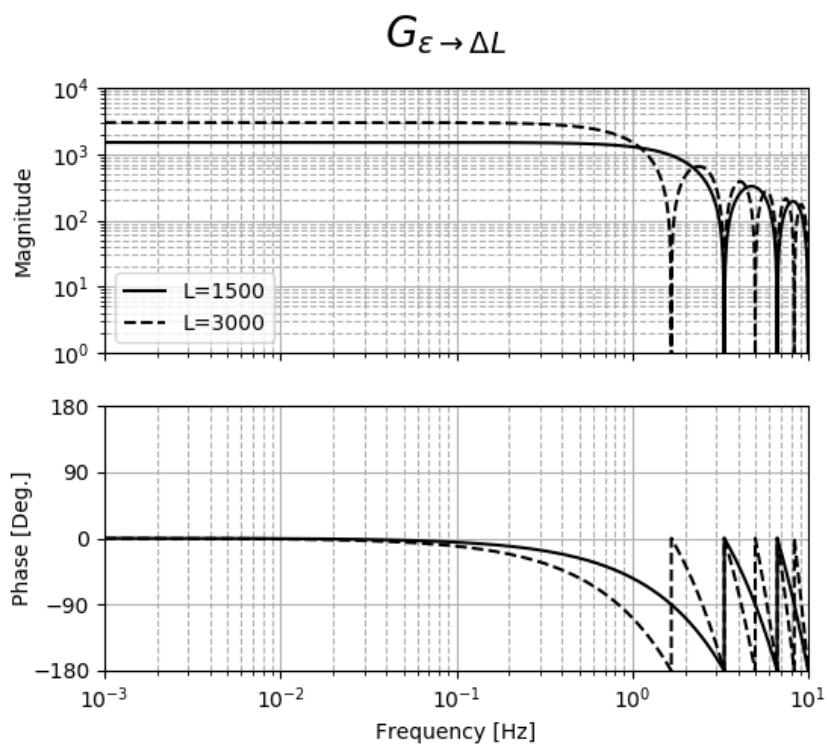


Figure 4.3

### 4.2.3 Signal Detection Scheme

#### Quadrature Phase Detection

### 4.2.4 Noise

どういふノイズが原理的に存在するか述べる。空気ゆらぎ、周波数雑音を述べる。

## 4.3 Optics Design

...

### 4.3.1 Overview

Optical layout is shown in Fig. 4.4. Optics of GIF is categorized by three category; mode-matching optics, core optics and frequency stabilized laser.



Figure 4.4

#### 4.3.2 Gaussian Beam Profile

#### 4.3.3 Input Output Optics

#### 4.3.4 Core Optics

#### 4.3.5 Frequency Stabilized Laser

### 4.4 Data Aquisition System

#### 4.4.1 Realtime Processing

#### 4.4.2 ...

### 4.5 Summary of the Chapter

本章で述べたパラメータを表にまとめる。





## Chapter 5

# Arm Length Compensation System for Global Seismic Control

### 5.1 Basics in Vibration Isolation and Control Technique

#### 5.1.1 Passive Vibration Isolation

Single Pendulum

Multi Pendulum

#### 5.1.2 Active Vibration Isolation

#### 5.1.3 Sensor Belnding Control Technique

#### 5.1.4 2 Types Feedforward Control Techniques

Feedforward at Feedback Point

Feedforward at Error Point

#### 5.1.5 Toward the Global Seismic Control

Overview

Suspension Point Interferometer

### 5.2 Difficulties in the Global Seismic Control

#### 5.2.1 Overview

#### 5.2.2 Actuator Range Limit

#### 5.2.3 ...

## Chapter 6

# Demonstration of Arm Length Compensation Control

### 6.1 Experimental Arrangement

#### 6.1.1 ...

### 6.2 Results

#### 6.2.1 ...

### 6.3 Discussion and Summary of the Chapter

#### 6.3.1 Discussion

#### 6.3.2 Summary



## Chapter 7

# Conculusion and Future Directions

### 7.1 Conclusion

### 7.2 Future Directions



## Appendix A

# Theory of Seismic Waves

### A.1 Body Wave

等方弾性体中では変位  $\mathbf{u}$  は以下の波動方程式に従う。

$$\rho \ddot{\mathbf{u}} = (\lambda + 2\mu) \nabla(\nabla \cdot \mathbf{u}) - \mu \nabla \times (\nabla \times \mathbf{u}) \quad (\text{A.1})$$

ここで  $\rho$  は媒質の密度、 $\lambda, \mu$  はラメ定数である。

この波動方程式は縦波である P 波と横波である S 波について解くことができる。そのためにまず Helmholtz decomposition をつかって変位  $\mathbf{u}$  を発散成分  $\mathbf{u}_{\text{div}}$  と回転成分  $\mathbf{u}_{\text{rot}}$  で表す。つまり、

$$\mathbf{u}_{\text{div}} = \nabla \phi \quad (\text{A.2})$$

$$\mathbf{u}_{\text{rot}} = \nabla \times \psi \quad (\text{A.3})$$

となるスカラーポテンシャル  $\phi$  とベクトルポテンシャル  $\psi$  が存在し、変位  $\mathbf{u}$  は

$$\mathbf{u} = \nabla \phi + \nabla \times \psi \quad (\text{A.4})$$

と表すことができる。式 (A.1) に式 (A.4) を代入し、かつベクトル解析の公式、 $\nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$  を使うと、

$$\ddot{\phi} = v_L^2 \nabla^2 \phi \quad (\text{A.5})$$

$$\ddot{\psi} = v_T^2 \nabla^2 \psi \quad (\text{A.6})$$

のように 2 つの波動方程式を得る。ここで  $v_L, v_T$  は、

$$v_L = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \quad v_T = \sqrt{\frac{\mu}{\rho}} \quad (\text{A.7})$$

である。

$v_L, v_T$  はそれぞれ縦波と横波の位相速度を表しているが、これを示す。まずスカラーポテンシャルとベクトルポテンシャルは式 (A.5)、式 (A.6) の波動方程式に従うので、これらの一般解は

$$\phi = \phi_0(\omega t - \mathbf{k} \cdot \mathbf{x}) \quad (\text{A.8})$$

$$\psi = \psi_0(\omega t - \mathbf{k} \cdot \mathbf{x}) \quad (\text{A.9})$$

で表すことができる。ここで  $\omega, \mathbf{k}$  は各周波数と波数ベクトルである。発散成分である  $\mathbf{u}_{\text{div}}$  は式 (A.2) に式 (A.8) を代入して、

$$\mathbf{u}_{\text{div}} = \nabla \phi_0(\omega t - \mathbf{k} \cdot \mathbf{x}) = -\mathbf{k} \phi \quad (\text{A.10})$$

となるので、変位の向きは波数ベクトルと平行である。つまり縦波であり P 波に相当する。一方で回転成分である  $\mathbf{u}_{\text{rot}}$  は式 (A.3) に式 (A.9) を代入して、

$$\mathbf{u}_{\text{rot}} = \nabla \times \psi_0(\omega t - \mathbf{k} \cdot \mathbf{x}) = -\mathbf{k} \times \psi \quad (\text{A.11})$$

となるので、変位の向きは波数ベクトルと直行している。つまり横波であり S 波に相当する。したがって  $v_L, v_T$  はそれぞれ縦波と横波の位相速度を示していることがわかった。また  $\lambda$  と  $\mu$  は正の定数なので、

$$v_L > v_T \quad (\text{A.12})$$

となって、縦波のほうが横波よりも速いことがわかる。

## A.2 Rayleigh 波

(レイリー波の導出。)

## A.3 Depth Dependence

(レイリー波の振幅が深さに依存していることを述べる。)



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