Seismic Noise Under the Ground

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Seismic Noise Under the Ground

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1.4.1 Introduction

The seismic noise shakes both the common and the differential motion of the arm cavity. The common motion moves the center of mass of that, and the differential motion moves the arm length. These motions are the same each other, when two mirrors in the cavity moves with no correlation. However, If there is the coherence, the differential motion is less than common motion.

A ratio of these motions depends on both the arm length and the wave length of seismic motion which propagates along the arm. If the arm length is much smaller than the wave length, two mirrors in the cavity moves together. It means that the common motion is greater than differential motion. This effect is remain as long as coherence of these mirrors is exist.

In fact, arm length fluctuation is caused by not only the differential motion of the ground but also the coupling from the common motion to the length. This coupling ratio is kwnon as the Common Mode Rejection Ratio (CMRR). This is defined as the ratio of the powers of the differential-mode response over the common-mode response in a system. For example, in the case of ideal arm cavity, CMRR is infinity because the common response from the ground to optics is 0. However, in the actual case, the commom response is not 0 because there are some differencies in the responses of two mirrors.

In this section, a ratio of the powers of the differential motion over common motion in the ground is defined as Common and Differential Motion Ratio (CDMR), and described in detail. This redcution effects could relax to control the arm cavity.

1.4.2 Differential Motion Reduction

Motions of two mirrors in the arm cavity can be represented as the differential and the common motion. These motions are defined and calculated with the coherence between these two mirrors. In folowing disscuss, the mirrors are fixed on the ground and these of CMRR is high enough to ignore the coupling from common motion of ground to differential motion of the mirrors.

Differential Motion and Common Motion

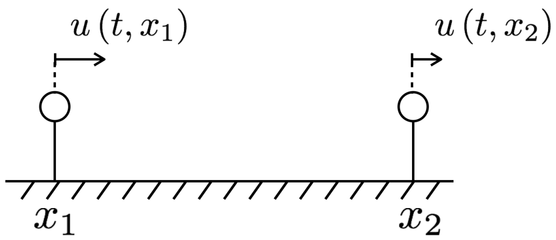


Figure 1.1: Displacements of two points,x_1, x_2. Displacements of each location are represented as $u(x_1, t)$, $u(x_2, t)$, in the case displacement field is given with $u(x, t)$, where x is location of the point and t is a time.

Displacement of both differential motion and common motion of two points shown in Figure1.1 is defined as

$$u_{\mathrm{diff}} \equiv \frac{u_1 - u_2}{\sqrt{2}},\ u_{\mathrm{comm}} \equiv \frac{u_1 + u_2}{\sqrt{2}} \tag{1.1}$$

, where $u_1(x,t)$ and $u_2(x,t)$ are the displacement in each points.

Common and Differential Motion Ratio (CDMR)

Common and Differential Motion Ratio (CDMR) is defined as the powers of common motion over the differential motion as bellow,

$$\mathrm{CDMR} \equiv \sqrt{\frac{\text{Common Motion}}{\text{Differential Motion}}} = \sqrt{\frac{P_{\mathrm{comm}}(\omega)}{P_{\mathrm{diff}}(\omega)}} \tag{1.2}$$

,where $P_{\mathrm{comm}}, P_{\mathrm{diff}}$ is the power spectrum densities (PSDs) of them. These are estimated by the autocorrelation function of these with the Wiener-Khinchin theorem.

Autocorrelation function C_{diff} is given by

$$\begin{aligned} C_{\mathrm{diff}}(\tau) &= \frac{1}{2}\left\langle \Big[x_1(t) - x_2(t)\Big]\Big[x_1(t+\tau) - x_2(t+\tau)\Big] \right\rangle & (1.3) \\ &= \frac{1}{2}\Big[C_{11}(\tau) - C_{12}(\tau) - C_{21}(\tau) + C_{22}(\tau)\Big], & (1.4) \end{aligned}$$

,where C_{ij} are the autocorrelation functions of each location and defined as $C_{ij} \equiv \langle x_i(t)x_j(t+\tau)\rangle$, $(i = 1,2,\ j = 1,2)$. Therefore, the power spectrum density of differential motion $P_{\mathrm{diff}}(\omega)$ can be computed as

$$\begin{aligned} P_{\mathrm{diff}}(\omega) &= \frac{1}{2}\Big[P_1(\omega) + P_2(\omega) - P_{12}(\omega) - P_{12}^*(\omega)\Big] & (1.5) \\ &= \frac{1}{2}\Big[P_1 + P_2 - \mathrm{Re}\,[\mathrm{coh}] \times 2\sqrt{P_1P_2}\Big] & (1.6) \end{aligned}$$

where $P_1(\omega), P_2(\omega)$ are the power spectrum densities of each locations, and $P_{12}(\omega)$ are the cross spectrum between two location. coh is the complex coherence between them defined below.

$$\mathrm{coh} \equiv \frac{P_{12}}{\sqrt{P_1P_2}} \tag{1.7}$$

Assuming that $P_1 = P_2 \equiv P$, one can compute the CDMR using Eq.(1.2) as

$$\mathrm{CDMR} = \sqrt{\frac{1 + \mathrm{Re}\,[\mathrm{coh}]}{1 - \mathrm{Re}\,[\mathrm{coh}]}}. \tag{1.8}$$

Eq.(1.8) indicate that CDMR can be expressed by only the coherence between of two locations.

Incidentally, If the coherence is known and the PSDs of two location are same, one can estimate the PSDs of differential motion using that of one location according to Eq.(1.6) as

$$P_{\mathrm{diff}} = P\sqrt{1 - \mathrm{Re}[\mathrm{coh}]}. \tag{1.9}$$

This expression is usefull to estimate a length fluctuation of the arm cavity even single point measurement because the coherence can be calculate using some models which is discussed following subsection.

Single Plane Wave Model

The CDMR when single plane wave propagates along the arm cavity is discussed. This model can be applied in the case the source of seismic motion is only one such as an earth quake. Assuming that the plane wave propagates with the azimuth angle θ along the direction of arm cavity, the wave length λ is $\lambda/\cos\theta$. In this situation, the coherence from x_1 to x_2 is denoted as

$$\mathrm{coh} = e^{i\frac{L\cos\theta\omega}{c}} \tag{1.10}$$

Therefore, one can compute the CDMR as

$$\mathrm{CDMR} = \sqrt{\frac{1+\cos(\frac{L\omega}{c}\cos\theta)}{1-\cos(\frac{L\omega}{c}\cos\theta)}}. \tag{1.11}$$

Uniform Plane Wave Model

The CDMR when the plane waves are distributed uniformly around the azimuth is discussed. This model can be applied in the case microseisms excite the ground. The coherence is equal to the integral over all direction.

$$\mathrm{coh} \;=\; \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{i\frac{\omega}{c}L\cos\theta}d\theta \tag{1.12}$$

where the coherence is normized azimuth angle. Therefore, the CDMR is given as

$$\mathrm{CDMR} = \sqrt{\frac{1+J_0(\frac{L\omega}{c})}{1-J_0(\frac{L\omega}{c})}}. \tag{1.13}$$

Reduction in the X-arm

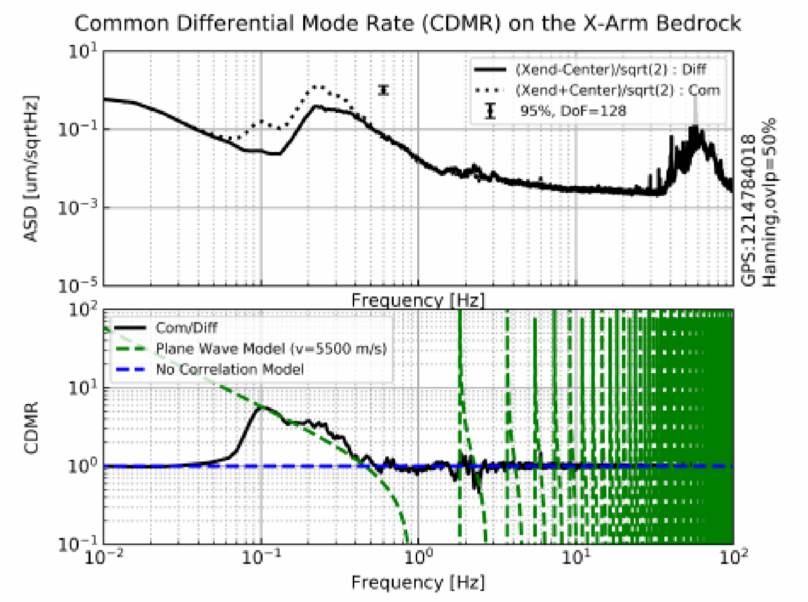
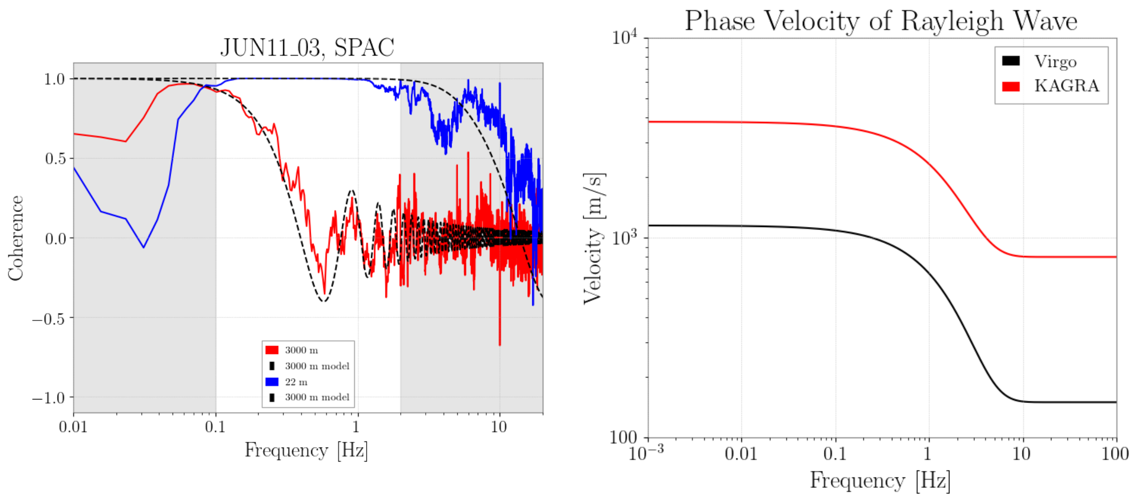


Figure 1.2: **(Caution!! This plot shows the case of single plane wave.)** CDMR in the case of the uniform plane wave model.(Upper) The amplitude spectrum densities of the differential and common motion in X arm cavity which is measured by seismometers near the each ITMX and ETMX chambers. (Lower) CDMR which is a ratio of the amplitudes of the common motion over differential motion, is shown in black line. Green Line is CDMR in the case the uniform plane wave model. Blue line is CDMR with no correlation between each mirrors.

1.4.3 Comparizon with surface detectors

I'm tired. I'll write tomorrow.



(a)

(b)

Figure 1.3: (a) The complex coherence between two locations. Red solid line is a complex coherence in 3000 m distance. Blue one is a coherence in 22 m distance. Black dashed line is given by Eq.(1.12) with asuming a profile of the phase velocity on Fig.(1.3b). (b) The phase velocity of the Rayleigh wave. Black solid line is sited from M.Beker's PhD thesis. Red one is taken by fitting the measured data on Fig.(1.3a)

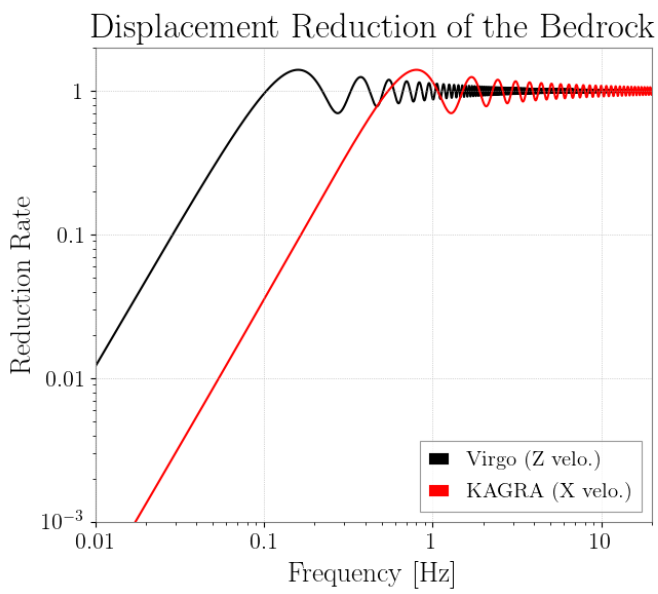


Figure 1.4: Comparizon of the Reduction effect between KAGRA and Virgo. Eq.(1.9) with the phase velocity on the Fig.(1.3b)

1.5 Summary of the Chapter