Under Ground Seismic Noise

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Contents

Chapter 1

Under Ground Seismic Noise

1.1 Introduction

1.1.1 Site on the Ground Surface

1.1.2 Site under the Ground

1.1.3 KAGRA Site

1.2 Theory of Seismic Waves

1.2.1 Seismic Waves

1.2.2 Depth Dependence

1.2.3 ...

1.3 Seismic Noise

1.3.1 Problematic Seismic Noise

1.3.2 The Microseisms

1.3.3 The Earth Quakes

1.3.4 The Earth Tide

1.3.5 ...

1.4 Seismic Noise Rejection in the Arm Cavity

1.4.1 Rejection of the Cavity Length

The seismic noise shakes both common and differential motion of the arm cavity. The common motion is the center of mass of that, and the differential motion is the arm length. These motions are same each other, when two points move with no correlation. However, If there is a coherence, the differential motion is less than common motion. This redcution effects relax to control the arm cavity.

A ratio of these motions depends on the arm length and the wave length of seismic motion which propagates along the arm. If wave length is much smaller than the wave length, two mirrors in the cavity are move together. It means that the common motion is greater than differential motion. This effect is remain as long as coherence of these mirrors is exist.

In fact, arm length fluctuation is caused by not only differential motion of the ground but also the coupling from the common motion. This coupling ratio is kwnon as the Common Mode Rejection Ratio (CMRR). This is defined as the ratio of the powers of the differential-mode response over the common-mode response in a system. For example, in the case of ideal arm cavity, CMRR is infinity because response of the differential motion of the arm is 1 and that of the common motion is 0. In the actual case, differencies in the ground responce of each mirrors causes the coupling.

In this section, a ratio of the powers of the differential motion over common motion in the ground is defined as Common and Differential Motion Ratio (CDMR), and described in detail.

1.4.2 Rejection Rate

Differential Motion and Common Motion

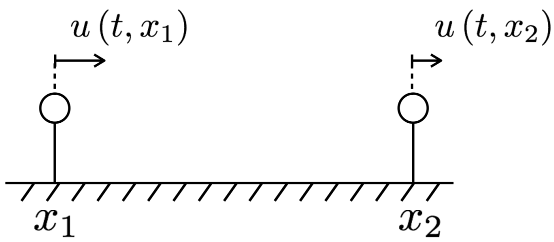


Figure 1.1: Displacements of two points,x_1, x_2. Displacements of each location are represented as $u(x_1, t)$, $u(x_2, t)$, in the case displacement field is given with $u(x, t)$, where x is location of the point and t is a time.

Displacement of both differential motion and common motion of two points shown in Figure1.1 is defined as

$$u_{\text{diff}} \equiv \frac{u_1 - u_2}{\sqrt{2}},\ u_{\text{comm}} \equiv \frac{u_1 + u_2}{\sqrt{2}} \tag{1.1}$$

, where $u_1(x, t)$ and $u_2(x, t)$ are the displacement in each points.

Common and Differential Motion Ratio (CDMR)

Common and Differential Motion Ratio (CDMR) is defined as the powers of common motion over the differential motion as bellow,

$$\mathrm{CDMR} \equiv \sqrt{\frac{\text{Common Motion}}{\text{Differential Motion}}} = \sqrt{\frac{P_{\mathrm{comm}}(\omega)}{P_{\mathrm{diff}}(\omega)}} \tag{1.2}$$

,where $P_{\mathrm{comm}}, P_{\mathrm{diff}}$ is the power spectrum densities (PSDs) of them. These are estimated by the autocorrelation function of these with the Wiener-Khinchin theorem.

Autocorrelation function C_{diff} is given by

$$C_{\mathrm{diff}}(\tau) = \frac{1}{2}\left\langle \Big[x_1(t) - x_2(t)\Big]\Big[x_1(t+\tau) - x_2(t+\tau)\Big]\right\rangle \tag{1.3}$$

$$= \frac{1}{2}\Big[C_{11}(\tau) - C_{12}(\tau) - C_{21}(\tau) + C_{22}(\tau)\Big], \tag{1.4}$$

,where C_{ij} are the autocorrelation functions of each location and defined as $C_{ij} \equiv \langle x_i(t)x_j(t+\tau)\rangle$, $(i = 1, 2,\ j = 1, 2)$. Therefore, the power spectrum density of differential motion $P_{\mathrm{diff}}(\omega)$ can be computed as

$$P_{\mathrm{diff}}(\omega) = \frac{1}{2}\Big[P_1(\omega) + P_2(\omega) - P_{12}(\omega) - P_{12}^*(\omega)\Big] \tag{1.5}$$

$$= \frac{1}{2}\Big[P_1 + P_2 - \mathrm{Re}\,[\mathrm{coh}] \times 2\sqrt{P_1P_2}\Big] \tag{1.6}$$

where $P_1(\omega), P_2(\omega)$ are the power spectrum densities of each locations, and $P_{12}(\omega)$ are the cross spectrum between two location. coh is the complex coherence between them defined below.

$$\mathrm{coh} \equiv \frac{P_{12}}{\sqrt{P_1P_2}} \tag{1.7}$$

Assuming that $P_1 = P_2 \equiv P$, one can compute the CDMR using Eq.(1.2) as

$$\mathrm{CDMR} = \sqrt{\frac{1 + \mathrm{Re}\,[\mathrm{coh}]}{1 - \mathrm{Re}\,[\mathrm{coh}]}}. \tag{1.8}$$

Eq.(1.8) indicate that CDMR can be expressed by only the coherence between of two locations.

Incidentally, If the coherence is known and the PSDs of two location are same, one can estimate the PSDs of differential motion using that of one location according to Eq.(1.6) as

$$P_{\mathrm{diff}} = P\sqrt{1 - \mathrm{Re}[\mathrm{coh}]}. \tag{1.9}$$

This expression is usefull to estimate a length fluctuation of the arm cavity even single point measurement because the coherence can be calculate using some models which is discussed following subsection.

Single Plane Wave Model

The CDMR when single plane wave propagates along the arm cavity is discussed. This model can be applied in the case the source of seismic motion is only one such as an earth quake. Assuming that the plane wave propagates with the azimuth angle θ along the direction of arm cavity, the wave length λ is $\lambda/\cos\theta$. In this situation, the coherence from x_1 to x_2 is denoted as

$$\mathrm{coh} = e^{i\frac{L\cos\theta\omega}{c}} \tag{1.10}$$

Therefore, one can compute the CDMR as

$$\mathrm{CDMR} = \sqrt{\frac{1+\cos(\frac{L\omega}{c}\cos\theta)}{1-\cos(\frac{L\omega}{c}\cos\theta)}}. \tag{1.11}$$

Uniform Plane Wave Model

The CDMR when the plane waves are distributed uniformly around the azimuth is discussed. This model can be applied in the case microseisms excite the ground. The coherence is equal to the integral over all direction.

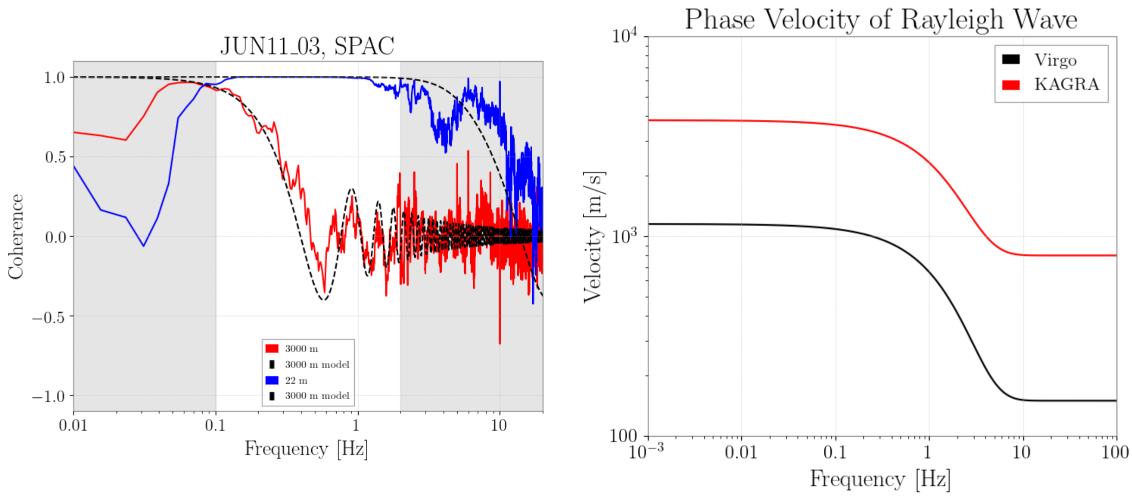
$$\mathrm{coh} \quad = \quad \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{i\frac{\omega}{c}L\cos\theta} d\theta \tag{1.12}$$

where the coherence is normized azimuth angle. Therefore, the CDMR is given as

$$\mathrm{CDMR} = \sqrt{\frac{1+J_0(\frac{L\omega}{c})}{1-J_0(\frac{L\omega}{c})}}. \tag{1.13}$$

1.4.3 Comparizon with surface detectors

I'm tired. I'll write tomorrow.



(a) **(b)**

Figure 1.2: (a) The complex coherence between two locations. Red solid line is a complex coherence in 3000 m distance. Blue one is a coherence in 22 m distance. Black dashed line is given by Eq.(1.12) with asuming a profile of the phase velocity on Fig.(1.2b). (b) The phase velocity of the Rayleigh wave. Black solid line is sited from M.Beker's PhD thesis. Red one is taken by fitting the measured data on Fig.(1.2a)

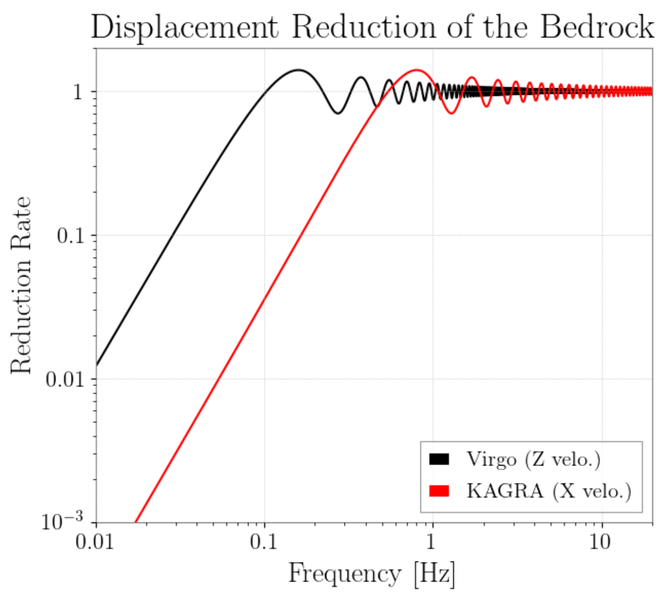


Figure 1.3: Comparizon of the Rejection effect between KAGRA and Virgo. Eq.(1.9) with the phase velocity on the Fig.(1.2b)

1.5 Summary of the Chapter