

STAT 8320 Spring 2015 Assignment 5

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► 1. Solution. (a).

$$\begin{aligned} f(\lambda|y_i) &= \frac{f(y_i|\lambda)f(\lambda)}{f(y_i)} \\ &= \frac{\frac{\lambda^{y_i+a-1}}{\Gamma(a)b^a y_i!} e^{-\lambda(1+1/b)}}{f(y_i)} \\ &\propto \frac{\lambda^{y_i+a-1}}{\Gamma(a)b^a y_i!} e^{-\lambda(1+1/b)} \\ &\propto \lambda^{y_i+a-1} e^{-\lambda(1+1/b)} \end{aligned}$$

So $\lambda|y_i \sim \text{GAM}(y_i + a - 1, \frac{1}{1+1/b})$, and

$$f(\lambda|y_i) = \frac{\lambda^{y_i+a-1}}{\Gamma(y_i + a) \left(\frac{b}{1+b}\right)^{y_i+a}} e^{-\lambda(1+1/b)}$$

Thus,

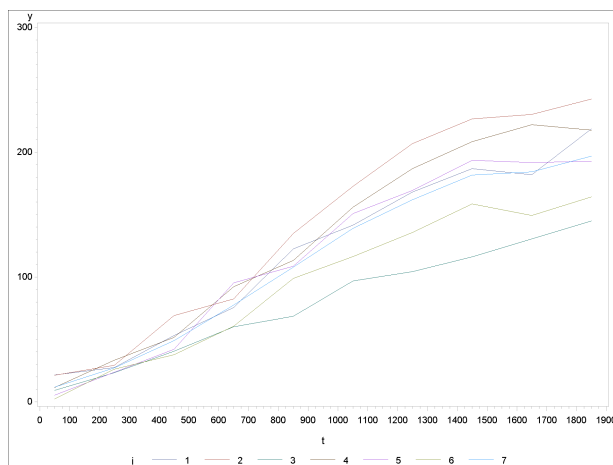
$$\begin{aligned} f(y_i) &= \int_0^\infty f(y_i|\lambda)f(\lambda)d\lambda = \frac{f(y_i|\lambda)f(\lambda)}{f(\lambda|y_i)} \\ &= \frac{\frac{\lambda^{y_i+a-1}}{\Gamma(a)b^a y_i!} e^{-\lambda(1+1/b)}}{\frac{\lambda^{y_i+a-1}}{\Gamma(y_i+a)\left(\frac{b}{1+b}\right)^{y_i+a}} e^{-\lambda(1+1/b)}} \\ &= \frac{\Gamma(y_i + a)}{\Gamma(a)y_i!} \left(\frac{1}{1+b}\right)^a \left(\frac{b}{1+b}\right)^{y_i} \\ &= \binom{a+y_i-1}{a-1} \left(\frac{b}{1+b}\right)^{y_i} \end{aligned}$$

We can conclude that $y_i \sim \text{NB}(\frac{1}{1+b}, a)$.

(b) From the theories in generalized linear model, we have already known that negative binomial distribution usually is used to fixed the over-dispersion problem of count data when Poisson distribution assumption or independence assumption are no longer valid. And we also know that in most time the over-dispersion may be caused by the dependence of data, like some repeated measurements in student attendance example. The GLMM essentially takes covariates between dependent data into model, so it also can model the over-dispersed count data. Or in other words, the derivation in part (a) just shows that negative binomial distribution can work well with over-dispersed count data.

► **2. Solution.** (a). Before we fit this nonlinear mixed model, we should firstly plot the profile of the data, trying to acquire some intuitive result from the plot. As the Figure 1 shows, different plants are label as 1 to 6, and number 7 represents the average profile. We can approximately know that the max value is between 150 and 250, and the inflection point should be between 500 and 1000. Without loss of generality, we set the initial value of (β_1, β_2) as $(200, 850)$. Then we can solve the β_3 based on the data, and it is about 350. Next, by using PROC MEANS in SAS, we can get the standard deviations at different time points. Because $1 + e^{-(t_{ij}-\beta_2)/\beta_3}$ is relatively large, it is reasonable to assume that the variance of Y is mostly from σ^2 . So we set the initial value of σ^2 as 40. As t_{ij} grows, the denominator becomes smaller and smaller, then the proportion of σ_u^2 in variance of Y becomes larger, and it seems that σ_u^2 should be between 400 to 1600, so we set the initial value of σ_u^2 as 900.

Figure 1: Profile of Plant Growth



Then we use the PROC NLIN and PROC NLMIXED to fit the nonlinear model and the nonlinear mixed model respectively. The results about parameters are listed as Figure 2 and Figure 3.

Figure 2: Parameters of Nonlinear Models

Parameter	Estimate	Approx Std Error	Approximate 95% Confidence Limits		Skewness
beta1	199.7	10.3827	178.9	220.4	0.5330
beta2	797.8	55.1103	687.4	908.1	0.3725
beta3	300.7	42.8631	214.8	386.5	0.4782

Figure 3: Parameters of Nonlinear Mixed Models

Parameter Estimates						
Parameter	Estimate	Standard Error	DF	t Value	Pr > t	Alpha
beta1	199.41	15.2372	5	13.09	<.0001	0.05
beta2	797.42	14.6250	5	54.52	<.0001	0.05
beta3	298.48	11.4146	5	26.15	<.0001	0.05
resvar	49.8315	9.5902	5	5.20	0.0035	0.05
varu	1346.95	784.96	5	1.72	0.1468	0.05
Parameter Estimates						
Parameter	Lower	Upper	Gradient			
beta1	160.24	238.58	4.83E-7			
beta2	759.82	835.01	-3.07E-6			
beta3	269.14	327.82	2.884E-6			
resvar	25.1791	74.4838	2.602E-6			
varu	-670.87	3364.76	-1.06E-8			

From these output, we can answer the questions like,

1. To test

$$H_0 : \beta_3 = 350 \quad \text{v.s.} \quad H_A : \beta_3 \neq 350$$

we can easily reject the null hypothesis because the Wald-type confidence intervals of both models do not contain 350, which is equivalent to a Wald test. We also can use the ESTIMATE statement in PROC NLMIXED to estimate $\beta_3 - 350$, as Figure 4

Figure 4: Testing " $\beta_3 = 350$ "

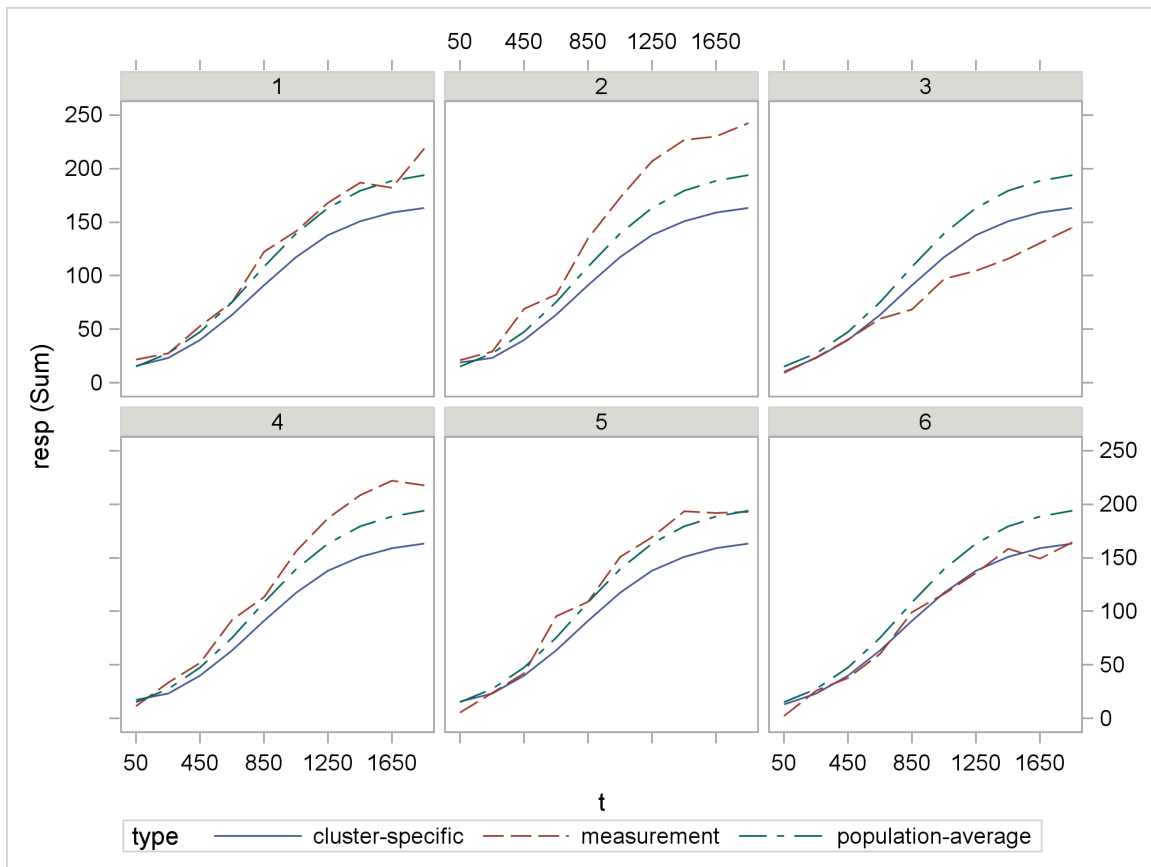
Additional Estimates						
Label	Estimate	Standard Error	DF	t Value	Pr > t	Alpha
Beta_3=350?	-51.5207	11.4146	5	-4.51	0.0063	0.05
Additional Estimates						
Label	Lower		Upper			
Beta_3=350?	-80.8627		-22.1786			

It is obviously that we should reject the null hypothesis, which is the same result as above. So β_3 does not equal 350.

2. To test whether the random effect is necessary. Because the method of parameter estimate of mixed model is not based on likelihood, we cannot use likelihood ratio test. So we still use the Wald-type confidence interval. Because the interval contains zero, we cannot reject the null hypothesis, that is, the random effect is not significant. Furthermore, we can see that the estimate of parameters of fixed effect does not change too much, so this also indicates that it is not necessary to introduce random effect into model. Finally, from the plots of prediction and measurements, we can see the cluster-specific prediction curve are almost same, which also indicates that the random effect is slight.

But, we should be cautious about the result, because from the parameter estimates of parameter of nonlinear model, skewness of all parameters is much greater than 0.25, which means all parameters have vary apparent skewness. This makes the inferences unreliable.

Figure 5: Plots of Prediction and Measurements



► 3. Solution.

Firstly, we fit the generalize linear model using PROC GENMOD, and parameter estimates are listed as Figure 6 (Because of the limitation of page width, the last column "P-value" of table is not included in page).

Figure 6: Results from PROC GENMOD

Analysis Of Maximum Likelihood Parameter Estimates							
Parameter	DF	Estimate	Standard Error	Wald 95% Confidence Limits		Wald Chi-Square	Pr
Intercept	1	-2.2769	0.1048	-2.4823	-2.0714	471.93	
sst*loc 1	1	-1.6597	0.4796	-2.5997	-0.7196	11.97	
sst*loc 2	1	-0.4395	0.6825	-1.7772	0.8981	0.41	
sst*loc 3	1	0.5892	0.6244	-0.6345	1.8129	0.89	
sst*loc 4	1	-1.0846	0.5885	-2.2380	0.0688	3.40	
sst*loc 5	1	-1.5915	0.4924	-2.5566	-0.6264	10.45	
sst*loc 6	1	0.5658	0.6286	-0.6663	1.7979	0.81	
sst*loc 7	1	1.1118	0.5135	0.1055	2.1182	4.69	
sst*loc 8	1	0.5834	0.6254	-0.6424	1.8092	0.87	
sst*loc 9	1	0.9666	0.5461	-0.1037	2.0369	3.13	
sst*loc 10	1	-0.5665	0.6695	-1.8787	0.7458	0.72	
sst*loc 11	1	1.0661	0.5238	0.0395	2.0927	4.14	
sst*loc 12	1	0.3029	0.6692	-1.0088	1.6146	0.20	
sst*loc 13	1	0.1068	0.6885	-1.2426	1.4561	0.02	
sst*loc 14	1	0.2370	0.6769	-1.0897	1.5636	0.12	
sst*loc 15	1	0.1418	0.6858	-1.2024	1.4860	0.04	
sst*loc 16	1	-1.0449	0.5957	-2.2125	0.1226	3.08	
sst*loc 17	1	1.0902	0.5184	0.0742	2.1061	4.42	
sst*loc 18	1	0.6014	0.6221	-0.6179	1.8207	0.93	
sst*loc 19	1	0.0945	0.6893	-1.2565	1.4455	0.02	
sst*loc 20	1	0.4467	0.6489	-0.8250	1.7185	0.47	
Scale	0	1.0000	0.0000	1.0000	1.0000		
NOTE: The scale parameter was held fixed.							

We want to test

$$H_0 : \text{All } \beta_{1,i} \text{ equal } 0 \quad \text{v.s.} \quad H_A : \text{At least one } \beta_{1,i} \text{ not equal } 0$$

From the output, I can see that the P-value for the slope at location 1 is 0.0005, which means that $\beta_{1,1}$ is significantly not equal to 0. So we can say that $\beta_{1,i}$ are significant. This means that SST effect is important to predict the tornado occurrence.

Then we fit the mixed models with random intercept and/or slope. Using PROC GLIMMIX, some results are listed in the Appendix B. Firstly, from Figure 10, 14 and 18, the type III tests show that the fixed effect of interaction between interaction and SST is significant, which support the result that $\beta_{1,i}$ are significant. Then to test the random effect, using COVTEST statement in PROC GLIMMIX, the results which are showed in Figure 11, 15 and 19 indicate that we cannot reject the null hypothesis of no random effect, neither random intercept nor random slope.

From Figure 21, we plot the predication of tornado occurrence frequency of different location versus the actual frequency. The reference line is a line with slope equal 1, which means the predications perfectly fit the facts. The distance between observation point and reference line represent the accuracy of the predication of the model; the point far away from the line has a very bad predication. We can see that the four plots are not different so much, that is to say, random effect(s) does not improve our prediction so much, which also can be seen from the sum of squared residuals in Figure 7. This is in accordance with the result of COVTEST, that is, it is not necessary to introduce random effect. Moreover, from the plots, the locations 7, 16, 3, 4, 18 and 12 have very good predication under the GL model; the predications of location 14 and 6 are still acceptable, not only because the differences are not so large, but also because they predict more tornado than actual occurrence, which is conservative and safe. However, location 19 has very bad predication. We should realize that location 19 is an outlier, so there may be some other factors that we do not notice or take into consideration.

► 4. **Solution.** (a)

$$\mathbf{Y} = \begin{pmatrix} X_1 \\ X_3 \end{pmatrix} \sim N \left(\begin{pmatrix} -3 \\ 2 \end{pmatrix}, \begin{pmatrix} 4 & 0 \\ 0 & 3 \end{pmatrix} \right)$$

(b)

$$\begin{aligned} \mathbf{Y}|X_2 &\sim N \left(\begin{pmatrix} -3 \\ 2 \end{pmatrix} + \begin{pmatrix} \frac{x_2}{2} - \frac{1}{2} \\ \frac{x_2}{2} - \frac{1}{2} \end{pmatrix}, \begin{pmatrix} 3.5 & -0.5 \\ -0.5 & 2.5 \end{pmatrix} \right) \\ &= N \left(\begin{pmatrix} \frac{x_2}{2} - \frac{7}{2} \\ \frac{x_2}{2} + \frac{3}{2} \end{pmatrix}, \begin{pmatrix} 3.5 & -0.5 \\ -0.5 & 2.5 \end{pmatrix} \right) \end{aligned}$$

(c)

$$\begin{aligned}
X_2|\mathbf{Y} &\sim N\left(2 + (1 \ 1) \begin{pmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{3} \end{pmatrix} \begin{pmatrix} x_1 + 3 \\ x_3 - 2 \end{pmatrix}, 2 - (1 \ 1) \begin{pmatrix} \frac{1}{4} & 0 \\ 0 & \frac{1}{3} \end{pmatrix} \begin{pmatrix} 1 \\ 1 \end{pmatrix}\right) \\
&= N\left(\frac{1}{4}x_1 + \frac{1}{3}x_3 + \frac{13}{12}, \frac{17}{12}\right).
\end{aligned}$$

(d)

$$Z \sim N(-3 + 3 \cdot 1, 4 + 3^3 \cdot 2 + 2 \cdot 3 \cdot 1) = N(0, 28)$$

► **5. Solution.** (a). The hypotheses are

$$H_0 : \mu_{11} = \mu_{12} = \mu_{13} \quad \text{v.s.} \quad H_a : \text{at least two means are not equal}$$

or we can write null hypothesis as

$$H_0 : \mathbf{C}_1 \boldsymbol{\mu}_1 = \mathbf{0}$$

where

$$\mathbf{C}_1 = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix}$$

The statistics are

$$T^2 = n_1(\mathbf{C}\bar{\mathbf{y}})'(\mathbf{C}\mathbf{S}\mathbf{C}')^{-1}(\mathbf{C}\bar{\mathbf{y}}) = 111.4286$$

$$F = \frac{n_1 - c}{(n_1 - 1)c} T^2 = 53.3929 \sim f_{c, n_1 - c}$$

where $c = 2$ and $n_1 = 25$. The critical value of F statistic is $f_{0.95, 2, 23} = 3.422$, so $F > f_{0.95, 2, 23}$ and P-value is 2.28E^{-09} . We will reject the null hypothesis, that is, the mean concentrations are significantly different at three time points.

(b).

i The common covariance is

$$\mathbf{S}_{pool} = \frac{(n_1 - 1)\mathbf{S} + (n_2 - 1)\mathbf{W}}{(n_1 - 1) + (n_2 - 1)} = \begin{pmatrix} 28.4 & 10.8 & 12.4 \\ 10.8 & 15.8 & 5.6 \\ 12.4 & 5.6 & 39.4 \end{pmatrix}$$

where $n_2 = 17$. The degree of freedom is $25+17-2=40$.

ii The hypotheses are

$$H_0 : \Sigma_A = \Sigma_B \quad \text{v.s.} \quad H_A : \Sigma_A \neq \Sigma_B$$

The statistic is

$$\begin{aligned} M &= (n_1 + n_2 - 2) \log |\mathbf{S}_{pool}| - (n_1 - 1) \log |\mathbf{S}| - (n_2 - 1) \log |\mathbf{W}| = 1.1948 \\ C^{-1} &= 1 - \frac{2 \times 3^2 + 3 \times 3 - 1}{6 \times (3 + 1) \times (2 - 1)} \left\{ \frac{1}{n_1 - 1} + \frac{1}{n_2 - 1} - \frac{1}{n_1 + n_2 - 2} \right\} = 0.9142 \\ MC^{-1} &= 1.1948 \times 0.9142 = 1.0923 \sim \chi_6^2 \end{aligned}$$

Because $MC^{-1} = 0.9142 < \chi_{0.95,6}^2 = 12.592$ and P-value is 0.9819, we cannot reject the null hypothesis, which means that the assumption of same population covariance are reliable.

iii The squared Mahalanobis distance between $\mathbf{y} - \mathbf{z}$ is

$$T^2 = (\mathbf{y} - \mathbf{z})' \left[\mathbf{S}_{pool} \left(\frac{1}{n_1} + \frac{1}{n_2} \right) \right]^{-1} (\mathbf{y} - \mathbf{z}) = 18.03155$$

iv The hypotheses are

$$H_0 : \mathbf{y} - \mathbf{z} = \mathbf{0} \quad \text{v.s.} \quad H_A : \mathbf{y} - \mathbf{z} \neq \mathbf{0}$$

The statistic can be computed from the Mahalanobis distance from part (iii)

$$F = \frac{n_1 + n_2 - 3 - 1}{(n_1 + n_2 - 2) \times 3} T^2 = 5.70999 \sim f_{3,38}$$

Because $F = 5.70999 > f_{0.95,3,38} = 2.851$ with P-value=0.0025. We will reject the null hypothesis, so drug A and drug B do not have equal means.

v To test the parallel profiles, the hypotheses are

$$H_0 : \mu_{11} - \mu_{21} = \mu_{12} - \mu_{22} = \mu_{13} - \mu_{23} \quad \text{v.s.} \quad H_A : \text{at least two difference not equal}$$

or we can rewrite the null hypothesis as

$$H_0 : \mathbf{C}_2(\mathbf{y} - \mathbf{z}) = \mathbf{0}$$

$$\text{where } \mathbf{C}_2 = \begin{pmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \end{pmatrix}.$$

The statistics are

$$T^2 = \frac{n_1 n_2}{n_1 + n_2} (C(\bar{\mathbf{y}} - \bar{\mathbf{z}}))' (C S C')^{-1} (C(\bar{\mathbf{y}} - \bar{\mathbf{z}})) = 14.2658$$

$$F = \frac{n_1 + n_2 - c - 1}{(n_1 + n_2 - 2)c} T^2 = 6.7849 \sim f_{c, n_1 + n_2 - c - 1}$$

Because $F = 6.7849 > f_{0.95, 2, 39} = 3.238$ with P-value=0.0030. We will reject the null hypothesis, so there is a significant interaction between drug and time.

Appendices

A SAS Code for Problem 3

```
libname da2 'C:\Users\psy6b\Desktop\8320 datasets';
ods graphics on;
options ls=70 ps=35;

/*To reading the data*/
data da2.h5q31;
  infile 'C:\Users\psy6b\Desktop\8320 datasets\ssttornado532001.dat';
  retain ss1-ss49;
  array ss{49} ss1-ss49;
  if _N_=1 then do;
    input ss1-ss49;
  end;
  loc+1;
  drop ss1-ss49;
  do t=1 to 49;
    sst=ss{t};
    input torn @;
    output;
  end;
run;
data da2.h5q32;
  infile 'C:\Users\psy6b\Desktop\8320 datasets\M0tornlatlon.dat';
```

```
loc+1;
input lat lon;
run;
proc sql;
  create table da2.h5q3
  as select * from da2.h5q31 as a, da2.h5q32 as b
  where a.loc=b.loc;
run;
quit;

/*Fitting different models*/
proc genmod data=da2.h5q3;
  class loc;
  model torn = sst*loc / dist=poisson link=log;
  output out=h5q3out1 resraw=Residual pred=Predicted lower=Lower
         upper=Upper;
run;
proc glimmix data=da2.h5q3 noitprint;
  class loc;
  model torn = sst sst*loc / dist=poisson link=log ddfm=betwithin
         solution;
  random intercept / subject=loc type=sp(exp)(lon lat);
  nloptions tech=newrap;
  covtest 'Random Int.' indep;
  output out=h5q3out2 pred(ilink)=predicted lcl(ilink)=lower
         ucl(ilink)=upper residual(ilink)=Residual;
run;
proc glimmix data=da2.h5q3 noitprint;
  class loc;
  model torn = sst sst*loc / dist=poisson link=log ddfm=betwithin
         solution;
  random sst / subject=loc type=sp(exp)(lon lat);
  nloptions tech=newrap;
  covtest 'Random Coef.' indep;
  output out=h5q3out3 pred(ilink)=predicted lcl(ilink)=lower
         ucl(ilink)=upper residual(ilink)=Residual;
run;
proc glimmix data=da2.h5q3 noitprint;
```

```
class loc;
model torn = sst sst*loc / dist=poisson link=log ddfm=betwithin
    solution;
random intercept sst / subject=loc type=sp(exp)(lon lat);
nloptions tech=newrap;
covtest 'Random Int. & Coef.' indep;
output out=h5q3out4 pred(ilink)=predicted lcl(ilink)=lower
    ucl(ilink)=upper residual(ilink)=Residual;
run;

/*Processing output*/
proc sort data=h5q3out1;
    by loc;
run;
data h5q3eval1;
    set h5q3out1;
    by loc;
    keep loc torn predicted residual lat lon;
    retain sumtorn sumpred sumres;
    if first.loc then do;
        sumtorn=0;
        sumpred=0;
        sumres=0;
    end;
    sumtorn+torn;
    sumpred+predicted;
    sumres+residual;
    if last.loc then do;
        torn=sumtorn;
        predicted=sumpred;
        residual=sumres;
        output;
    end;
run;
proc sort data=h5q3out2;
    by loc;
run;
```

```
data h5q3eval2;
  set h5q3out2;
  by loc;
  keep loc torn predicted residual lat lon;
  retain sumtorn sumpred sumres;
  if first.loc then do;
    sumtorn=0;
    sumpred=0;
    sumres=0;
  end;
  sumtorn+torn;
  sumpred+predicted;
  sumres+residual;
  if last.loc then do;
    torn=sumtorn;
    predicted=sumpred;
    residual=sumres;
    output;
  end;
run;
proc sort data=h5q3out3;
  by loc;
run;
data h5q3eval3;
  set h5q3out3;
  by loc;
  keep loc torn predicted residual lat lon;
  retain sumtorn sumpred sumres;
  if first.loc then do;
    sumtorn=0;
    sumpred=0;
    sumres=0;
  end;
  sumtorn+torn;
  sumpred+predicted;
  sumres+residual;
  if last.loc then do;
    torn=sumtorn;
```

```
        predicted=sumpred;
        residual=sumres;
        output;
    end;
run;
proc sort data=h5q3out4;
    by loc;
run;
data h5q3eval4;
    set h5q3out4;
    by loc;
    keep loc torn predicted residual lat lon;
    retain sumtorn sumpred sumres;
    if first.loc then do;
        sumtorn=0;
        sumpred=0;
        sumres=0;
    end;
    sumtorn+torn;
    sumpred+predicted;
    sumres+residual;
    if last.loc then do;
        torn=sumtorn;
        predicted=sumpred;
        residual=sumres;
        output;
    end;
run;
data h5q3eval;
    set h5q3eval1(in=a) h5q3eval2(in=b) h5q3eval3(in=c)
        h5q3eval4(in=d);
    length model $23;
    if a then do;
        model='Independent';
    end;
    if b then do;
        model='Random Int.';
    end;
end;
```

```
    if c then do;
        model='Random Coef.';
    end;
    if d then do;
        model='Random Int. & Coef.';
    end;
    label torn='Actual Measurements';
run;

/*Evaluating models*/
proc sort data=h5q3eval;
    by torn;
run;
proc sgpanel data=h5q3eval noautolegend;
    panelby model/columns=2 rows=2 spacing=5;
    scatter x=torn y=predicted/ datalabel=loc;
    series x=torn y=torn;
    keyword "Observations" "Reference Line";
run;
proc sql;
    title 'Model Comparison';
    select model,sum(residual*residual) label='Model Type' as SSR
        label='Sum of Squared Residual'
    from h5q3eval
    group by model;
quit;

/*Plotting the profile*/
data panelplot2;
    set h5q3out2;
    length type $20;
    keep loc t type resp;
    t=t+1952;
    type='measurement';
    resp=torn;
    output;
```

```
type='cluster-specific';
resp=predicted;
output;
type='lower bound';
resp=lower;
output;
type='upper bound';
resp=upper;
output;
run;
proc sgpanel data=panelplot2;
  where loc le 4 and loc ge 1;
  panelby loc/rows=2 columns=2 spacing=5;
  vline t/response=resp group=type;
  colaxis fitpolicy=thin alternate;
  rowaxis alternate;
run;
proc sgpanel data=panelplot2;
  where loc le 8 and loc ge 5;
  panelby loc/rows=2 columns=2 spacing=5;
  vline t/response=resp group=type;
  colaxis fitpolicy=thin alternate;
  rowaxis alternate;
run;
proc sgpanel data=panelplot2;
  where loc le 12 and loc ge 9;
  panelby loc/rows=2 columns=2 spacing=5;
  vline t/response=resp group=type;
  colaxis fitpolicy=thin alternate;
  rowaxis alternate;
run;
proc sgpanel data=panelplot2;
  where loc le 16 and loc ge 13;
  panelby loc/rows=2 columns=2 spacing=5;
  vline t/response=resp group=type;
  colaxis fitpolicy=thin alternate;
  rowaxis alternate;
run;
```



```

proc sgpanel data=panelplot2;
  where loc le 20 and loc ge 17;
  panelby loc/rows=2 columns=2 spacing=5;
  vline t/response=resp group=type;
  colaxis fitpolicy=thin alternate;
  rowaxis alternate;
run;

```

B Some Outputs for Promblem 3

Figure 7: Goodness of Fit of Independent Model

Criteria For Assessing Goodness Of Fit			
Criterion	DF	Value	Value/DF
Deviance	959	530.4118	0.5531
Scaled Deviance	959	530.4118	0.5531
Pearson Chi-Square	959	1252.0239	1.3056
Scaled Pearson X2	959	1252.0239	1.3056
Log Likelihood		-346.7726	
Full Log Likelihood		-365.5408	
AIC (smaller is better)		773.0817	
AICC (smaller is better)		774.0462	
BIC (smaller is better)		875.7203	

Figure 8: Results of Random Intercept Model

Fit Statistics	
-2 Res Log Pseudo-Likelihood	5209.11
Generalized Chi-Square	1158.20
Gener. Chi-Square / DF	1.21

Figure 9: Results of Random Intercept Model

Covariance Parameter Estimates			
Cov Parm	Subject	Estimate	Standard Error
Variance	loc	0.1122	0.09420
SP(EXP)	loc	1.0000	.

Figure 10: Results of Random Intercept Model

Type III Tests of Fixed Effects				
Effect	Num DF	Den DF	F Value	Pr > F
sst	1	940	0.19	0.6607
sst*loc	19	940	2.04	0.0053

Figure 11: Results of Random Intercept Model

Tests of Covariance Parameters Based on the Residual Pseudo-Likelihood						
Label	DF	-2 Res Log P-Like	ChiSq	Pr > ChiSq	Note	
Random Int.	2	5212.01	2.89	0.2352	--	
--: Standard test with unadjusted p-values.						

Figure 12: Results of Random Coefficient Model

Estimated G matrix is not positive definite.	
Fit Statistics	
-2 Res Log Pseudo-Likelihood	5271.30
Generalized Chi-Square	1252.02
Gener. Chi-Square / DF	1.31

Figure 13: Results of Random Coefficient Model

Covariance Parameter Estimates			
			Standard
Cov Parm	Subject	Estimate	Error
Variance	loc	0	.
SP(EXP)	loc	1.0000	.

Figure 14: Results of Random Coefficient Model

Type III Tests of Fixed Effects				
Effect	Num DF	Den DF	F Value	Pr > F
sst	1	940	0.31	0.5781
sst*loc	19	940	2.16	0.0028

Figure 15: Results of Random Coefficient Model

Tests of Covariance Parameters Based on the Residual Pseudo-Likelihood						
Label	DF	-2 Res Log P-Like	ChiSq	Pr > ChiSq	Note	
Random Coef.	2	2854.49	.	1.0000	--	
--: Standard test with unadjusted p-values.						

Figure 16: Results of Random Int. and Coef. Model

Estimated G matrix is not positive definite.	
Fit Statistics	
-2 Res Log Pseudo-Likelihood	5209.11
Generalized Chi-Square	1158.20
Gener. Chi-Square / DF	1.21

Figure 17: Results of Random Int. and Coef, Model

Covariance Parameter Estimates			
Cov Parm	Subject	Estimate	Standard Error
Variance	loc	0.1122	0.09420
SP(EXP)	loc	1.0000	.

Figure 18: Results of Random Int. and Coef. Model

Type III Tests of Fixed Effects				
Effect	Num DF	Den DF	F Value	Pr > F
sst	1	940	0.15	0.6975
sst*loc	19	940	1.97	0.0077

Figure 19: Results of Random Int. and Coef. Model

Tests of Covariance Parameters Based on the Residual Pseudo-Likelihood					
Label	DF	-2 Res Log P-Like	ChiSq	Pr > ChiSq	
Random Int. & Coef.	2	2854.49	.	1.0000	
Tests of Covariance Parameters Based on the Residual Pseudo-Likelihood					
Label	Note				
Random Int. & Coef.	--				
--: Standard test with unadjusted p-values.					

Figure 20: Goodness of Fit of Independent Model

Model Comparison	
model	Sum of Squared Residual
-----	-----
Independent	146.3692
Random Coef.	146.3692
Random Int.	56.18219
Random Int. & Coef.	56.18219

Figure 21: Scatterplot: Predication v.s. Measurements

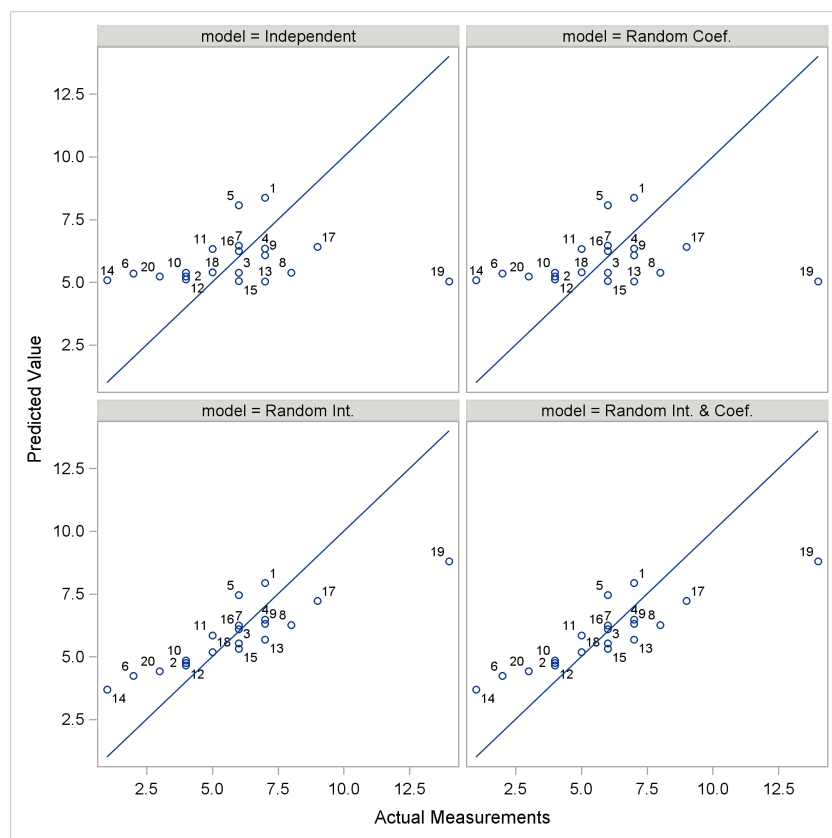


Figure 22: Plots of Prediction For Random Intercept Model

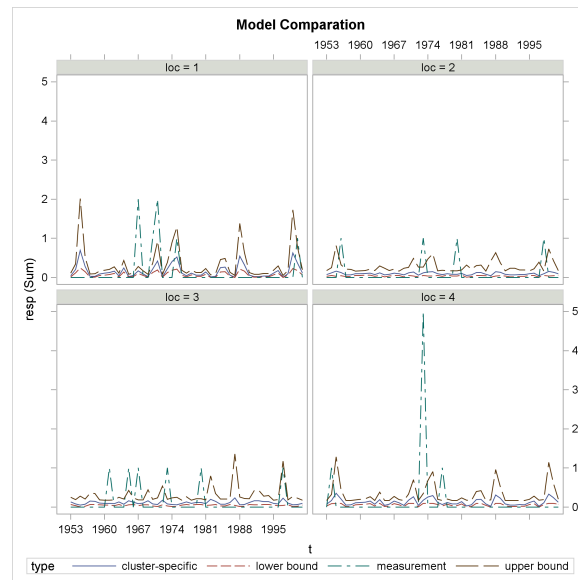


Figure 23: Plots of Prediction For Random Intercept Model

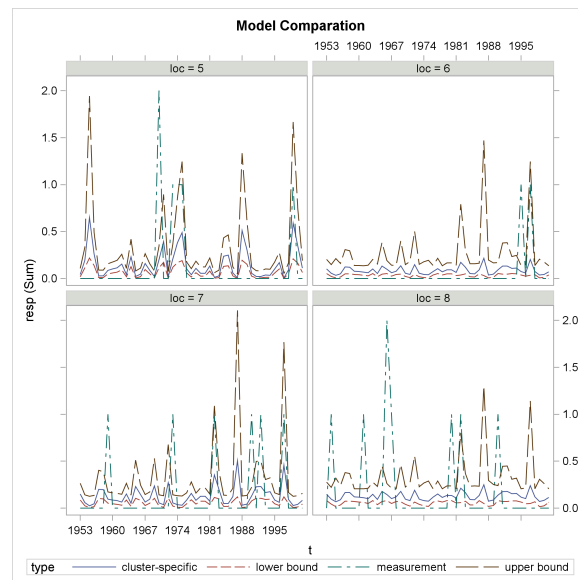


Figure 24: Plots of Prediction For Random Intercept Model

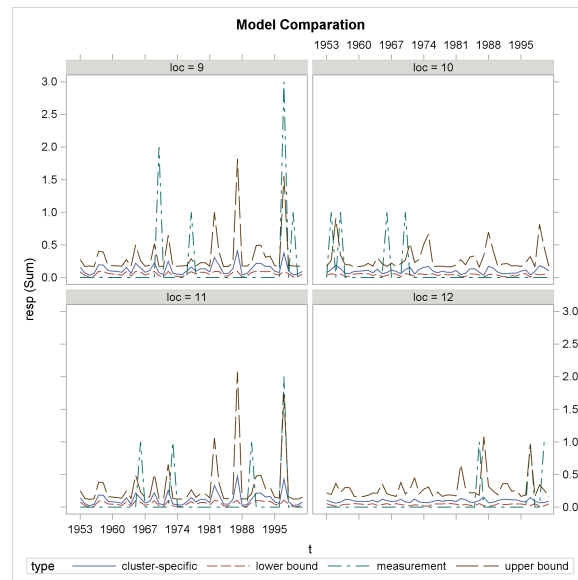


Figure 25: Plots of Prediction For Random Intercept Model

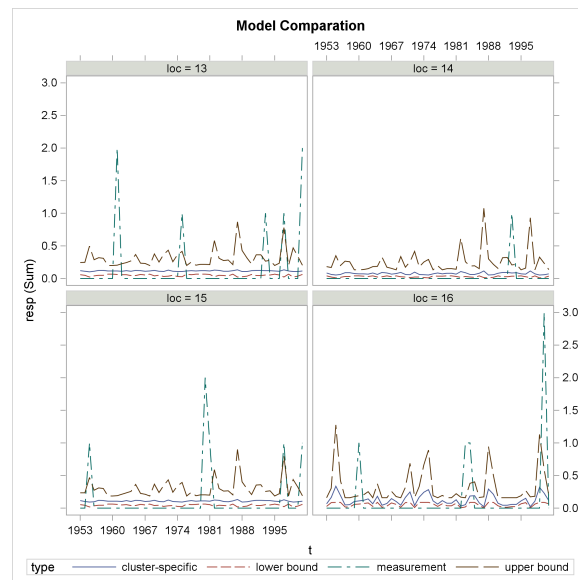


Figure 26: Plots of Prediction For Random Intercept Model

