

# STAT 8320 Spring 2015 Assignment 6

Peng Shao 14221765

April 16, 2015

► **1. Solution.** (a). We can know that  $n_1 = 16$ ,  $n_2 = 11$  and  $p = 4$ . Then to test the hypothesis

$$H_0 : \boldsymbol{\mu}_1 - \boldsymbol{\mu}_2 = \mathbf{0} \quad \text{v.s.} \quad H_A : \boldsymbol{\mu}_1 - \boldsymbol{\mu}_2 \neq \mathbf{0}$$

The statistic is

$$F = \frac{n_1 + n_2 - p - 1}{(n_1 + n_2 - 2) \cdot p} T^2 = \frac{16 + 11 - 4 - 1}{(16 + 11 - 2) \cdot 4} \cdot 16.5 = 3.63 \sim f_{4,22}$$

Because  $F = 3.63 > f_{0.95,4,22} = 2.816708$  with P-value=0.0237. We will reject the null hypothesis, so the mean profiles are significantly different.

(b). To test the parallel profiles, the hypotheses are

$$H_0 : \mathbf{C}(\mathbf{y} - \mathbf{z}) = \mathbf{0} \quad \text{v.s.} \quad H_A : \mathbf{C}(\mathbf{y} - \mathbf{z}) \neq \mathbf{0}$$

where  $\mathbf{C} = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$ .

The statistics are

$$F = \frac{n_1 + n_2 - c - 1}{(n_1 + n_2 - 2)c} T^2 = 2.699 \sim f_{3,23}$$

Because  $F = 2.699 < f_{0.95,3,23} = 3.027998$  with P-value=0.06934349. We cannot reject the null hypothesis, so there is not significant interaction effect, i.e., the mean profiles are parallel.

(c). Since the mean profiles are parallel, then we want to test whether they are identical or not. Then the  $\mathbf{C}$  matrix becomes  $\mathbf{C} = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}$ . This test should give us the same result of part (a).

We also want to test whether there is a significant time effect, i.e. each profile is horizontal respectively. Then the hypotheses are

$$H_0 : \mathbf{C}\boldsymbol{\mu}_1 = 0 \quad \text{v.s.} \quad H_A : \mathbf{C}\boldsymbol{\mu}_1 \neq 0$$

where  $\mathbf{C} = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \end{pmatrix}$ .

► **2. Solution.** (a). Mauchly's test examines the form of the common covariance matrix. A spherical matrix has equal variances and covariances equal to zero. The common covariance matrix of the transformed within-subject variables must be spherical. So the purpose of Mauchly test is to help us decide which output to use. If we can use the univariate output, we may have more power to reject the null hypothesis in favor of the alternative hypothesis. However, the univariate approach is appropriate only when the sphericity assumption is not violated. If the sphericity assumption is violated, then in most situations you are better off staying with the multivariate output.

From the SAS output (Figure 1), the second line shows that the Mauchly's criterion is 0.7343,  $\chi^2$  value is 7.29 with degree of freedom 5, and the P-value is 0.1997 > 0.05. Thus, we cannot reject the null hypothesis of sphericity, that is, the sphericity assumption has not been violated. Then we should use univariate approach to make inferences, because it is more powerful to reject the null hypothesis.

Figure 1: Mauchly's Sphericity Test

Sphericity Tests				
Variables	DF	Mauchly's Criterion	Chi-Square	Pr > ChiSq
Transformed Variates	5	0.7353334	7.2929515	0.1997
Orthogonal Components	5	0.7353334	7.2929515	0.1997

(b).

(i). To test the parallel profiles, the null hypothesis is

$$H_0 : \mu_{11} - \mu_{21} = \mu_{12} - \mu_{22} = \mu_{13} - \mu_{23} = \mu_{14} - \mu_{24}$$

In Figure 2, the Wilks' Lambda value is 0.7399, the F value is 2.70, the degree of freedom of numerator is 3, the degree of freedom of denominator is 23, and the P-value is  $0.0696 > 0.05$ . So we cannot reject the null hypothesis, that is, the mean profiles are parallel, or we can say that there is no significant interaction effect.

Figure 2: Parallel Profiles Test

MANOVA Test Criteria and Exact F Statistics for the Hypothesis of no AGE*Gender Effect H = Type III SSCP Matrix for AGE*Gender E = Error SSCP Matrix						
	S=1	M=0.5	N=10.5			
Statistic	Value	F Value	Num DF	Den DF	Pr > F	
Wilks' Lambda	0.73988739	2.70	3	23	0.0696	
Pillai's Trace	0.26011261	2.70	3	23	0.0696	
Hotelling-Lawley Trace	0.35155702	2.70	3	23	0.0696	
Roy's Greatest Root	0.35155702	2.70	3	23	0.0696	

(ii). To test the parallel profiles, the null hypothesis is

$$H_0 : \mu_{11} = \mu_{12} = \mu_{13} = \mu_{14}$$

In Figure 3, the Wilks' Lambda value is 0.1948, the F value is 31.69, the degree of freedom of numerator is 3, the degree of freedom of denominator is 23, and the P-value is less than  $0.0001 < 0.05$ . So we will reject the null hypothesis, that is, the mean profiles are not horizontal, or we can say that there is a significant age effect.

Figure 3: Horizontal Profiles Test

MANOVA Test Criteria and Exact F Statistics for the Hypothesis of no AGE Effect H = Type III SSCP Matrix for AGE E = Error SSCP Matrix						
S=1      M=0.5      N=10.5						
Statistic	Value	F Value	Num DF	Den DF	Pr > F	
Wilks' Lambda	0.19479424	31.69	3	23	<.0001	
Pillai's Trace	0.80520576	31.69	3	23	<.0001	
Hotelling-Lawley Trace	4.13362211	31.69	3	23	<.0001	
Roy's Greatest Root	4.13362211	31.69	3	23	<.0001	

(iii). To test the parallel profiles, the null hypothesis is

$$H_0 : \boldsymbol{\mu}_1 - \boldsymbol{\mu}_2 = \mathbf{0}$$

In Figure 4, there is only one approach for this test. The F value is 9.29, the degree of freedom of numerator is 1, the degree of freedom of denominator is 25, and the P-value is  $0.0054 < 0.05$ . So we will reject the null hypothesis, that is, the mean profiles are not identical, or we can say that there is a significant gender effect.

Figure 4: Coincident Profiles Test

The GLM Procedure				
Repeated Measures Analysis of Variance				
Tests of Hypotheses for Between Subjects Effects				
Source	DF	Type III SS	Mean Square	F Value
Gender	1	140.4648569	140.4648569	9.29
Error	25	377.9147727	15.1165909	
Source		Pr > F		
Gender		0.0054		
Error				

(c). From the SAS output (Figure 5), we can see the F value of interaction is 2.36 with P-value=0.0797>0.05. So we cannot reject the null hypothesis of no interaction. Because of no interaction, it is appropriate to test the main effect. The F value of age is 35.35 with P-value less than 0.0001. So we can conclude that there is a significant effect of age.

Figure 5: Univariate Approach

The GLM Procedure				
Repeated Measures Analysis of Variance				
Univariate Tests of Hypotheses for Within Subject Effects				
Source	DF	Type III SS	Mean Square	F Value
AGE	3	209.4369739	69.8123246	35.35
AGE*Gender	3	13.9925295	4.6641765	2.36
Error(AGE)	75	148.1278409	1.9750379	
Adj Pr > F				
Source	Pr > F	G - G	H-F-L	
AGE	<.0001	<.0001	<.0001	
AGE*Gender	0.0781	0.0878	0.0797	
Error(AGE)				

(d). From the SAS output (Figure 6), the Greenhouse-Geisser adjustment is 0.8672, and the Huynh-Feldt adjustment is 0.9769.

Figure 6: Univariate Approach Adjustment

Greenhouse-Geisser Epsilon	0.8672
Huynh-Feldt-Lecoutre Epsilon	0.9769

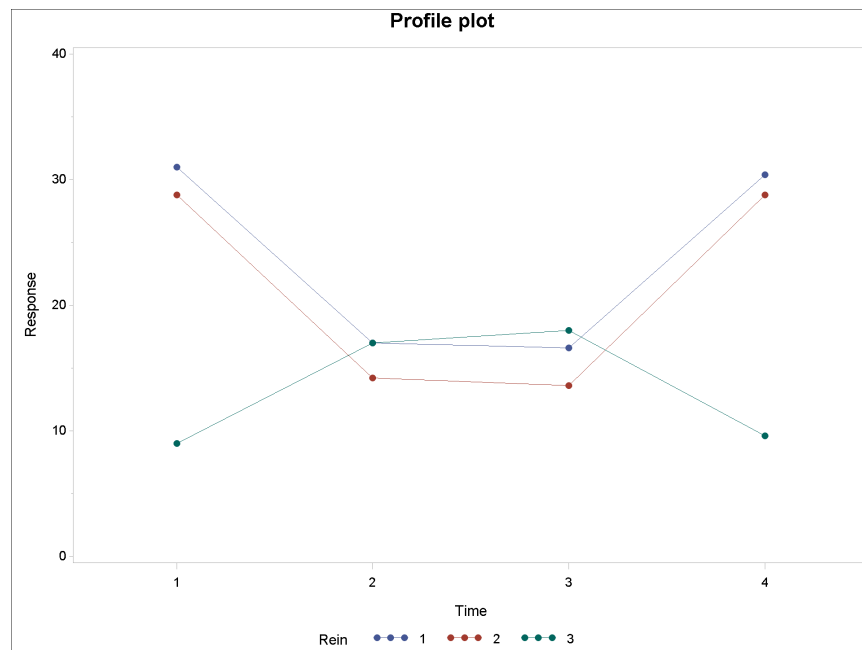
(e). From the final SAS output of PROC MIXED (Figure 7), we have the exactly same result that there is no significant interaction, but the gender effect and age effect are both significant as univariate approach shows, especially the P-values of these three effects. This also support the result of Mauchly's test that the sphericity assumption is valid, which ensures the univariate approach is appropriate.

Figure 7: Result of PROC MIXED

Type 3 Tests of Fixed Effects				
Effect	Num DF	Den DF	F Value	Pr > F
Gender	1	25	9.29	0.0054
age	3	75	35.35	<.0001
Gender*age	3	75	2.36	0.0781

► **3. Solution.** (a). From the plot (Figure 8), firstly we can see that there are some intersections among the profiles, which means that there may be some significant interaction between reinforcement schedules and conditions. Then we can see that the profile of schedule 1 is parallel with the profile of schedule 2, and the difference is so small that they may behave in same way. Finally, the behavior of schedule 3 seems totally different from the other two's.

Figure 8: Plots of Mean Profiles



(b). From the SAS output (Figure 9), P-value of the test is  $0.0119 < 0.05$ , so we can say that there is a significant schedule effect. The next step we should do is to find where the differences lie, so we will follow up with several post-hoc tests in part (c).

Figure 9: Overall Reinforcement Schedule Effect

MANOVA Test Criteria and F Approximations for the Hypothesis of No Overall Rein Effect H = Type III SSCP Matrix for Rein E = Error SSCP Matrix						
S=2      M=0.5      N=3.5						
Statistic	Value	F Value	Num DF	Den DF	Pr > F	
Wilks' Lambda	0.14954207	3.57	8	18	0.0119	
Pillai's Trace	0.99249428	2.46	8	20	0.0488	
Hotelling-Lawley Trace	4.73727283	5.09	8	10.78	0.0081	
Roy's Greatest Root	4.52748560	11.32	4	10	0.0010	
NOTE: F Statistic for Roy's Greatest Root is an upper bound.						
NOTE: F Statistic for Wilks' Lambda is exact.						

(b). We will begin with the multivariate test of schedule 1 versus the average of schedule 2. The null hypothesis is

$$H_0 : \mathbf{LBM} = \mathbf{0}$$

where  $\mathbf{L} = \begin{pmatrix} 1 & -1 & 0 \end{pmatrix}$ , and

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \end{pmatrix}$$

. From the SAS output (Figure 10), P-value of the test is  $0.9979 > 0.05$ , so we cannot reject  $H_0$  that schedule 1 and schedule 2 behave the same.



Figure 10: Overall Reinforcement Schedule Effect

MANOVA Test Criteria and Exact F Statistics for the Hypothesis of No Overall Compare Schedule 1 v.s. Schedule 2 Effect on the Variables Defined by the M Matrix Transformation H = Contrast SSCP Matrix for Compare Schedule 1 v.s. Schedule 2 E = Error SSCP Matrix						
S=1      M=0.5      N=4						
Statistic	Value	F Value	Num DF	Den DF	Pr > F	
Wilks' Lambda	0.99619584	0.01	3	10	0.9979	
Pillai's Trace	0.00380416	0.01	3	10	0.9979	
Hotelling-Lawley Trace	0.00381869	0.01	3	10	0.9979	
Roy's Greatest Root	0.00381869	0.01	3	10	0.9979	

We will begin with the multivariate test of schedule 3 versus the average of schedule 1 and 2. The null hypothesis is

$$H_0 : \mathbf{LBM} = \mathbf{0}$$

where  $\mathbf{L} = (-1 -1 +2)$ , and  $\mathbf{M}$  is same as above.

From the SAS output (Figure 11), P-value of the test is  $0.0030 < 0.05$ , so we will reject  $H_0$  that schedule 3 behaves like the other 2 schedules under different conditions.

Figure 11: Overall Reinforcement Schedule Effect

MANOVA Test Criteria and Exact F Statistics for the Hypothesis of No Overall Compare Schedule 1 & 2 v.s. Schedule 3 Effect on the Variables Defined by the M Matrix Transformation H = Contrast SSCP Matrix for Compare Schedule 1 & 2 v.s. Schedule 3 E = Error SSCP Matrix						
	S=1	M=0.5	N=4			
Statistic	Value	F Value	Num DF	Den DF	Pr > F	
Wilks' Lambda	0.26293170	9.34	3	10	0.0030	
Pillai's Trace	0.73706830	9.34	3	10	0.0030	
Hotelling-Lawley Trace	2.80326909	9.34	3	10	0.0030	
Roy's Greatest Root	2.80326909	9.34	3	10	0.0030	