STAT 8330 FALL 2015 ASSIGNMENT 1

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September 6, 2015

▶ **Exercises 2.5. Solution.**

(1).

- advantage: can fit many different functional forms; low bias; usually predict more accurately
- disadvantage: overfitting problem; sually hard to interpret; high variance

(2). If our goal is to predict more accurately, it will usually be best to choose a more flexible approach.

(3). If our goal is to make some inferences, we prefer choosing a less flexible approach because the relation between response and predictor is more explicit.

▶ **Exercises 2.6. Solution.**

(1). The essential difference between parametric and non-parametric approach is that, the parametric make an assumption of the form of f, which can reduce problem of estimating f down to one of estimating a set of parameter, but non-parametric do not make explicit assumptions about the functional form of f.

(2).

- advantage: it is easier to estimate parameter; the relation between response and predictor is more explicit;
- disadvantage: the model we choose will usually not match the true unknown form of f; sometimes need more assumption.

▶ **Exercises 2.10. Solution.**

▶ **Exercises 3.5. Solution.**

▶ **Exercises 3.15. Solution.**

▶ **Exercises 4.3. Solution.**

We know that we classify X into kth class based on Bayes' classifier if

$$p_k(x) = \frac{f_k(x)\pi_k}{\sum_{l=1}^{K} f_l(x)\pi_l}$$

is largest among all $p_l(x)$, $l = 1, 2, ..., K$. For 1 dimension, the density of x from kth class is

$$f_k(x) = \frac{1}{\sqrt{2\pi}\sigma_k} e^{-\frac{(x-\mu_k)^2}{2\sigma_k^2}}$$

In comparing two classes k and l, it is sufficient to look at the log-ratio, and we see that

$$\begin{aligned}
\log\left(\frac{p_k(x)}{p_l(x)}\right) &= \log\left(\frac{\pi_k}{\pi_l}\right) + \log\left(\frac{f_k(x)}{f_l(x)}\right) \\
&= \log\left(\frac{\pi_k}{\pi_l}\right) + \log\left(\frac{\sigma_l}{\sigma_k}\right) - \frac{(x-\mu_k)^2}{2\sigma_k^2} + \frac{(x-\mu_l)^2}{2\sigma_l^2} \\
&= \left(-\frac{(x-\mu_k)^2}{2\sigma_k^2} - \log\sigma_k + \log\pi_k\right) - \left(-\frac{(x-\mu_l)^2}{2\sigma_l^2} - \log\sigma_l + \log\pi_l\right) \\
&= \delta_k(x) - \delta_l(x)
\end{aligned}$$

Then the Beyes' classifier can be be defined as

$$C(x) = \arg\max_k \delta_k(x)$$

where $\delta_k(x) = -\frac{(x-\mu_k)^2}{2\sigma_k^2} - \log \sigma_k + \log \pi_k$.

It is obvious that the decision boundary between each pair of classes k and l is described by a quadratic equation $\{x : \delta_k(x) = \delta_l(x)\}$.

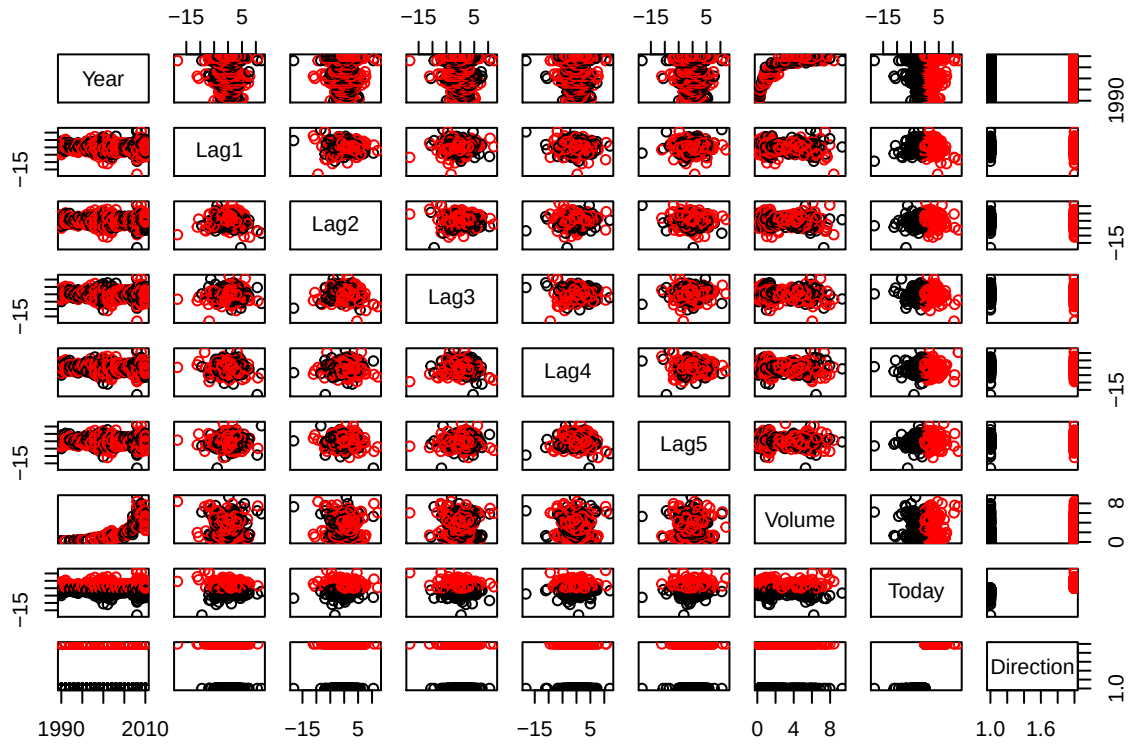
► **Exercises 4.10. Solution.**

(a). From the output, we can see that (1) the variable Volume is increased as the Year increased, and the increase rates become larger and larger; (2) the variable Today is highly, but not complete, correlated with the indicator variable Direction, so we may guess that Direction is transformed from Today. Except this two pairs, no other pairs show any obvious patterns.

```
cor(Weekly[, -9])
```

```
##                Year         Lag1        Lag2        Lag3         Lag4
## Year    1.00000000 -0.032289274 -0.03339001 -0.03000649 -0.031127923
## Lag1   -0.03228927  1.000000000 -0.07485305  0.05863568 -0.071273876
## Lag2   -0.03339001 -0.074853051  1.00000000 -0.07572091  0.058381535
## Lag3   -0.03000649  0.058635682 -0.07572091  1.00000000 -0.075395865
## Lag4   -0.03112792 -0.071273876  0.05838153 -0.07539587  1.000000000
## Lag5   -0.03051910 -0.008183096 -0.07249948  0.06065717 -0.075675027
## Volume  0.84194162 -0.064951313 -0.08551314 -0.06928771 -0.061074617
## Today  -0.03245989 -0.075031842  0.05916672 -0.07124364 -0.007825873
##                Lag5      Volume        Today
## Year   -0.030519101  0.84194162 -0.032459894
## Lag1   -0.008183096 -0.06495131 -0.075031842
## Lag2   -0.072499482 -0.08551314  0.059166717
## Lag3    0.060657175 -0.06928771 -0.071243639
## Lag4   -0.075675027 -0.06107462 -0.007825873
## Lag5    1.000000000 -0.05851741  0.011012698
## Volume -0.058517414  1.00000000 -0.033077783
## Today   0.011012698 -0.03307778  1.000000000
```

```
pairs(Weekly, col = Direction)
```



(b). Fitting the model as below, the summary result shows that only intercept and coefficient of Lag2 is significant.

```
logit.fit <- glm(Direction ~ Lag1 + Lag2 + Lag3 + Lag4 + Lag5 + Volume,
                 family = binomial, data = Weekly)
summary(logit.fit)
```

```
##
## Call:
## glm(formula = Direction ~ Lag1 + Lag2 + Lag3 + Lag4 + Lag5 +
##     Volume, family = binomial, data = Weekly)
##
## Deviance Residuals:
##     Min       1Q   Median       3Q      Max
## -1.6949  -1.2565   0.9913   1.0849   1.4579
##
## Coefficients:
##             Estimate Std. Error z value Pr(>|z|)
## (Intercept)  0.26686    0.08593   3.106   0.0019 **
## Lag1        -0.04127    0.02641  -1.563   0.1181
## Lag2         0.05844    0.02686   2.175   0.0296 *
## Lag3        -0.01606    0.02666  -0.602   0.5469
## Lag4        -0.02779    0.02646  -1.050   0.2937
## Lag5        -0.01447    0.02638  -0.549   0.5833
## Volume      -0.02274    0.03690  -0.616   0.5377
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##     Null deviance: 1496.2  on 1088  degrees of freedom
```

```
## Residual deviance: 1486.4  on 1082  degrees of freedom
## AIC: 1500.4
##
## Number of Fisher Scoring iterations: 4
```

(c). Using threshold = 0.5,

```
glm.probs <- predict(logit.fit,type="response")
glm.pred <- rep("Down",nrow(Weekly))
glm.pred[glm.probs > 0.50]="Up"
ct <- table(glm.pred, Direction)
ct
```

```
##         Direction
## glm.pred Down  Up
##     Down   54  48
##     Up    430 557
```

```
ConfusionTable(ct)
```

```
## $Accuracy
## [1] 0.5610652
##
## $`True Positive Rate`
## [1] 0.9206612
##
## $`False Posistive Rate`
## [1] 0.8884298
##
## $Precision
## [1] 0.5643364
##
## $`Total Error Rate`
## [1] 0.4389348
```

► **Exercises 4.13. Solution.**

► **Appendices**

Code of function ConfusionTable()

```
ConfusionTable <- function(ct){
    accuracy <- (ct[1, 1] + ct[2, 2]) / (sum(ct))
    TPr <- ct[2, 2] / (ct[1, 2] + ct[2, 2])
    FPr <- ct[2, 1] / (ct[1, 1] + ct[2, 1])
    precision <- ct[2, 2] / (ct[2, 1] + ct[2, 2])
    error <- 1 - accuracy
    result <- list(accuracy, TPr, FPr, precision, error)
    names(result) <- c("Accuracy", "True Positive Rate",
                       "False Posistive Rate", "Precision",
                       "Total Error Rate")
    return(result)
}
```