STAT 8330 FALL 2015 ASSIGNMENT 4

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▶ **Exercises 7.1. Solution.**

(a). For all $x \leq \xi$, $(x-\xi)_+^2 = 0$, then

$$\begin{aligned} f(x) &= \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 \\ &= a_1 + b_1 x + c_1 x^2 + d_1 x^3 \end{aligned}$$

Thus,

$$\begin{aligned} a_1 &= \beta_0 \\ b_1 &= \beta_1 \\ c_1 &= \beta_2 \\ d_1 &= \beta_3 \end{aligned}$$

(b). For all $x > \xi$,

$$\begin{aligned} f(x) &= \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \beta_4 (x-\xi)^3 \\ &= (\beta_0 - \beta_4 \xi^3) + (\beta_1 + 3\xi^2 \beta_4) x + (\beta_2 - 3\beta_4 \xi) x^2 + (\beta_3 + \beta_4) x^3 \\ &= a_1 + b_1 x + c_1 x^2 + d_1 x^3 \end{aligned}$$

Thus,

$$\begin{aligned} a_2 &= \beta_0 - \beta_4 \xi^3 \\ b_2 &= \beta_1 + 3\beta_4 \xi^2 \\ c_2 &= \beta_2 - 3\beta_4 \xi \\ d_2 &= \beta_3 + \beta_4 \end{aligned}$$

(c). Since

$$\begin{aligned} f_1(\xi) &= \beta_0 + \beta_1 \xi + \beta_2 \xi^2 + \beta_3 \xi^3 \\ f_2(\xi) &= (\beta_0 - \beta_4 \xi^3) + (\beta_1 + 3\xi^2 \beta_4)\xi + (\beta_2 - 3\beta_4 \xi)\xi^2 + (\beta_3 + \beta_4)\xi^3 \\ &= \beta_0 + \beta_1 \xi + \beta_2 \xi^2 + \beta_3 \xi^3 \end{aligned}$$

Then, $f_1(x) = f_2(x)$ at $x = \xi$, i.e. f(x) is continuous at ξ.

(d). Since

$$\begin{aligned} f_1'(\xi) &= \beta_1 + 2\beta_2 \xi + 3\beta_3 \xi^2 \\ f_2'(\xi) &= \beta_1 + 3\xi^2 \beta_4 + 2(\beta_2 - 3\beta_4 \xi)\xi + 3(\beta_3 + \beta_4)\xi^2 \\ &= \beta_1 + 2\beta_2 \xi + 3\beta_3 \xi^2. \end{aligned}$$

Then, $f_1'(x) = f_2'(x)$ at $x = \xi$, i.e. f'(x) is continuous at ξ.

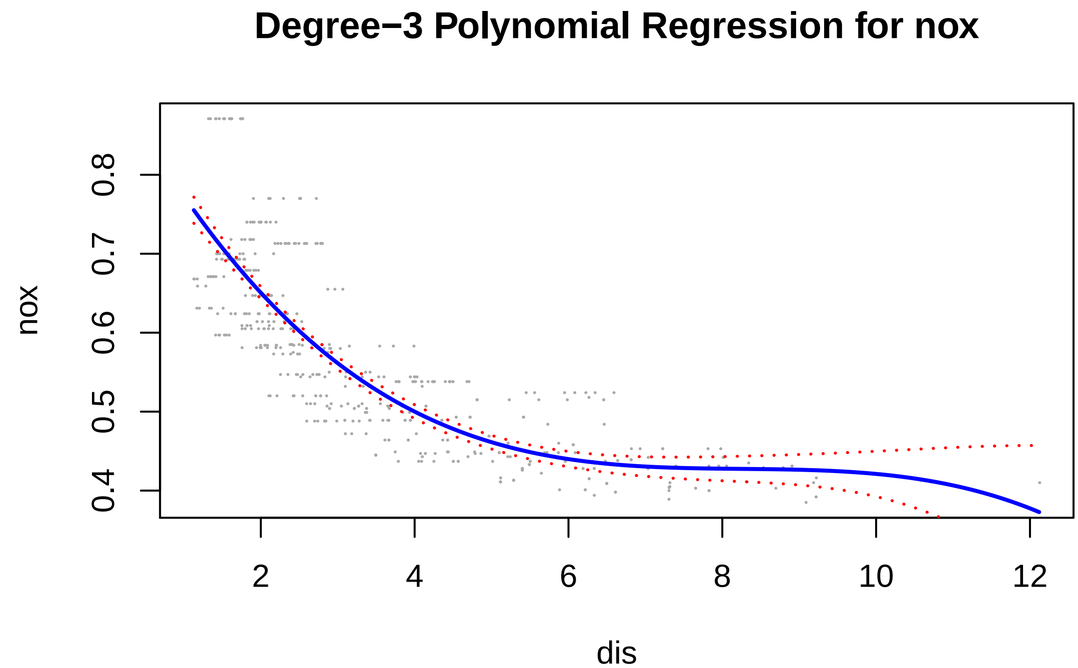
(e).

$$\begin{aligned} f_1''(\xi) &= 2\beta_2 + 6\beta_3 \xi \\ f_2''(\xi) &= 2(\beta_2 - 3\beta_4 \xi) + 6(\beta_3 + \beta_4)\xi \\ &= 2\beta_2 + 6\beta_3 \xi.. \end{aligned}$$

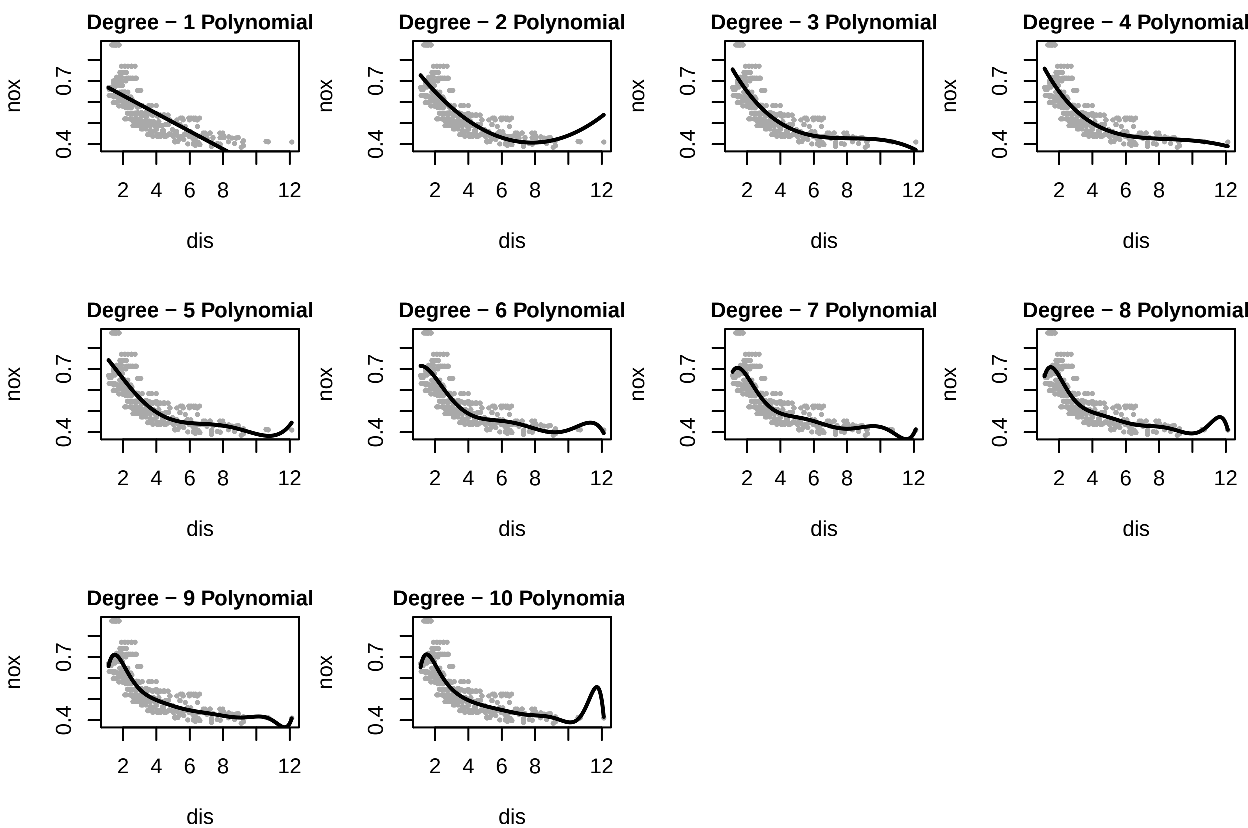
Then, $f_1''(x) = f_2''(x)$ at $x = \xi$, i.e. f"(x) is continuous at ξ.

▶ **Exercises 7.1. Solution.**

(a).



(b). From the plots we can see that all fitted curves are similar within the range of data, while high order curves seem much more wiggle when they come to the out of the bonudary of data. It is no surprising that the highest order polynomial has the lowest residual sum of square since we did not apply any smoothing or regularization method on this approximation.




```
## [1] "RSS for Degree - 1 Polynomial is 2.769"
## [1] "RSS for Degree - 2 Polynomial is 2.035"
```

```
## [1] "RSS for Degree - 3 Polynomial is 1.934"
## [1] "RSS for Degree - 4 Polynomial is 1.933"
## [1] "RSS for Degree - 5 Polynomial is 1.915"
## [1] "RSS for Degree - 6 Polynomial is 1.878"
## [1] "RSS for Degree - 7 Polynomial is 1.849"
## [1] "RSS for Degree - 8 Polynomial is 1.836"
## [1] "RSS for Degree - 9 Polynomial is 1.833"
## [1] "RSS for Degree - 10 Polynomial is 1.832"
```

(c). The 10-fold cross-validation error for each degree is

```
cv.error
```

```
##  Degree - 1  Degree - 2  Degree - 3  Degree - 4  Degree - 5  Degree - 6
## 0.009491648 0.030264200 0.009465320 0.023763427 0.025487441 0.004264861
##  Degree - 7  Degree - 8  Degree - 9 Degree - 10
## 0.010033157 0.013344953 0.006236745 0.004148996
```

So the best mode is the Degree - 10 Polynomial Regression with CV error 0.004149. It is a little that the Degree - 10 Polynomial Regression is still not overfitting based on cross-validation. Actually, this result highly depends on the seed of random number. The best model above is selected based on seed=1, but if we change the seed to 5, then the best model will be Degree - 5 Polynomial Regression. So to be careful, maybe we should consider the model based on one standard rule as a better model since it is usually more stable.