TASTY NUMBERS

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1. Abstract

This is the summary for my capstone focusing on the definition, generation, and patterns of tasty numbers. This document consists of the definition of tasty numbers, the mechanical methods for finding tasty numbers, and the patterns that I found for tasty numbers with associated proofs.

2. Definitions

- **Projection**. For a given natural number, the **projection** is defined as an ordered list of natural numbers in the form $x_0 \to ... \to x_a \to ... \to x_n$, where $x_0$is the original number, $x_n$is the digit root of x_0, and $x_a$is the digit sum of x_{a-1}. The function $p(n,r)$ returns the projection of a number n for a given base r.
- **Weight**. Weight is defined as the number of times that a digit appears in a given projection. The function $w(p,d)$ returns the weight of a digit where p is a projection and d is the digit in question.
- **Tasty**. A number is classified as **tasty** in a given base if all digits of that base have the same **weight**. In other words, a number n is tasty in base r if $w(p(n,r),0) = ... = w(p(n,r),r-1)$. From now on, the base of a tasty number will be assumed to be the base in which the number is tasty. Two common examples of tasty numbers are 100_2 and 1034568_{10}.
- **Order**. The order of a tasty number is a measure of how many tasty numbers appear in its projection. Order is represented by the function $\sigma(n,r)$ where n is a tasty number and r is a given base. If n has only one tasty number in its projection (such as 100_2) then$\sigma(n,r) = 0$. An example of a higher order tasty number is 11110000_2. For $p(11110000, 2)$, we get $11110000_2 \to 100_2 \to 1_2$. 11110000_2 is a tasty number, as is 100_2, so $\sigma(11110000, 2) = 1$.
- **Confer Set Notation (CSN)**. Tasty numbers are tested by addition. Since addition is commutative, the permutation of digits in x_0 are irrelevant (that is, 1034568_{10} and 1054368_{10} have the same projection after x_0 of $x_0 \to 27_{10} \to 9_{10}$). With this in mind, a notation was developed to leverage this and make larger tasty numbers more convenient to read. The notation is as follows: $[f(r-1),...,f(0)]_r$where $f(x)$ is the number of times a digit appeares in a number. Using this notation, 100_2 becomes $[1,2]_2$ and 1034568_{10} becomes $[0,1,0,1,1,1,1,0,1,1]_{10}$. If a base is not specified, the cardinality of the CSN serves as the base. While results in a loss of information, it provides all of the information that we need to work with tasty numbers. From now on, all tasty numbers (but not the contents of

their projections) will be written in this notation. Addition and subtraction in CSN is handled as such: $[a, b, c] + [d, e, f] = [a + d, b + e, c + f]$ $[a, b, c] - [d, e, f] = [a - d, b - e, c - f]$. The function $Z(n, r)$ returns a CSN of base r with a 1 in digit n and the rest of the digits being 0 (ie. $Z(1, 3) = [0, 1, 0]_3$, $Z(0, 4) = [0, 0, 0, 1]_4$).

3 Initial Proofs

Before sweeping patterns were seen in tasty numbers, the infinite cardinality of tasty numbers in a base were found in specific instances.

Theorem. *Infinite Tasties and Boundless Order in Base 2*

Proof. To prove that there are infinite tasties in base 2, we start with the first tasty of the base, $[1, 2]_2$, and show that a tasty number of infinite order can be generated from it.

We start by choosing the decimal value of one of the permutations. For this first tasty, there is only one option: $100_2 = 4_{10}$. Next, we start with a base 2 number of all 1's and a length of the decimal value we just chose; here that is 1111_2 or $[4, 0]_2$. Finally, we must take this value and make a tasty number. We know that the digit sum of $[4, 0]_2$ is a tasty number. From this, we know that the weight of 1 and 0, excluding x_0, are equal. It then follows that if we add an equal number of 0's to the 1's that we already have, then the result will be a tasty number. After doing this, we have $[4, 4]_2$, $p([4, 4]_2, 2) = [4, 4]_2 \to 100_2 \to 1_2$, and $w(p, 1) = w(p, 0) = 6$.

We now have a second tasty in base 2. If we repeat the for 10000111_2, we get $[135, 135]_2$. It becomes apparent that we can continue doing this indefinitely. Therefore, there are infinite tasty numbers in base 2. By using this method, we also have proven that there is no upper bound to the order of base 2 tasties. $\square$

4 The Weights and Bases of Tasties

The first major pattern of tasties and the cornerstone of all progress made after the initial proofs came from analyzing the smallest tasty number of base 2-25.

Base	x_0	x_1	x_2	x_3
2	[1 2]	1	-	-
3	[3 1 3]	2 1	1 0	1
4	[0 1 1 1]	3	-	-
5	[2 1 1 0 1]	2 3	1 0	1
6	[1 2 1 1 2 2]	3 2	5	-
7	[1 0 0 1 1 0 1]	1 4	5	-
8	[0 0 1 1 1 1 0 1]	1 6	7	-
9	[1 1 1 1 0 0 1 0 1]	3 1	4	-
10	[0 1 0 1 1 1 1 0 1 1]	2 7	9	-
11	[1 1 1 1 1 0 0 1 1 0 1]	4 1	5	-
12	[0 1 1 0 1 1 1 1 0 1 1 1]	3 8	11	-
13	[1 1 0 1 1 1 0 1 0 1 1 1 1]	4 6	10	-
14	[0 1 1 1 0 1 1 1 1 0 1 1 1 1]	4 9	13	-
15	[1 1 1 1 1 1 1 0 0 1 1 1 1 0 1]	6 1	7	-
16	[0 1 1 1 1 0 1 1 1 1 0 1 1 1 1 1]	5 10	15	-
17	[1 1 1 1 1 1 1 1 0 0 1 1 1 1 1 0 1]	7 1	8	-
18	[0 1 1 1 1 1 0 1 1 1 1 0 1 1 1 1 1 1]	6 11	17	-
19	[1 1 1 0 1 1 1 1 1 1 0 0 1 1 1 1 1 1 1]	7 8	15	-
20	[0 1 1 1 1 1 1 0 1 1 1 1 0 1 1 1 1 1 1 1]	7 12	19	-
21	[1 1 1 1 1 1 1 1 1 1 0 0 1 1 1 1 1 1 1 1 0 1]	9 1	10	-
22	[0 1 1 1 1 1 1 1 0 1 1 1 1 0 1 1 1 1 1 1 1 1]	8 13	21	-
23	[1 1 1 1 1 1 1 1 1 1 1 0 0 1 1 1 1 1 1 1 1 1 0 1]	10 1	11	-
24	[0 1 1 1 1 1 1 1 1 0 1 1 1 1 0 1 1 1 1 1 1 1 1 1]	9 14	23	-
25	[1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 0 1 1 1 1 1 1 1 0 0 0 1]	10 17	1 2	3

The pattern is not immediately recognizable, but becomes apparent when we restrict our view to even bases greater than 8.

Base	x_0	x_1	x_2	x_3
8	[0 0 1 1 1 1 0 1]	1 6	7	-
10	[0 1 0 1 1 1 1 0 1 1]	2 7	9	-
12	[0 1 1 0 1 1 1 1 0 1 1 1]	3 8	11	-
14	[0 1 1 1 0 1 1 1 1 0 1 1 1 1]	4 9	13	-
16	[0 1 1 1 1 0 1 1 1 1 0 1 1 1 1 1]	5 10	15	-
18	[0 1 1 1 1 1 0 1 1 1 1 0 1 1 1 1 1 1]	6 11	17	-
20	[0 1 1 1 1 1 1 0 1 1 1 1 0 1 1 1 1 1 1 1]	7 12	19	-
22	[0 1 1 1 1 1 1 1 0 1 1 1 1 0 1 1 1 1 1 1 1 1]	8 13	21	-
24	[0 1 1 1 1 1 1 1 1 0 1 1 1 1 0 1 1 1 1 1 1 1 1 1]	9 14	23	-

From here we see a pattern that is dependent on the base: $[0, O(\frac{r}{2}-4), 0, 1, 1, 1, 1, 0, O(\frac{r}{2}-3)]_r \rightarrow (\frac{r}{2}-3) * r + (\frac{r}{2}+2)_r \rightarrow (r-1)_r$ where $O(n)$ represents a string of 1's of length n. This pattern appears for all bases r where $(\frac{r}{2}-4) \geq 0$.

Theorem. *The above pattern holds for all bases greater than or equal to 8.*

Proof. This pattern can be shown by induction.

Let r=8. The digit sum of this pattern is found by the summation of consecutive integers minus the two exceptions:

$$(\sum_{i=1}^{r-2} i) - (\frac{r}{2}+2) - (\frac{r}{2}-3) = \frac{(r-1)(r-2)}{2} - r + 1$$

For r=8, this formula gives us 14_{10} or 16_8. The digit sum of 16_8 is 7_8, showing that the pattern holds for r=8.

Now let r=k; the formula gives $\frac{(k-1)(k-2)}{2} - k + 1$ or $\frac{k^2-5k+4}{2}$. Now consider two of the remaining digits, $\frac{k}{2} - 3$ and $\frac{k}{2} + 2$. Multiply $\frac{k}{2} - 3$ by the base to get $\frac{k^2-6k}{2}$. This gives a two digit number where the first digit is one of the three missing digits and the second digit is 0. Adding $\frac{k}{2} + 2$ then gives a two digit number comprised of two of the remaining digits. Simplifying $\frac{k^2-6k}{2} + \frac{k}{2} + 2$ gives $\frac{k^2-5k+4}{2}$, showing that the first formula is true. Finally, we take the digit sum of the result. $\frac{k}{2} - 3 + \frac{k}{2} + 2$ gives $k - 1$. All digits of the base, k, have been produced exactly once within this projection, therefore the original number is tasty. □

This proof establishes the pattern of base k weight 1 tasty numbers where k is even. The next pattern noticed was that of base k weight 2 tasties where k is greater than 5.

Theorem. *There is a tasty number in all bases greater than 5 where the weight is 2 that abide by the following formula:*

$$[1, 2, 1, T(r-6), 1, 2, 2]_r$$

where T(N) is a string of 2's of length N.

Proof. This formula can also be proven directly.

Let us establish the function for the first digit sum first. $2*\frac{r(r-1)}{2} = r(r-1)$ is the sum of the digits in base r multiplied by 2. Now we subtract the 3 other digits: $r-1$, $r-3$, and 2, giving $r(r-1)-(r-1)-(r-3)-2 = r^2-r-r+1-r+3-2 = r^2-3r+2$. This time, multiply $r-3$ by r and add 2 to get the same form of a two digit number from the previous proof: $r^2 - 3r + 2$. From this, we see that the formula holds for the first digit sum. Add the digits $r - 3$ and 2 to get the digit root of the number, which is $r - 1$. All digits have appeared twice in this projection, therefore the number is tasty. □

A similar pattern was found for weight 4. These patterns are valid for all bases where functions such as $T(n)$ and $O(n)$ do not get passed a negative value for n.

Theorem. *There is a tasty number in all bases greater than 6 where the weight is 4 that abide by the following formula:*

$$[3, 4, 4, 3, F(r-7), 3, 3, 4]_r$$

where F(N) is a string of 4's of length N.

Proof. We will prove this formula in the same manner as the previous one.

The digit sum for this number is found by the same manner as weight 2: $4 * \frac{r(r-1)}{2} - (r-1) - (r-4) - 2 - 1 = 2r^2 - 4r + 2$. x_1 for this pattern should take the form of $1 * r^2 + (r-4) * r + 2 = r^2 + r^2 - 4r + 2 = 2r^2 - 4r + 2$, so the formula holds for x_1. Finally, $x_2$should be $r - 1$. The digit sum of x_1 is $1 + 2 + (r-4) = 3 + r - 4 = r - 1$. All digits of base r have appeared 4 times in this projection, therefore the formula holds. □

There is a similar pattern that emerges in bases greater than 7 for weight 6: $[5,6,6,6,5,S(r-8),4,6,6]_r$ and for weight 8: $[7,8,8,8,8,7,E(r-10),7,7,8,8]_r$. After observing these patterns, rather than trying to prove them individually, it is more prudent to look for a general pattern that will generate a tasty number for any base and weight that it is valid for. Consider this table:

Weight	Digits subtracted from x_0	Minimum base for the formula
4	$(r-1),(r-4),2,1$	7
6	$(r-1),(r-5),2,2$	8
8	$(r-1),(r-6),2,3$	9
10	$(r-1),(r-7),2,5$	10
12	$(r-1),(r-7),2,6$	11

There is a clear linear progression for the parameters of the forumulas. We will use this to generate a general theorem.

Theorem. *For a given even weight w where $w > 2$, there is a tasty number in all bases $r \in N$ such that $r \geq \frac{w}{2} + 5$ that abides by the following formula:*

$$[W(r)] - Z(r-1) - Z(r - (\frac{w}{2}+2)) - Z(1 + (\frac{w}{2}-2)) - Z(2)$$

where $W(n)$ returns a string of the value of w of length n.

Proof. Step 1: Prove the base case of $r = 7, w = 4$ which are the minimum values of the formula.

The formula gives: $[4,4,4,4,4,4,4]-[1,0,0,0,0,0,0]-[0,0,0,0,1,0,0]-[0,0,0,0,0,1,0] = [3,4,4,3,3,3,4]$. The projection we get from this number is $[3,4,4,3,3,3,4] \to 132_7 \to 6_7$. The weight of all base 7 digits is 4, therefore the number is tasty and the base case for the formula holds.

Step 2: Prove the first general case with a variable base r and a constant weight of 4.

This was proven for the general formula of $w = 4$.

Step 3: Prove the second general case with a variable base $r = k$ and a variable weight $w = l$.

For this, we assume that k is a valid base for l. The digit sum of $[L(k)]$ where $L(n)$ returns a string of l's of length n is $l\frac{k^2-k}{2}$. From this, we subtract the digits specified by the formula giving: $l\frac{k^2-k}{2} - (k-1) - (k-(\frac{l}{2}+2)) - (1+(\frac{l}{2}-2)) - 2 = \frac{lk^2-k(l+4)+4}{2}$. To check this digit sum, we repeat the method of the previous proofs of multiplying the missing digits by increasing powers of k. This gives: $k^2 * (1 + (\frac{l}{2}-2)) + k * (k - (\frac{l}{2}+2)) + 2 = k^2 + \frac{lk^2}{2} - 2k^2 + k^2 - \frac{lk}{2} - 2k + 2 = \frac{lk^2-k(l+4)+4}{2} = x_1$. The sums are equal, therefore the formula holds for x_1. For the formula to be proven correct, $x_1 = k-1$ must be true. From this, we get: $1 + \frac{l}{2} - 2 + k - \frac{l}{2} - 2 + 2 = k - 1 = x_2$. The sum equals $k-1$, hence the formula holds. □

From here we have proven that there are infinite tasty numbers in all even weights, now we look at the patterns for tasty numbers in odd weights. The odd weights differ from the even weights in one crucial manner: for a given odd weight, tasty numbers of odd bases and tasty numbers of even bases do not follow the same pattern. Consider the following tables for base 14 and base 15 tasties:

Weight	Tasty in base 14	Weight	Tasty in base 15
1	$[0,1,1,1,0,1,1,1,1,0,1,1,1,1]$	1	$[1,1,1,1,1,1,1,0,0,1,1,1,1,0,1]$
3	$[2,3,3,3,2,3,3,3,3,3,2,3,2,3]$	3	$[3,3,3,3,3,3,3,2,3,2,3,3,3,1,3]$
5	$[4,5,5,5,4,5,5,5,5,5,5,3,5,5]$	5	$[5,5,5,5,5,5,5,4,5,5,4,5,4,4,5]$
7	$[6,7,7,7,6,7,7,7,7,7,6,7,6,7]$	7	$[7,7,7,7,7,7,7,6,7,7,7,5,7,6,7]$
9	$[8,9,9,9,8,9,9,9,9,8,9,9,9,8]$	9	$[9,9,9,9,9,9,9,8,9,9,8,9,8,8,9]$

There is a clear discrepency between the two and, once again, the lowest value is an outlier. The common trait is that both even and odd bases have 4 digits missing from x_0, implying a 3 digit x_1, and a 1 digit x_2. We will start with the even bases.

Theorem. *For a given odd weight w where $w > 1$, there is a tasty number in all bases $r \in E$ such that $r \geq w+5$ that abides by the following formula:*

$$[W(r)] - Z(r-1) - Z(\frac{r}{2}+2) - Z(\frac{r}{2} - (\frac{w+1}{2}+2)) - Z(\frac{w-1}{2})$$

where $W(n)$ returns a string of the value of w of length n.

Proof. We will prove this formula using the same 3 step process as the general even weight formula.

Step 1: Prove the base case of $w = 3$ and $r = 8$.

Start the digit sum of $[2,2,3,3,3,3,2,2] \rightarrow 3 * \frac{8*7}{2} - 7 - 6 - 1 = 3 * 28 - 14 = 84 - 14 = 70_{10} = 106_8 \rightarrow 7_8$. The projection has a uniform weight of 3, so the formula holds for the base case as $[2,2,3,3,3,3,2,2]$ is tasty.

Step 2: Prove the general case of $w = 3$ and a variable weight r.

We start by calculating the formula for the general digit sum of $x_0$and compare it with the expected value as we have in previous proofs. The formula for x_0 works out to be $3*\frac{r(r-1)}{2} - (r-1) - (\frac{r}{2}+2) - (\frac{r}{2} - (\frac{3+1}{2}+2)) - (\frac{3-1}{2}) = \frac{3r^2-7r+4}{2}$. Now we calculate the expected x_1: $r^2 * \frac{3-1}{2} + r*(\frac{r}{2} - (\frac{3+1}{2}+2)) + (\frac{r}{2}+2) = r^2 + \frac{r^2}{2} - 4r + \frac{r}{2} + 2 = \frac{3r^2-7r+4}{2}$. Now that x_1 has been shown to hold, we do the same for x_2. The formula for x_2 found to be: $\frac{3-1}{2} + (\frac{r}{2} - (\frac{3+1}{2}+2)) + (\frac{r}{2}+2) = r-1$ which is the expected value of x_2. Therefore, the number is tasty and this formula holds.

Step 3: Prove the overall general case of r and w.

Again, we start with formula of x_1: $w\frac{r^2-r}{2} - (r-1) - (\frac{r}{2}+2) - (\frac{r}{2} - (\frac{w+1}{2}+2)) - (\frac{w-1}{2}) = \frac{wr^2 - r(w+4) + 4}{2}$ and the expected value of x_1: $r^2(\frac{w-1}{2}) + r(\frac{r}{2} - (\frac{w+1}{2}+2)) + (\frac{r}{2}+2) = \frac{wr^2-r(w+4)+4}{2}$. Now that x_1 has been verified, we move on to the formula for x_2: $(\frac{w-1}{2}) + (\frac{r}{2} - (\frac{w+1}{2}+2)) + (\frac{r}{2}+2) = r-1$ which is the expected value. Having verified the values of x_0,x_1, and x_2, we know that the number is tasty and the formula holds. □

Theorem. *For a given odd weight w where $w > 1$, there is a tasty number in all bases $r \in O$ such that $r \geq w+2$ that abides by the following formula:*

$$[W(r)] - Z(1) - Z(\frac{r-1}{2} - \frac{w+1}{2}) - Z(\frac{r-1}{2}) - Z(\frac{w-1}{2})$$

where $W(n)$ returns a string of the value of w of length n.

Proof. Once again, we will use the same 3 step method to prove the formula.

Step 1: Prove the base case of $r = 5$ and $w = 3$.

The projection is $[3,3,2,1,2] \rightarrow 101_5 \rightarrow 2_5$. This has a uniform weight of 3, therefore the number is tasty and the pattern holds for the base case.

Step 2: Prove the general case of r and $w = 3$.

First we get the formula for x_1: $3\frac{r^2-r}{2} - 1 - \frac{r-1}{2} - \frac{3-1}{2} - \frac{r-5}{2} = \frac{3r^2-5r+2}{2}$. Then we calculate the expected value for x_1: $r^2(\frac{3-1}{2}) + r(\frac{r-5}{2}) + 1 = \frac{3r^2-5r+2}{2}$. Now that we have confirmed x_1, we check the formula for x_2 against the expected value of $\frac{r-1}{2}$: $\frac{3-1}{2} + \frac{r-5}{2} + 1 = \frac{r-1}{2}$. The check for x_1 and x_2 hold, therefore x_0 is tasty and the first general case is true.

Step 3: Prove the overall general case of r and w.

First we get the formula for x_1: $w\frac{r^2-r}{2} - 1 - \frac{r-1}{2} - \frac{w-1}{2} - \frac{r-w-2}{2} = \frac{wr^2-r(w+2)+2}{2}$. Then we calculate the expected value for x_1: $r^2(\frac{w-1}{2}) + r(\frac{r-w-2}{2}) + 1 = \frac{wr^2-r(w+2)+2}{2}$. Now that we have confirmed x_1, we check the formula for x_2 against the expected value of $\frac{r-1}{2}$: $\frac{w-1}{2} + \frac{r-w-2}{2} + 1 = \frac{r-1}{2}$. The check for x_1 and x_2 hold, therefore x_0 is tasty and the theorem is true. □

5 The Orders of Tasties

We have now gone through an extensive examination of the weights of tasties and proven the patterns for them. However, we have, as of yet, only looked at order 0 tasty numbers. The first question to ask is how to generate a tasty number of a higher order given a known tasty number. We will use the same method that we used to prove the infinite order of base 2 tasties and we will start with base 10.

We will not be using CSN for some tasty numbers in this process. The goal is to find a tasty number who's digit sum equals another tasty number. The easiest way to find numbers like this is to look at numbers that have an equal number of every digit in the base. This is because numbers like this do not throw off the weights of a tasty number.

The sum of digits in base 10 is 45. This means that every number in base 10 that has an equal number of every digit will have a digit sum that is a multiple of 45. For this, we will look at standard tasty number in base 10: $[0, 1, 0, 1, 1, 1, 1, 0, 1, 1]$. Given that $x_1 = 27$, all permutations of this number is divisible by 9 so we only need a permutation that is divisible by 5. For the sake of using minimal values, we will use 1034685_{10}.

We will now divide this tasty number by 45 to get 22993. 22993 will then be the weight we use for the weights in our new tasty number. We now have the number:

$$[22993, 22993, 22993, 22993, 22993, 22993, 22993, 22993, 22993, 22993] \rightarrow$$

$$1034685_{10} \rightarrow 27_{10} \rightarrow 9_{10}$$

which, given that we used the smallest possible tasty number to generate this number, is the smallest order 1 tasty number in base 10. An interesting pair of facts become apparent from this: this number is obviously divisble by 9 given how it was generated and has numerous permuations that are divisible by 5. This means that this number can, infact, be used for this same process. It is also clear that the number that results from this will also be divisible by 9 and will have permutations that are divisible by 5. From this, we can see that, not only can we find a higher order tasty number by this method but we can actually find infinite tasty numbers by this method. Now the question that we need to ask is obvious: can we do this for all bases?

Theorem. *Given any natural base r, there is a tasty number for which we can generate an infinite number of higher order tasty numbers of boundless order.*

Proof. For this theorem, we need to prove one major lemma: every natural base has a tasty number that has a permutation which is divisible by the sum of digits for that base.

It has been proven previously that if the digit root of a number is the largest digit of the base, that number is divisible by the largest digit. This means that any tasty number that terminates in the largest digit of its base is immediately a candidate for this.

Now let us look at the sum of digits for a base r. This can be found by $\frac{r(r-1)}{2}$. In the event of an even base, $\frac{r}{2}$ is an integer and for odd bases, $\frac{r-1}{2}$ is an integer. These integers are what we need to look at for our candidates. It is clear that a number x_r is divisible by r if the number ends in 0. In the case of even bases, the number is divisible by $\frac{r}{2}$ if it ends in $\frac{r}{2}$. The significance of this is that these cases effect the divisibility of a number. Therefore, if the digit root of a x_r is $r-1$ (or $\frac{r-1}{2}$ in the case of odd bases) and x_r contains the digit 0 (or $\frac{r}{2}$ in the case of even bases), x_r is then divisible by the sum of digits in r.

We know that we can generate these higher order tasty numbers in any base given a valid tasty number and we now have a method for identifying valid tasty numbers. What we need now is a valid tasty number in every natural base. Conveniently, we already have this for the majority of bases. We have a formula for even weight tasty numbers in all bases greater than 7. In this formula, every tasty number x_r terminates in $r-1$ and every tasty number also contains $\frac{r}{2}$ and 0 so every valid base for this method has at least one tasty number that can generate infinite tasty numbers of boundless orders. We also have shown that base 2 works for this, so the only bases that need to be shown to have a valid tasty number are $r = 3, 4, 5, 6, 7$. Rather than using any general methods to find, we will simply list valid examples for these 5 bases. The examples and their projections are as follows: $[269, 269, 269] \to 1002220_3 \to 21_3 \to 10_3 \to 1_3$, $[3, 3, 3, 3] \to 102_4 \to 3_4$,$[7390, 7390, 7390, 7390, 7390] \to 4331100_5 \to 22_5 \to 4_5$,$[112749, 112749, 112749, 112749, 112749, 112749] \to 100125443_6 \to 32_6 \to 5_6$,$[379239, 379239, 379239, 379239, 379239, 379239, 379239] \to 124456500_7 \to 36_7 \to 12_7 \to 3_7$.

We have established that ever natural base has a valid tasty number for which and infinite number of tasty numbers can be generated, therefore the theorem is true. □

6 Unanswered Questions

Over the course of this project, we extensively explored the nature of tasty numbers in natural bases and the weights of these tasty numbers. There were certain areas that have not been analyzed yet.

The first and most obvious of these are tasty numbers that do not exist within any of the patterns listed here. An example of this is $[0, 2, 3, 3, 3, 3, 3, 3, 2, 3]_{10}$. This number is a valid base 10 tasty number that does not abide by any formulas that we have developed here. It is currently unknown whether or not these numbers exist in a seperate pattern, whether or not they exist in all bases, or the cardinality of these tasty numbers. What is known about them is that they do not exist for all weights and, for the samples that we have analyzed, they appear far less frequently than the tasty numbers that exist within the patterns that we have described.

The second is less obvious but poses interesting opportunities: what tasty numbers exist for bases that are not natural? Namely, we can ask this of non-integer bases and negative integer bases. This is a question that is, as of yet, poorly explored and we do not have any substantial data on the matter. What is known is that tasty numbers (at least trivial examples) do exist in negative bases. The main example that we know of is $[1, 2]_{-2} \to 1_{-2}$. This number is a valid tasty number in base -2.