CS 7750: Solutions to homework 3

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3.14

a. True, in the best case scenario depth-first search and A^* search with the heuristic function $h(n) = C^*$ will only need to expand d nodes to reach the goal, while in the worst case both search algorithms will need b^m to find the goal node and they will not be complete in infinite state space.

b. True, $h(n) = 0$ is an admissible heuristic for 8-puzzle since it's always true that for any 8-puzzle instance you will need zero or more steps to solve. In addition, the average solution cost of the 8-puzzle problem is about 22 which makes $h(n) = 0$ to never overestimate the true cost of its solution thus it is admissible if not too optimistic.

3.21

a. Uniform-cost search will select the node with the lowest path cost $g(n)$, however, if all step costs happen to be the same the uniform-cost search will expand all the nodes in the same level before moving down the search tree since the nodes at the lower levels will always have higher path cost $g(n)$ than its parent and this is exactly the behavior of breath-first search "expanding the shallowest node".

b. Greedy best-first tree search select the node for expansion based on heuristic function $h(n)$ and if $h(n)$ yields the same estimate for every node to the goal node, best-first search will explore one branch until the bottom of the tree after another which is the same behavior found in depth-first search. Thus, depth-first search is a special case of best-first tree search when $h(n) = h(n + 1)$.

c. A^* search evaluates the estimated cost of the cheapest solution through n $f(n)$ by the cost to reach the node $g(n)$ and the estimated cost to the goal from the node $h(n)$.

$$f(n) = g(n) + h(n)$$

In the case that $h(n) = 0$, $f(n) = g(n) + 0$ or $f(n) = g(n)$ which is identical to the uniform-cost search algorithm. Thus, uniform-cost search is a special case of A^* search where its heuristic function equals to zero.

3.23

The sequence of nodes ($f = g + h$) to be considered by A^* search algorithm:

Lugoj ($244 = 0 + 244$), Mehadia ($311 = 70 + 241$), Drobeta ($387 = 145 + 242$), Croiova ($430 = 265 + 160$), Timisoara ($440 = 111 + 329$), Pitesti ($503 = 403 + 100$), Bucharest ($504 = 403 + 101$).

3.26

a. An unbounded rectangular grid has the branching factor $b = 4$ since there are 4 successors at the origin $(0, 0)$.

b. Distinct states at depth k = $2k^2 \quad (k > 0)$.

c. The maximum number of nodes expanded by breath-first tree search $= 2 \times 4^{d+1}$.

d. The maximum number of nodes expanded by breath-first graph search $= 4^{d+1}$.

e. $h = |u - x| + |v - y|$ is an admissible heuristic for a state at (u, v). $|u - x|$ and $|v - y|$ are the changes in horizontal and vertical direction respectively from (u, v) so the sum of them is just the **Manhattan distance** to the goal state (x, y) and each move using the heuristic will get it one step closer the goal node.