CS 8790: Report for assignment 7

Chanmann Lim

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Report:

The state of a target moving with constant velocity in 2D coordinate is observed. A sequence of the 2D measurements come with the corresponding *timestamp* at which each observation was taken, the mean (x, y) and the upper triangular elements of the square root of the covariance $sqrtm(R)$ thus observation is now in the form of (t_z, z, R).

One way to start is to initialize our filter with time zero, zero mean and velocity and infinite covariance.

$$t = 0, x = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, P = \begin{bmatrix} 100000000 & 0 & 0 & 0 \\ 0 & 100000000 & 0 & 0 \\ 0 & 0 & 100000000 & 0 \\ 0 & 0 & 0 & 100000000 \end{bmatrix}$$

Yet, combining the estimate and the observation is not feasible unless both measurements are valid at the same *timestamp* which lead us to make prediction of the state of the estimate into the same timestamp as the observation using constant velocity model F.

$$F \quad (x, P) \rightarrow (Fx, FPF')$$

, where

$$F = \begin{bmatrix} 1 & 0 & \Delta t & 0 \\ 0 & 1 & 0 & \Delta t \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

And since the observation comes in lower dimension (without velocity) than the state estimate we have to use transformation matrix H to project down the state estimate before combining them using innovation fusion equation.

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

1.1 The state of the target at the time of the final observation is :

$$\text{mean with standard deviation} = \begin{pmatrix} 245.277904 & , & 0.002747 \\ 510.449761 & , & 0.001239 \\ 0.280005 & , & 0.000007 \\ 0.600000 & , & 0.000002 \end{pmatrix}$$

$$x\_final = \begin{bmatrix} 245.277904 \\ 510.449761 \\ 0.280005 \\ 0.600000 \end{bmatrix}$$

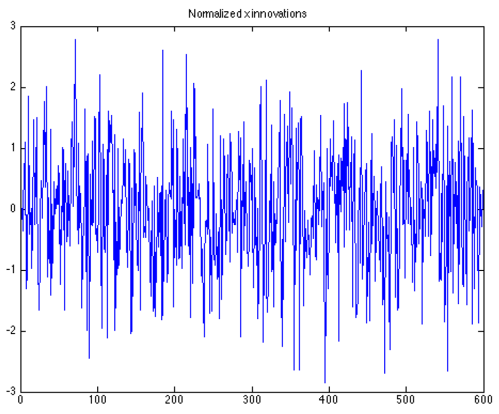
$$P\_final = \begin{bmatrix} 0.00000754 & -0.00000074 & 0.00000002 & -0.00000000 \\ -0.00000074 & 0.00000153 & -0.00000000 & 0.00000000 \\ 0.00000002 & -0.00000000 & 0.00000000 & -0.00000000 \\ -0.00000000 & 0.00000000 & -0.00000000 & 0.00000000 \end{bmatrix}$$

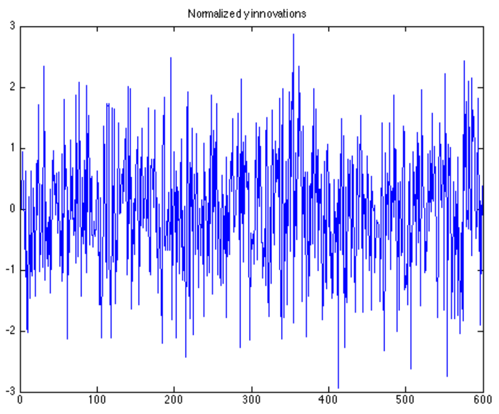
1.2 The prediction of the state of the target one hour after the time of the final observation ($t = 927.969649836$) :

$$x(4527.969650) = \begin{bmatrix} 1253.296210 \\ 2670.451308 \\ 0.280005 \\ 0.600000 \end{bmatrix}$$

$$P(4527.969650) = \begin{bmatrix} 0.00082509 & -0.00013653 & 0.00000021 & -0.00000003 \\ -0.00013653 & 0.00008861 & -0.00000004 & 0.00000002 \\ 0.00000021 & -0.00000004 & 0.00000000 & -0.00000000 \\ -0.00000003 & 0.00000002 & -0.00000000 & 0.00000000 \end{bmatrix}$$

1.3 The plots of the sequence of normalized x and y innovations separately :

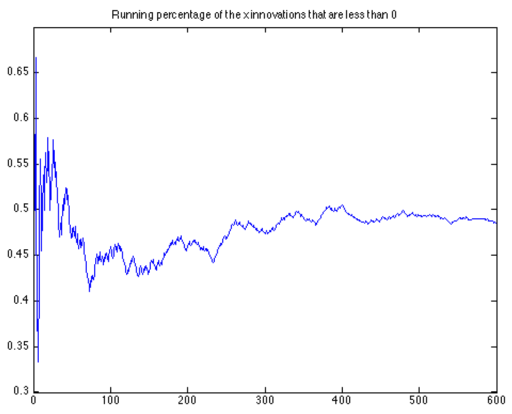


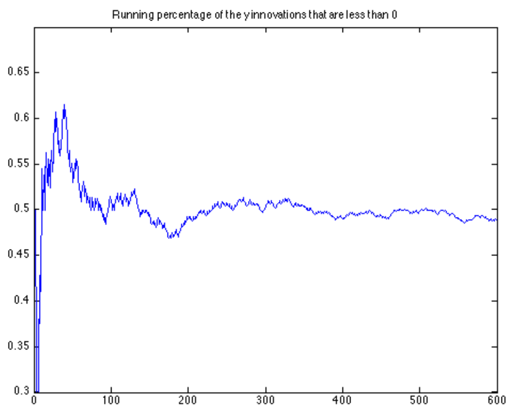


The normalized x innovations is obtained by $(z - Hx)(1)/\sqrt{S(1,1)}$ and y innovations $= (z - Hx)(2)/\sqrt{S(2,2)}$, where S is the innovation covariance.

1.4 The Percentage of x and y innovations that are less than 0 is $[48.50\%, 49.00\%]$

The plot of the running percentage of the x and y innovations that are less than zero :





For the second dataset, `Target2.txt`, the motion of the target deviates slightly from true constant velocity

Appendix:

assignment_7.m

```
% Run filter_7 on Target1.txt with q = 0
filter_7 ('Target1.txt', 0);

% Run filter_7 on Target2.txt with q = 0
filter_7 ('Target2.txt', 0);

% Run filter_7 on Target2.txt with q = 0.15
filter_7 ('Target2.txt', 0.15);

% Run filter_7 on Target2.txt with q = 1
filter_7 ('Target2.txt', 1);
```

filter_7.m

```
function filter_7 (data_file, q)
    % initialize filter
    t = 0;
    x = zeros(4,1);
    P = eye(4) * 10^8; % large covariance P
    result = report();

    % open data file
    fid = fopen(data_file);
    while ~feof(fid)
        tline = fgetl(fid);
        data = sscanf(tline, '%f');

        if ~isempty(data)
            [t_new, z, R] = get_observation(data);
            [t, x, P] = predict(t, x, P, t_new, q);
            [x, P, vx] = update(x, P, z, R);
            result = result.add_data(vx);
        end
    end
    % close file
    fclose(fid);

    result = result.update_estimate(t, x, P);

    result.print_final_estimate();
    result.print_prediction(t + 3600, q);
    result.plot();
    result.print_innovations_percentage();
end
```

get_observation.m

```
function [ t_z, z, R ] = get_observation( data )
%GET_OBSERVATION get observation from data
%   data - column vector containing observation data

    t_z = data(1);
    z = data(2:3);
    R = [data(4:5)'; data(5:6)']; % sqrtm(R) not R
    R = R * R;
end
```

predict.m

```
function [ t, x, P ] = predict( t, x, P, t_new, q )
%PREDICT - constant velocity precition for (x, P) at time "t_new"
%   F : constant velocity transformation matrix respected to "delta_t"
%   Q : process noice covariance matrix to ensure that P is conservative

    delta_t = (t_new-t);
    F = [1 0 delta_t 0; 0 1 0 delta_t; 0 0 1 0; 0 0 0 1];
    Q = q * [1 0 0 0; 0 1 0 0; 0 0 0 0; 0 0 0 0] * delta_t; % velocity should not be inflated
    % (Fx, FPF' + Q)
    t = t_new;
    x = F * x;
    P = (F * P * F') + Q;
end
```

update.m

```
function [ x, P, vx ] = update( x, P, z, R )
%UPDATE - update sensor estimate
%   incorporate information from new observation (z, R)
%   returns
%       x  - updated mean x
%       P  - updated covariance P
%       vx - normalized unit innovations


    % Use dimensionality transformation matrix H
    %   to project down the state dimension
    %   since the observation doesn't come with velocity
    H = eye(2, 4);

    S = (H * P * H') + R;
    W = (P * H') / S;
    P = P - (W * S * W');
    innovation = (z - H * x);
    x = x + W * innovation;

    vx = innovation ./ sqrt(S([1;4]));
end
```

report.m

```
classdef report

    properties
        normalized_unit_innovations;
        running_percentages;
        t;
        x;
        P;
    end

    methods
        function obj = report()
            obj.normalized_unit_innovations = [];
            obj.running_percentages = [];
        end

        function obj = add_data(obj, vx)
        % vx - normalized unit innovation vector
        % vs - innovation size
            obj.normalized_unit_innovations(:, end + 1) = vx;
            obj.running_percentages(:, end + 1) = ...
                sum(obj.normalized_unit_innovations < 0, 2) / length(obj.normalized_unit_innovations);
        end

        function obj = update_estimate(obj, t, x, P)
            obj.t = t;
            obj.x = x;
            obj.P = P;
        end

        function print_prediction(obj, t, q)
            [~, x_new, P_new] = predict(obj.t, obj.x, obj.P, t, q);
            fprintf('x(%f) = \n\n', t);
            fprintf('%14f \n', x_new);
            fprintf('\n');

            fprintf('P(%f) = \n\n', t);
            fprintf('%14.8f %14.8f %14.8f %14.8f \n', P_new);
            fprintf('\n');
        end

        function print_final_estimate(obj)
```

```
            % print final position
            standard_deviation = sqrt(diag(obj.P));
            fprintf('The final position of the target with standard deviation =\n');
            fprintf('\n%14f, %14f\n%14f, %14f\n%14f, %14f\n%14f, %14f\n', ...
                obj.x(1), standard_deviation(1), ...
                obj.x(2), standard_deviation(2), ...
                obj.x(3), standard_deviation(3), ...
                obj.x(4), standard_deviation(4));
            fprintf('\n');

            fprintf('x_final = \n\n');
            fprintf('%14f \n', obj.x);
            fprintf('\n');

            fprintf('P_final = \n\n');
            fprintf('%14.8f %14.8f %14.8f %14.8f \n', obj.P);
            fprintf('\n');
        end

        function plot(obj)
            % plot x innovations
            figure;
            plot(obj.normalized_unit_innovations(1,:));
            ylim([-3, 3]);
            title('Normalized x innovations');

            figure;
            plot(obj.running_percentages(1, :));
            ylim([.3, .7]);
            title('Running percentage of the x innovations that are less than or equal to 0');

            % plot y innovations
            figure;
            plot(obj.normalized_unit_innovations(2,:));
            ylim([-3, 3]);
            title('Normalized y innovations');

            figure;
            plot(obj.running_percentages(2, :));
            ylim([.3, .7]);
            title('Running percentage of the y innovations that are less than or equal to 0');
        end

        function print_innovations_percentage(obj)
            template = 'Percentage of the innovations that are less than 0 = [ %.2f%%, %.2f%% ]\n';
            percent = ...
                sum(obj.normalized_unit_innovations < 0, 2) / length(obj.normalized_unit_innovations);
            fprintf(template, percent * 100);
        end
    end
end
```