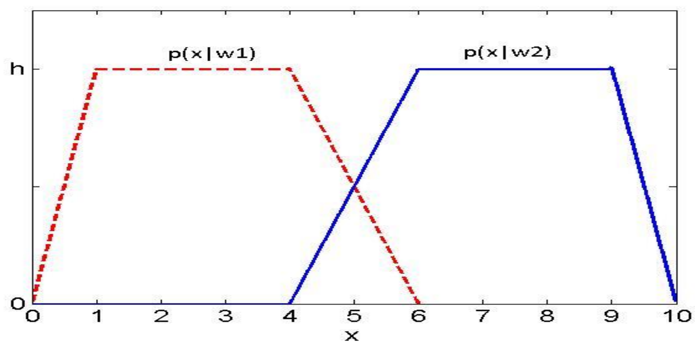
CS-ECE4720/7720 Spring 2015
Introduction to Machine Learning and Pattern Recognition

Homework Assignment 2

due Thursday 2/26/2015

Problem 1

The class-conditional probability density functions (pdf) of a feature x are shown in the figure below:

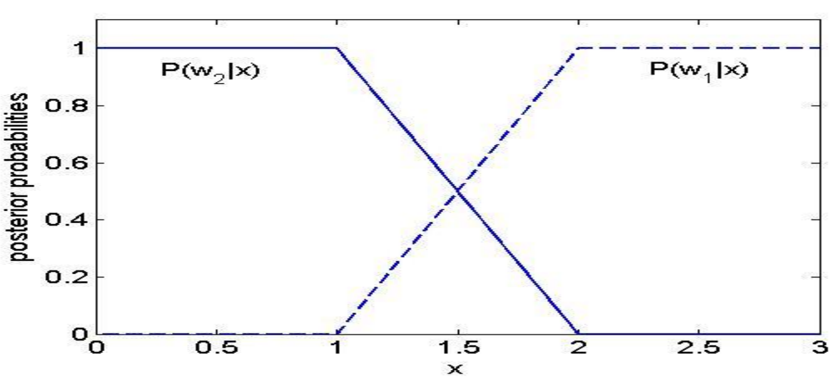


The prior probabilities of the two classes are P(w1) = 2/3 and P(w2) = 1/3.
Do the followings:

a) Determine the value of h.
b) Sketch the pdfs p(x, w1) and p(x,w2) in one figure.
c) Sketch the evidence p(x).
d) Sketch the posterior probability distributions P(w1|x) and P(w2|x) in one figure.
e) Determine P(w1|x=5) and P(w2|x=5).
f) For the minimum error classifier, determine the decision regions for the two classes w0 and w1.
g) Compute the Bayes error P(error) (note: it is easier to compute the Bayes error based on the figure produced in part b).

Problem 2.

The posterior probability distributions of two classes are shown below. The pdf p(x) = 1/3 for $x \in [0,\ 3]$.



The classification costs are given as $\lambda_{11} = \lambda_{22} = 0$, $\lambda_{12} = 1$, and $\lambda_{21} = 2$.

a) Design the minimum risk classifier, and compute the expected classification risk and classification error.
b) Design the minimum error classifier, and compute the expected classification risk and classification error.

Problem 3 Textbook Prob. 31 of Section 2.7 (page 72-73)

Problem 4

Assume that $\underline{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ comes from two 2-D Gaussian distributions:

$$\underline{\mu}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix},\ \underline{\mu}_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix},\ \Sigma_1 = \Sigma_2 = \begin{bmatrix} 1.21 & 0 \\ 0 & 1.21 \end{bmatrix}$$

with $P(\omega_1) = P(\omega_2)$.

Part I:

a. Derive the discriminant function $g(\underline{x}) = g_1(\underline{x}) - g_2(\underline{x})$ (with all irrelevant terms removed) for the minimum error classifier;
b. Implement the minimum error classifier in MATLAB;
c. Use your code to classify the 50 data samples provided in the data file **2DGaussianDataset1.txt.**
d. Plot the 50 data samples and the decision boundary $g(\underline{x}) = 0$ in one figure, where the first and the last 25 data samples should be marked by two different colors as the former comes from the distribution 1 and the latter from the distribution 2.
e. Make a plot that shows the $g(\underline{x}_n)$ values as a function of the index n of the data samples $\underline{x}_n$.
f. Compute the classification error rate, i.e., (#misclassified samples/total samples)×100%.

Part II:

Fifty new data samples are generated from the two 2-D Gaussian distributions

$$\underline{\mu}_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix},\ \underline{\mu}_2 = \begin{bmatrix} -2 \\ 1 \end{bmatrix},\ \Sigma_1 = \Sigma_2 = \begin{bmatrix} 1.21 & 0.8 \\ 0.8 & 1.21 \end{bmatrix}$$

with the first 25 samples from the distribution 1 and the last 25 samples from the distribution 2. The data file is **2DGaussianDataset2.txt.**

Using the classifier designed in Part I, repeat the work specified in c), d), e), and f) on the new data set, and explain the difference between the error rates on the two different datasets.

Part III:

Redesign the classifier for the dataset **2DGaussianDataset2.txt** as in the steps a) and b) of Part I, and using this new classifier, repeat the work specified in c), d), e), and f) on the new data set, and explain the difference between the error rates in Part II and Part III on this dataset.

Note: please include in the report your MATLAB code.

Problem 5

Consider the Bayesian belief network for fish as discussed in the textbook.

a. Given that a medium, wide fish is caught in the winter, classify the fish.
b. What is your chance of making an error in this classification?