STAT 7750: Solutions to homework set 2

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September 18, 2014

Solution 1: Chapter 1, Exercise 18

(a)

Since $A \cap B$ and $A \cap B'$ are mutually exclusive, and $A = (A \cap B) \cap (A \cap B')$

$$P(A) = P(A \cap B) + P(A \cap B')$$
$$P(A \cap B') = P(A) - P(A \cap B)$$

(b)

Since $S = (A \cup B) \cap (A' \cap B')$

$$1 = P(A \cap B) + P(A' \cap B')$$
$$P(A \cap B) = 1 - P(A' \cap B')$$

Solution 2: Chapter 1, Exercise 19

(a)

$$P(B') = 1 - P(B)$$
$$= 1 - \frac{1}{3}$$
$$= \frac{2}{3}$$

(b)

$$P(A \cup B') = 1 - P(B) + P(A \cap B)$$
$$= 1 - \frac{1}{3} + \frac{1}{10}$$
$$= \frac{30 - 10 + 3}{30}$$
$$= \frac{23}{30}$$

(c)

$$P(B \cup A') = P(B) - P(B \cap A)$$
$$= \frac{1}{3} - \frac{1}{10}$$
$$= \frac{10 - 3}{30}$$
$$= \frac{7}{30}$$

(d)

$$P(A' \cup B') = P(S) - P(A \cup B)$$
$$= 1 - (P(A) + P(B) - P(A \cap B))$$
$$= 1 - (\frac{1}{3} + \frac{1}{3} - \frac{1}{10})$$
$$= 1 - \frac{10 + 10 - 3}{30}$$
$$= 1 - \frac{17}{30}$$
$$= \frac{13}{30}$$

Solution 3: Chapter 1, Exercise 20

(a)

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$
$$= \frac{1}{2} + \frac{1}{8} + \frac{1}{4}$$
$$= \frac{4 + 1 + 2}{8}$$
$$= \frac{7}{8}$$

(b)

$$P(A' \cap B' \cap C') = 1 - P(A \cup B \cup C)$$
$$= 1 - \frac{7}{8}$$
$$= \frac{1}{8}$$

Solution 4: Chapter 1, Exercise 23

(a)

$$P(Both\ are\ on) = P(A \cap B)$$
$$= P(A) + P(B) - P(A \cup B)$$
$$= 0.4 + 0.3 - 0.5$$
$$= 0.2$$

(b)

$$P(Color\ set\ on\ and\ other\ off) = P(A \cap B')$$
$$= P(A) - P(A \cap B)$$
$$= 0.4 - 0.2$$
$$= 0.2$$

(c)

$$P(Exactly\ one\ is\ on) = P((A \cap B') \cup (B \cap A'))$$
$$= P(A \cap B') + P(B \cap A')$$
$$= 0.2 + (P(B) - P(B \cap A))$$
$$= 0.2 + (0.3 - 0.2)$$
$$= 0.3$$

(d)

$$P(Neither\ set\ is\ on) = P(A' \cap B')$$
$$= 1 - P(A \cup B)$$
$$= 1 - 0.5$$
$$= 0.5$$

Solution 5: Chapter 1, Exercise 25

(a)

$$P(A\ good) = \frac{3}{5}$$

(b)

$$P(B\ good|A\ good) = \frac{P(A\ good \cap B\ good)}{P(A\ good)}$$
$$= \frac{(3 \cdot 2)/(5 \cdot 4)}{(3/5)}$$
$$= \frac{1}{2}$$

(c)

$$P(B\ good|A\ bad) = \frac{P(A\ good \cap B\ bad)}{P(A\ bad)}$$
$$= \frac{(3 \cdot 2)/(5 \cdot 4)}{(2 \cdot 1)/(5 \cdot 4) + (2 \cdot 3)/(5 \cdot 4)}$$
$$= \frac{3}{4}$$

(d)

$$P(B\ good \cap A\ good) = P(A\ good)P(B\ good|A\ good)$$
$$= \frac{3}{5} \cdot \frac{1}{2}$$
$$= \frac{3}{10}$$

(e)

	A good	A bad	
B good	3 . 2	2 . 3	5 . 3
B bad	3 . 2	2 . 1	5 . 2
	3 . 4	2 . 1	5 . 4

$$P(B\ good \cap A\ good) = \frac{3 \cdot 2}{5 \cdot 4}$$
$$= \frac{3}{10}$$

$$P(B\ good|A\ good) = \frac{3 \cdot 2}{3 \cdot 4}$$
$$= \frac{1}{2}$$

(f)

$$P(B\ good) = P(B\ good \cap A\ good) + P(B\ good \cap A\ bad)$$
$$= \frac{3 \cdot 2}{5 \cdot 4} + \frac{3 \cdot 2}{5 \cdot 4}$$
$$= \frac{3}{5}$$

(g)

$$P(A\ good|B\ good) = \frac{P(A\ good)P(B\ good|A\ good)}{P(B\ good)}$$
$$= \frac{(3/5) \cdot (1/2)}{(3/5)}$$
$$= \frac{1}{2}$$

Solution 6: Chapter 1, Exercise 32

(a)

$$P(team\ wins\ game) = P(W) = P(A) \cdot (0.4) + P(B) \cdot (0.6) + P(C) \cdot (0.8)$$
$$= (0.2) \cdot (0.4) + (0.3) \cdot (0.6) + (0.5) \cdot (0.8)$$
$$= 0.66$$

(b)

$$P(A \text{ } pitched \text{ } game | team \text{ } wins \text{ } game) = P(A|W) = \frac{P(W|A)P(A)}{P(W)}$$
$$= \frac{(0.4) \cdot (0.2)}{0.66}$$
$$= \frac{4}{33}$$

Solution 7: Chapter 1, Exercise 37

(a)

If A and B are mutually exclusive,

$$P(A \cup B) = P(A) + P(B)$$
$$P(B) = P(A \cup B) - P(A)$$
$$= 0.6 - 0.4$$
$$= 0.2$$

(b)

If A and B are independent,

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \cap B) = P(A)P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A)P(B)$$
$$0.6 = 0.4 + P(B) - (0.4)P(B)$$
$$0.2 = (0.6)P(B)$$
$$P(B) = \frac{1}{3}$$

Solution 8: Chapter 1, Exercise 46

(a)

If A, B and C are mutually exclusive,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$
$$= \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$$
$$= \frac{20 + 15 + 12}{60}$$
$$= \frac{47}{60}$$

(b)

If A, B, and C are independent,

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

$$P(A \cap B) = P(A)P(B) = (1/3) \cdot (1/4) = \frac{1}{12}$$

$$P(A \cap C) = P(A)P(C) = (1/3) \cdot (1/5) = \frac{1}{15}$$

$$P(B \cap C) = P(B)P(C) = (1/4) \cdot (1/5) = \frac{1}{20}$$

$$P(A \cap B \cap C) = P(A)P(B)P(C) = (1/3) \cdot (1/4) \cdot (1/5) = \frac{1}{60}$$

$$P(A \cup B \cup C) = \frac{1}{3} + \frac{1}{4} + \frac{1}{5} - \frac{1}{12} - \frac{1}{15} - \frac{1}{20} + \frac{1}{60}$$
$$= \frac{20 + 15 + 12 - 5 - 4 - 3 + 1}{60}$$
$$= \frac{36}{60}$$
$$= \frac{3}{5}$$

Solution 9:   Chapter 1, Exercise 41

Let A be the event that the first system "fails" and B be the event that the second system "fails".

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Let A_1 and A_2 be the events that the components of the first system "fails".

$$P(A) = P(A_1)P(A_2)$$
$$= (0.1) \cdot (0.2)$$
$$= 0.02$$

Let B_1, B_2 and B_3 be the events that the components of the second system "fails".

$$P(B) = P(B_1)P(B_2)P(B_3)$$
$$= (0.1) \cdot (0.2) \cdot (0.3)$$
$$= 0.006$$

$$P(A \cup B) = 0.02 + 0.006 - (0.02)(0.006)$$
$$= 0.02588$$

The probability the system does not malfunction is $1 - 0.02588 = 0.97412$.