STAT 7750: Solutions to homework set 3

Chanmann Lim

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Solution 1: Chapter 2, Exercise 3

(a)

$$f(x) = P[X = x|normal\ coin] + P[X = x|2heads\ coin]$$

$$\begin{aligned}
P[X = 0|normal\ coin] &= \binom{3}{0} \cdot \frac{1}{8} \cdot \frac{2}{3} = \frac{1}{12} \\
P[X = 1|normal\ coin] &= \binom{3}{1} \cdot \frac{1}{8} \cdot \frac{2}{3} = \frac{1}{4} \\
P[X = 2|normal\ coin] &= \binom{3}{2} \cdot \frac{1}{8} \cdot \frac{2}{3} = \frac{1}{4} \\
P[X = 3|normal\ coin] &= \binom{3}{3} \cdot \frac{1}{8} \cdot \frac{2}{3} = \frac{1}{12}
\end{aligned}$$

$$P[X = 0|2heads\ coin] = P[X = 1|2heads\ coin] = P[X = 2|2heads\ coin] = 0$$

$$P[X = 3|2heads\ coin] = \binom{3}{3} \cdot (1) \cdot \frac{1}{3} = \frac{1}{3}$$

x	0	1	2	3
$f(x)$	$1/12$	$1/4$	$1/4$	$5/12$

(b)

Solution 2: Chapter 2, Exercise 8

(a)

$$\begin{aligned}
f(x) &= F(x) - F(x-1) \\
&= 1 - (1/2)^{x+1} - \left(1 - (1/2)^{x+1-1}\right) \\
&= (1/2)^x - (1/2)^{x+1} \\
&= 2 \times (1/2)^{x+1} - (1/2)^{x+1} \\
&= (1/2)^{x+1}
\end{aligned}$$

(b)

$$\begin{aligned} P[10 < x \leq 20] &= F(20) - F(10) \\ &= 1 - \left(\frac{1}{2}\right)^{20+1} - \left(1 - \left(\frac{1}{2}\right)^{10+1}\right) \\ &= -\left(\frac{1}{2}\right)^{21} + \left(\frac{1}{2}\right)^{11} \\ &= \frac{2^{10} - 1}{2^{21}} \\ &= 0.0004878 \end{aligned}$$

(c)

Solution 3: Chapter 2, Exercise 11

$$X = \textit{the amount received}$$

$$EX = \frac{1 + 2 + 3 + 4 + 5 + 6}{6} = 3.5$$

The amount the player should pay for rolling to make it a fair game is \$3.5.