STAT 7750: Solutions to homework set 2

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Solution 1: Chapter 1, Exercise 18

(a)

Since $A \cap B$ and $A \cap B'$ are mutually exclusive, and $A = (A \cap B) \cap (A \cap B')$

$$\begin{aligned} P(A) &= P(A \cap B) + P(A \cap B') \\ P(A \cap B') &= P(A) - P(A \cap B) \end{aligned}$$

(b)

Since $S = (A \cup B) \cap (A' \cap B')$

$$\begin{aligned} 1 &= P(A \cap B) + P(A' \cap B') \\ P(A \cap B) &= 1 - P(A' \cap B') \end{aligned}$$

Solution 2: Chapter 1, Exercise 19

(a)

$$\begin{aligned} P(B') &= 1 - P(B) \\ &= 1 - \frac{1}{3} \\ &= \frac{2}{3} \end{aligned}$$

(b)

$$\begin{aligned} P(A \cup B') &= 1 - P(B) + P(A \cap B) \\ &= 1 - \frac{1}{3} + \frac{1}{10} \\ &= \frac{30 - 10 + 3}{30} \\ &= \frac{23}{30} \end{aligned}$$

(c)

$$\begin{aligned} P(B \cup A') &= P(B) - P(B \cap A) \\ &= \frac{1}{3} - \frac{1}{10} \\ &= \frac{10-3}{30} \\ &= \frac{7}{30} \end{aligned}$$

(d)

$$\begin{aligned} P(A' \cup B') &= P(S) - P(A \cup B) \\ &= 1 - (P(A) + P(B) - P(A \cap B)) \\ &= 1 - (\frac{1}{3} + \frac{1}{3} - \frac{1}{10}) \\ &= 1 - \frac{10+10-3}{30} \\ &= 1 - \frac{17}{30} \\ &= \frac{13}{30} \end{aligned}$$

Solution 3: Chapter 1, Exercise 20

(a)

$$\begin{aligned} P(A \cup B \cup C) &= P(A) + P(B) + P(C) \\ &= \frac{1}{2} + \frac{1}{8} + \frac{1}{4} \\ &= \frac{4+1+2}{8} \\ &= \frac{7}{8} \end{aligned}$$

(b)

$$\begin{aligned} P(A' \cap B' \cap C') &= 1 - P(A \cup B \cup C) \\ &= 1 - \frac{7}{8} \\ &= \frac{1}{/}8 \end{aligned}$$

Solution 4: Chapter 1, Exercise 23

(a)

$$\begin{aligned} P(\text{"}BothAreOn\text{"}) &= P(A \cap B) \\ &= P(A) + P(B) - P(A \cup B) \\ &= 0.4 + 0.3 - 0.5 \\ &= 0.2 \end{aligned}$$

(b)

$$\begin{aligned} P(\text{"}ColorSetOnAndOtherOff\text{"}) &= P(A \cap B') \\ &= P(A) - P(A \cap B) \\ &= 0.4 - 0.2 \\ &= 0.2 \end{aligned}$$

(c)

$$\begin{aligned} P(\text{"}ExactlyOneIsOn\text{"}) &= P((A \cap B') \cup (B \cap A')) \\ &= P(A \cap B') + P(B \cap A') \\ &= 0.2 + (P(B) - P(B \cap A)) \\ &= 0.2 + (0.3 - 0.2) \\ &= 0.3 \end{aligned}$$

(d)

$$\begin{aligned} P(\text{"}NeitherSetIsOn\text{"}) &= P(A' \cap B') \\ &= 1 - P(A \cup B) \\ &= 1 - 0.5 \\ &= 0.5 \end{aligned}$$

Solution 5: Chapter 1, Exercise 25

Solution 6: Chapter 1, Exercise 32

Solution 7: Chapter 1, Exercise 37

Solution 8: Chapter 1, Exercise 46

Solution 9: Chapter 1, Exercise 41