STAT 7750: Solutions to homework set 2

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$$\begin{bmatrix} x & y \\ z & v \end{bmatrix}$$

Solution 1: Chapter 1, Exercise 18

(a)

Since $A \cap B$ and $A \cap B'$ are mutually exclusive, and $A = (A \cap B) \cap (A \cap B')$

$$P(A) = P(A \cap B) + P(A \cap B')$$
$$P(A \cap B') = P(A) - P(A \cap B)$$

(b)

Since $S = (A \cup B) \cap (A' \cap B')$

$$1 = P(A \cap B) + P(A' \cap B')$$
$$P(A \cap B) = 1 - P(A' \cap B')$$

Solution 2: Chapter 1, Exercise 19

(a)

$$\begin{aligned} P(B') &= 1 - P(B) \\ &= 1 - \frac{1}{3} \\ &= \frac{2}{3} \end{aligned}$$

(b)

$$\begin{aligned} P(A \cup B') &= 1 - P(B) + P(A \cap B) \\ &= 1 - \frac{1}{3} + \frac{1}{10} \\ &= \frac{30 - 10 + 3}{30} \\ &= \frac{23}{30} \end{aligned}$$

(c)

$$\begin{aligned}
P(B \cup A') &= P(B) - P(B \cap A) \\
&= \frac{1}{3} - \frac{1}{10} \\
&= \frac{10 - 3}{30} \\
&= \frac{7}{30}
\end{aligned}$$

(d)

$$\begin{aligned}
P(A' \cup B') &= P(S) - P(A \cup B) \\
&= 1 - (P(A) + P(B) - P(A \cap B)) \\
&= 1 - (\frac{1}{3} + \frac{1}{3} - \frac{1}{10}) \\
&= 1 - \frac{10 + 10 - 3}{30} \\
&= 1 - \frac{17}{30} \\
&= \frac{13}{30}
\end{aligned}$$

Solution 3: Chapter 1, Exercise 20

(a)

$$\begin{aligned}
P(A \cup B \cup C) &= P(A) + P(B) + P(C) \\
&= \frac{1}{2} + \frac{1}{8} + \frac{1}{4} \\
&= \frac{4 + 1 + 2}{8} \\
&= \frac{7}{8}
\end{aligned}$$

(b)

$$\begin{aligned}
P(A' \cap B' \cap C') &= 1 - P(A \cup B \cup C) \\
&= 1 - \frac{7}{8} \\
&= \frac{1}{8}
\end{aligned}$$

Solution 4: Chapter 1, Exercise 23

(a)

$$\begin{aligned}
P(Both\ are\ on) &= P(A \cap B) \\
&= P(A) + P(B) - P(A \cup B) \\
&= 0.4 + 0.3 - 0.5 \\
&= 0.2
\end{aligned}$$

(b)

$$\begin{aligned}P(Color\ set\ on\ and\ other\ off) &= P(A \cap B') \\ &= P(A) - P(A \cap B) \\ &= 0.4 - 0.2 \\ &= 0.2\end{aligned}$$

(c)

$$\begin{aligned}P(Exactly\ one\ is\ on) &= P((A \cap B') \cup (B \cap A')) \\ &= P(A \cap B') + P(B \cap A') \\ &= 0.2 + (P(B) - P(B \cap A)) \\ &= 0.2 + (0.3 - 0.2) \\ &= 0.3\end{aligned}$$

(d)

$$\begin{aligned}P(Neither\ set\ is\ on) &= P(A' \cap B') \\ &= 1 - P(A \cup B) \\ &= 1 - 0.5 \\ &= 0.5\end{aligned}$$

Solution 5: Chapter 1, Exercise 25

(a)

$$P(A\ good) = \frac{3}{5}$$

(b)

$$\begin{aligned}P(B\ good|A\ good) &= \frac{P(A\ good \cap B\ good)}{P(A\ good)} \\ &= \frac{(3 \cdot 2)/(5 \cdot 4)}{(3/5)} \\ &= \frac{1}{2}\end{aligned}$$

(c)

$$\begin{aligned}P(B\ good|A\ bad) &= \frac{P(A\ good \cap B\ bad)}{P(A\ bad)} \\ &= \frac{(3 \cdot 2)/(5 \cdot 4)}{(2 \cdot 1)/(5 \cdot 4) + (2 \cdot 3)/(5 \cdot 4)} \\ &= \frac{3}{4}\end{aligned}$$

(d)

$$\begin{aligned}
P(B\ good \cap A\ good) &= P(A\ good)P(B\ good|A\ good) \\
&= \frac{3}{5} \cdot \frac{1}{2} \\
&= \frac{3}{10}
\end{aligned}$$

(e)

	A good	A bad	
B good	3 . 2	2 . 3	5 . 3
B bad	3 . 2	2 . 1	5 . 2
	3 . 4	2 . 1	5 . 4

$$\begin{aligned}
P(B\ good \cap A\ good) &= \frac{3 \cdot 2}{5 \cdot 4} \\
&= \frac{3}{10}
\end{aligned}$$

$$\begin{aligned}
P(B\ good|A\ good) &= \frac{3 \cdot 2}{3 \cdot 4} \\
&= \frac{1}{2}
\end{aligned}$$

(f)

$$\begin{aligned}
P(B\ good) &= P(B\ good \cap A\ good) + P(B\ good \cap A\ bad) \\
&= \frac{3 \cdot 2}{5 \cdot 4} + \frac{3 \cdot 2}{5 \cdot 4} \\
&= \frac{3}{5}
\end{aligned}$$

(g)

$$\begin{aligned}
P(A\ good|B\ good) &= \frac{P(A\ good)P(B\ good|A\ good)}{P(B\ good)} \\
&= \frac{(3/5) \cdot (1/2)}{(3/5)} \\
&= \frac{1}{2}
\end{aligned}$$

Solution 6: Chapter 1, Exercise 32

(a)

$$\begin{aligned}
P(team\ wins\ game) = P(W) &= P(A) \cdot (0.4) + P(B) \cdot (0.6) + P(C) \cdot (0.8) \\
&= (0.2) \cdot (0.4) + (0.3) \cdot (0.6) + (0.5) \cdot (0.8) \\
&= 0.66
\end{aligned}$$

(b)

$$\begin{aligned} P(A\, pitched\ game|team\ wins\ game) = P(A|W) &= \frac{P(W|A)P(A)}{P(W)} \\ &= \frac{(0.4)\cdot(0.2)}{0.66} \\ &= \frac{4}{33} \end{aligned}$$

Solution 7: Chapter 1, Exercise 37

(a)

If A and B are mutually exclusive,

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) \\ P(B) &= P(A \cup B) - P(A) \\ &= 0.6 - 0.4 \\ &= 0.2 \end{aligned}$$

(b)

If A and B are independent,

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A \cap B) \\ P(A \cap B) &= P(A)P(B) \end{aligned}$$

$$\begin{aligned} P(A \cup B) &= P(A) + P(B) - P(A)P(B) \\ 0.6 &= 0.4 + P(B) - (0.4)P(B) \\ 0.2 &= (0.6)P(B) \\ P(B) &= \frac{1}{3} \end{aligned}$$

Solution 8: Chapter 1, Exercise 46

(a)

If A, B and C are mutually exclusive,

$$\begin{aligned} P(A \cup B \cup C) &= P(A) + P(B) + P(C) \\ &= \frac{1}{3} + \frac{1}{4} + \frac{1}{5} \\ &= \frac{20 + 15 + 12}{60} \\ &= \frac{47}{60} \end{aligned}$$

(b)

If A, B, and C are independent,

$$
\begin{aligned}
P(A \cup B \cup C) &= P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C) \\
P(A \cap B) &= P(A)P(B) = (1/3) \cdot (1/4) = \frac{1}{12} \\
P(A \cap C) &= P(A)P(C) = (1/3) \cdot (1/5) = \frac{1}{15} \\
P(B \cap C) &= P(B)P(C) = (1/4) \cdot (1/5) = \frac{1}{20} \\
P(A \cap B \cap C) &= P(A)P(B)P(C) = (1/3) \cdot (1/4) \cdot (1/5) = \frac{1}{60}
\end{aligned}
$$

$$
\begin{aligned}
P(A \cup B \cup C) &= \frac{1}{3} + \frac{1}{4} + \frac{1}{5} - \frac{1}{12} - \frac{1}{15} - \frac{1}{20} + \frac{1}{60} \\
&= \frac{20 + 15 + 12 - 5 - 4 - 3 + 1}{60} \\
&= \frac{36}{60} \\
&= \frac{3}{5}
\end{aligned}
$$

Solution 9: Chapter 1, Exercise 41

Let A be the event that the first system "fails" and B be the event that the second system "fails".

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Let A_1 and A_2 be the events that the components of the first system "fails".

$$
\begin{aligned}
P(A) &= P(A_1)P(A_2) \\
&= (0.1) \cdot (0.2) \\
&= 0.02
\end{aligned}
$$

Let B_1, B_2 and B_3 be the events that the components of the second system "fails".

$$
\begin{aligned}
P(B) &= P(B_1)P(B_2)P(B_3) \\
&= (0.1) \cdot (0.2) \cdot (0.3) \\
&= 0.006
\end{aligned}
$$

$$
\begin{aligned}
P(A \cup B) &= 0.02 + 0.006 - (0.02)(0.006) \\
&= 0.02588
\end{aligned}
$$

The probability the system does not malfunction is $1 - 0.02588 = 0.97412$.