CS 8725: Report for assignment 1

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Let $\hat{\theta}$ be the estimation of the probability of the coin show up head $P(Head)$, and θ^* be the true value of $P(Head)$. If there is a constraint on the error bound ϵ to guarantee that the accuracy of the estimation is larger than $1-\delta$ where δ denotes the failure probability and from the theory of probability we understand that the accuracy is just the complement the probability of error. Then we obtain

$$P(|\hat{\theta}-\theta^*|<\epsilon) \geq 1-\delta \tag{1}$$

$$1-P(|\hat{\theta}-\theta^*|\geq\epsilon) \geq 1-\delta \tag{2}$$

$$P(|\hat{\theta}-\theta^*|\geq\epsilon) \leq \delta \tag{3}$$

According to Hoeffding's inequality we have

$$P(|\hat{\theta}-\theta^*|\geq\epsilon) \leq 2e^{-2n\epsilon^2} \tag{4}$$

And in order to guarantee that $P(|\hat{\theta}-\theta^*|\geq\epsilon)$ is always less than or equal to δ, the upper bound of the probability of error $2e^{-2n\epsilon^2}$ must be less than or equal to δ.

$$\begin{aligned}
2e^{-2n\epsilon^2} &\leq \delta \\
ln(2)-2n\epsilon^2 &\leq ln(\delta) \\
ln(2)-ln(\delta) &\leq 2n\epsilon^2 \\
ln(2/\delta) &\leq 2n\epsilon^2 \\
\frac{ln(2/\delta)}{2\epsilon^2} &\leq n
\end{aligned}$$

Therefore,

$$n \geq \frac{ln(2/\delta)}{2\epsilon^2}$$