Some Remarks on Radio-Frequency Holography

Abstract—**The extension of holographic techniques to radio frequencies offers new possibilities. The recording can be done by scanning an area with a probe. The reference wave can be replaced by a signal whose phase is varied with the probe position according to some program. Thus, "slow" reference waves ($k > k_0$) can be simulated. Complex modulation schemes can also be used. A product detector can multiply the probe and the reference signal together and the recorded product can be used in the same manner as an intensity hologram. With two such "correlators," the probe signal can be obtained in phase and magnitude. Optical reconstruction can be done from this information.**

This letter discusses some unique possibilities offered by the application and extension of holographic techniques to radio frequencies.

At optical frequencies, the recording of a hologram is done photographically. It consists of the *intensity* map of some interference field. It is recorded all at once. After development, the photographic plate, illuminated by some reference beam, is used directly in the reconstruction process.

In contrast to this situation, photographic plates sensitive to radio waves, although conceivable, are not yet available. The recording must be done a point at a time by moving a probe and detector over the surface to be explored. The record must then be converted to a variable transparency if the reconstruction is to be done optically. Compensating for this inconvenience, a number of possibilities arise due to the larger wavelength and to the existence of waveguides and other radio components.

A first observation is that, at radio frequencies, it becomes possible to produce the interference pattern without actually producing a reference plane wave, or a reference spherical wave. This can be done by adding before detection a locally produced reference signal to that picked up by the probe (Fig. 1).

In order to simulate a plane wave, the phase of the locally produced signal must vary linearly with the displacements of the probe in some direction. This can be most easily achieved by linking the motions of the probe, in that particular direction, to the rotations of a phase shifter. Realizing a true reference plane wave would not be as easy: either the source must be at large distances and it may be difficult to have enough field strength or a reflector must be used which produces an imperfect plane wave because of diffraction effects. Furthermore, it would be difficult to prevent the plane wave from illuminating the object, an unwanted effect if we wish to observe, for instance, the currents on a transmitting antenna.

The simulation of a spherical wave would be somewhat more complicated. Within the Fresnel approximation, however, the phase should be made to vary quadratically with position and this is relatively simple.

More generally, this method of using a local reference signal with a programmed phase opens the way to many other possibilities. Using a plane reference wave is sometimes compared to modulating a carrier with the signal wave. With a programmed phase, other known modulation schemes become possible.

One instance of this possibility is that a "slow wave" (k larger than the free space k_0) may be used as a reference. It would be extremely difficult to generate such a wave with reliable characteristics over a sufficient region of space. It is extremely simple to rotate the phase shifter linked to the probe position at any desired rate, thus simulating, if desired, an arbitrarily large wave vector. This makes it possible to shift the spectrum of the object, even when it occupies a wide band, enough to avoid overlap with the "image" spectrum. This is essential to separate real and virtual images in the reconstruction. Another instance is the simulation of a noise-like reference wave. The phase is made to vary, in some arbitrary manner, $\phi(x, y)$, as a function of position. Of course, for the purpose of reconstruction, the function $\phi(x, y)$ must be remembered either by being stored or generated in a well defined pseudo-random fashion.

The method can be compared to that in which some diffuse light is used as a reference wave and the *same* diffuse light is used in the reconstruction. Some advantages have been claimed for that system.

In brief, the advantages of using a programmed reference wave are: easier instrumentation even in the simplest case of a plane wave, more flexibility, and the possibility of simulating complex modulation schemes.

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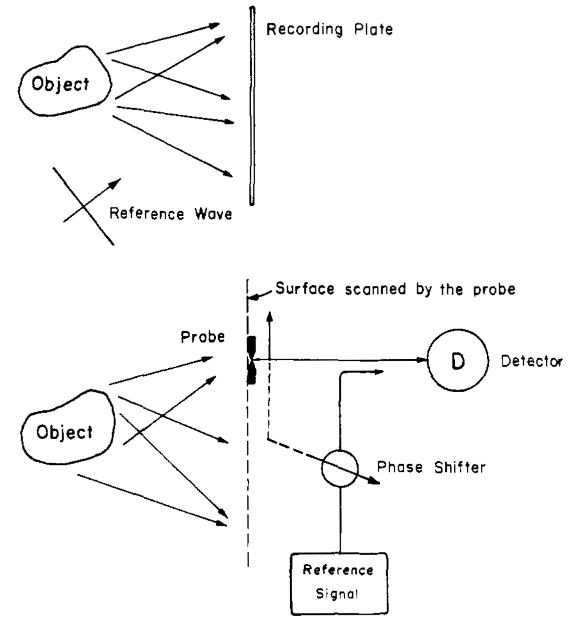

Fig. 1. Comparison of optical holography and radiowave holography with locally produced reference wave.

Another possibility which is unique to radio frequencies is that the detector does not have to be the usual intensity-measuring device or square-law detector. A *product detector* can be used instead. This is a multiplier followed by a low-pass filter. It is also called a correlator and is widely used in interferometry, and, in particular, radio astronomy.

If R and S are the two input signals, S from the object and R the reference, the output of the product detector is Re R^*S. This output recorded as a function of the probe position (x, y) will be called a *product hologram*. It can be transcribed to a photographic plate and used exactly as the intensity hologram in the reconstruction. The only difference is that the product hologram does not contain the unwanted terms $|R|^2$ and $|S|^2$. These have to be filtered out in intensity holography and, in some cases, the spectrum of $|S|^2$ could overlap those of R^*S and RS^*, resulting in distortions.

The product hologram is not always positive; it must be recorded and transcribed with its sign.

At radio frequencies, it is relatively easy to record both phase and amplitude of the field instead of just the intensity. This can be done by a combination of two product detectors, designated by the letter C in Fig. 2. One detector correlates R and S and the other, jR (R shifted in phase by 90°) and S. The two outputs are the real and imaginary parts of R^*S. Recorded as two functions, $H_1(x, y)$ and $H_2(x, y)$, they form the *complex hologram*.

For this system, we can dispense with shifting the phase of the reference signal. The function of this phase shift was only to make it possible to recover the phase after measuring only one intensity pattern. This recovery of the phase can be understood by comparison with the complex hologram: if the phase of the reference wave varies very rapidly compared to the signal S, that is, if the signal S does not change appreciably as R goes through a complete phase rotation, two intensity measurements taken at points where the phase of R differs by 90° will be equivalent to a measurement of R^*S.

The complex hologram consists of two maps $H_1(x, y) = \text{Re } R^*S$ and $H_2(x, y) = \text{Im } R^*S$ as functions of position in the plane of the record.

If we wish to reproduce S at some point, we can multiply the signal R by H_1, the signal jR by H_2, and add the results. If $|R|^2$ is taken as unity the output is precisely S (Fig. 3).

The optical reconstruction is simplified if we assume that R has a constant phase over the recording plane. Then the two optical reference waves

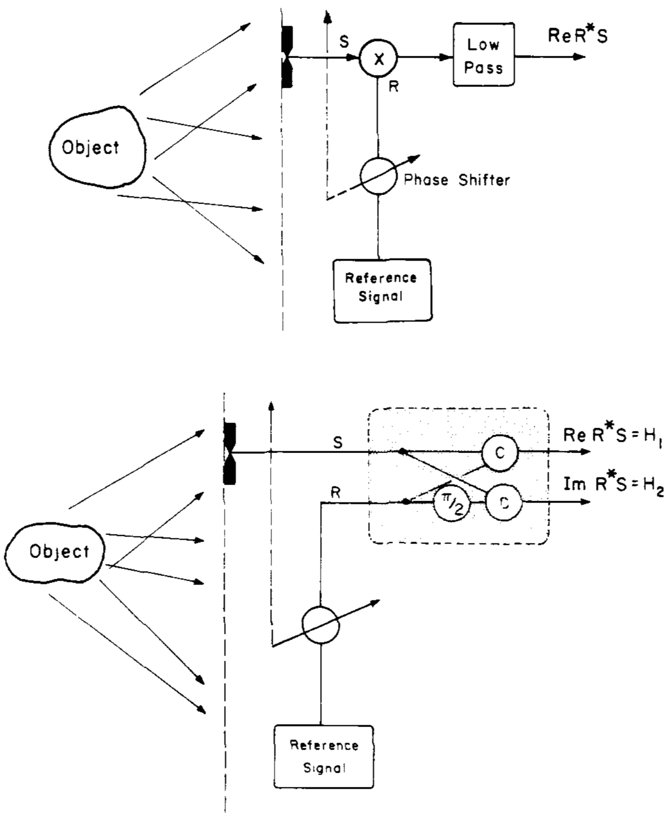

Fig. 2. Recording of a product hologram and of a complex hologram $H_1 + jH_2$.

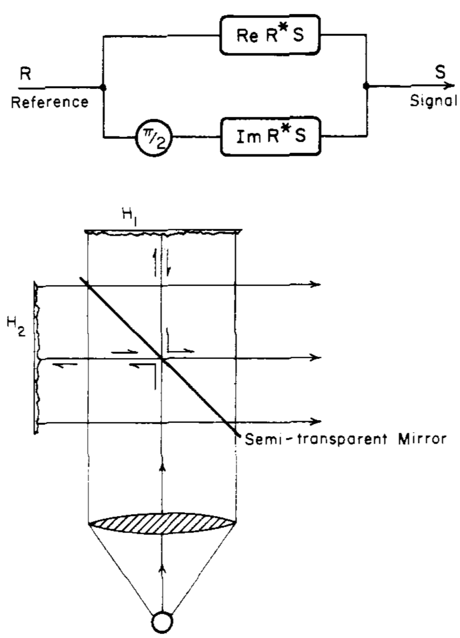

Fig. 3. Reconstruction of the field from a complex hologram.

may be a plane wave, normally incident on the transparency H_1, and the same wave shifted by a quarter-wave plate incident on the transparency H_2. The two outputs must be superimposed, which can be realized, for instance, by the arrangement shown in Fig. 3.

It is interesting to note that in contrast with the intensity hologram, only one image is produced: the virtual image. There is no need to separate the other. If we want the real image, all we need do is replace the $+90°$ phase shift by a $-90°$ phase shift. In Fig. 3, this can be done by moving one of the variable reflectivity mirrors, H_1, or H_2.

The reconstruction could also be done at microwave. Using a similar device, a field recorded at one frequency can be reconstructed at a higher frequency.

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The Induced Voltage for Conducting Ring Interaction with Axially Symmetric Magnetic Fields

Abstract—**The interaction of a conducting ring with an axially symmetric magnetic field is considered theoretically in order to derive the induced voltage expression. The results are suitable for experimental applications and may be extended to disks and slugs by computer integration.**

Lin et al.[1] have considered an axially symmetric magnetic field perturbed by a shock wave, to determine the electrical conductivity of a shock plasma. Lewis[2] used an axially symmetric magnetic field interaction to study the diffusion of shock produced plasmas in magnetic fields. Koyama et al.[3] have considered a similar interaction in an investigation of inductively coupled ac MHD converters.

Consider a fixed conducting ring (α) of radius a_1 with a current I producing a magnetic field in which a smaller conducting ring (β) of radius b moves with a constant velocity u coaxially through the first ring. Let a third conducting fixed ring (γ) of radius a_2, located at a distance d from the first ring, be considered a search coil as shown in Fig. 1. The mutual inductance between the first ring (α) and the moving ring (β) as a function of axial position, can be shown to be:[4]

$$M_1 = \frac{2\mu_0(a_1 b)^{1/2}}{k_1}\left[\left(1 - \frac{k_1^2}{2}\right)K(k_1) - E(k_1)\right] \tag{1}$$

where $K(k_1)$ and $E(k_1)$ are elliptic integrals of the first and second kind and k_1 is given by: $k_1^2 = 4a_1 b/[(a_1+b)^2 + z^2]$. Equation (1) can be expressed in series form as

$$M_1 = \pi\mu_0(a_1 b)^{1/2} \sum_1^\infty C_i k_1^{2i+1} \tag{2}$$

where

$$C_1 = 1/16 \quad \text{and} \quad C_{i+1} = \frac{(2i+1)^2}{4i(i+2)}. \tag{3}$$

Assuming a small magnetic Reynold's number, the induced voltage e in the search coil having radius a_2 is given by

$$e = \frac{\sigma A u^2 I}{2\pi b}\left[\frac{dM_2}{dz}\frac{dM_1}{dz} + M_2\frac{d^2 M_1}{dz^2}\right] \tag{4}$$

where M_2 is the mutual inductance between the moving ring and the search coil given by

$$M_2 = \pi\mu_0(a_2 b)^{1/2} \sum_1^\infty C_i k_2^{2i+1} \tag{5}$$

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