RECURSION AND BACKTRACKING

Introduction to Recursion

Recursion → Essey function ko recursion bolty hai! jo kudhi ko call krty hai!

Math	Java
$f(x) = x^2$	`p s v fun (int x){`
	`return x*x;`
$f(2) = 2^2 = 4$	`}`
	`int i = fun(2);`

// Programming langauge me algebra ko expresss krne ke tool ko function kahty hai!

Principle of Mathematical Induction

$$\boxed{\sum_{i=1}^{n} i = \frac{n(n+1)}{2}}$$

Let us Assume the formula is true for $n=k$

We need to prove $n=k+1$

- checking for $n=1 \rightarrow \boxed{LHS = RHS}$
- Assuming that formula is true for $n=k$.

$$\therefore \boxed{\sum_{i=1}^{k} i = \frac{k(k+1)}{2}}$$

- To prove that formula is true for $n=k+1$.

$$\sum_{i=1}^{k+1} i = \frac{(k+1)((k+1)+1)}{2} = \frac{(k+1)(k+2)}{2} = RHS$$

$$\sum_{i=1}^{k+1} i = \underbrace{1+2+3+\ldots+k}_{\sum_{i=1}^{k} i} + k+1 = \frac{k(k+1)}{2} + (k+1) \Rightarrow \frac{k(k+1)+2(k+1)}{2} = LHS$$

$$\therefore \boxed{RHS = LHS}$$

So, $\boxed{n=1}$ ke liye maine proff krida → $\boxed{n=k}$ ke liye Assume kreia. jisko use krke $\boxed{n=k+1}$ ko bhi proff krdia!

↓

for all natural numbers

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

PRINT Decreasing

Input : 5 → PD(5)
Output : 5
4
3
2
1

Expectation
↳ Yeh chaiye hume!

Faith → bina ek bhi line likhe mai yeh Faith rakhunga PD yeh assume krunga already chalna janta hai 5 ke liye nahi isse choti value ke liye!

PD(4) = 4
3
2
1

LINK

AGAR PD(4) apna kaam krna janta hai!
toh,
PD(5) ko PD(4) (Faith) ki terms (Expectation) express kro!

∴ PD(5) = 5 print krke PD(4) ko invoke krdey!
Kaam hoyega!

High-level Thinking

- Establise the expectation.
- Establise the Faith.
- LINK Expectation and Faith (LINK)

GENERLIZATION

PD(n) = n
n−1
⋮
3
2
1

EXPECTATION

PD(n−1) = n−1
n−2
⋮
3
2
1

FAITH

$\boxed{P(n) = n \quad PD(n-1)}$

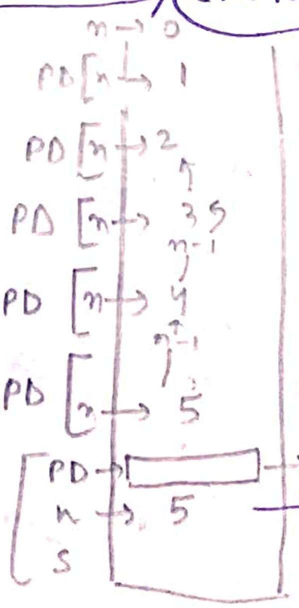
LINKED

Code

```
p s v m (S[] a) {
scanner s = new Scanner (System.in);
int n = s.nextInt();
printDecreasing (n);
}

p s v printDecreasing (int n){
if (n==0) { //BASE CASE
return;
}
Syso(n); // Expectation meeting Faith
printDecreasing(n-1); //Faith
}
```

↓ Recursive Call

CONSOLE : 5
4
3
2
1



isske baad rokhna hoga isko. Agar n=0 ajaye toh kuch na karey laut jaye!

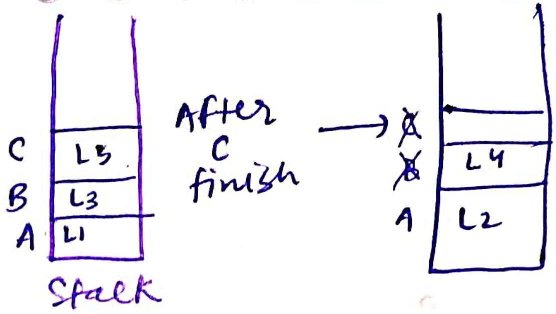
Pheli ⊙ pheli line
Dusri ⊙ dusri line

PD func invoke hogaya!

user se input

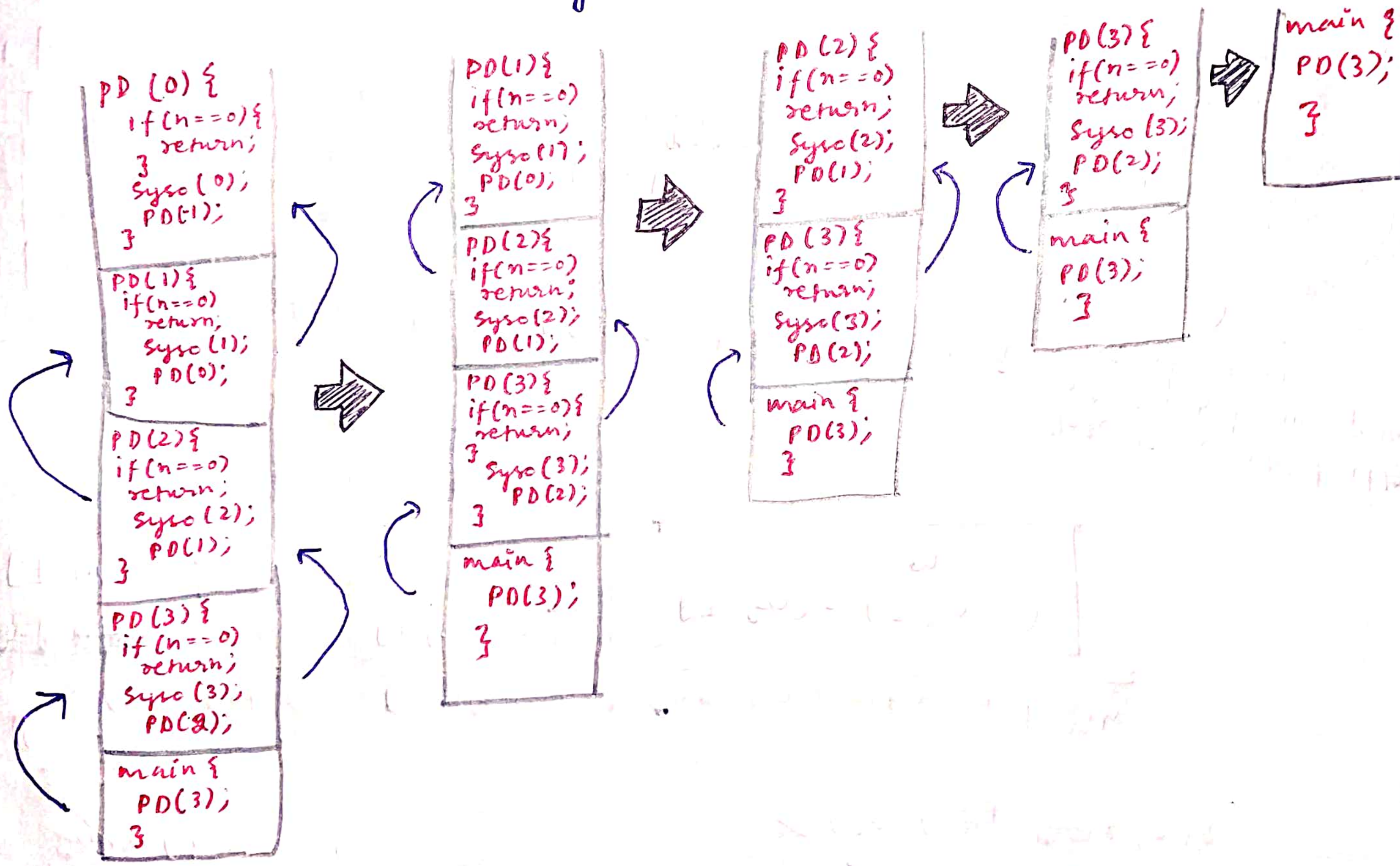
Low-Level Thinking

```
A(){
  L1 ✓
  B() ✓
  L2
}

B(){
  L3 ✓
  C() ✓
  L4
}

C(){
  ✓ L5
}
```



APPROACH

- High-level Thinking
  - Expectation
  - Faith
  - Metting Expectation with Faith
- Low-level Thinking — Dry run and draw Stack Frame



Time-complexity : $\underline{O(n)}$

Space-complexity : $\underline{O(1)}$