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Homework

Geometric Modeling
November 4, 2019

Subdivision

The techniques to create curves and surfaces through a process of subdi vision are relatively recent. Informally, subdivision surfaces can be thought of as a polygon mesh plus a set of rules that enable it to be refined (through subdivision) to better approximate the surface it represents. In this way sub division surfaces have the same properties as polygon meshes while alleviating the problem of discrete approximation. This motivates the growing interest in subdivision methods in geometric modeling, subdivision bridges the gap be tween discrete and continuous representations of curves and surfaces. In the next section, as for the description of parametric surfaces, we begin to present an example of curves generation through subdivision, and then we describe subdivision surfaces.

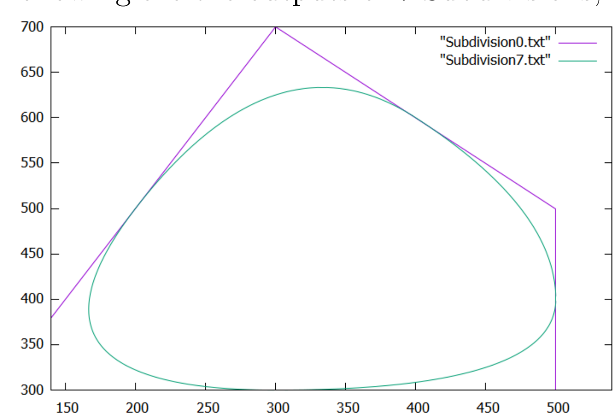
Chaikin's Algorithm

The Chaikin's algorithm is a method to generate curves starting from a set of control points by means of subdivision. Let us asaume tat the initial curve P^0 is represented by the sequence of vertices $\{p_1^p, p_2^0, \ldots, p_n^0\}$ (the superseript of the points indicates the level of subdivision of the curve). The zero level corresponds to the origina leotitrel potygen. At each pair of consecutive ones using scheme creates two new vertices between each pair of consecutive ones using the following subdivision rules:

$$q_{2i}^{k+1} = \frac{3}{4}p_i^k + \frac{1}{4}p_{i+1}^k$$
$$q_{2i+1}^{k+1} = \frac{1}{4}p_i^k + \frac{3}{4}p_{i+1}^k$$

where q_i^{k+1} are the new generated points at level of subdivision $k+1$. After the generation of the new points the old vertices are discarded and only the new points q_i^{k+1} define the curve at level $k+1$.

Following are the outputs of 7 Subdivisions, Distance and Curvature plots.




```
Distance Subdivision: 111.803399
Distance Subdivision: 55.901699
Distance Subdivision: 27.950850
Distance Subdivision: 13.975425
Distance Subdivision: 6.987712
Distance Subdivision: 3.493856
Distance Subdivision: 1.746928
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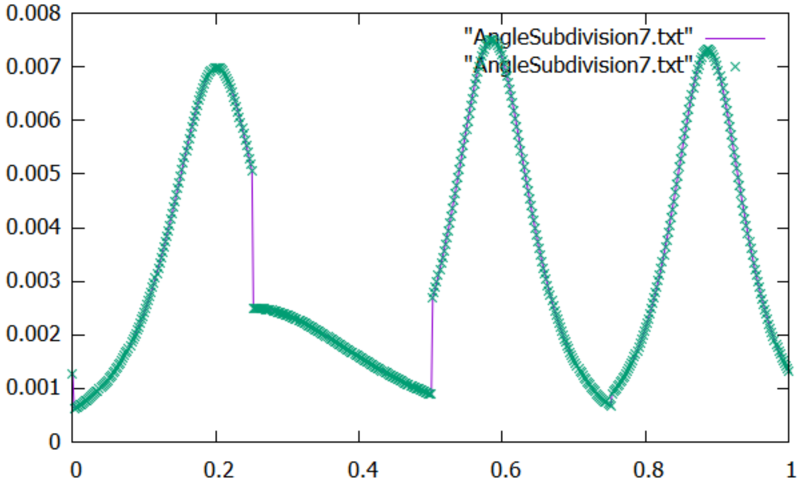


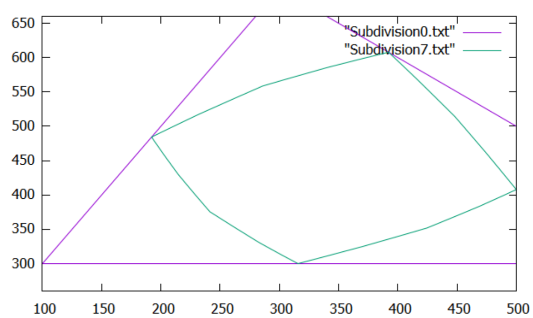
The above plot shows the limit of the curve is C1 and not C2.

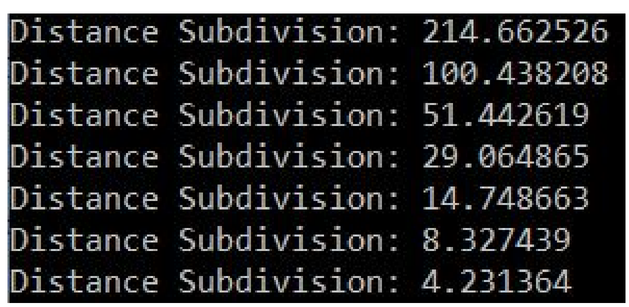
Corner Cutting Algorithm

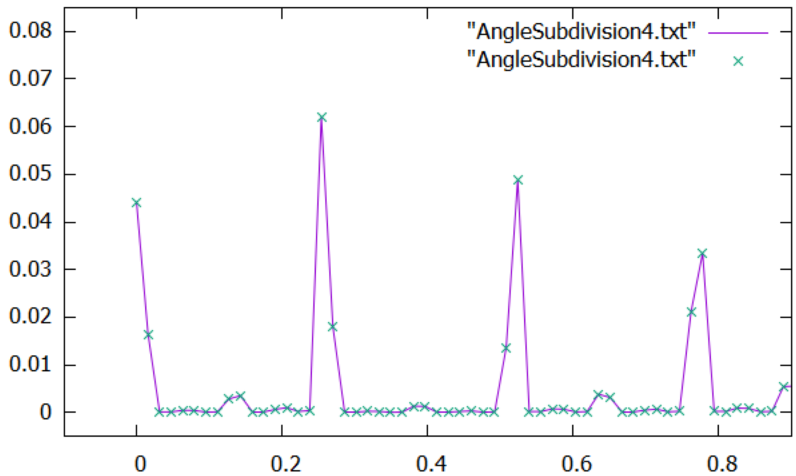
Cornor cutting Algorithm is a generalization of Chaikin's algorithm with two parameters $0 < a < b < 1$.

$\mathbf{x}_{2i}^{k+1} = (1-a)\mathbf{x}_i^k + a\mathbf{x}_{i+1}^k$
$\mathbf{x}_{2i+1}^{k+1} = (1-b)\mathbf{x}_i^k + b\mathbf{x}_{i+1}^k$
Following are the outputs of the Subdivisions, Distance and Curvature plots.

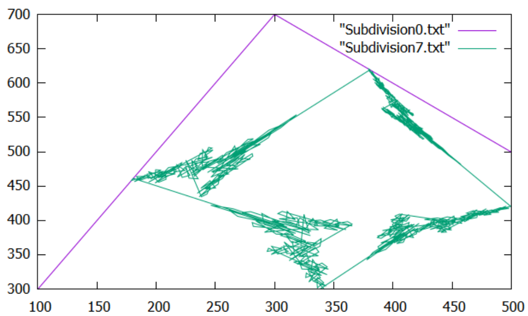
For **a = 0.52 and b = 0.41**:

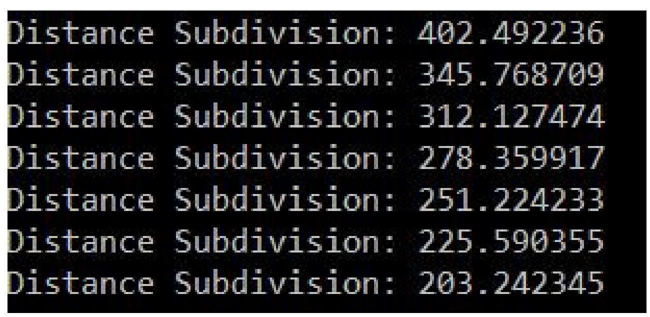


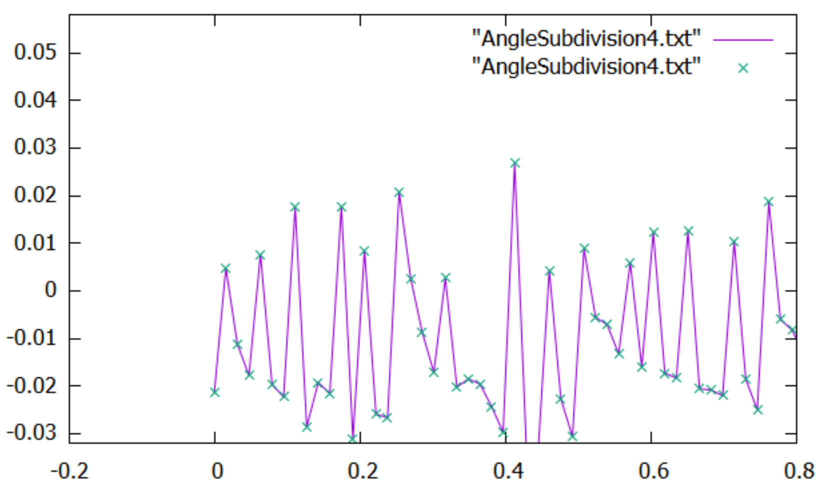

Distance Subdivision: 214.662526
Distance Subdivision: 100.438208
Distance Subdivision: 51.442619
Distance Subdivision: 29.064865
Distance Subdivision: 14.748663
Distance Subdivision: 8.327439
Distance Subdivision: 4.231364



For **a = 0.1 and b = 0.6 i.e, b = a +1/2** :

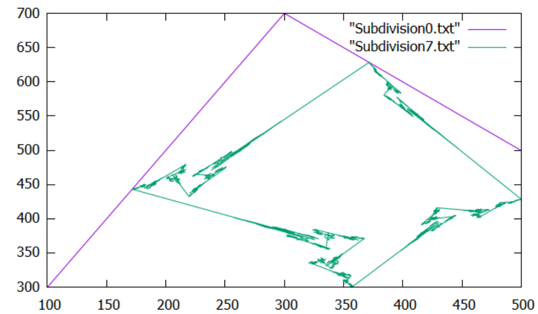


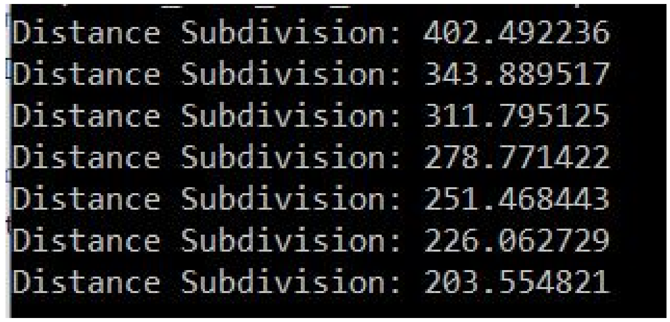




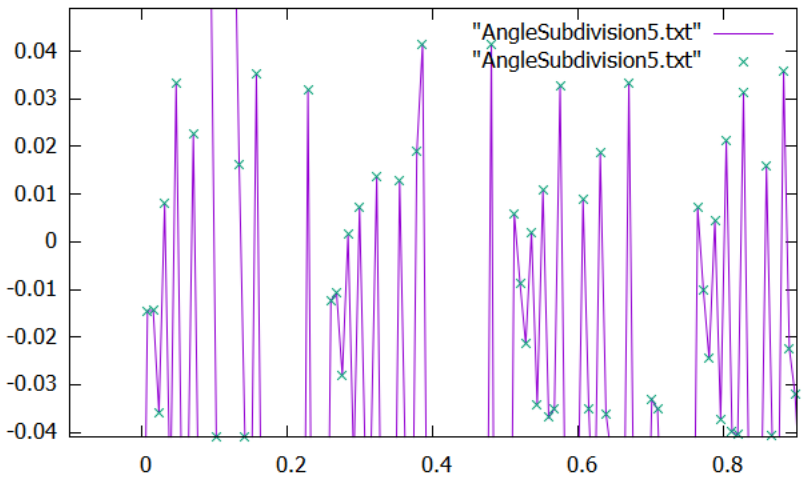
The above plot shows the limit of the curve is C0 not C1 since discontinuities.

For **a = 0.1 and b = 0.5 i.e, b != a +1/2** :





Distance Subdivision: 402.492236
Distance Subdivision: 343.889517
Distance Subdivision: 311.795125
Distance Subdivision: 278.771422
Distance Subdivision: 251.468443
Distance Subdivision: 226.062729
Distance Subdivision: 203.554821



The above plot shows the limit of the curve is C0. For values a = 0.25 and b = 0.75 we get the same as chaikin's output.

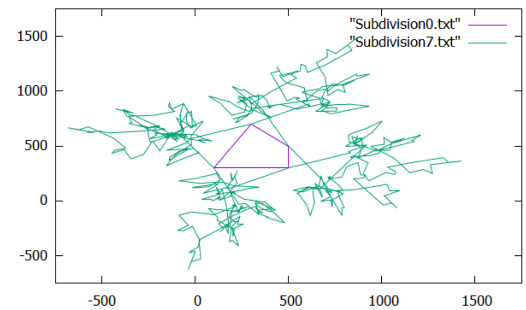
Four point and generalized four point Algorithm

Following equations were used to perform four point algorithm. Here **w** value is different for Generalized four point and normal four point algorithm.

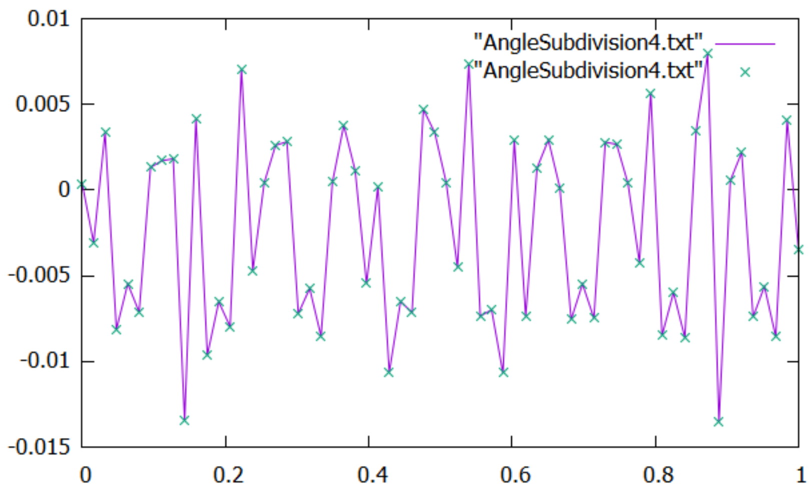
$$p_{2i}^{k+1} = p_i^k$$

$$p_{2i+1}^{k+1} = \left(p_i^k + p_{i+1}^k\right)(w + 1/2) - w\left(p_{i-1}^k + p_{i+2}^k\right)$$

For $\mathbf{w} = \sqrt{5}+1)/8 + \sqrt{5}-1)/16$ i.e, for **four point algorithm**, following are the outputs of the Subdivisions, Distance and Curvature plots.

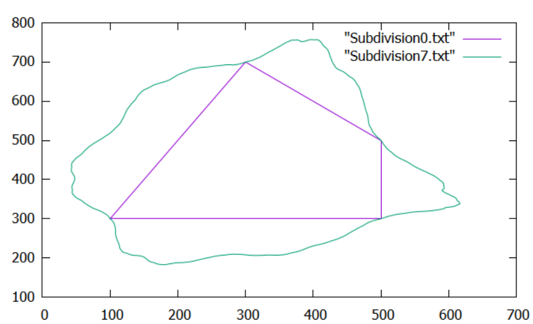



```
Distance Subdivision: 304.693508
Distance Subdivision: 301.361490
Distance Subdivision: 240.137975
Distance Subdivision: 223.253446
Distance Subdivision: 209.917437
Distance Subdivision: 185.885830
Distance Subdivision: 183.040970
```

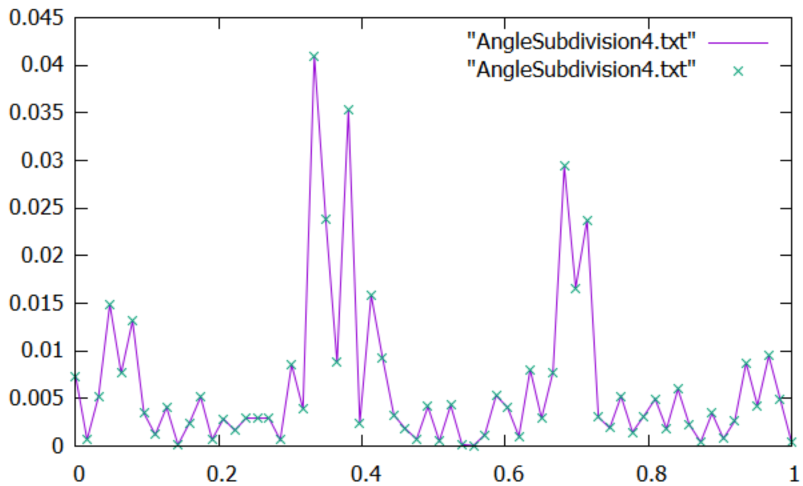


The above plot shows the limit of the curve is C0.

For $\mathbf{w} = \sqrt{5}-1)/8$ i.e, for **Generalized four point algorithm**, we observe fractal curve, following are the outputs of the Subdivisions, Distance and Curvature plots.



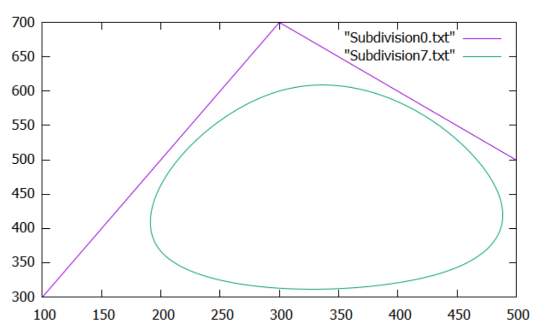

```
Distance Subdivision: 97.719756
Distance Subdivision: 62.084542
Distance Subdivision: 21.946785
Distance Subdivision: 9.940200
Distance Subdivision: 4.357569
Distance Subdivision: 1.947912
Distance Subdivision: 0.828571
```

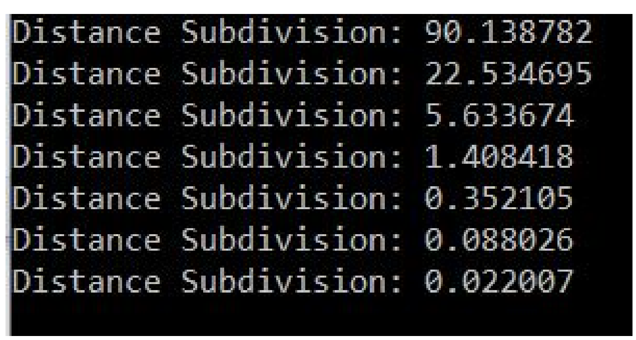


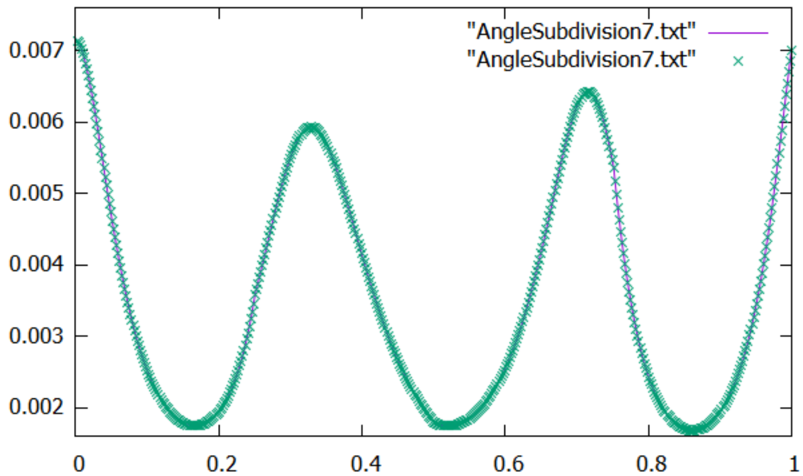
The above plot looks like C1. The curve is fractal for this specific w value.

Uniform splines of degree 3

Following equations were used to perform Uniform splines of degree 3.
$\mathbf{x}_{2i}^{k+1} = 1/8\mathbf{x}_{i-1}^{k} + 6/8x_{i1}^{k} + 1/8x_{i+1}^{k}$
$\mathbf{x}_{2i+1}^{k+1} = 1/2x_{i1}^{k} + 1/2x_{i+1}^{k}$
Following are the outputs of the Subdivisions, Distance and Curvature plots.







It can be differentiable twice therefore C2 is the limit of the curve.