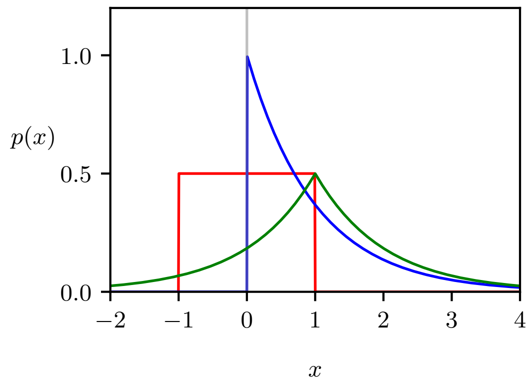

Figure 2.7 Plots of a uniform distribution over the range $(-1, 1)$, shown in red, the exponential distribution with $\lambda = 1$, shown in blue, and a Laplace distribution with $\mu = 1$ and $\gamma = 1$, shown in green.

Another simple form of density is the *exponential distribution* given by

$$p(x|\lambda) = \lambda \exp(-\lambda x), \qquad x \geqslant 0. \tag{2.34}$$

A variant of the exponential distribution, known as the *Laplace distribution*, allows the peak to be moved to a location μ and is given by

$$p(x|\mu, \gamma) = \frac{1}{2\gamma} \exp\left(-\frac{|x-\mu|}{\gamma}\right). \tag{2.35}$$

The constant, exponential, and Laplace distributions are illustrated in Figure 2.7.

Another important distribution is the *Dirac delta function*, which is written

$$p(x|\mu) = \delta(x - \mu). \tag{2.36}$$

This is defined to be zero everywhere except at $x = \mu$ and to have the property of integrating to unity according to (2.28). Informally, we can think of this as an infinitely narrow and infinitely tall spike located at $x = \mu$ with the property of having unit area. Finally, if we have a finite set of observations of x given by $\mathcal{D} = \{x_1, \ldots, x_N\}$ then we can use the delta function to construct the *empirical distribution* given by

$$p(x|\mathcal{D}) = \frac{1}{N} \sum_{n=1}^{N} \delta(x - x_n), \tag{2.37}$$

which consists of a Dirac delta function centred on each of the data points. The probability density defined by (2.37) integrates to one as required.

Exercise 2.6

2.2.2 Expectations and covariances

One of the most important operations involving probabilities is that of finding weighted averages of functions. The weighted average of some function $f(x)$ under a probability distribution $p(x)$ is called the *expectation* of $f(x)$ and will be denoted by $\mathbb{E}[f]$. For a discrete distribution, it is given by summing over all possible values of x in the form

$$\mathbb{E}[f] = \sum_{x} p(x) f(x) \tag{2.38}$$