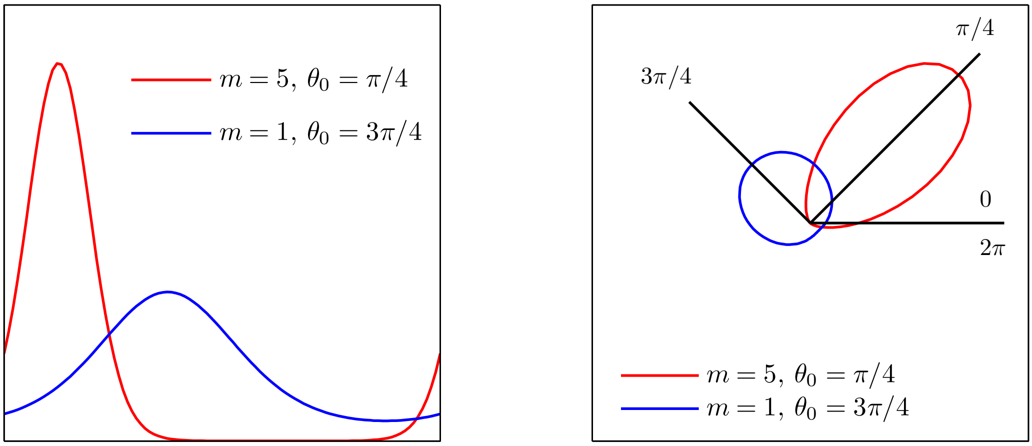


**Figure 3.11** The von Mises distribution plotted for two different parameter values, shown as a Cartesian plot on the left and as the corresponding polar plot on the right.

which is called the *von Mises* distribution or the *circular normal*. Here the parameter $\theta_0$ corresponds to the mean of the distribution, whereas $m$, which is known as the *concentration* parameter, is analogous to the inverse variance (i.e. the precision) for the Gaussian. The normalization coefficient in (3.129) is expressed in terms of $I_0(m)$, which is the zeroth-order modified Bessel function of the first kind (Abramowitz and Stegun, 1965) and is defined by

$$I_0(m) = \frac{1}{2\pi} \int_0^{2\pi} \exp\{m \cos\theta\}\, \mathrm{d}\theta. \tag{3.130}$$

*Exercise 3.31*

For large $m$, the distribution becomes approximately Gaussian. The von Mises distribution is plotted in Figure 3.11, and the function $I_0(m)$ is plotted in Figure 3.12.

Now consider the maximum likelihood estimators for the parameters $\theta_0$ and $m$ for the von Mises distribution. The log likelihood function is given by

$$\ln p(\mathcal{D}|\theta_0, m) = -N\ln(2\pi) - N \ln I_0(m) + m \sum_{n=1}^{N} \cos(\theta_n - \theta_0). \tag{3.131}$$

Setting the derivative with respect to $\theta_0$ equal to zero gives

$$\sum_{n=1}^{N} \sin(\theta_n - \theta_0) = 0. \tag{3.132}$$

To solve for $\theta_0$, we make use of the trigonometric identity

$$\sin(A - B) = \cos B \sin A - \cos A \sin B \tag{3.133}$$

*Exercise 3.32*

from which we obtain