and that the determinant of a diagonal matrix is given by the product of the elements on the leading diagonal. Thus, for a 2×2 matrix, the determinant takes the form

$$|\mathbf{A}| = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}. \tag{A.11}$$

The determinant of a product of two matrices is given by

$$|\mathbf{AB}| = |\mathbf{A}||\mathbf{B}| \tag{A.12}$$

as can be shown from (A.10). Also, the determinant of an inverse matrix is given by

$$\left|\mathbf{A}^{-1}\right| = \frac{1}{|\mathbf{A}|}, \tag{A.13}$$

which can be shown by taking the determinant of (A.2) and applying (A.12).

If $\mathbf{A}$ and $\mathbf{B}$ are matrices of size $N \times M$, then

$$\left|\mathbf{I}_N + \mathbf{A}\mathbf{B}^{\mathrm{T}}\right| = \left|\mathbf{I}_M + \mathbf{A}^{\mathrm{T}}\mathbf{B}\right|. \tag{A.14}$$

A useful special case is

$$\left|\mathbf{I}_N + \mathbf{a}\mathbf{b}^{\mathrm{T}}\right| = 1 + \mathbf{a}^{\mathrm{T}}\mathbf{b} \tag{A.15}$$

where $\mathbf{a}$ and $\mathbf{b}$ are N-dimensional column vectors.

A.3. Matrix Derivatives

Sometimes we need to consider derivatives of vectors and matrices with respect to scalars. The derivative of a vector $\mathbf{a}$ with respect to a scalar x is a vector whose components are given by

$$\left(\frac{\partial \mathbf{a}}{\partial x}\right)_i = \frac{\partial a_i}{\partial x} \tag{A.16}$$

with an analogous definition for the derivative of a matrix. Derivatives with respect to vectors and matrices can also be defined, for instance

$$\left(\frac{\partial x}{\partial \mathbf{a}}\right)_i = \frac{\partial x}{\partial a_i} \tag{A.17}$$

and similarly

$$\left(\frac{\partial \mathbf{a}}{\partial \mathbf{b}}\right)_{ij} = \frac{\partial a_i}{\partial b_j}. \tag{A.18}$$

The following is easily proven by writing out the components:

$$\frac{\partial}{\partial \mathbf{x}}\left(\mathbf{x}^{\mathrm{T}}\mathbf{a}\right) = \frac{\partial}{\partial \mathbf{x}}\left(\mathbf{a}^{\mathrm{T}}\mathbf{x}\right) = \mathbf{a}. \tag{A.19}$$