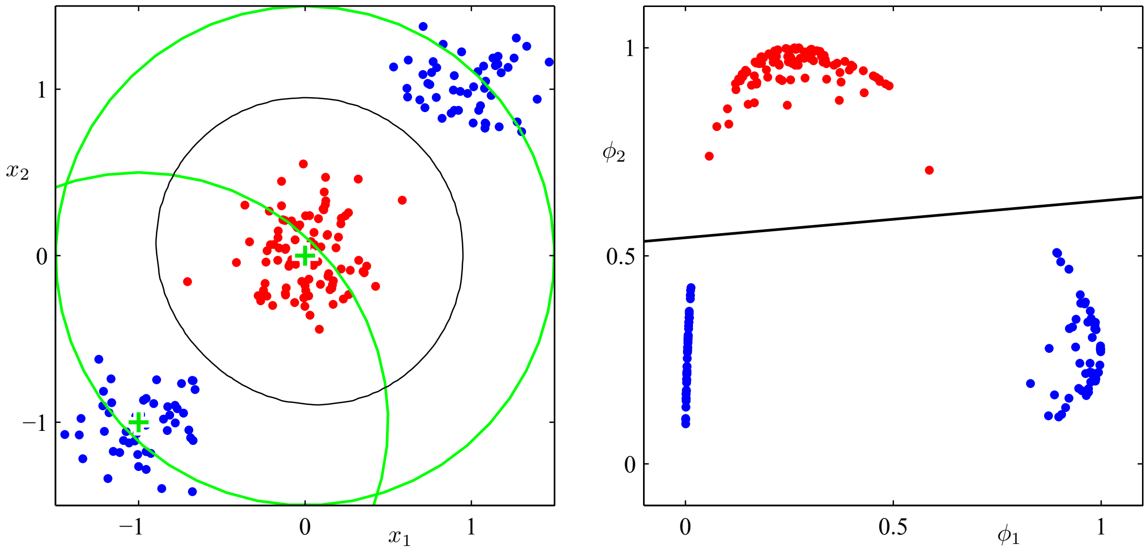

Figure 5.15 Illustration of the role of nonlinear basis functions in linear classification models. The left-hand plot shows the original input space (x_1, x_2) together with data points from two classes labelled red and blue. Two 'Gaussian' basis functions $\phi_1(\mathbf{x})$ and $\phi_2(\mathbf{x})$ are defined in this space with centres shown by the green crosses and with contours shown by the green circles. The right-hand plot shows the corresponding feature space (ϕ_1, ϕ_2) together with the linear decision boundary obtained given by a logistic regression model of the form discussed in Section 5.4.3. This corresponds to a nonlinear decision boundary in the original input space, shown by the black curve in the left-hand plot.

5.4.3 Logistic regression

We first consider the problem of two-class classification. In our discussion of generative approaches in Section 5.3, we saw that under rather general assumptions, the posterior probability of class $\mathcal{C}_1$ can be written as a logistic sigmoid acting on a linear function of the feature vector $\boldsymbol{\phi}$ so that

$$p(\mathcal{C}_1|\boldsymbol{\phi}) = y(\boldsymbol{\phi}) = \sigma\left(\mathbf{w}^{\mathrm{T}}\boldsymbol{\phi}\right) \tag{5.71}$$

with $p(\mathcal{C}_2|\boldsymbol{\phi}) = 1 - p(\mathcal{C}_1|\boldsymbol{\phi})$. Here $\sigma(\cdot)$ is the *logistic sigmoid* function defined by (5.42). In the terminology of statistics, this model is known as *logistic regression*, although it should be emphasized that this is a model for classification rather than for continuous variable.

For an M-dimensional feature space $\boldsymbol{\phi}$, this model has M adjustable parameters. By contrast, if we had fitted Gaussian class-conditional densities using maximum likelihood, we would have used $2M$ parameters for the means and $M(M+1)/2$ parameters for the (shared) covariance matrix. Together with the class prior $p(\mathcal{C}_1)$, this gives a total of $M(M+5)/2+1$ parameters, which grows quadratically with M, in contrast to the linear dependence on M of the number of parameters in logistic regression. For large values of M, there is a clear advantage in working with the logistic regression model directly.