

Figure 5.5 Schematic illustration of the joint probabilities  $p(x,\mathcal{C}_k)$  for each of two classes plotted against x, together with the decision boundary  $x=\widehat{x}$ . Values of  $x\geqslant\widehat{x}$  are classified as class  $\mathcal{C}_2$  and hence belong to decision region  $\mathcal{R}_2$ , whereas points  $x<\widehat{x}$  are classified as  $\mathcal{C}_1$  and belong to  $\mathcal{R}_1$ . Errors arise from the blue, green, and red regions, so that for  $x<\widehat{x}$ , the errors are due to points from class  $\mathcal{C}_2$  being misclassified as  $\mathcal{C}_1$  (represented by the sum of the red and green regions). Conversely for points in the region  $x\geqslant\widehat{x}$ , the errors are due to points from class  $\mathcal{C}_1$  being misclassified as  $\mathcal{C}_2$  (represented by the blue region). By varying the location  $\widehat{x}$  of the decision boundary, as indicated by the red double-headed arrow in (a), the combined areas of the blue and green regions remains constant, whereas the size of the red region varies. The optimal choice for  $\widehat{x}$  is where the curves for  $p(x,\mathcal{C}_1)$  and  $p(x,\mathcal{C}_2)$  cross, as shown in (b) and corresponding to  $\widehat{x}=x_0$ , because in this case the red region disappears. This is equivalent to the minimum misclassification rate decision rule, which assigns each value of x to the class having the higher posterior probability  $p(\mathcal{C}_k|x)$ .