from which we can read off the functional derivative by comparison with (B.3). Requiring that the functional derivative vanishes then gives

$$\frac{\partial G}{\partial y} - \frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\partial G}{\partial y'}\right) = 0, \tag{B.8}$$

which are known as the *Euler–Lagrange* equations. For example, if

$$G = y(x)^2 + (y'(x))^2 \tag{B.9}$$

then the Euler–Lagrange equations take the form

$$y(x) - \frac{\mathrm{d}^2 y}{\mathrm{d}x^2} = 0. \tag{B.10}$$

This second-order differential equation can be solved for $y(x)$ by making use of the boundary conditions on $y(x)$.

Often, we consider functionals defined by integrals whose integrands take the form $G(y, x)$ and that do not depend on the derivatives of $y(x)$. In this case, stationarity simply requires that $\partial G/\partial y(x) = 0$ for all values of x.

If we are optimizing a functional with respect to a probability distribution, then we need to maintain the normalization constraint on the probabilities. This is often most conveniently done using a Lagrange multiplier, which then allows an unconstrained optimization to be performed.

Appendix C

The extension of the above results to a multi-dimensional variable $\mathbf{x}$ is straightforward. For a more comprehensive discussion of the calculus of variations, see Sagan (1969).