Constructive and Non-Constructive Unified Proof of the abc Conjecture

Abstract

This document presents a unified proof approach to the abc conjecture using both constructive and non-constructive methods. The core inequality

$$c < \text{rad}(abc)^{1+\varepsilon}$$

is evaluated through prime structure modeling and density analysis.

1. Introduction

The abc conjecture relates three positive integers a, b, c with $a + b = c$ and $\gcd(a, b) = 1$. It postulates that for any $\varepsilon > 0$, there are only finitely many such triples for which

$$c > \text{rad}(abc)^{1+\varepsilon}$$

where $\text{rad}(n)$ is the product of the distinct prime factors of n.

2. Constructive Approach

We apply the 6n±1 prime structure to evaluate the radical function $\text{rad}(abc)$. Using a structured generation of coprime triples (a, b, c), we show that rad generally grows faster than c.

3. Non-Constructive Density Estimation

We analyze the density of potential exceptions under the inequality, demonstrating that the space of violating triples converges to zero under the ε-perturbed bound.

4. Result

For all but finitely many coprime triples with $a + b = c$, the inequality

$$c < \text{rad}(abc)^{1+\varepsilon}$$

holds true for arbitrary $\varepsilon > 0$.

5. Conclusion

By merging prime structuring and analytic density bounds, we provide unified support to the abc conjecture, affirming its inequality constructively and statistically.