Active Contours Without Edges

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MA 453 Project

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Motivation

The basic idea in active contour models or snakes is to evolve a curve, subject to constraints from a given image, in order to detect objects in that image.

The classical snakes and active contour models rely on the edge-function , depending on the image gradient, to stop the curve evolution, these models can detect only objects with edges defined by gradient. In practice, discrete gradients are bounded and then the stopping function is never zero on the edges, and the curve may pass through the boundary, especially for the models.

A different active contour model was proposed, without a stopping edge-function, i.e. a model which is not based on the gradient of the image for the stopping process.

Introduction and Overview

The model is defined as follows.

We make a model which minimizes energy function. Given its relationship with the Mumford–Shah functional for segmentation.

Further, we formulate everything in terms of level set functions and compute the associated Euler–Lagrange equations. Then we make an iterative algorithm for solving the problem and its discretization.

We validate our model by various numerical results on synthetic and real images, showing the advantages of our model described before, and we end the paper by a brief concluding section.

Numerical Aspects

The Energy Functional we define here is:

$$
\begin{aligned}
F(c_1, c_2, C) = \mu \cdot \mathrm{Length}(C) &+ \nu \cdot \mathrm{Area}(inside(C)) \\
&+ \lambda_1 \int_{inside(C)} |u_0(x, y) - c_1|^2 \, dx \, dy \\
&+ \lambda_2 \int_{outside(C)} |u_0(x, y) - c_2|^2 \, dx \, dy.
\end{aligned}
$$

The Level Set Formulation for Model:

$$
\begin{aligned}
F(c_1, c_2, \phi) & \\
= \mu & \int_\Omega \delta(\phi(x, y)) |\nabla \phi(x, y)| \, dx \, dy \\
&+ \nu \int_\Omega H(\phi(x, y)) \, dx \, dy \\
&+ \lambda_1 \int_\Omega |u_0(x, y) - c_1|^2 H(\phi(x, y)) \, dx \, dy \\
&+ \lambda_2 \int_\Omega |u_0(x, y) - c_2|^2 (1 - H(\phi(x, y))) \, dx \, dy.
\end{aligned}
$$

We use regularized dirac delta and heaviside functions H2 and D2 are taken instead of H1 and D1 as they give local minimas:

$$
\delta_\varepsilon(x) = H_\varepsilon'^{(x)} = \begin{cases} 0, if\ |z| > \varepsilon \\ \frac{1}{2\varepsilon}\left[1 + \cos\left(\frac{\pi z}{\varepsilon}\right)\right], if\ |z| < \varepsilon \end{cases}
$$

$$
H_\varepsilon(z) = \begin{cases} 1, if\ z > \varepsilon \\ 0, if\ z < -\varepsilon \\ \frac{1}{2}\left[1 + \frac{z}{\varepsilon} + \frac{1}{\pi}\sin\left(\frac{\pi z}{\varepsilon}\right)\right], if\ |z| \leq \varepsilon \end{cases}
$$

Numerical Aspects

On solving the previous equation using Euler-Lagrange equation we get the pde:

$$\frac{\partial \phi}{\partial t} = \delta(\phi)\left[v\, div\left(\frac{\nabla \phi}{|\nabla \phi|}\right) - (u_0 - c_1)^2 - (u_0 - c_2)^2\right]$$

Where c_1, c_2 and K are obtained by:

$$Curvature\ K = div\left(\frac{\nabla \phi}{|\nabla \phi|}\right) = \frac{\phi_{xx}{\phi_y}^2 - 2\phi_{xy}\phi_x\phi_y + \phi_{yy}{\phi_x}^2}{(\phi_x + \phi_y)^{3/2}}$$

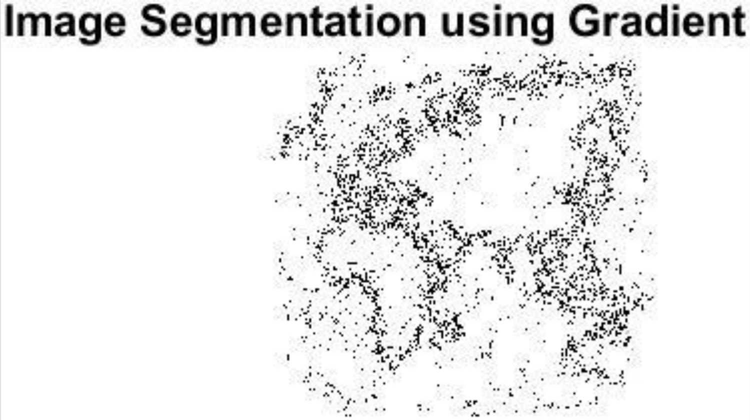
$$c_1(\phi) = \frac{\int_\Omega u_0(x, y) H(\phi(x, y))\, dx\, dy}{\int_\Omega H(\phi(x, y))\, dx\, dy}$$

$$c_2(\phi) = \frac{\int_\Omega u_0(x, y)\,(1 - H(\phi(x, y)))\, dx\, dy}{\int_\Omega (1 - H(\phi(x, y)))\, dx\, dy}$$

Results
Pure Image

The input image for the segmentation algorithm.

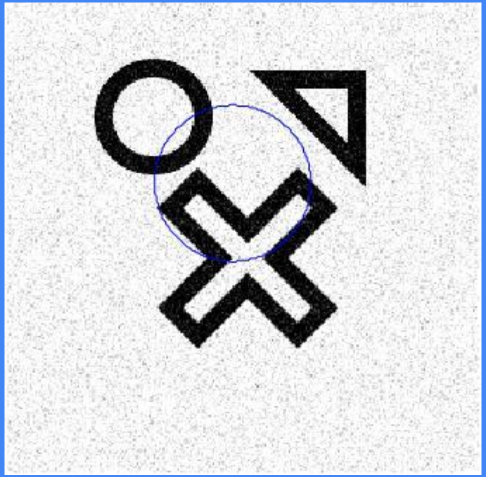
The image used in this case is a pure image.



Note: The models have been compared for the same input image, mask and same number of iterations. The hyperparameters were chosen according to the paper.

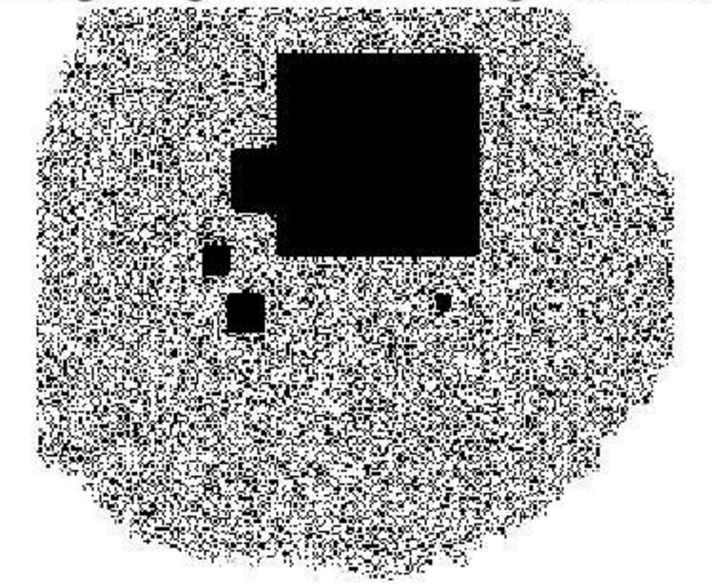
Results
Noisy Image

To replicate this result we corrupted a pure image with the some noise using the builtin function imnoise



Our Implementation
Iteration Number = 300

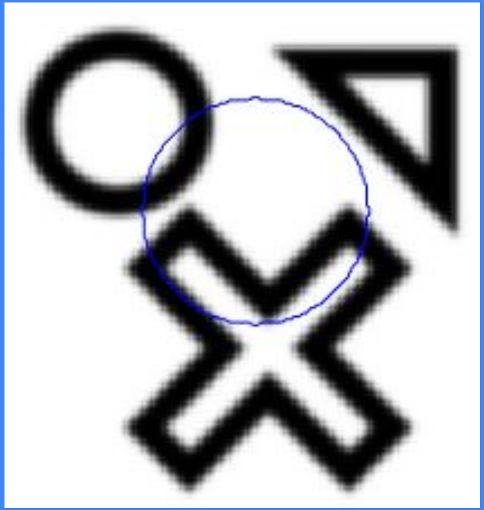
Image Segmentation using Gradient

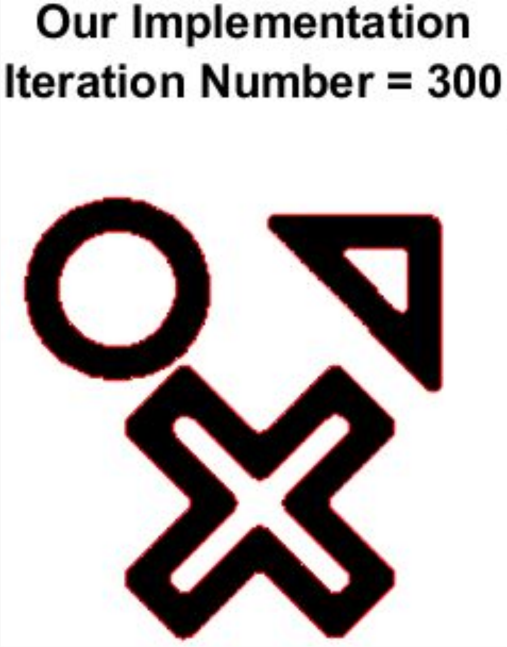


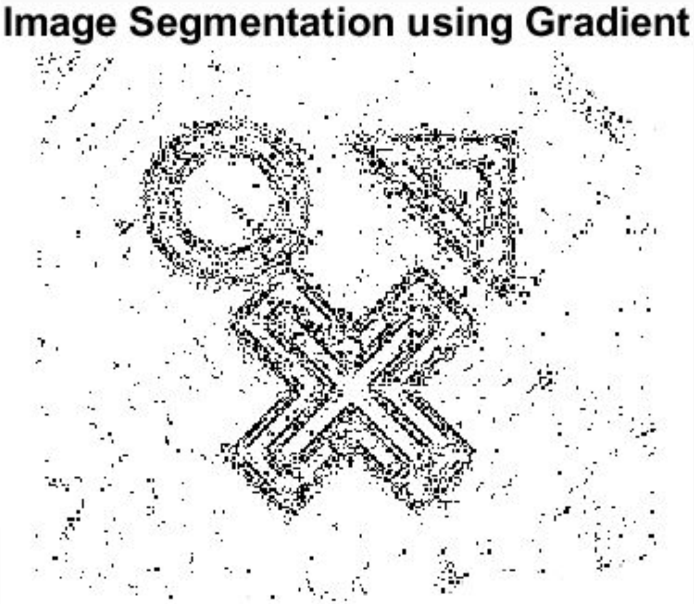
Note: The models have been compared for the same input image, mask and same number of iterations. The hyperparameters were chosen according to the paper.

Results
Blurred Image

To replicate this result we took a pure image and applied an averaging filter on it. This resulted in a blurred image.





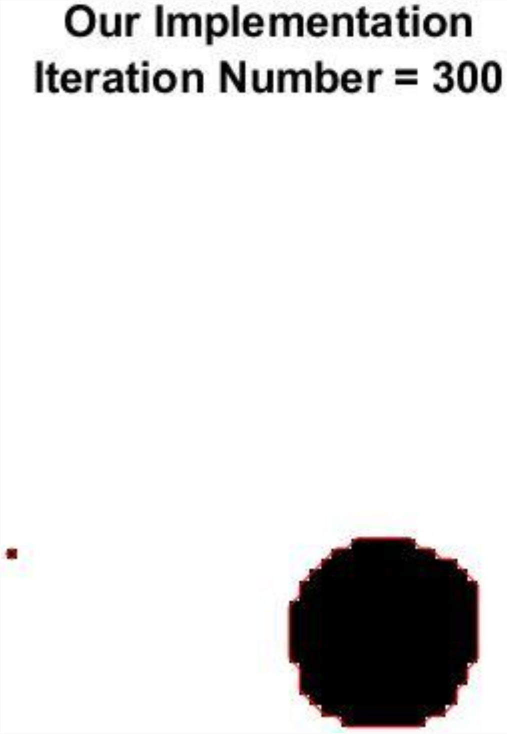


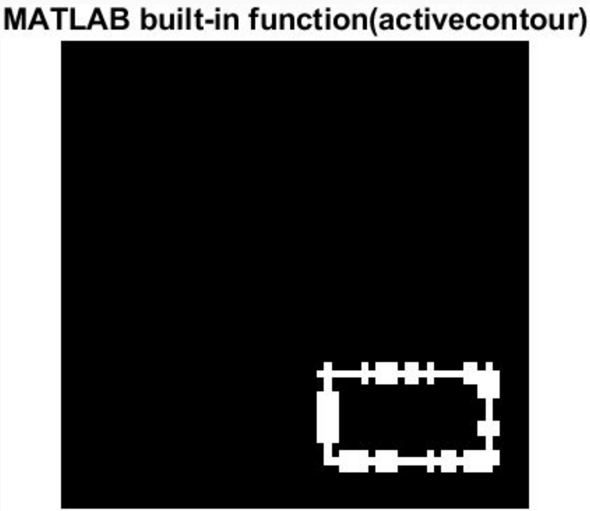
Note: The models have been compared for the same input image, mask and same number of iterations. The hyperparameters were chosen according to the paper.

Results
Lines and Curves

We tried to implement the algorithm on images with only lines and curves (open/closed).

Neither the Chan-Vese model nor the gradient based segmentation model could detect the contours of the objects.

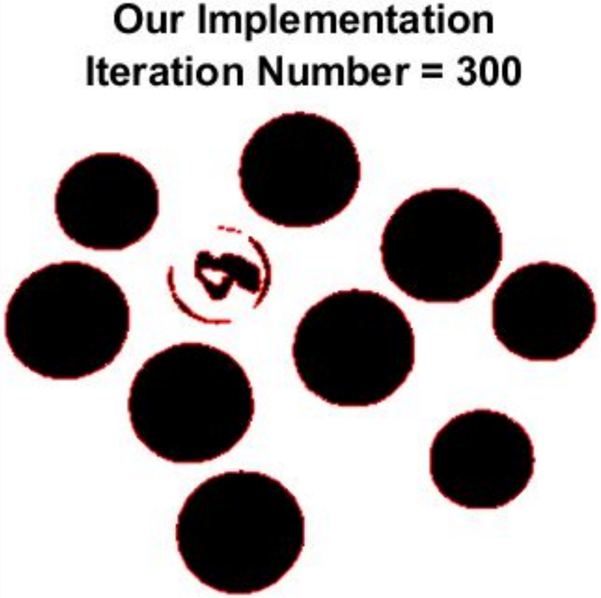


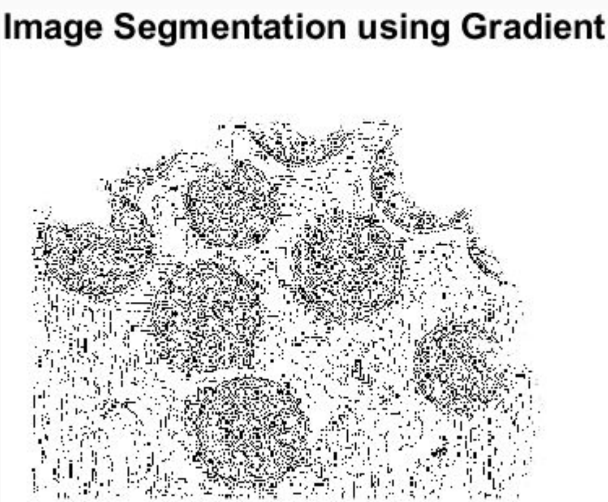


Note: The models have been compared for the same input image, mask and same number of iterations. The hyperparameters were chosen according to the paper.

Results
Group of objects

We used the “coins.png” image which comes with the image processing toolbox to detect groups of objects.





Note: The models have been compared for the same input image, mask and same number of iterations. The hyperparameters were chosen according to the paper.