Max Kramer

I affirm that I have adhered to the honor code on this assignment.

*Hello again, scientist! I'll write in italics, and problems for you will always be in **bold**. As a general rule, I expect you to do at least as much writing as I do. Code should be part of your solution, but I expect variables to be clear and explanation to involve complete sentences. Cite your sources; if you work with someone in the class on a problem, that's an extremely important source.*

Problem 9.1.

Consider the following linear transformations.

```
T1 = [1 1 1 1; 0 4 3 4; 4 0 1 0].'
T2 = [2 -2 -1 -2; 1 2 0 1].'
T3 = [ 1 0 -4 3; 0 1 0 -1]
```

```
T1 =

     1     0     4
     1     4     0
     1     3     1
     1     4     0


T2 =

     2     1
    -2     2
    -1     0
    -2     1


T3 =

     1     0    -4     3
     0     1     0    -1
```

Let A = Col(T1), B = Col(T2), and C = Nul(T3). ***Show me that A, B, and C are all 2-dimensional subspaces of R^4 by showing me a basis for each.***

```
A = T1(:,1:2)
rref(A)
B = T2
rref(B)
C_prep = rref(horzcat(T3,[0;0]))
C = [4 -3;0 1;1 0;0 1]
rref(C)
```

```
A =
```

```
     1     0
     1     4
     1     3
     1     4


ans =

     1     0
     0     1
     0     0
     0     0


B =

     2     1
    -2     2
    -1     0
    -2     1


ans =

     1     0
     0     1
     0     0
     0     0


C_prep =

     1     0    -4     3     0
     0     1     0    -1     0


C =

     4    -3
     0     1
     1     0
     0     1


ans =

     1     0
     0     1
     0     0
     0     0
```

The first two columns of T1 are linearly independent and form a basis for Col(T1). Therefore, Col(T1) is 2 dimensional. Each column of T1 is comprised of 4 elements. By the basis theorem, Col(T1) is a 2 dimensional subspace of R^4. The two columns of T2 are linearly independent and form a basis for Col(T2). Therefore, Col(T2) is 2 dimensional. Each column of T2 is comprised of 4 elements. By the basis theorem, Col(T2) is a 2 dimensional subspace of R^4. To find a basis for Nul(T3), we row reduce the augmented system T3 * x = 0. We hand calculated the parametric vector form and found a general solution of two vectors C. C is comprised of 2 linearly independent vectors with 4 elements each. By the basis theorem, C is a 2 dimensional subspace of R^4.

A, B, and C are all isomorphic -- geometrically, they're planes. But are they the same planes? In general, to show that two sets X and Y are equal, you have to show that every x in X is in Y, and then show that every y in Y is in X. But dimensionality is powerful; here's a theorem. If V and W are vector spaces of the same dimension, and {v_1, ..., v_n} is a basis for V, then V = W if and only if all of v_1, ..., v_n are in W.

Using the above theorem, ***show me that A is not equal to B.*** *Hint: to show that something's not in the span of some other stuff, get a pivot in the last column.*

```
spantest = [A B];
rref(spantest)


ans =

     1     0     2     0
     0     1    -1     0
     0     0     0     1
     0     0     0     0
```

By creating a matrix spantest from sets A and B, we can test if B is contained in span{A}. The resulting row reduced form has a pivot in the last column, so the columns of B are not within span{A}. Since the above theorem requires each vector in B to be in A, B is not equal to A.

Show that each basis vector for A is in the nullspace of T3.

```
spantest_2 = [A(:,1) C];
spantest_3 = [A(:,2) C];
rref(spantest_2)
rref(spantest_3)


ans =

     1     0     1
     0     1    -1
     0     0     0
     0     0     0


ans =

    1.0000         0    0.2500
         0    1.0000   -0.7500
         0         0         0
         0         0         0
```

By creating matrices spantest_2 and spantest_3, we can test each basis vector for A in terms of whether or not it is contained in span{C}. As both row reduced matrices do NOT have pivots in the last column, both basis vectors for A are contained in span{C}.

Why does this prove that A = C? Explain.

As each basis vector in A is in C and both A and C are vector spaces of the same dimension, A must equal C.

Does B = C? *Either do the calculations, or explain why you don't need to.*

We do not need to perform this calculation as we already know A = C and A does not equal B. Therefore, B does not equal C.

Published with MATLAB® R2019b