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I affirm that I have adhered to the honor code on this assignment.

Hello again, scientist! I'll write in italics, and problems for you will always be in ***bold****. As a general rule, I expect you to do at least as much writing as I do. Code should be part of your solution, but I expect variables to be clear and explanation to involve complete sentences. Cite your sources; if you work with someone in the class on a problem, that's an extremely important source.*

Problem 10.1.

You can write a linear transformation with respect to whatever set of bases you want, but some bases are better than others. Check it out.

```
A = [0 3 -6 6 4 -5; 3 -7 8 -5 8 9; 3 -9 12 -9 6 15];
size(A)
rank(A)
```

```
ans =

     3     6


ans =

     3
```

As a linear map, A takes R^6 (in the standard basis) onto R^3 (in the standard basis). That means that Col(A) is a basis for R^3. Lay Chapter 6 Theorem 4 says that the pivot columns of A form a basis for R^3. ***Find that basis, and call it B = [b1 b2 b3].***

```
rref(A)
B = A(:,[1 2 5])
```

```
ans =

     1     0    -2     3     0   -24
     0     1    -2     2     0    -7
     0     0     0     0     1     4


B =

     0     3     4
     3    -7     8
     3    -9     6
```

The pivot columns of A are 1, 2, and 5. The above command creates a matrix B from the 1st, 2nd, and 5th columns of A.

Show me the matrix C which represents the transformation A as a map from R^6 (in the standard basis) to R^3 in the B basis. *Here's a good way to do that: compose the matrix A with an appropriate change of basis.*

```
C = B \ A
rref(A)
```

```
C =

    1.0000    0.0000   -2.0000    3.0000         0  -24.0000
         0    1.0000   -2.0000    2.0000         0   -7.0000
         0   -0.0000    0.0000   -0.0000    1.0000    4.0000


ans =

     1     0    -2     3     0   -24
     0     1    -2     2     0    -7
     0     0     0     0     1     4
```

The matrix C is a linear map from R^6 to R^3 created by composing the matrix A with the inverse of the change of basis matrix from the B basis to the standard basis. The result is a linear map from R^6 in the standard basis to R^3 in the B basis.

If you did the previous part right, then C = rref(A). (Wait, what!?!?!) Actually, that's what row reduction is: it's a change of basis of the codomain. This is why row reduction can (and often will) change Col(A), but it doesn't change Row(A) or Null(A): those live in the domain, and the basis for the domain hasn't changed.

Here's one more.

```
K = [1 2 3 4; 4 5 6 7; 6 7 8 9]
```

```
K =

     1     2     3     4
     4     5     6     7
     6     7     8     9
```

Find a basis M = [m1 m2 m3] for which L=rref(K) is the coordinate matrix for K, where K takes the standard basis of R^4 to the standard basis of R^3 and L takes the standard basis of R^4 to the M-basis of R^3.

```
M = horzcat(K(:,[1 2]),[5;7;4])
rref(M)
L = M \ K
rref(K)
```

```
M =

     1     2     5
     4     5     7
```

```
     6     7     4


ans =

     1     0     0
     0     1     0
     0     0     1


L =

    1.0000         0   -1.0000   -2.0000
   -0.0000    1.0000    2.0000    3.0000
    0.0000   -0.0000   -0.0000   -0.0000


ans =

     1     0    -1    -2
     0     1     2     3
     0     0     0     0
```

The matrix M is created by combining 2 vectors in Col(K) with a third linearly independent vector m3, testing by row reducing the resulting matrix. The columns of the matrix are linearly independent. Therefore, the set {m1,m2,m3} is a set of three linearly independent vectors with 3 elements each, forming the M basis for R^3. When the inverse of M is multiplied by K, the resultant matrix L is row equivalent to rref(K).

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