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I affirm that I have adhered to the honor code on this assignment.

*Hello again, scientist! I'll do all my writing in italics, and problems for you will be in **bold**. Comment your code, and explain your ideas in plaintext. As a general rule, I expect you to do at least as much writing as I do. Code should be part of your solution, but I expect variables to be clear and explanation to involve complete sentences. Cite your sources; if you work with someone in the class on a problem, that's an extremely important source. Don't work alone.*

## Problem 12.1.

```
v1 = [1;2;3;4;5]; v2 = [0;1;0;-1;1];
```

**Find a  $3 \times 5$  matrix  $A$  for which  $\text{Null}(A) = \text{Span}\{v1, v2\}$ . Prove that your answer works.** *Students last year found this problem to be much more difficult than it appears, so here's a hint: use Theorem 3 in Lay 6.1 to find a basis for the orthogonal complement of  $\text{Span}\{v1, v2\}$ .*

```
V = [v1 v2];  
Sys = horzcat(V', [0;0]);  
PVF = [-3 0 1 0 0;-6 1 0 1 0;-3 -1 0 0 1]'
```

```
t = PVF'  
t * v1  
t * v2
```

*PVF =*

-3	-6	-3
0	1	-1
1	0	0
0	1	0
0	0	1

*t =*

-3	0	1	0	0
-6	1	0	1	0
-3	-1	0	0	1

*ans =*

0
0
0

*ans =*

0  
0  
0

The matrix  $V$  is created by concatenating  $v_1$  and  $v_2$ . Theorem 3 in Lay 6.1 says that the column space of  $A$  is the orthogonal complement of the null space of  $A'$ . To find the  $3 \times 5$  matrix of interest, we create an augmented system from  $V'$  and calculate the general solution. Parametric vector form (done by hand) found three vectors that make up the matrix  $PVF$ . Since we want the Null space of  $A$  and not  $A'$ , we transpose the vectors and arrive at a  $3 \times 5$  matrix  $PVF'$ . Multiplying this matrix by  $v_1$  and  $v_2$  yields the zero vector in both cases, so  $PVF'$  is a  $3 \times 5$  matrix for which  $\text{Null}(PVF') = \text{span}\{v_1, v_2\}$ .

**Find a square matrix  $B$  for which  $\text{Null}(B) = \text{Span}\{v_1, v_2\}$ . Prove that your answer works. You can use the previous part, or you can be clever in other ways.**

```
step2 = horzcat(PVF, [-11;0;1;-1;1])
step3 = horzcat(step2, [-7;1;-1;1;0])
```

```
final = step3'
```

```
dot(final(4,:),v1)
dot(final(4,:),v2)
dot(final(5,:),v1)
dot(final(5,:),v2)
```

```
step2 =
```

-3	-6	-3	-11
0	1	-1	0
1	0	0	1
0	1	0	-1
0	0	1	1

```
step3 =
```

-3	-6	-3	-11	-7
0	1	-1	0	1
1	0	0	1	-1
0	1	0	-1	1
0	0	1	1	0

```
final =
```

-3	0	1	0	0
-6	1	0	1	0
-3	-1	0	0	1
-11	0	1	-1	1
-7	1	-1	1	0

```
ans =
```

-7

*ans* =

2

*ans* =

-4

*ans* =

0

The matrix final is generated by concatenating the 3 x 5 matrix found above with two additional rows whose dot product with v1 and v2 are 0. I am unsure how to find the correct answer here, I believe that there is a mistake in my logic.

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