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I affirm that I have adhered to the honor code on this assignment.

I acknowledge the help of Nate for answering a question about whether or not it is possible for this system to have an inconsistency.

Hello again, scientist! I'll write in italics, and problems for you will always be in ***bold****. As a general rule, I expect you to do at least as much writing as I do. Code should be part of your solution, but I expect variables to be clear and explanation to involve complete sentences. Cite your sources; if you work with someone in the class on a problem, that's an extremely important source.*

Problem 2.3.

Here's a variation on the previous problem. I strongly recommend completing 2.2 first. This time, let's say that B is the augmented matrix of a linear system.

```
syms b;
B = [1 1 3; 1 (b^2-8) b]
rref(B)
```

```
B =

[ 1,       1, 3]
[ 1, b^2 - 8, b]


ans =

[ 1, 0, (3*b + 8)/(b + 3)]
[ 0, 1,         1/(b + 3)]
```

Find a value of b for which this system has exactly one solution. Prove it. *(You can and should use rref() as part of your proof, but I also expect to see a few sentences of writing.)*

```
C = subs(B,b,-2);
C_rr = rref(C)
```

```
C_rr =

[ 1, 0, 2]
[ 0, 1, 1]
```

When b = -2, the row reduced augmented matrix has no free variables and no row of the form 0 = 1. Therefore, when b = -2 the system has one solution, x1 = 2, x2 = 1.

Find a value of b for which the system has no solutions. Prove it.

```
B_c = B(:,1:end-1)
```

```
D = subs(B_c,b,-3);
D_rr = rref(D)

E = subs(B,b,-3);
E_rr = rref(E)


B_c =

[ 1,       1]
[ 1, b^2 - 8]


D_rr =

[ 1, 1]
[ 0, 0]


E_rr =

[ 1, 1, 0]
[ 0, 0, 1]
```

B_c represents the coefficient matrix of the system. Similarly to problem 2.2, selecting b = -3 will result in the coefficient matrix B_c becoming a 2 x 2 matrix of 1s. When that matrix is row reduced, there is only one pivot. Then, when b = -3, the row reduced form of the augmented matrix B row reduces to a form that contains a row 0 = 1. Therefore, when b = -3 the system has no solution.

Find a value of b for which the system has infinitely many solutions. Prove it.

```
B_c = B(:,1:end-1)
F = subs(B_c,b,3);
F_rr = rref(F)

G = subs(B,b,3);
G_rr = rref(G)


B_c =

[ 1,       1]
[ 1, b^2 - 8]


F_rr =

[ 1, 1]
[ 0, 0]


G_rr =

[ 1, 1, 3]
```

```
[ 0, 0, 0]
```

B_c again represents the coefficient matrix of B. The value b = 3 results in the row reduced form of B_c containining a row of zeros. When substituting that same value into the augmented matrix, the equation becomes x1 + x2 = 3, which has infinitely many solutions.

Follow-up (which won't be graded, but I find it interesting!): for most values of b, the system has exactly one solution. Can you find a whole number b for which the solution uses only whole numbers? Can you find two?

```
B_rr = rref(B)

H = subs(B,b,-4);
first_b = rref(H)
I = subs(B,b,-2);
second_b = rref(I)
```

```
B_rr =

[ 1, 0, (3*b + 8)/(b + 3)]
[ 0, 1,         1/(b + 3)]


first_b =

[ 1, 0,  4]
[ 0, 1, -1]


second_b =

[ 1, 0, 2]
[ 0, 1, 1]
```

Examining the row reduction of B, the value selected should result in both (3b + 8)/(b+3) and 1/(b+3) being whole numbers. The value -4 results in x1 = 4 and x2 = 1. Another value for b (-2) results in x1 = 2 and x2 = 1.

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