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I affirm that I have adhered to the honor code on this assignment

*Hello again, scientist! I'll write in italics, and problems for you will always be in **bold**. As a general rule, I expect you to do at least as much writing as I do. Code should be part of your solution, but I expect variables to be clear and explanation to involve complete sentences. Cite your sources; if you work with someone in the class on a problem, that's an extremely important source.*

Problem 11.2.

Here's another matrix.

```
F = [1 1; 1 0]
```

```
F =

     1     1
     1     0
```

And here's its characteristic polynomial.

```
syms x;
poly = charpoly(F,x)
```

```
poly =

x^2 - x - 1
```

Compute the exact eigenvalues of F. *(Quadratic formula.)*

```
solve(poly)
```

```
ans =

 1/2 - 5^(1/2)/2
 5^(1/2)/2 + 1/2
```

The exact eigenvalues of F are (1-sqrt(5))/2 and (1+sqrt(5))/2.

We're going to find the largest one, which also happens to be the golden ratio, using what's called the "power method." The eig() command is really just a fancier version of this. Start with a random vector, like this one.

```
v = [1;1];
```

Divide by its length so it has length 1. (The length of a vector is called its "norm." More on that next week.)

```
v = v/norm(v)
```

```
v =

    0.7071
    0.7071
```

The power method is really simple: ***start with a norm 1 vector, multiply by F, divide by the length so it has norm 1 again, and repeat. Do that until you get the same vector twice in a row.*** *(You may want to write a loop.)*

```
vec_i = v;

for i = 1:10
    if i == 1
        vec_prev = vec_i;
    end
    vec = F * vec_prev;
    vec = vec/norm(vec);
    if vec == vec_prev
        break
    end
    vec_prev = vec;
end
```

Congratulations, you've found an eigenvector! And because it has norm 1, its associated eigenvalue will be the norm of the vector after multiplying by F. ***Do that.*** *If everything's gone right, you should get the golden ratio.*

```
eigenv = F * vec;
norm(eigenv)
```

```
ans =

    1.6180
```

multiplying the resultant vector by F and then taking the norm results in the golden ratio.

If you've taken discrete, you should seriously consider doing this: prove that the first column of F^n is given by [f{n+1}; f{n}], where f{n} is the nth Fibonacci number. (Since a linear transformation is determined by its action on a basis, and we know that F^n[1;0] is approximately an eigenvector of eigenvalue equal to the golden ratio, this proves that f{n+1}/f{n} is approximately equal to the golden ratio.)*

Published with MATLAB® R2019b