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I affirm that I have adhered to the honor code on this assignment.

Hello again, scientist! I'll write in italics, and problems for you will always be in ***bold****. As a general rule, I expect you to do at least as much writing as I do. Code should be part of your solution, but I expect variables to be clear and explanation to involve complete sentences. Cite your sources; if you work with someone in the class on a problem, that's an extremely important source.*

Problem 5.3.

First, ***let Q be a 3x3 matrix whose entries are randomly drawn from the one-digit numbers 0-9. Show me Q and det(Q).*** *Note: your output should be different every time you publish your m file.*

```
Q = randi(10,3)-1
detQ = det(Q)
```

```
Q =

     9     0     8
     8     9     4
     8     8     0


detQ =

  -352
```

The randi(r,n) command creates a square matrix of size n made up of random integers between 1 and r. By using 10 as the seed and then subtracting 1, we get a matrix of random integers betwen 0 and 9. The det() command is then used to calculate the determinant.

Uncomment the following code and explain what property of determinants is being demonstrated with each example.

```
det(2*Q)

det(Q^2)

det(Q^(-1))

det(Q.')

R = [2 0 0; 0 1 0; 0 0 3]; det(R*Q)

S = Q; S(1,:) = S(1,:)+S(2,:); det(S)
```

```
ans =
```

```
       -2816


ans =

   1.2390e+05


ans =

   -0.0028


ans =

  -352


ans =

       -2112


ans =

  -352
```

The first, second, and last lines demonstrate the property that if one row of A is multiplied by k to produce B, then det(B) = k *det(A). The third line demonstrates that a sqaure matrix is invertible iff its determinant is not equal to 0. The fourth line demonstrates that for a square matrix A, det(A)is equal to det(A'). The fifth line demonstrates the multiplicative property of determinants (det(A)*det(B)=det(AB)).

There's actually a small chance that one of the above will throw an error when you uncomment it. ***Which one, and why?***

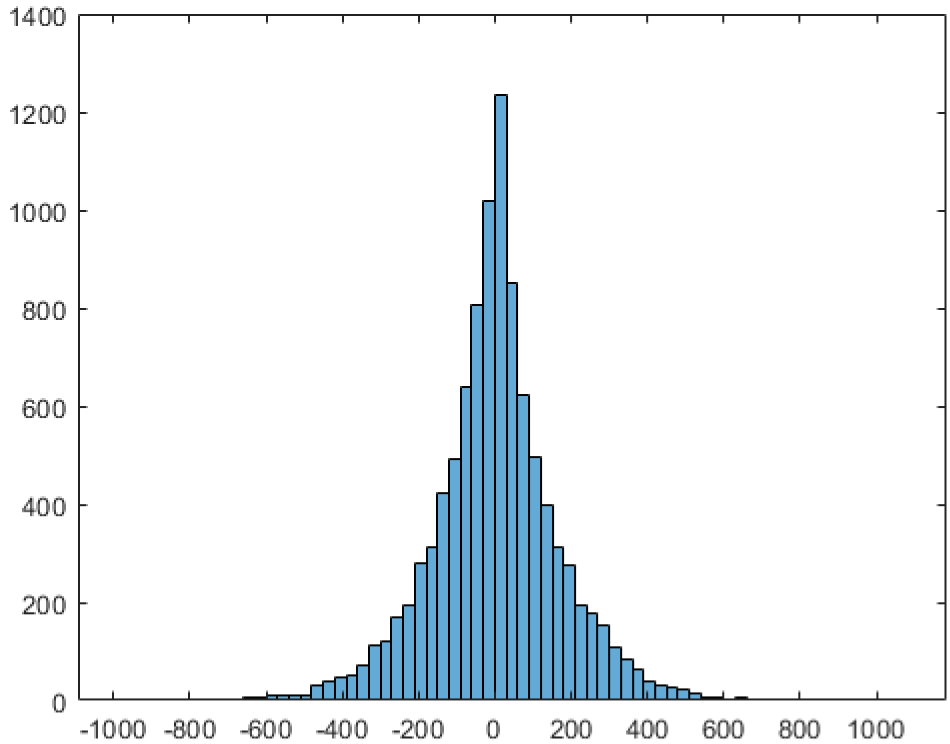
det(Q^(-1)) may throw an error as it is entirely possible that the randomly generated matrix Q will be a singular matrix and thus not be invertible.

Well yeah okay, but what are the chances of that? ***Put abunchofdets.m in the same path as this m file and use it to generate the determinants of 10^4 random 3x3 matrices.*** *Use a dang semicolon.*

```
B = abunchofdets(3,10^4);
% help abunchofdets % this'll show you how to use abunchofdets()
```

Show me a histogram of the output.

```
histogram(B)
```



How many of these 10^4 matrices are not invertible? What's that as a percentage of the total?

```
zerodet = length(B)-nnz(B)
zerodetpercent = zerodet / length(B)
```

```
zerodet =

   212


zerodetpercent =

    0.0212
```

The nnz command calculates the number of nonzero entries in a vector. By subtracting this number from the total determinants calculated, we are left with the number of 0 entries which is the number of non-invertible matrices.

Of the matrices that you found, what's the largest determinant?

```
biggestdet = max(B)
```

```
biggestdet =
```

```
        1080
```

The max() command returns the largest entry in a vector.

Bonus! CS majors take note, but this isn't part of the problem. rng(seed), where seed is a number between 1 and 2^32, makes the random number generator return a repeatable output. Uncomment the following code, then change n around until you find a value that makes L as large as you can get it. Because there's absolutely no way to shortcut this other than brute force, this is a good example of what's called a 'proof of work' task. You're basically mining cryptocurrency, except that you can't sell it. But I'll bring candy or balloons or something if anyone finds an n that gives L = 1458.

```
bigL = [];
tic;
for n = 1:1000000
    rng(n);
    L = abunchofdets(3,1);
    bigL(1,n) = L;
end
toc;
max(bigL)
```

```
Elapsed time is 195.576812 seconds.

ans =

        1080
```

By checking every seed between 1 and 1,000,000 I found a maximum determinant of 1080. This computation took 207.67 seconds.

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