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I affirm that I have adhered to the honor code on this assignment.

*Hello again, scientist! I'll write in italics, and problems for you will always be in **bold**. As a general rule, I expect you to do at least as much writing as I do. Code should be part of your solution, but I expect variables to be clear and explanation to involve complete sentences. Cite your sources; if you work with someone in the class on a problem, that's an extremely important source.*

Problem 9.1.

Consider the following linear transformations.

`T1 = [1 1 1 1; 0 4 3 4; 4 0 1 0].'`

`T2 = [2 -2 -1 -2; 1 2 0 1].'`

`T3 = [1 0 -4 3; 0 1 0 -1]`

`T1 =`

1	0	4
1	4	0
1	3	1
1	4	0

`T2 =`

2	1
-2	2
-1	0
-2	1

`T3 =`

1	0	-4	3
0	1	0	-1

Let $A = \text{Col}(T1)$, $B = \text{Col}(T2)$, and $C = \text{Nul}(T3)$. Show me that A , B , and C are all 2-dimensional subspaces of \mathbb{R}^4 by showing me a basis for each.

`A = T1(:, 1:2)`

`B = T2`

`C = [4 -3; 0 1; 1 0; 0 1]`

`A =`

1	0
1	4

$$\begin{array}{cc} 1 & 3 \\ 1 & 4 \end{array}$$

$B =$

$$\begin{array}{cc} 2 & 1 \\ -2 & 2 \\ -1 & 0 \\ -2 & 1 \end{array}$$

$C =$

$$\begin{array}{cc} 4 & -3 \\ 0 & 1 \\ 1 & 0 \\ 0 & 1 \end{array}$$

The first two columns of $T1$ are linearly independent and form a basis for $\text{Col}(T1)$. Therefore, $\text{Col}(T1)$ is 2 dimensional. Each column of $T1$ is comprised of 4 elements. By the basis theorem, $\text{Col}(T1)$ is a 2 dimensional subspace of \mathbb{R}^4 . The two columns of $T2$ are linearly independent and form a basis for $\text{Col}(T2)$. Therefore, $\text{Col}(T2)$ is 2 dimensional. Each column of $T2$ is comprised of 4 elements. By the basis theorem, $\text{Col}(T2)$ is a 2 dimensional subspace of \mathbb{R}^4 . By calculating the parametric vector form of $T3 * x = 0$, we find a basis for $\text{Nul}(T3)$ that comprises of 2 linearly independent vectors with 4 elements each. By the basis theorem, $\text{Nul}(T3)$ is a 2 dimensional subspace of \mathbb{R}^4 .

A, B, and C are all isomorphic -- geometrically, they're planes. But are they the same planes? In general, to show that two sets X and Y are equal, you have to show that every x in X is in Y, and then show that every y in Y is in X. But dimensionality is powerful; here's a theorem. If V and W are vector spaces of the same dimension, and $\{v_1, \dots, v_n\}$ is a basis for V, then $V = W$ if and only if all of v_1, \dots, v_n are in W.

*Using the above theorem, **show me that A is not equal to B**. Hint: to show that something's not in the span of some other stuff, get a pivot in the last column.*

```
spantest = [A B];
rref(spantest)
```

$ans =$

$$\begin{array}{cccc} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{array}$$

By creating a matrix spantest from sets A and B, we can test if B is contained in $\text{span}\{A\}$. The resulting row reduced form has a pivot in the last column, so the columns of B are not within $\text{span}\{A\}$. Since the above theorem requires each vector in B to be in A, B is not equal to A.

Show that each basis vector for A is in the nullspace of T3.

```
spantest_2 = [A(:,1) C];
```

```
spantest_3 = [A(:,2) C];  
rref(spantest_2)  
rref(spantest_3)
```

ans =

1	0	1
0	1	-1
0	0	0
0	0	0

ans =

1.0000	0	0.2500
0	1.0000	-0.7500
0	0	0
0	0	0

By creating matrices spantest_2 and spantest_3, we can test each basis vector for A in terms of whether or not it is contained in $\text{span}\{C\}$. As both row reduced matrices do NOT have pivots in the last column, both basis vectors for A are contained in $\text{span}\{C\}$.

Why does this prove that $A = C$? Explain.

As each basis vector in A is in C and both A and C are vector spaces of the same dimension, A must equal C.

Does $B = C$? Either do the calculations, or explain why you don't need to.

We do not need to perform this calculation as we already know $A = C$ and A does not equal B. Therefore, B does not equal C.

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