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I affirm that I have adhered to the honor code on this assingment.

*Hello again, scientist! I'll write in italics, and problems for you will always be in **bold**. As a general rule, I expect you to do at least as much writing as I do. Code should be part of your solution, but I expect variables to be clear and explanation to involve complete sentences. Cite your sources; if you work with someone in the class on a problem, that's an extremely important source.*

Problem 6.1.

This week's MATLAB problems are all about getting used to function space. Unlike with vectors in R^n, where it's usually easy to see if two vectors are the same, two functions are equal iff they evaluate to the same output for every input. So there's potentially infinitely many places for things to go wrong.

Let's start with sets of two vectors in function space. Theorem 4 in Lay's Chapter 4 says the same thing for these as for vectors in R^n: two functions are linearly dependent if and only if one is a multiple of the other.

```
syms x;
f = x+1;
g = 3*x+3; % for example!
```

But here's the thing: that multiple needs to be exactly the same for every real value of x (assuming that these are functions from R to R). Consider!

```
h = 2*x+4;
A = [f;h];
subs(A,x,0)
subs(A,x,1)
```

```
ans =

 1
 4


ans =

 2
 6
```

Explain why the above computation proves that f(x) and h(x) are linearly independent.

When x = 0, the value of f(0) = 1 and h(0) = 4. When x = 1, f(1) = 2 and h(1) = 6. The ratio of 1 to 4 is not the same as 2 to 6, so the multiple relating f(x) to h(x) is NOT the same at every real value of x. Therefore, f(x) and h(x) are linearly independent.

Prove that the following pairs of functions are linearly independent.

```
a = x;
b = x^2;
```

```
B = [a;b];
subs(B,x,2)
subs(B,x,3)


ans =

 2
 4


ans =

 3
 9
```

The ratio of 2 to 4 is not the same as 3 to 9, so the multiple relating a(x) to b(x) is NOT the same at every real value of x. Therefore, a(x) and b(x) are linearly independent.

```
c = x;
d = abs(x);

C = [c;d];
subs(C,x,1)
subs(C,x,-1)


ans =

 1
 1


ans =

 -1
  1
```

The ratio of 1 to 1 is not the same as -1 to 1, so the multiple relating c(x) to d(x) is NOT the same at every real value of x. Therefore, c(x) and d(x) are linearly independent.

```
k = heaviside(x-3.43);
l = heaviside(x-3.44);

D = [k;l];
subs(D,x,3.43)
subs(D,x,4)


ans =

 1/2
```

```
    0


ans =

  1
  1
```

The ratio of 1/2 to 0 is not the same as 1 to 1, so the multiple relating k(x) and l(x) is NOT the same at every real value of x. Therefore, k(x) and l(x) are linearly independent.

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