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I affirm that I have adhered to the honor code on this assignment

*Hello again, scientist! I'll write in italics, and problems for you will always be in **bold**. As a general rule, I expect you to do at least as much writing as I do. Code should be part of your solution, but I expect variables to be clear and explanation to involve complete sentences. Cite your sources; if you work with someone in the class on a problem, that's an extremely important source.*

Problem 11.2.

Here's another matrix.

```
F = [1 1; 1 0]
```

```
F =

     1     1
     1     0
```

And here's its characteristic polynomial.

```
syms x;
poly = charpoly(F,x)
```

```
poly =

x^2 - x - 1
```

Compute the exact eigenvalues of F. *(Quadratic formula.)*

```
solve(poly)
```

```
ans =

 1/2 - 5^(1/2)/2
 5^(1/2)/2 + 1/2
```

The exact eigenvalues of F are (1-sqrt(5))/2 and (1+sqrt(5))/2.

We're going to find the largest one, which also happens to be the golden ratio, using what's called the "power method." The eig() command is really just a fancier version of this. Start with a random vector, like this one.

```
v = [1;1];
```

Divide by its length so it has length 1. (The length of a vector is called its "norm." More on that next week.)

```
v = v/norm(v)
```

```
v =

    0.7071
    0.7071
```

The power method is really simple: ***start with a norm 1 vector, multiply by F, divide by the length so it has norm 1 again, and repeat. Do that until you get the same vector twice in a row.*** *(You may want to write a loop.)*

```
vec_i = v;

for i = 1:10
    if i == 1
        vec_prev = vec_i;
    end
    vec = F * vec_prev;
    vec = vec/norm(vec);
    if vec == vec_prev
        break
    end
    vec_prev = vec;
end
```

The loop structure above begins with the vector v. On each run through the loop, the vector is multiplied by the matrix F and then divided by its norm to get the vector with a norm of 1. The code then checks if the length 1 vector is the same as the length 1 vector seen in the last iteration, and ends the loop if so.

Congratulations, you've found an eigenvector! And because it has norm 1, its associated eigenvalue will be the norm of the vector after multiplying by F. ***Do that.*** *If everything's gone right, you should get the golden ratio.*

```
eigenv = F * vec;
norm(eigenv)
```

```
ans =

    1.6180
```

multiplying the resultant vector by F and then taking the norm results in the golden ratio.

If you've taken discrete, you should seriously consider doing this: prove that the first column of F^n is given by [f{n+1}; f{n}], where f{n} is the nth Fibonacci number. (Since a linear transformation is determined by its action on a basis, and we know that F^n[1;0] is approximately an eigenvector of eigenvalue equal to the golden ratio, this proves that f{n+1}/f{n} is approximately equal to the golden ratio.)*

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