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Investigating the Evolution of a Globular Cluster

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This work aims to demonstrate the findings from allowing a large cluster of stars to collapse freely under its own gravity. Results from two isolated clusters are compared, with one cluster being allowed to evolve for much longer than the other to understand how the energy profiles of the systems develop with time.

1. Background

Globular clusters are centrally concentrated and gravitationally bound systems of stars [1]. Containing up to one million stars, they are compact with their tightly bound spherical appearance being significantly shaped by the internal two body gravitational interactions between their stars [2].

Newton's law of gravitation gives the force, F_i, on each body, i, due to each other body, j, as:

$$\boldsymbol{F_i = -Gm_i \sum_{j \neq i} m_j \frac{r_i - r_j}{\left[|r_i - r_j|^2 + \epsilon^2\right]^{3/2}}}, \qquad (1)$$

where G is the gravitational constant, m and r represent the masses and position vectors of the bodies in the system and ϵ represents the softening parameter. The use of ϵ ensures that the bodies cannot come within one hundredth of the radius of the cluster of each other, thus preventing problems with infinities caused by collisions within the cluster.

Using this law, one can simulate the interactions between the constituent stars and thus investigate the evolution of a globular cluster over time.

2. Assumptions

To simplify the complexity of the celestial mechanics involved in many-body systems [3], several necessary assumptions were made:

- All bodies are represented by point masses
- All bodies have equal mass of one solar mass
- No external forces act on the system other than the gravitational force between the bodies
- The system follows a radial distribution – this mimics the fact that globular clusters have a higher concentration of stars towards their centres

3. Method

To begin the simulation, the initial conditions of the system needed to be set.

A random, radial distribution of 1000 particles within the radius of the cluster (~20 pc, where 1 pc = 3.09 x 10^{16} m) was first generated using spherical coordinates – this coordinate system naturally simulated the higher concentration of particles near the centre of the cluster since it is based on the combination of the radial distance, r, and the inclination, θ, and azimuthal, φ, angles.

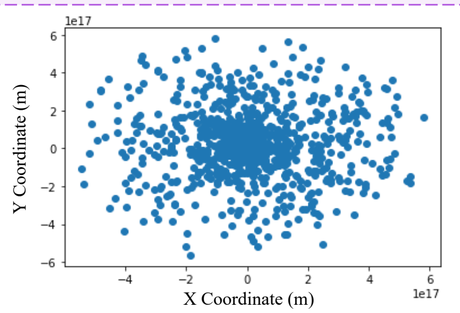

Figure 1: A 2-dimensional representation of the 3-dimensional distribution generated.

All particles were initially assumed to be stationary – this both kept the simulation simple and allowed tests on whether the cluster reached virial equilibrium to be conducted.

The leapfrog finite difference approximation method was then used to calculate the positions and velocities of each body at time steps, $n = 1000$, of size $dt = 10^5$ years. For this method to work, the positions and velocities had to be calculated half a time step out of phase using:

$$\boldsymbol{v_i^{n+1/2} = v_i^{n-1/2} + F_i \frac{dt}{m_i}}, \qquad (2) \qquad \boldsymbol{x_i^{n+1} = x_i^n + v_i^{n+1/2} dt}, \qquad (3)$$

where x_i and v_i are the particle positions and velocities, respectively. The method was used instead of other, more accurate, integration schemes as it is lightweight, stable and does not produce systematic errors [4].

From here the evolution of the cluster through time could be tracked with particle positions being plotted periodically to visualize the evolution. Kinetic, E_K, and potential, E_{PE}, energies of the system were calculated:

$$E_K = \sum_i^n m_i v_i^2, \qquad (4) \qquad E_{PE} = -Gm_i \sum_{j \neq i}^n \frac{m_j}{\left[|r_i - r_j|^2 + \epsilon^2\right]^{1/2}}, \qquad (5).$$

4. Discussion of Results

After evolving the cluster for 1000 timesteps of size $dt = 10^5$ years, most stars in cluster A, collapsed towards the centre of the cluster under gravity, as seen in Figure 2, with the final state being more densely packed. This was expected as the only force in the system was gravity meaning that the attractive force between particles drew them to the high concentration of mass in the centre of the radial distribution.

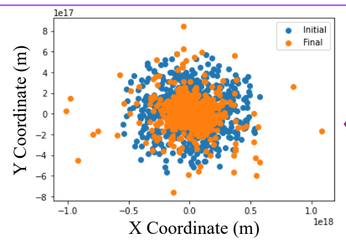

Figure 2: Initial and Final positions of stars in cluster A every 10^5 years for 1000 timesteps.

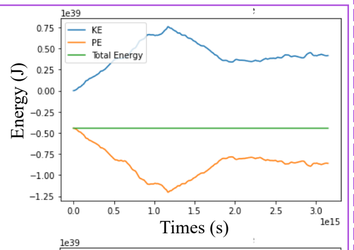

Figure 3: KE, PE and total energy of the system every 10^5 years for 1000 timesteps.

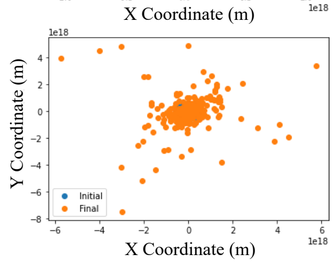

Figure 4: Initial and Final positions of stars in cluster B every 5 x 10^5 years for 1000 timesteps.

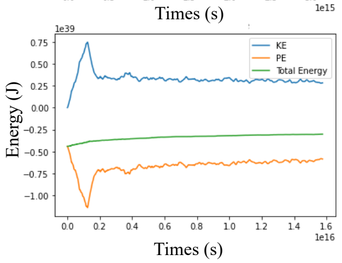

Figure 5: KE, PE and total energy of the system every 5 x 10^5 years for 1000 timesteps.

Cluster B produced a different outcome (as seen in Figure 4) where the initial state is almost entirely covered by the final state, implying a decreased cluster density and thus seeming to suggest that B did not collapse but instead exploded. However, analysis throughout the evolutions demonstrates both clusters first collapse (as seen by the peak in kinetic and potential energies in Figures 3 and 5) but after a certain time, which only B reaches (due to both A and B having the same number of timesteps but B's being larger at $dt = 5$ x 10^5 years), more particles are ejected than collapsed into the centre. Ejection of stars occurs in A too, as seen by the outer orange particles in Figure 2, but here it is on a smaller scale since there is less time for stars to get close enough for the gravitational force between them to cause them to fire out of the cluster with a high velocity.

Despite the particles starting with zero velocity, both clusters end in a state approximately in virial equilibrium - a concept stating that for a system of particles in equilibrium, double the kinetic energy divided by the potential energy should be equal to minus unity. Both clusters result in a final state with this ratio at -0.97 (2SF), which is close to the desired value suggesting that the simulation is valid. A reason for these values not being perfect could be due to the softening parameter which could explain why both systems reach the same value when checking the virial theorem.

Out of the two systems, A had a better energy conservation, losing only 0.067% (2SF) which could be due to the softening parameter affecting more stars in B than A. Evolving the cluster for longer means that the stars have more time to become close enough after the initial collapse for the softening parameter to prevent collisions and thus energy is not conserved as well in B as it is in A where less of the particles become close enough for the softening parameter to have a large affect.

5. Conclusions & Future Work

- Both clusters produce results in virial equilibrium suggesting a valid model
- Both models suggest that a globular cluster with no external forces acting on it collapses under the gravitational force between its constituent stars
- The softening parameter limits the quality of the model as it prevents any collisions between particles
- Evolving a cluster for longer results in worse energy conservation due to the softening parameter preventing more of the particles from colliding - therefore the evolution of cluster A is more stable, reliable and comparable to a real cluster as it conserves energy well with a 0.067% change between the initial and final states

Therefore, this simulation is suitable for clusters collapsing, but simulating collisions instead of using of a softening parameter would make this comparable to real clusters.

Now that a suitable model which conserves energy has been developed, this model will be further extended to investigate the collision of two globular clusters.

6. References

[1] M. A. Beasley, "Globular cluster systems and galaxy formation", *Reviews in Frontiers of Modern Astrophysics*, p. 245-277, Springer, Cham, 2020.

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[3] L. Greengard, "The Numerical Solution of the N-Body Problem", *Computers in Physics*, Vol 4, 142, 1990

[4]U.Mutze, *An Asynchronous Leap-frog Method*, p. 4