Quiz 3: Relations and Functions II

Q1 Consider $R \subseteq \mathbb{N} \times \mathbb{N}$ given by $(x, y) \in R$ if $x - y \geq 7$. Which of the properties Reflexivity (R) and Transitivity (T) does R have?

Answer: $x - x = 0 < 7$ for all $x \in \mathbb{N}$ so $(x, x) \notin R$ for all $x \in \mathbb{N}$. Therefore R is antireflexive (so not reflexive).

If $(x, y) \in R$ and $(y, z) \in R$ then $x - y \geq 7$ and $y - z \geq 7$. So $x - z = (x - y) + (y - z) \geq 14 \geq 7$. So $(x, z) \in R$. Therefore R is transitive.

Q2 Suppose R is a partial order. True or false: $R \cup R^{\leftarrow}$ is an equivalence relation.

Answer: Consider the partial order $R = \{(1,1), (2,2), (3,3), (1,2), (3,2)\}$. We have $(1,2) \in R$ and $(2,3) \in R^{\leftarrow}$ so $(1,2), (2,3) \in R \cup R^{\leftarrow}$, however $(1,3)$ is neither in R nor $R^{\leftarrow}$, so $(1,3) \notin R \cup R^{\leftarrow}$. Hence $R \cup R^{\leftarrow}$ is not an equivalence relation.

Q3 Consider the poset $(\{1, 3, 5, 9, 15, 45\}, |)$. What is $\text{glb}(15, 9)$?

Answer: The lower bounds of 15 and 9 are all the numbers in the set which divide both 15 and 9: $\{1, 3\}$. Of these, 3 is divisible by every element in $\{1, 3\}$ so it is the maximum element of the set of lower bounds. Hence $\text{glb}(15, 9) = 3$.

Q4 Suppose R is a symmetric relation. True or false: $R = R^{\leftarrow}$?

Answer: $(x, y) \in R$ if and only if $(y, x) \in R$ (because R is symmetric), and $(y, x) \in R$ if and only if $(x, y) \in R^{\leftarrow}$ (by the definition of converse). So $R = R^{\leftarrow}$.

Q5 Which of the following is the lexicographic ordering of: $01, 101, 1001, 11100, 01111, 0011$?

Answer: The lexicographic (i.e. dictionary) ordering is: 0011, 01, 01111, 1001, 101, 11100.