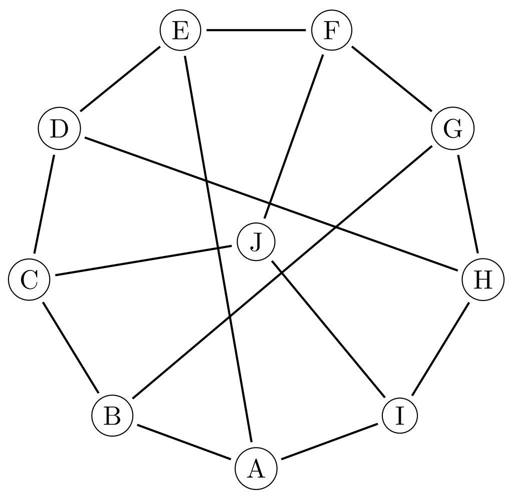
Quiz 4: Graphs

All questions refer to the following graph, G, which is the Petersen graph:



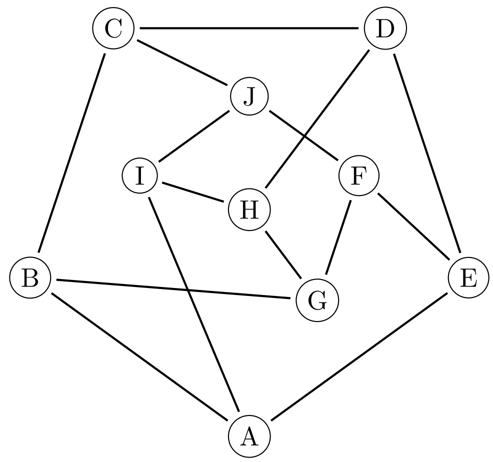
Q1 Which of the following is **not** isomorphic to G?

Answer: In G, every vertex has degree 3. In the graph specified by the adjacency matrix, the vertex in the fifth row has degree 4. So the adjacency matrix is not isomorphic to G. To see that the other representations are isomorphic to G:

Adjacency list: This is just the adjacency list of G.

A:	B, E, I
B:	A, C, G
C:	B, D, J
D:	C, E, H
E:	D, A, F
F:	E, G, J
G:	B, F, H
H:	D, G, I
I:	A, H, J
J:	C, F, I

Graph: Here is a labelling of the vertices showing the isomorphism.



Incidence matrix: Here is how we can label the columns and rows to get the incidence matrix of G:

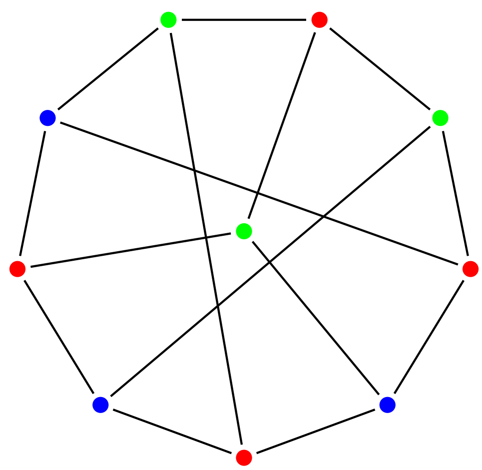
	AB	BC	CD	DE	EF	FG	GH	HI	AI	AE	BG	CJ	DH	FJ	IJ
A	1	0	0	0	0	0	0	0	1	1	0	0	0	0	0
B	1	1	0	0	0	0	0	0	0	0	1	0	0	0	0
C	0	1	1	0	0	0	0	0	0	0	0	1	0	0	0
D	0	0	1	1	0	0	0	0	0	0	0	0	1	0	0
E	0	0	0	1	1	0	0	0	0	1	0	0	0	0	0
F	0	0	0	0	1	1	0	0	0	0	0	0	0	1	0
G	0	0	0	0	0	1	1	0	0	0	1	0	0	0	0
H	0	0	0	0	0	0	1	1	0	0	0	0	1	0	0
I	0	0	0	0	0	0	0	1	1	0	0	0	0	0	1
J	0	0	0	0	0	0	0	0	0	0	0	1	0	1	1

Q2 Does G have an Euler path, Hamiltonian path, both, or neither?

Answer: There are more than two vertices with odd degree, so G does not have an Euler path. A-B-C-D-E-F-G-H-I-J is an example of a Hamiltonian path.

Q3 What is the chromatic number, $\chi(G)$, of G?

Answer: G contains an odd-length cycle (A-B-C-D-E-F-G-H-I-A) so it requires at least 3 colours, i.e. $\chi(G) \geq 3$. On the other hand, here is a 3-colouring showing $\chi(G) \leq 3$:



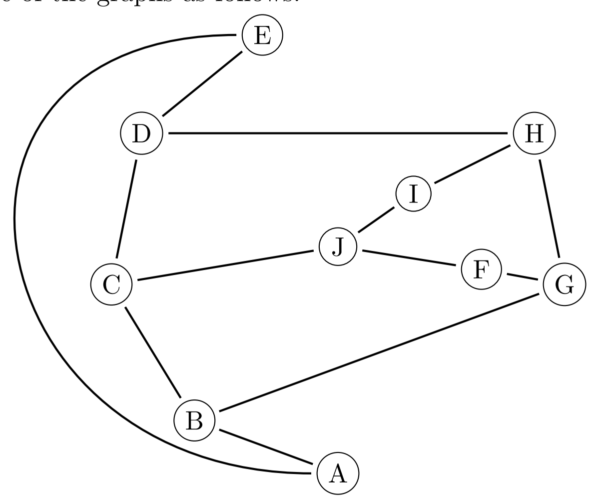
Q4 What is the clique number, $\kappa(G)$, of G?

Answer: There are edges so $\kappa(G) \geq 2$. There are no cycles with 3 vertices (i.e. 3-cliques), so $\kappa(G) \leq 2$.

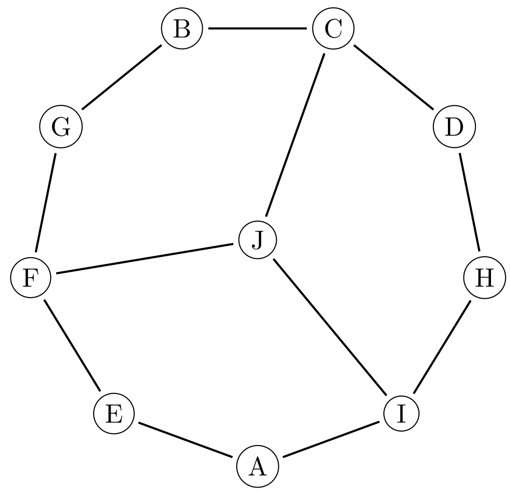
Q5 Which edges, when removed from G, result in a subdivision of $K_{3,3}$?

Answer: We can "untangle" three of the graphs as follows:

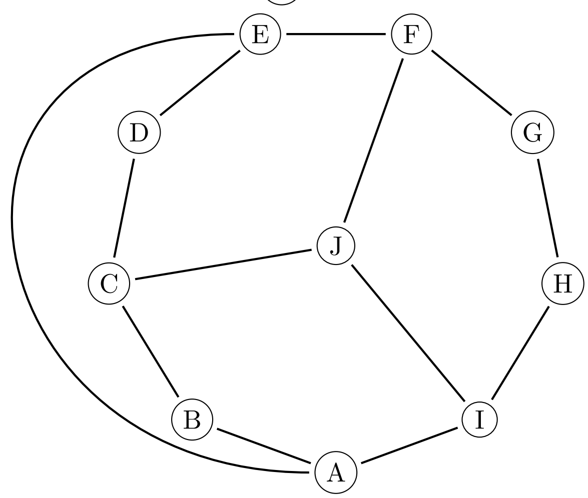
Remove $\{E, F\}$ and $\{A, I\}$



Remove $\{A, B\}, \{D, E\}, \{G, H\}$



Remove $\{B, G\}$ and $\{D, H\}$



This shows that these graphs are planar, so none of them can be a subdivision of $K_{3,3}$. For the remaining option we see that we can partition the six degree three vertices as $\{B, F, H\}$ and $\{D, G, I\}$ such that between each pair of vertices in separate partitions there is either an edge or a subdivided edge, and between each pair of vertices in the same partition there is neither an edge nor a subdivided edge.