1 Introduction

The sand particles can be electrified due to the transportation back to date ??[1]. The charges carried by the particles moving in the air will form a electric field, which is call the Aeolian electric field [2].

In 1913, Rudge observed the atmospheric field due to the presence of dust, and he found that the Aeolian electric field can reach up to 10kV/m in the severe dust storm [3]. Actually before Rudge's observation, M. Smith already observed the same thing during dust storms happened in India, and he found that the potential would often run up so rapidly and was impossible to measure [4]. Smidth, et al. experimentally investigated the charge- mass ratio of the sand particles and the Aeolian electric field near the sand dune of White state in USA, and they found that the charge-mass ratio of the sand particles with the particle radius 60μm is about -60μC/kg, and the Aeolian electric field can reach up to 166kV/m at the height 1.7cm above the sand dune surface [5]. Zheng's research group investigated the charge-mass ration and Aeolian electric field in the sand flux in a wind tunnel in detailed, and they not only found that the Aeolian electric field generated by the charges carried by the mixed sand particles is much more higher than the one generated by the charges carried by the uniform sand particles, but also presented the trend of the dependence of the Aeolian electric field and the charge-mass ratio on the wind speed particle size at the different height above the sand bed [6]. And the conclusions about the experimental results on the wind sand electrification and the further the numerical simulation of the Aeolian electric field can be seed in Refs. [7-11]. Apart from the wind speed and particle size, actually there are other environment conditions such as temperature and relative humidity also have an effect on the Aeolian electric field [12-14]. The one of the earliest work on the effect of the temperature and the relative humidity on the Aeolian electric field is carried out by ?? [12], he ???, but based on the experimental results he thought ??? [12]. However, some works on the contact electrification of the particles indicated that the relative humidity has a significant effect on the contact electrification, and sometimes the relative humidity determines if the contact electrification can be happened [15-17]. Even it was thought the H^+ ions in water film absorbed around the particles are the transferred charge carriers [15-16]. The relative humidity determines the thickness of the water film, that means the relative humidity has an important role to determine if the contact electrification can be happened. Xie and Han also carried out an experiment to investigate the dependence of the Aeolian electric field in a sand flux blown by the wind on the relative humidity in the wind tunnel in Lanzhou University of China, and they found that there exists a critical relative humidity, where the Aeolian electric field reaches up to its maximum value. With the increase of relative humidity, the Aeolian electric field linearly increases when the relative humidity is lower than the critical value, but exponentially decreases when the relatively humidity is higher than the critical value [18].

In order to figure out why the Aeolian electric field has such dependence on the relative humidity, Xie et al. carried out a experiment to observe the charges carried by a single glass particle owing to a single collision with other glassy particle under different relative humidity, they found that The experimental results indicate that the net charge on a sphere from a single collision is significantly altered by varying the RH level; the charge increases with increasing RH at low humidity, and then decreases at high RH conditions. The net charge reaches a maximum in the 20%–40% RH range [19]. To explain the dependence of the CE on RH, a model is proposed, which yields predictions in agreement with the experimental data. The model also reveals how CE can be affected by temperature and surface absorption energy [19]. By using this charging model of the particles, the Aeolian electric field can be numerically simulated in this paper. The simulation process is introduced in section 2 and the charge-mass ratio and the Aeolian electric field under different relative humidity in the developing wind sand flux and in the steady wind sand flux are discussed in section 3; a conclusion is drawn in section 4.

2 Simulation of wind blown sand movement

By using the AFT-MEP (advanced front technique and minimum energy principle) method proposed in Ref. [20], a 2D sand bed is generated by N sand particles shown as in Fig.1 . The height and the length of the sand bed are respectively named as H and L. The sand particles are taken as 3D spheres with radius R, and the sand size distribution function is $P(R)$. A rectangular coordinate system is established shown as in Fig.1, and the original of the coordinate, O, is located at the left lower angle of the sand bed, and x-axis and y-axis are along the left side and the bottom side of the sand bed. When the wind is blown over the sand bed with the wind speed $u(y) = u_* \ln\left(\frac{y}{y_0}\right)/k$ in which u_*, y_0 and k are respective friction wind speed, roughness and Karman constant, and meanwhile n sand particles collide with sand bed as an initial disturbance, and then the sand flux will be developed. Here, we adopt a direct numerical method to simulate the wind sand flux, further to calculate the Aeolian electric filed and the charge-mass ratio.

2.1 Motion equations of sand particles in sand bed

In the simulation, firstly the number of the sand particles contacting with a sand particle, named as i, need to be counted, and then identify where the particle i is located. If the particle i is located at the sand bed, only the normal force and tangential force between the sand particles due to the normal compression deformation and the tangential relative motion are considered, and the drag force is without consideration. Assume the position of the particle i is (x_i, y_i), by using the distinct element method (DEM) [21], and the motion equation of the particle i with mass m_i is expressed as following,

$$\begin{aligned} m_i\ddot{x}_i &= \sum_j \boldsymbol{F}_{nij,x} + \sum_j \boldsymbol{F}_{\tau ij,x} \\ m_i\ddot{y}_i &= \sum_j \boldsymbol{F}_{nij,y} + \sum_j \boldsymbol{F}_{\tau ij,y} - m_i g \end{aligned} \tag{1}$$

where g is the gravity acceleration , and g=9.8ms^{-2}. $\boldsymbol{F}_{nij}$ and $\boldsymbol{F}_{\tau ij}$ are the normal force and the tangential force between the particles i and j, and the subscripts x and y stand for the components along the x and y axis. According to the Hertz contact, the normal force can be calculated as following [22]

$$\boldsymbol{F}_{nij} = \begin{cases} k\delta_{ij}\boldsymbol{n}_{ij} & \text{if } \delta_{ij} > 0 \\ 0 & \text{if } \delta_{ij} \le 0 \end{cases} \tag{2}$$

in which k is the stiffness coefficient and δ_{ij} is the compression between the particles i and j, and $\delta_{ij} = \left(R_i + R_j\right) - \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$.

In the calculation of the tangential force, two situations are considered. When there is a relative tangential motion between the particles i and j happened at the contact point, the tangential force will be the sliding friction force which is proportional to the normal force, that means $\boldsymbol{F}_{\tau ij} = \mu \boldsymbol{F}_{nij}$, in which μ is the friction coefficient. If no relative tangential motion happened at the contact point of the particles i and j, the tangential force is considered to be proportional relative tangential deformation as,

$$\boldsymbol{F}_{\tau ij} = -k_t \delta_{t,ij} \boldsymbol{\tau}_{ij}, \quad \text{if } \left|\boldsymbol{F}_{\tau ij}\right| < \left|\mu \boldsymbol{F}_{nij}\right| \tag{3}$$

Where $\delta_{t,ij} = \int_0^{t_{ij}} \left|\boldsymbol{v}_{i,\tau} - \boldsymbol{v}_{j,\tau}\right| dt$, and $\boldsymbol{v}_{i,\tau}$ and $\boldsymbol{v}_{j,\tau}$ are the respective tangential velocities of the particles i and j at the contact point. $\boldsymbol{\tau}_{ij}$ is the relative tangential direction.

2.2 Motion equations of sand particles in sand flux

When the sand particle moving in the air, the drag force, electrostatics force and the gravity force are considered in the trajectory equation as

$$\begin{aligned} m_i \ddot{x}_i &= \boldsymbol{F}_{Ei,x} + \boldsymbol{F}_{Di,x} \\ m_i \ddot{y}_i &= \boldsymbol{F}_{Ei,y} + \boldsymbol{F}_{Di,y} - m_i g \end{aligned} \tag{4}$$

Where $\boldsymbol{F}_D$ is the drag force operating in the direction of the relative velocity of the sand particle and the local wind velocity [23], and may be written as

$$\boldsymbol{F}_{Di} = \frac{1}{2} \rho C_D \pi R_i^2 (\boldsymbol{v}_i - u)^2 \tag{5}$$

in which ρ is the air density, 1.22kgm^{-3}, and $\boldsymbol{v}_i = \dot{\boldsymbol{x}}_i + \dot{\boldsymbol{y}}_i$ is the particle velocity. $C_D = \frac{24}{R_e} + \frac{6}{1.0+\sqrt{R_e}} + 0.4$ is the drag coefficient, and $R_e = 2|\boldsymbol{v}_i - u|/\gamma$, is the Reynold's number where γ is the kinematic viscosity of air (=1.5×10^{-5}m^{2}s) [24].

The electrostatic force, $\boldsymbol{F}_{Ei}$, is calculated by the Aeolian electric field, $\boldsymbol{E}$, and the charges carried by the sand particle, q_i, as $\boldsymbol{F}_{Ei} = \boldsymbol{E}q_i$. Here we consider the collisions between the saltation particles and the sand particles in sand bed make the charge carriers transferred between the sand particles, and after separated, the sand particles are electrified and carry the net surface [25]. According to the collision contact charging model [26-28], assume that the sand particle surfaces are totally covered by the donors and the acceptors, the numbers of which are named as T_D and T_A, and when two sand particles are brought into collision contact, the donors belong to one surface can transfer to the other contact surface, which makes the sand particle carries net charges due to the areas of the collision contact surfaces are not equal. For any saltation sand particle *i*, due to the gravity, it will fall down and collides with the sand particles in the sand bed. In the sand flux, the particle *i* collides with the sand particle not only once. Therefore the amount of the net charges transferred that means the charges carried the particle *i* will cumulate due to multiple collisions with the sand bed. The amount of the net transferred charges due to n^{th} collision between the particle *i* and the bed particle *j* is,

$$\Delta Q^i(n) = \alpha\rho N_D^j \frac{N_A^i(n-1)}{N^i(n-1)} - \alpha^i(n)\rho N_D^i(n-1)\frac{N_A^j}{N^j} \tag{6}$$

in which α and $\alpha^i(n)$ are the transferred probabilities of the donors from the particle *j* to the particle *i* and from the particle *i* to the particle *j*. Here, we taken as $\alpha^i(n) = \alpha \times 0.8^n$, which is ??? [??] and α=0.132. ρ denotes the surface charge of a donor on particle *i* or *j*. $\rho=\rho_0 A_0$, where A_0 is the area of the donor/acceptor and ρ_0 denotes the charge density of a donor, which is approximate equal to that of a single electron charge. Based on the testing results, Baytekin determined the value for a single electron to be 10 nm^2 [???]. In order to guarantee the transferred charge is no less than a basic electron charge, we assume that ρ_0=0.02C/m^2. $N_{D/A}^i(n-1)$ and $N_{D/A}^j$ are respective numbers of donors/acceptors in the contact surfaces of particles *i* and *j*. Therefore, $N^i(n-1) = N_D^i(n-1) + N_A^i(n-1)$ and $N^j = N_D^j + N_A^j$. Assume the areas of the involved contact surfaces of the particles i and j as $S^i(n-1)$ and S^j, and thus $N^i(n-1) = S^i(n-1)/A_0$ and $N^j = S^j/A_0$ which are calculated by $S^i(n-1)$ and S^j when the incident velocities $v^i(n-1)$ and $v^i(n)$ of the particle i for the $(n\text{-}1)^{\text{th}}$ and the n^{th} collisions with the sand bed. The mean value of the net transferred charges can be calculated as

$$\left\langle \Delta Q^i(n) \right\rangle = \alpha\rho \frac{\left\langle N_D^j N_A^i(n-1) \right\rangle}{N^i(n-1)} - \alpha^i(n)\rho \frac{\left\langle N_D^i(n-1) N_A^j \right\rangle}{N^j} \tag{7}$$

Assume N_D^j and $N_A^i(n-1)$ are independent, and the number of the donor obeys the binomial distribution, and thus $\left\langle N_D^j N_A^i(n-1) \right\rangle = \left\langle N_D^j \right\rangle \left\langle N_A^i(n-1) \right\rangle = N^j P_D(0) N^i(n-1)\left[1 - P_D^i(n-1)\right]$ and $\left\langle N_A^j N_D^i(n-1) \right\rangle = \left\langle N_A^j \right\rangle \left\langle N_D^i(n-1) \right\rangle = N^j\left[1 - P_D(0)\right] N^i(n-1) P_D^i(n-1)$, in which $P_D(0)$ and $P_D^i(n-1)$ are the probabilities of the donors in the contact surfaces of the particle j and the particle i before their contact. As we know, every collision between the sand particles will change the number of the donors on the particle surface, so the number of the donors after n collision is

$$\left\langle T_D^i(n) \right\rangle = \left\langle T_D^i(n-1) \right\rangle + \frac{\left\langle \Delta Q^i(n) \right\rangle}{\rho} \tag{8}$$

Because the each collision occurs locally on small surfaces of the contact particles, which makes the distribution of the donors change only in some small surfaces of the particles, and thus the distribution of the donors in different zone of the particle surface will different. For simplification, we assume that the donors will redistribute on the whole particle surface after each collision, and still obeys the binomial distribution, therefore we have

$$P_D^i(n) = P_D^i(n-1) + \frac{\left\langle \Delta Q^i(n) \right\rangle}{T^i\rho} \tag{9}$$

in which $T^i = 4\pi R_i^2/A_0$. $P_D^i(0) = P_D(0)$. According to the model proposed in Ref. [??], $P_D(0) = \frac{1}{\left(k_B T/\lambda^3\right)\left(\Pi/P\right)e^{-\xi_0/k_B T}+1}$, where k_B is Boltzmann's constant (k_B =1.38×10^{-23}JK^{-1}), and T is the temperature. $\lambda^3 = \left[2\pi m k_B/h^2\right]^{-3/2}$, in which m is the mass of a charge and h is Planck's constant (h=6.63×10^{-34}Js). $\Pi = \int_1^{\infty} e^{-\frac{q^2 t}{4\pi z_0 \varepsilon_0 k_B T}}/t^2 dt$, where z_0 is the thickness of the water film and ε_0 is the permittivity. ξ_0 is the adsorption energy and P is the vapor pressure. P can be calculated by the relative humidity, RH, and the saturated vapor pressure, P_s, when T is given, and that is $P = RH \times P_s/100$.

Here, we assume that all sand particles before the simulation are neutralized, and in the simulation the sand particles in the sand bed and the saltation sand particles fallen back into the sand bed without lifting off again are neutralized. Therefore, the charges carried by the a sand particle are accumulated experiencing multiple collisions with the sand bed,

$$Q^i = \sum_n \left\langle \Delta Q^i(n) \right\rangle \tag{10}$$

2.3 Sand transportation rate

When the sand particle moving in air, the sand mass transportation rate including the horizontal mass transportation rate, $M_h(x, y, t)$, and the vertical mass transportation rate, $M_v(x, y, t)$. Here we assume that the sand particles uniformly arranged along the lateral direction, therefore the number of the sand particles in a cube (x, x+△x; y, y+△y; 0, 1) is related to the one in the region (x, x+△x; y, y+△y). However, there is not an exact relationship between them. If the sand particles arrange closely along the lateral direction and the sand particles are equal in size, the number of the sand particles in a cube (x, x+△x; y, y+△y; 0, 1) is $1/\bar{D}$ times of the one in the region (x, x+△x; y, y+△y), in which $\bar{D}$ is the mean particle diameter which is the sand particles' diameter for the uniform sand flux. For the mixed-size sand flux, we still assume the number of the sand particles in a cube (x, $x+\triangle x$; y, $y+\triangle y$; 0, 1) is $1/\bar{D}$ times of the one in the region (x, $x+\triangle x$; y, $y+\triangle y$). But, usually the sand particles can not be arranged closely along the lateral direction, so it can be assumed that the number of the sand particles in a cube (x, $x+\triangle x$; y, $y+\triangle y$; 0, 1) is $\eta 1/\bar{D}$ times of the one in the region (x, $x+\triangle x$; y, $y+\triangle y$), in which parameter 0<η<1, and thus

$$
\begin{aligned}
M_h(x,y,t) &= \frac{\eta\left[\sum_{x}^{x+\Delta x}\sum_{y}^{y+\Delta y}\left(m_{l,\rightarrow}\dot{x}_{l\rightarrow} - m_{l\leftarrow}\dot{x}_{l\leftarrow}\right)\right]}{\Delta x\Delta y\bar{D}} \\
M_v(x,y,t) &= \frac{\eta\left[\sum_{x}^{x+\Delta x}\sum_{y}^{y+\Delta y}\left(m_{l,\uparrow}\dot{y}_{l\uparrow} - m_{l\downarrow}\dot{y}_{l\downarrow}\right)\right]}{\Delta x\Delta y\bar{D}}
\end{aligned}
\tag{11}
$$

2.4 Wind sand electrification

According to the sand particles occurring in the cub (x, $x+\triangle x$; y, $y+\triangle y$; 0, 1) at time t, the charges carried on these sand particles can be calculated simultaneously, and thus the charge-mass ratio per volume also can be calculated as

$$
q(x,y,t) = \frac{\left[\sum_{x}^{x+\Delta x}\sum_{y}^{y+\Delta y} Q_l / m_l\right]}{\Delta x\Delta y\bar{D}}
\tag{12}
$$

Due to the charges carried by the sand particles, an electric field will be generated. Assume that the charges carried by a sand particle can be thought as a point charge, and the electric field generated by this charged sand particle is inversely proportional to the distance, r, between the position of the charged sand particle and any other position as $\frac{Q_i \boldsymbol{r}}{4\pi\varepsilon r^2}$, in which ***r*** **pints to ???** and ε is the permittivity of the atmosphere in the sand flux. Because the sand flux is not uniform along the wind direction, that means x-direction in the development of the sand flux, here we only investigate the Aeolian electric field at ($x=L/2$, y). Consider the sand flux uniform along the lateral direction, so the Aeolian electric field has two components, Ex and Ey, along the wind direction and vertical sand bed, respectively. Thus,

$$Ex(L/2,y,t)=\frac{2\pi|x_i-L/2|\eta}{\overline{D}}\sum\frac{Q_i(y-y_i)}{4\pi\varepsilon\left[(x_i-L/2)^2+(y_i-y)^2\right]}$$
$$Ey(L/2,y,t)=\frac{2\pi|x_i-L/2|\eta}{\overline{D}}\sum\frac{Q_i(L/2-x_i)}{4\pi\varepsilon\left[(x_i-L/2)^2+(y_i-y)^2\right]} \tag{13}$$

2.5 Wind filed

As the sand particles lifting off away from sand bed increasing, these sand particle will exert force to the wind due to the wind accelerating the sand particles. Here, we only consider the horizontal and lateral uniform wind and the horizontal body force, $Fx(y, t)$, acting in the upwind direction, and the Navier-Stokes equation is

$$\frac{\partial u}{\partial t}=\nu\frac{\partial^2 u}{\partial y^2}-Fx(y,t)/\rho_a \tag{14}$$

in which ν and ρ_a are the kinematic viscosity and the density of air, and usually ν $=1.5\times10^{-5}$m^{2}s^{-1} and ρ_a=1.029kgm^{-3} at 20°C [Anderson, 1991]. According to McEwan et al., at the height far above the sand flux, the wind cannot be affected by the saltating sand particles, therefore the far-field shear stress is determined by the friction wind speed as $\rho_a k^2 y^2 \left(\frac{\partial u}{\partial y}\right)^2 = \rho_a u_*^2$ [???].

2.6 Simulation process

According to ???

1) Firstly a sand bed including N sand particles is generated, and the particle size distribution is P(r); all sand bed are still without initial velocities and the positions are already obtained during the sand bed generating process.

2) Choose Ni sand particles incident on the sand bed and collide with the sand particles in the sand bed, and the collision positions are uniformly randomly selected; the collision velocities are obeyed the initial ??? and the collision angles are uniformly selected from the region (0°, 15°).

3) Usually the collision time between any two sand particles is no beyond 10^{-4}s, while the saltation time is ~1s. There is 10^4 order in these two time scales. It is necessary to

simulate the collision process to obtain the maximum area of contact surface to calculate the charges carried by the sand particle. So, we select two time step lengths, and one is named as Δt_1 and the other is named as Δt_2. Considering that the sand particle cannot move two far, so we only simulate the collision process between sand particles and calculate the maximum areas of the contact surfaces of the sand particles, to calculate the charges carried by the sand particles in time step Δt_2. In time step Δt_1, we calculate the trajectories of sand particles by (4), to sand flux transportation rates by (??), the charges distribution by (??), the Aeolian electric field by (??) and the drag force of the air exerting upon the sand particles (??).

4) Calculate the air born force per volume exerting upon the sand particles by (??), and thus the force of the sand particles exerting upon the wind can be obtained, and then calculate the wind field by (14).

Here, the period condition is considered, and repeat (3)-(4) until the sand flux reaches to steady.

3 Results and discussions