

UNIT I: CLASSICAL MECHANICS

Module 3: Work, Energy, and Momentum



Introduction

Energy is an essential and fundamental concept in physics that plays a crucial role in our daily lives. It is present in various forms such as food, fuel for cars, and the warmth we feel under the sun. Moreover, energy exists in different scientific phenomena and is a vital component of many natural processes. The conservation of energy is a fundamental law in physics, which states that the total amount of energy in the universe remains constant, but it can be transformed from one form to another. In this module, we will explore the concept of energy, which is loosely defined as the ability to do work. We will begin with the discussion of work and move on to learn about momentum and collisions, which are closely related to energy.

By the end of this module, you will have a better understanding of energy and its significance in various physical phenomena. You may even be able to apply these concepts to improve your skills in activities such as billiards, bowling, and weightlifting.

Objectives

- After working on this module, you should be able to:
 - Define work,
 - Differentiate kinetic and potential energy,
 - Explain work-energy theorem,
 - Relate conservative and non-conservative forces to mechanical energy,
 - Understand the law of conservation of energy,
 - Explain momentum,
 - Calculate the momentum of moving objects,
 - Understand the law of conservation of momentum, and
 - Solve collision problems
-

TOPIC 8:***Work and Energy***

Work and energy are fundamental concepts in physics. In this module, we will discuss the definition and types of work, as well as the concepts of kinetic and potential energy and their relationship.

Work:

Work is defined as the transfer of energy that occurs when a force is applied over a distance. The mathematical formula for work is:

$$W = Fd \cos (\theta)$$

where W is work, F is the force applied, d is the distance over which the force is applied, and θ is the angle between the force and the direction of motion. Work is measured in units of joules (J).

Types of Work:

There are two main types of work: positive work and negative work. Positive work is done when the force applied and the direction of motion are in the same direction. Negative work is done when the force applied and the direction of motion are in opposite directions. No work is done when the force applied is perpendicular to the direction of motion.

Kinetic Energy:

Kinetic energy is the energy that an object possesses due to its motion. The mathematical formula for kinetic energy is:

$$K = (1/2)mv^2$$

where K is the kinetic energy, m is the mass of the object, and v is its velocity. Kinetic energy is measured in units of joules (J).

Potential Energy:

Potential energy is the energy that an object possesses due to its position or configuration. The two main types of potential energy are gravitational potential energy and elastic potential energy.

Gravitational Potential Energy:

Gravitational potential energy is the energy that an object possesses due to its position relative to a gravitational field. The mathematical formula for gravitational potential energy is:

$$U_g = mgh$$

Where U_g is the gravitational potential energy, m is the mass of the object, g is the acceleration due to gravity, and h is the height of the object above a reference level.

Elastic Potential Energy:

Elastic potential energy is the energy that an object possesses due to its deformation from its original shape. The mathematical formula for elastic potential energy is:

$$U_e = (1/2)kx^2$$

Where U_e is the elastic potential energy, k is the spring constant, and x is the displacement of the object from its equilibrium position.

Conservation of Energy:

The law of conservation of energy states that energy cannot be created or destroyed, only transformed from one form to another. In other words, the total energy of a closed system remains constant over time. This principle is often used to analyze physical systems and predict their behavior.

Applications of Work and Energy:

Work and energy have numerous practical applications in our daily lives, including:

Roller coasters: Roller coasters rely on the transfer of potential energy to kinetic energy to provide thrills and excitement.

Electrical power generation: Electrical power plants convert the potential energy of falling water or steam into kinetic energy, which is then converted to electrical energy.

Sports: Sports such as baseball, basketball, and soccer involve the transfer of kinetic energy from the athlete to the ball.

Transportation: Cars, bicycles, and other forms of transportation rely on the conversion of chemical energy to kinetic energy to provide motion.

Work and energy are fundamental concepts in physics that have numerous practical applications in our daily lives. Understanding the concepts of work, kinetic energy, potential energy, and conservation of energy is important in analyzing physical systems and predicting their behavior. By understanding work and energy, we can design and optimize systems that are more efficient and effective.

Here are some recommended YouTube videos on work and energy:

- **"Work, Energy, and Power" by Khan Academy:** This video provides a clear and detailed explanation of the concepts of work, energy, and power. It also includes several examples to help solidify your understanding.
- **"Work-Energy Principle" by The Physics Classroom:** This video focuses on the work-energy principle, which is an important concept in physics. It



provides a step-by-step explanation of how to apply the principle to solve problems.

- **"Potential Energy" by Flipping Physics:** This video provides an in-depth explanation of potential energy, including gravitational potential energy and elastic potential energy. It also includes several examples to help you understand the concept better.
- **"Conservation of Energy" by Michel van Biezen:** This video discusses the principle of conservation of energy, which is another important concept in physics. It explains how energy can be transformed from one form to another, but cannot be created or destroyed.
- **"Kinetic Energy" by Bozeman Science:** This video provides an explanation of kinetic energy and how it relates to work and potential energy. It also includes several examples to help you understand the concept better.



If you prefer to have a more detailed discussion on this topic, you can read the following chapters in textbooks:

- Chapter 7 of College Physics by Paul Peter Urone & Roger Hinrichs (pp. 241-278)
- Chapter 7-8 of University Physics Vol. 1 by Samuel J. Ling, Jeff Sanny & William Moebs (pp. 327-386)
- Chapter 8 of Physics for Dummies by Steven Holzner (pp. 119-136)

Here are some tips to keep in mind solving problems about work and energy:

1. Identify the system: Before attempting to solve a problem, it's important to identify the system you're working with. This includes identifying the objects or particles involved, the forces acting on them, and the energy forms present in the system.
2. Draw a diagram: Drawing a diagram of the system can help you visualize the problem and identify the forces acting on the objects. This can help you determine which forces are doing work on the system, and which forces are not.
3. Determine the energy involved: Work and energy problems involve different forms of energy, such as kinetic energy, potential energy, and gravitational energy among others. It's important to determine which energy forms are involved in the problem and how they are changing.
4. Apply conservation of energy: The law of conservation of energy states that energy cannot be created or destroyed, only transferred from one form to another. This means that the total energy of a system remains constant, and any changes in energy must come from transfers between different forms of energy.
5. Use equations and formulas: There are various equations and formulas that can be used to solve work and energy problems, such as the work-energy theorem, the conservation of mechanical energy equation, and the equations for calculating potential energy and kinetic energy.
6. Pay attention to units: Make sure to use consistent units throughout your calculations and convert units if necessary. It's also important to pay attention to the units of the answer, and make sure it makes sense in the context of the problem.

7. Check your answer: Once you've solved the problem, make sure to check your answer to ensure it makes sense and is reasonable. For example, the answer should not be negative if the question asks for work or energy, and the units should be correct.

By following these tips, you can improve your problem-solving skills and become more proficient in solving work and energy problems in physics.

Example 1: A 2-kg block is lifted 3 meters vertically against gravity. What is the work done by the person lifting the block, and what is the potential energy of the block at the end of the lift?

$$\text{Work done} = \text{force} \times \text{distance} = mg \times h = 2 \text{ kg} \times 9.8 \text{ m/s}^2 \times 3 \text{ m} = 58.8 \text{ J}$$

$$\text{Potential energy} = mgh = 2 \text{ kg} \times 9.8 \text{ m/s}^2 \times 3 \text{ m} = 58.8 \text{ J}$$

Example 2: A car with a mass of 1000 kg accelerates from rest to 20 m/s in 10 seconds. What is the work done by the car's engine, and what is its kinetic energy at the end of the acceleration?

$$\text{Work done} = \text{change in kinetic energy} = 1/2 mv^2 - 1/2 m(0)^2 = 1/2 \times 1000 \text{ kg} \times (20 \text{ m/s})^2 = 200,000 \text{ J}$$

$$\text{Kinetic energy} = 1/2 mv^2 = 1/2 \times 1000 \text{ kg} \times (20 \text{ m/s})^2 = 200,000 \text{ J}$$

Example 3: A spring with a spring constant of 100 N/m is compressed by 0.1 meters. What is the potential energy stored in the spring?

$$\text{Potential energy} = 1/2 kx^2 = 1/2 \times 100 \text{ N/m} \times (0.1 \text{ m})^2 = 0.5 \text{ J}$$

Example 4: A roller coaster car with a mass of 500 kg is released from a height of 50 meters above the ground. What is its potential energy at the top of the hill, and what is its kinetic energy at the bottom of the hill (assuming no energy loss due to friction)?

$$\text{Potential energy at the top of the hill} = mgh = 500 \text{ kg} \times 9.8 \text{ m/s}^2 \times 50 \text{ m} = 245,000 \text{ J}$$

$$\text{Kinetic energy at the bottom of the hill} = \text{potential energy at the top of the hill} = 245,000 \text{ J.}$$

These examples illustrate the application of work and energy concepts in different scenarios.

- The first example shows how the work done by a person lifting a block against gravity results in an increase in the potential energy of the block, which is the energy stored in an object due to its position in a gravitational field.
- The second example shows how the work done by a car's engine results in an increase in its kinetic energy, which is the energy an object possesses due to its motion.

- The third example shows how the potential energy stored in a spring is directly proportional to the square of its displacement, and the constant of proportionality is the spring constant.
- The fourth example shows how potential energy can be converted to kinetic energy as the roller coaster car descends the hill, and the total mechanical energy (potential energy plus kinetic energy) of the car remains constant in the absence of energy loss due to friction.

Overall, these examples demonstrate the relationship between work, energy, and forces, and how they play a role in various physical phenomena.

TOPIC 9

Momentum

Momentum is a fundamental concept in physics that describes the motion of an object. In this module, we will discuss the definition of momentum, its conservation, and its applications.

Momentum:

Momentum is defined as the product of an object's mass and velocity. The mathematical formula for momentum is:

$$p = m \cdot v$$

where p is momentum, m is the mass of the object, and v is its velocity. Momentum is measured in units of kilogram-meters per second ($\text{kg} \cdot \text{m/s}$).

Conservation of Momentum:

The law of conservation of momentum states that the total momentum of a closed system remains constant over time, provided that there are no external forces acting on the system. This principle is often used to analyze physical systems and predict their behavior.

Types of Collisions:

There are two main types of collisions: elastic and inelastic. In an elastic collision, the kinetic energy of the system is conserved, while in an inelastic collision, the kinetic energy is not conserved.

Elastic Collision:

In an elastic collision, the momentum and kinetic energy of the system are conserved. This means that the total momentum of the system before the collision is equal to the total momentum of the system after the collision, and the total kinetic energy of the system before the collision is equal to the total kinetic energy of the system after the collision.

Inelastic Collision:

In an inelastic collision, the momentum of the system is conserved, but the kinetic energy is not. This means that the total momentum of the system before the collision is equal to the total momentum of the system after the collision, but the total kinetic energy of the system before the collision is greater than the total kinetic energy of the system after the collision.

Applications of Momentum:

Momentum has numerous practical applications in our daily lives, including:

- *Automotive safety*: The design of car airbags and other safety features are based on the principles of momentum and its conservation.
- *Sports*: The transfer of momentum is important in many sports, such as football and basketball, where players must use their momentum to push or block opponents.
- *Rocket propulsion*: The principles of momentum and conservation of momentum are used in rocket propulsion to provide thrust and propel spacecraft into space.
- *Pool*: The principles of momentum are important in the game of pool, where players must use the momentum of the cue ball to hit other balls and score points.

Momentum is a fundamental concept in physics that describes the motion of an object. Understanding the principles of momentum and conservation of momentum is important in analyzing physical systems and predicting their behavior. By understanding momentum, we can design and optimize systems that are more efficient and effective.

Here are some excellent YouTube videos on momentum:

- **"Momentum and Impulse" by Khan Academy** - This video provides a comprehensive introduction to momentum and impulse and explains how they are related to force and acceleration.
- **"Conservation of Momentum" by Crash Course Physics** - This video explains the principle of conservation of momentum, which states that the total momentum of a closed system remains constant.
- **"Elastic and Inelastic Collisions" by Michel van Biezen** - This video explains the differences between elastic and inelastic collisions, and how they affect the momentum of objects involved.
- **"Momentum: Definition and Units" by The Science Asylum** - This video provides a clear definition of momentum and explains its units of measurement.
- **"Momentum and Kinetic Energy" by Flipping Physics** - This video explains the relationship between momentum and kinetic energy, and how they can be used to solve problems in physics.

If you prefer to have a more detailed discussion on this topic, you can read the following chapters on textbooks from OpenStax:

- Chapter 8 of College Physics by Paul Peter Urone & Roger Hinrichs (pp. 287-295)
- Chapter 9 of University Physics Vol. 1 by Samuel J. Ling, Jeff Sanny & William Moebs (pp. 395-423)
- Chapter 9 of Physics for Dummies by Steven Holzner (pp. 137-147)

Example 5: A 2-kg mass moving at 3 m/s collides with a stationary 3-kg mass. After the collision, the 2-kg mass moves in the opposite direction at 1 m/s. What is the velocity of the 3-kg mass after the collision?
Solution: To solve this problem, we can use the conservation of momentum. Before the collision, the total momentum is:

$$2 \text{ kg} \times 3 \text{ m/s} + 0 \text{ kg} \times 0 \text{ m/s} = 6 \text{ kg m/s}$$

After the collision, the total momentum is:

$$-2 \text{ kg} \times 1 \text{ m/s} + 3 \text{ kg} \times v = 0 \text{ kg m/s}$$

Solving for v , we get:

$$v = (2/3) \text{ m/s}$$

So, the velocity of the 3-kg mass after the collision is $2/3$ m/s.

Example 6: A 0.1-kg bullet traveling at 500 m/s strikes a 5-kg block of wood at rest. If the bullet becomes embedded in the block, what is the velocity of the block after the collision?

Solution: To solve this problem, we can use the conservation of momentum. Before the collision, the total momentum is:

$$0.1 \text{ kg} \times 500 \text{ m/s} + 0 \text{ kg} \times 0 \text{ m/s} = 50 \text{ kg m/s}$$

After the collision, the total momentum is:

$$(0.1 \text{ kg} + 5 \text{ kg}) \times v = 50 \text{ kg m/s}$$

Solving for v , we get:

$$v = 10 \text{ m/s}$$

So, the velocity of the block after the collision is 10 m/s.

Example 7: A 0.5-kg cart traveling at 2 m/s collides with a 1-kg cart traveling in the opposite direction at 1 m/s. After the collision, the 0.5-kg cart moves in the opposite direction at 1 m/s. What is the velocity of the 1-kg cart after the collision?

Solution: To solve this problem, we can use the conservation of momentum. Before the collision, the total momentum is:

$$0.5 \text{ kg} \times 2 \text{ m/s} + 1 \text{ kg} \times (-1 \text{ m/s}) = 0 \text{ kg m/s}$$

After the collision, the total momentum is:

$$-0.5 \text{ kg} \times 1 \text{ m/s} + 1 \text{ kg} \times v = 0 \text{ kg m/s}$$

Solving for v , we get:

$$v = 0.5 \text{ m/s}$$

So, the velocity of the 1-kg cart after the collision is 0.5 m/s in the opposite direction.

Example 8: A 20-kg cannon is mounted on a frictionless platform. The cannon fires a 2-kg cannonball with a velocity of 100 m/s. What is the recoil velocity of the cannon?

Solution: To solve this problem, we can use the conservation of momentum. Before firing, the total momentum is:

$$20 \text{ kg} \times 0 \text{ m/s} = 0 \text{ kg m/s}$$

After firing, the total momentum is:

$$-2 \text{ kg} \times 100 \text{ m/s} + 20 \text{ kg} \times v = 0 \text{ kg m/s}$$

Solving for v , we get:

$$v = 10 \text{ m/s}$$

So, the recoil velocity of the cannon is 10 m/s in the opposite direction.

From these examples, we can learn how to apply the conservation of momentum principle in solving problems related to collisions and motion. The conservation of momentum states that the total momentum of a system is conserved before and after the collision, provided that there are no external forces acting on the system. We can use this principle to solve for unknown variables such as velocity or mass in the system. We can also learn how to use the formula for momentum, which is the product of mass and velocity, to calculate the total momentum of the system before and after the collision. These examples demonstrate that the conservation of momentum is a fundamental concept in physics that can be used to explain and predict the behavior of objects in motion.

TOPIC 10:

Collisions

Collisions are a fundamental concept in physics that describes the interaction between two or more objects. In this module, we will discuss the types of collisions, conservation of momentum, and applications of collisions. As stated in Topic 9, there are two types of collisions:

② ***Elastic Collisions:***

In an elastic collision, the total kinetic energy of the system is conserved, as well as the momentum. During an elastic collision, the objects involved bounce off each other without any loss of energy.

② ***Inelastic Collisions:***

In an inelastic collision, the total kinetic energy of the system is not conserved, but the momentum is still conserved. During an inelastic collision, the objects involved may stick together or deform upon impact, resulting in a loss of energy.

Applications of Collisions:

Collisions have numerous practical applications in our daily lives, including:

Automotive safety: The design of car airbags and other safety features are based on the principles of collisions and their conservation.

Sports: Collisions are important in many sports, such as basketball and soccer, where players collide with each other or with objects like balls.

Industrial machinery: The principles of collisions are important in designing and optimizing industrial machinery, such as conveyor belts and packaging equipment.

Particle physics: The study of collisions between subatomic particles is important in particle physics, which seeks to understand the fundamental building blocks of the universe.

Collisions are a fundamental concept in physics that describe the interaction between two or more objects. Understanding the types of collisions, conservation of momentum, and applications of collisions is important in analyzing physical systems and predicting their behavior. By understanding collisions, we can design and optimize systems that are more efficient and effective.



If you prefer to have a more detailed discussion on this topic, you can read the following chapters on textbooks from OpenStax:

- Chapter 8 of College Physics by Paul Peter Urone & Roger Hinrichs (pp. 295-304)
- Chapter 9 of Physics for Dummies by Steven Holzner (pp. 148-152)

Here are some excellent YouTube videos on collisions:



- **"Elastic and Inelastic Collisions" by Khan Academy** - This video provides a clear explanation of the differences between elastic and inelastic collisions and how to calculate the momentum and kinetic energy of objects involved in a collision.
- **"Introduction to Collision Theory" by The Organic Chemistry Tutor** - This video provides an introduction to the concept of collision theory in chemistry, explaining the factors that affect the rate of a chemical reaction.
- **"Elastic Collisions: A Real Head Banger" by Veritasium** - This video explores the concept of elastic collisions and the conservation of momentum in a fun and engaging way.
- **"Inelastic Collisions and Explosions" by Doc Schuster** - This video explains the concept of inelastic collisions and how to calculate the velocity and momentum of objects involved in an inelastic collision.
- **"Momentum and Collisions" by Bozeman Science** - This video provides a comprehensive overview of the concepts of momentum and collisions in physics, including examples and practice problems.

Example 9: A 0.1-kg ball is traveling east at 2 m/s and collides with a 0.2-kg ball that is at rest. After the collision, the 0.1-kg ball moves south at 1 m/s. What is the velocity of the 0.2-kg ball after the collision?

Solution: Using conservation of momentum, we can calculate the velocity of the 0.2-kg ball after the collision. The initial momentum of the system is $0.1 \text{ kg} * 2 \text{ m/s} = 0.2 \text{ kg m/s}$. After the collision, the momentum of the system is $(0.1 \text{ kg} * (-1 \text{ m/s})) + (0.2 \text{ kg} * v)$, where v is the velocity of the 0.2-kg ball. Equating the two momenta, we get:

$$0.2 \text{ kg m/s} = -0.1 \text{ kg m/s} + 0.2 \text{ kg} * v$$

$$v = 1.5 \text{ m/s}$$

Therefore, the 0.2-kg ball moves east at 1.5 m/s after the collision.

Example 10: A 1,000-kg car is traveling north at 20 m/s and collides with a 2,000-kg truck that is traveling east at 10 m/s. After the collision, the car and truck stick together and move in a direction that makes an angle of 37 degrees east of north. What is the velocity of the car and truck after the collision?

Solution: Using conservation of momentum and conservation of kinetic energy, we can calculate the velocity of the car and truck after the collision. The initial momentum of the system is $(1,000 \text{ kg} * 20 \text{ m/s}) + (2,000 \text{ kg} * 10 \text{ m/s}) = 40,000 \text{ kg m/s}$. The initial kinetic energy of the system is $(1/2 * 1,000 \text{ kg} * (20 \text{ m/s})^2) + (1/2 * 2,000 \text{ kg} * (10 \text{ m/s})^2) = 500,000 \text{ J}$.

After the collision, the momentum of the system is $(1,000 \text{ kg} + 2,000 \text{ kg}) * v$, where v is the velocity of the car and truck after the collision. The kinetic energy of the system is $(1/2 * (1,000 \text{ kg} + 2,000 \text{ kg}) * v^2)$.

Using conservation of momentum and conservation of kinetic energy, we get two equations:

$$40,000 \text{ kg m/s} = (1,000 \text{ kg} + 2,000 \text{ kg}) * v * \sin(37 \text{ degrees})$$

$$500,000 \text{ J} = (1/2 * (1,000 \text{ kg} + 2,000 \text{ kg}) * v^2)$$

Solving these equations, we get:

$$v = 18.6 \text{ m/s}$$

Therefore, the car and truck move in a direction that makes an angle of 37 degrees east of north at a velocity of 18.6 m/s after the collision.

Example 11: Two cars are driving toward each other on a straight road. Car A has a mass of 1200 kg and is traveling at 25 m/s, while Car B has a mass of 900 kg and is traveling at 30 m/s. They collide head-on, and after the collision, they stick together and move off at 12 m/s. What is the velocity of the combined cars just before the collision?

Solution: Using the conservation of momentum, we can find the velocity of the combined cars just before the collision. Momentum before the collision is equal to momentum after the collision, so we have:

$$(1200 \text{ kg}) * (25 \text{ m/s}) + (900 \text{ kg}) * (30 \text{ m/s}) = (2100 \text{ kg}) * (12 \text{ m/s})$$

Rearranging and solving for the velocity of the combined cars just before the collision, we get:

$$v = (1200 \text{ kg} * 25 \text{ m/s} + 900 \text{ kg} * 30 \text{ m/s}) / (1200 \text{ kg} + 900 \text{ kg})$$

$$v = 27.5 \text{ m/s}$$

Therefore, the velocity of the combined cars just before the collision was 27.5 m/s.

Example 12: A 0.3 kg hockey puck slides along the ice at 5 m/s and collides with a stationary 0.2 kg hockey puck. After the collision, the first puck moves off at an angle of 60 degrees to its original path with a speed of 3 m/s, while the second puck moves off at an angle of 30 degrees to the original path with a speed of 4 m/s. What is the speed and direction of the combined system after the collision?

Solution: Using the conservation of momentum and the conservation of kinetic energy, we can find the velocity and direction of the combined system after the collision. First, we find the velocity of each puck after the collision using the momentum conservation equation:

$$0.3 \text{ kg} \times 5 \text{ m/s} = 0.3 \text{ kg} \times 3 \text{ m/s} \cos 60 + 0.2 \text{ kg} \times 4 \text{ m/s} \cos 30$$

$$v_1 = 2.5 \text{ m/s}$$

$$v_2 = 2.45 \text{ m/s}$$

Next, we use the conservation of kinetic energy equation to find the angle between the combined system and the original path:

$$0.5 \times 0.3 \text{ kg} \times 5^2 + 0.5 \times 0.2 \text{ kg} \times 0^2 = 0.5 \times (0.3 \text{ kg} + 0.2 \text{ kg}) \times v^2$$

$$v = 3.63 \text{ m/s}$$

Finally, we use the law of sines to find the direction of the combined system:

$$\sin \theta / 3.63 = \sin 60 / 2.5$$

$$\sin \theta = 0.863$$

$$\theta = 60.6 \text{ degrees}$$

Therefore, the speed of the combined system after the collision is 3.63 m/s, and its direction is 60.6 degrees from the original path.

From these examples, we can learn that the conservation of momentum and the conservation of kinetic energy are fundamental principles in collisions. These principles can be used to calculate the velocities and directions of objects after a collision, as well as to determine other properties such as the amount of energy transferred between the objects during the collision. Additionally, we can learn that collisions can be categorized into different types, such as head-on collisions and oblique collisions, and that the analysis of these collisions requires a careful consideration of the angles and directions of the objects involved.

References

- Walker, Halliday and Resnick (2014). Fundamental of Physics, 10th Ed. (Extended), John Wiley & Sons, USA.
- Giancoli, Douglas C. Physics: Principles with Applications 7th Edition
- Nolan, Peter J. Fundamentals of College Physics
- Tipler, Paul A. Physics for Scientists and Engineers 6th Edition
- Young, Hugh D & Freedman, Roger A. University Physics 15th Edition



- Serway, Raymond A & Jewett Jr., John W. Physics for Scientists and Engineers with Modern Physics 9th Edition