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Driven harmonic oscillator under conditions

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1.Abstract

A harmonic oscillator is a system that, when displaced from its equilibrium position, experiences a restoring force F proportional to the displacement x. The experiment is performed based on what we performed in the experiment “harmonic oscillator”. We add an external harmonic force to the second cart that connects to the harmonic oscillator. We measure the displacement of the two carts with respect to time and want to find out the motion of the oscillating cart under external harmonic driving force while the oscillator transforms to harmonic steady state. At steady state we find the response function and phase behavior as function of the driving frequency of the oscillating system and compare our findings with the theoretical models.

2.Theoretical background

Q factor: It is a dimensionless parameter which indicates the relationship of the between stored energy and rate of energy loss in oscillating systems, thus indicating their efficiency. An oscillating system could be anything from a mechanical pendulum, an element in a mechanical structure, or within electronic circuit such as a RLC circuit. While the Q factor of an element relates to the losses, this links directly in to the bandwidth of a resonator with respect to its natural frequency

$$Q = \frac{w_0}{\Delta w} = w_0 * \tau$$

$$\Delta w = \frac{1}{\tau}$$

Where $\Delta\omega$ *is the width whose endpoints have the value of* $1\sqrt{2}$ *of the maxmum amplitude*

Harmonic oscillation

The equation for the motion of the oscillator on a frictionless track is defined by

$$m_{osc}\frac{d^2x_{osc}}{dt^2} = -(k_1 + k_2)x_{osc} \qquad (eq. 1.1)$$

m_{osc} is the weight of the cart.

$k_1 k_2$ are the elastic constant of two springs.

x_{osc} is the displacement of the cart from the equilibrium point.

We can adapt from the upper equation for the displacement of harmonic oscillator

$$x_{osc} = A * \cos(\omega_o * t + \emptyset) \qquad (eq. 1.2)$$

We can develop from the equation of motion (eq. 1.1) that the motion of driven harmonic oscillator

$$x_{osc} + \frac{1}{\tau}x_{osc} + w_0 * x_{osc} = \frac{F_0}{m_{osc}}\cos(w_d * t) \qquad \text{(eq. 1.3)}$$

Usually the frequency of the oscillator with two springs is w_0 which is describes as

$$w_0 = \sqrt{\frac{k1+k2}{m_{osc}}} \qquad \text{(eq. 1.4)}$$

The general solution of "eq 2.1" can be divided into two parts,

$$x_{osc} = x_h + x_p \qquad \text{(eq. 1.5)}$$

Where the underdamped harmonic motion equation is given by:

$$x_{osc}(t) = Ae^{-\frac{t}{2\tau}}\cos(w_0 t + \emptyset_1) + Bsin\left(w_d t + \emptyset_p\right) \qquad \text{(eq. 1.6)}$$

At resonance when $w_d \approx w_0$, the amplitude can be obtained:

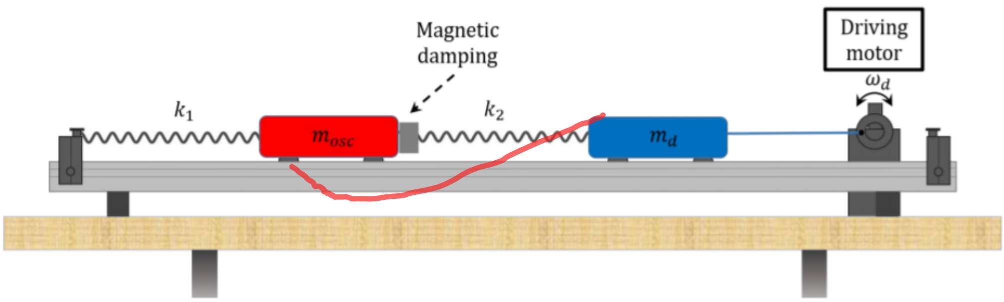
$$x_{osc}(t) = (1 - e^{-\frac{t}{2\tau}})\frac{F_0*\tau}{w_0*m_{osc}} sin\left(w_d t + \frac{\pi}{2}\right) \qquad \text{(eq. 1.7)}$$

$$A = \frac{F_0*\tau}{w_0*m_{osc}} \qquad \text{(eq. 1.8)}$$

The phase difference is given by：

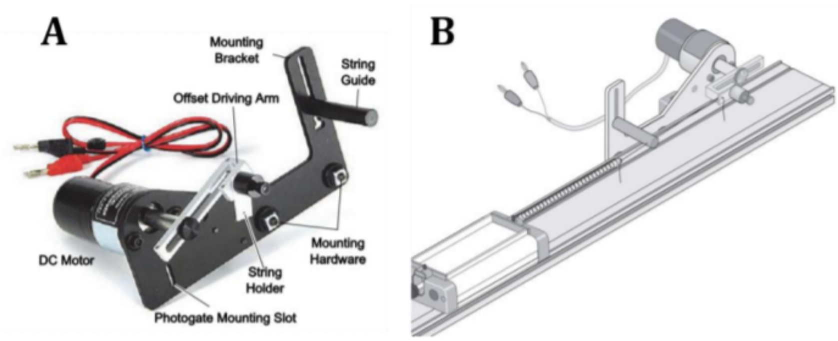
$$\tan\left(\emptyset_p\right) = \frac{w_d}{\tau} * \left(\frac{1}{w_0^2 - w_d^2}\right) \qquad \text{(eq. 1.9)}$$

3.Methods and material



(Figure 2.1 system of driven harmonic oscillator)

As is shown in figure 2.1, the oscillating cart with the mass of m(osc), whose one end connected to the driving cart with the mass of m(d) with a spring(k2) and the other end is attached from one end by a spring(k1) to the end of the track, is placed on the track with a magnetic damping device on it. The driving fore of the motor is described as $F_d = F_0 * \cos\left(w_d * t\right)$



(figure 2.2 A mechanical driving motor, B motor attachment)

Figure 2.2 shows the details of the driving motor we use in this experiment.

4.Results and discussion

1. we would like to find out τ, w_0 *and Q factor*

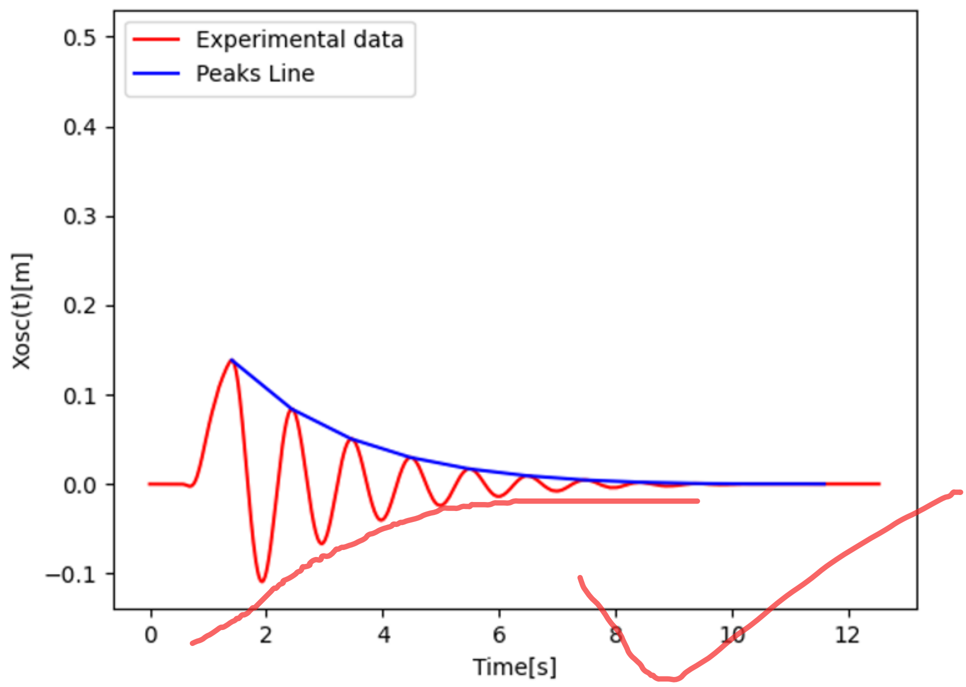


Figure 3.1 X-T curve of the damped oscillator

Using curve fitting we can obtain that $\tau = 1.26865 \pm 0.01565\ [s]$, $w_0 = 5.44 \pm 0.02000[1/s]$ and $Q = 6.773 \pm 0.00312$ (see "eq 1.4" and "eq 1.6")

2. X-T curve with different frequency.

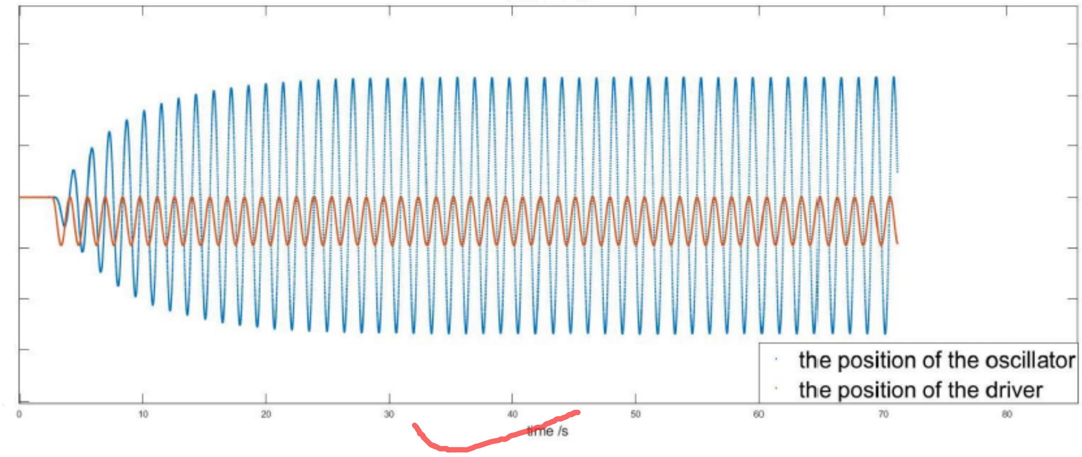


Figure 3.2 X-T curve at resonance frequency (5.04V)

By applying different voltage of the motor, we can obtain different driving frequency.

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We can see the transient phenomenon after which the oscillator transfers to a steady state. The resonance frequency in our experiment is w_d=5.3247 and $w_0 = 5.44$. Our phase difference $\emptyset = 1.53$ compared with the theoretical one $\emptyset = \frac{\pi}{2}$. The reason for this slight difference, we believe is that we should do more experiment with different frequency. The resonance frequency we had is just the one that are closed to the theoretical one.

We also have the curve when $w_0 \ll w_d$ *and* $w_0 \gg w_d$

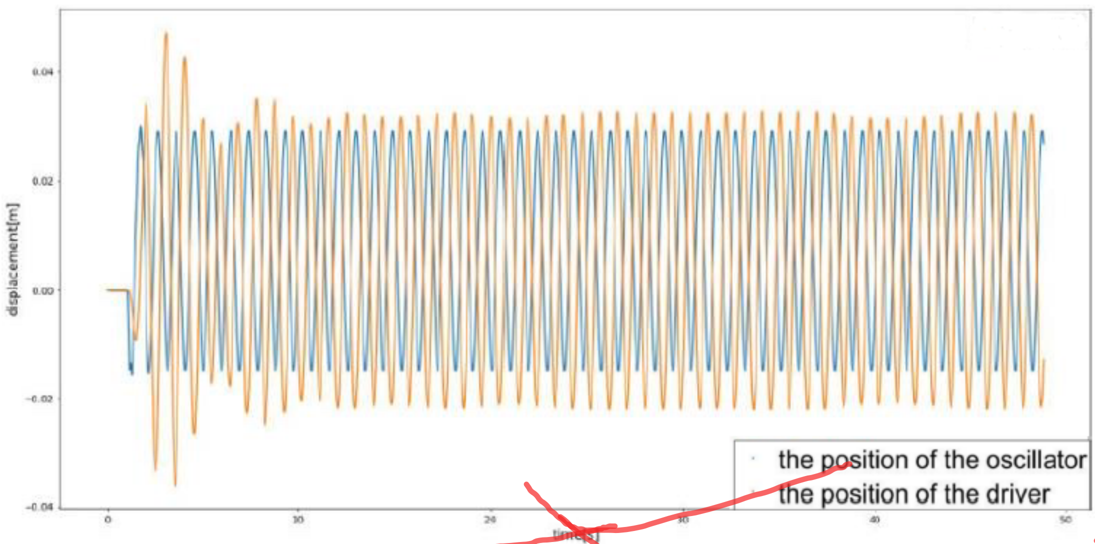

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Figure 3.4 X-T curve when $w_0 \ll w_d$

When we choose $w_0 \ll w_d$, the experimental results are A=0.027 and $\emptyset$=2.7 we can see that A and $\emptyset$ are very close to zero, and it takes much longer time to approach the resonance state.

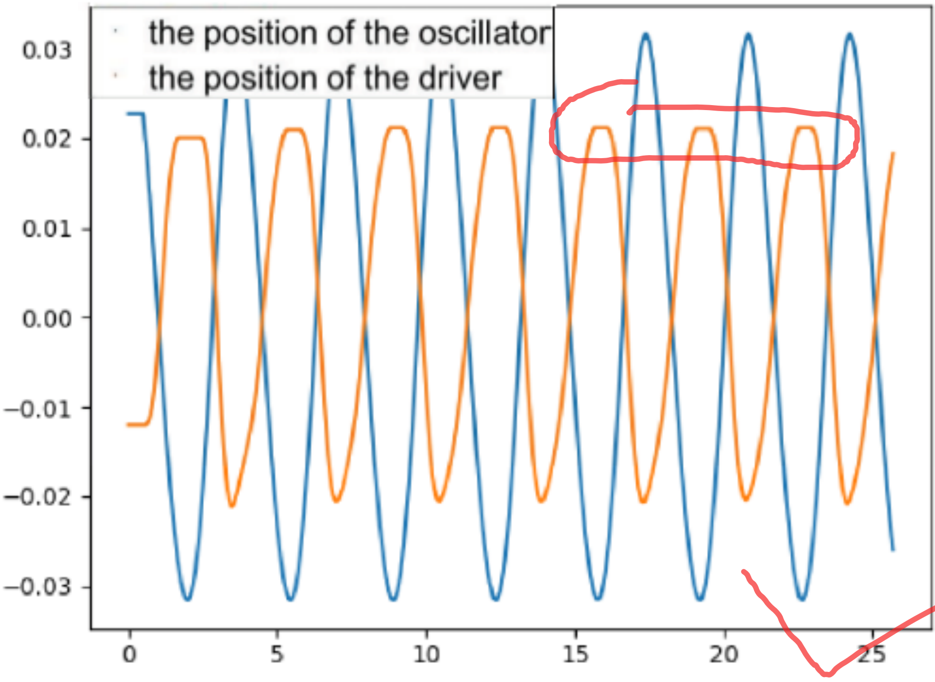

Figure 3.5 X-T curve when $w_0 \gg w_d$

When we choose $w_0 \gg w_d$, we can also observe that the time to approach the resonance state is longer. The experimental results are as follows: A=0.0235 Ø=0.034

3. Response function and phase difference

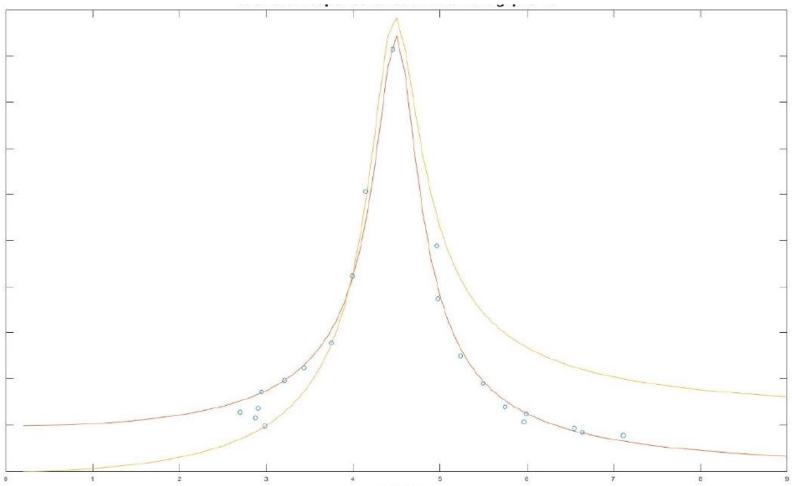

Figure 3.6 theoretical & experimental response function

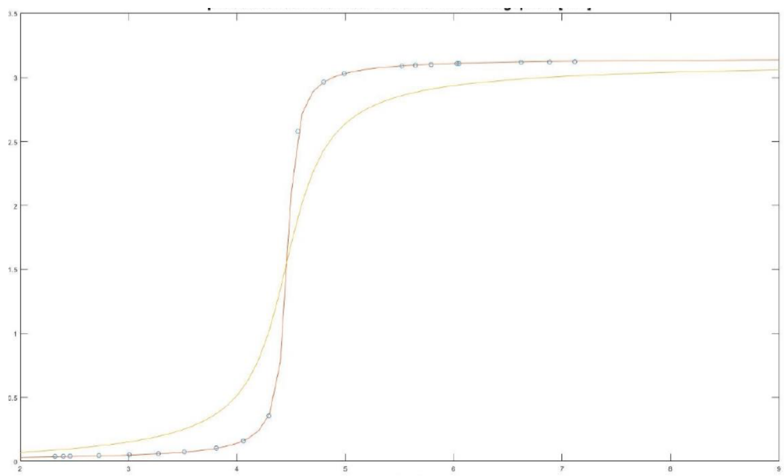

Figure 3.7 theoretical & experimental phase difference

We can see from the figure 3.6 that the experimental curve largely fits the theoretical one, the only problem occurs at the two sides of the curve. We believe this is resulted by the inaccurate measurement of the springs' s elastic constant, we only measured once with our eyes and a ruler. We should measure for more times and use caliper in order to improve accuracy.

5.Conclusion

In this experiment, we want to find out how a system response to an outer force. We applied sinusoidal force to harmonic oscillator. We discuss the three condition of the driven harmonic oscillator ($w_0 \ll w_d$, $w_0 \gg w_d$ and $w_0 = w_d$). w_0 is the natural frequency of the oscillator which we measured without outer force. We also obtain that $\tau = 1.26865$. The oscillator undergoes transient state to steady state in all cases. At last we find the response function where $w_0 = 5.32$ and $w_{the} = 5.44$. There are some diffference between the experimental data and the theoretical expectation. We believe we can do more experiment to obtain the more accurate frequency, also we should be more careful when measuring the elastic constant of the springs.

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6.Appendix

Q = ?

1.Use curve fitting to get τ

$$x(t) = Ae^{\frac{-t}{2\tau}}\cos(\omega_1 t)$$

$$peaks = ae^{\frac{-t}{b}}$$

$$\tau = 1.26865[s]$$

$$\delta\tau = \frac{\delta\tau}{\delta b} * \delta b = 0.01565[s]$$

2.Calculate the ω_0

$$\omega_0 = \sqrt{\frac{K_s + K_l}{m_{osc}}} = 5.44[\frac{1}{s}]$$

$$\delta\omega = \sqrt{\left(\frac{\delta\omega}{\delta K_s} * K_s\right)^2 + \left(\frac{\delta\omega}{\delta m_{osc}} * m_{osc}\right)^2 + \left(\frac{\delta\omega}{\delta K_l} * K_l\right)^2} = 0.02000[\frac{1}{s}]$$

3.Q factor

$$Q = \omega_0\tau = 6.773$$

$$\delta Q = Q\sqrt{\left(\frac{\delta\omega_0}{\omega_0}\right)^2 + \left(\frac{\delta\tau}{\tau}\right)^2} = 0.00312$$

7.Reference

1.The experiment of simple harmonic oscillation:
http://www.ehu.eus/acustica/english/basic/masen/masen.html

2.Literature about driven harmonic oscillator:
http://people.physics.tamu.edu/agnolet/Teaching/Phys_221/MathematicaWebPages/5_DrivenHarmonicOscillator.pdf