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Factors and the ways thay affect the simple pendulum motion

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Abstract

Consider a weight hanging from a cord. Releasing the weight away of its rest position will lead to a periodic motion of the weight, which is a pendulum. We would like to find the parameters affecting the motion of simple pendulum and how. We first derive the motion of simple pendulum with newton's law and then perform experiment. We measured the time period T of simple pendulum motion within two situations: Fixed length with changing releasing angles(I)Fixed releasing angle $\theta = 5^{\circ}$ with changing length (II).We finally obtain the result that the time period T follows a linear relationship with the square root of the length of the simple pendulum in situation (I) And a quadratic relationship with the squared of releasing angle in situation (II)

Theoretical background

Simple pendulum: simple pendulum can be theoretically considered as a pendulum where the mass can be considered as a point, and the string can be considered as massless. Practically, this was reached under following conditions. the thread mass-$m_1 \ll m_2$-the mass of pendulum. The length of the thread-$l_1 \gg l_2$-the diameter of the pendulum.

Dimension analysis: Dimensional analysis follows from the basic assumption that in any equation describing a physical phenomenon, both sides of the equation must have the same dimensions, also assumed that the dimensions of any physical quantity, can be expressed as the product of powers of the fundamental dimensions. In this situation, the affecting factors of the period are mass, angle, length of the thread and gravitational acceleration. Therefore, the period can be written as:

$$T(\theta, l) = C(\theta) * \sqrt{\frac{l}{g}} \qquad \text{(eq 2.1)}$$

Theoretical model: for a simple pendulum system, the pendulum oscillates under the gravity. Using newton's second law, its motion can be described ideally by:

$$\cancel{\frac{d^2\theta}{dt^2} + \sin(\theta) * \frac{g}{l} = 0} \qquad \text{(eq 2.2)}$$

By using Taylor's series:

$$\cancel{\sin(x) = x + \left(-\frac{x^3}{3!}\right) + \frac{x^5}{5!} + \cdots + \sin\left(n * \frac{pi}{2}\right) * \frac{x^n}{n!} + \sin[u + (n+1) * pi] * \frac{x^{n+1}}{(n+1)!}} \qquad \text{(eq 2.3)}$$

We can see that when x is small enough (θ <5°), $\sin(\theta) \approx \theta$, insert this to "eq 2.1" and we can simplify the equation to this:

$$\frac{d^2\theta}{dt^2} + \theta * \frac{g}{l} = 0 \qquad \text{(eq 2.4)}$$

This is a linear differential equation. By solving this equation, we can obtain:

$$T = 2\pi * \sqrt{\frac{l}{g}} \qquad \text{(eq 2.5)}$$

$T_0 = ?$ $T_2 = ?$

Method and materials

the experiment is conducted under the system shown below

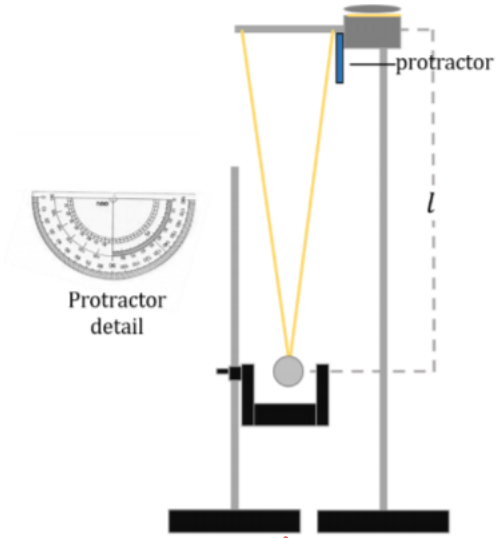

Figure 3.1 the simple pendulum experimental system

The oscillator is released at a certain angle, which is measured by the protractor. A photogate is set at the lowest position of the oscillation. Every time the oscillator passes through the photogate, time is recorded. The length between the oscillator and pivot is measured by ruler.

The first part of the experiment is conducted by releasing the oscillator at the same angle ($\theta < 5°$), with 8 various distances from the pivot (the length must satisfy the simple pendulum condition). When data is recorded, we use curve fit and goodness of test it. We would like to see whether it fits the assumption "eq 2.5". The gravitational constant g_{exp} is calculated and compared with the official local value.

The second part, the oscillator is released from various angles with the same distance from the pivot. We would like to obtain the plot of the period of oscillation with respect to angle, and try to find a function that fits. The limit of "small angles" in the system is defined as the error of measured angles.

4.Result and discussion

4.1

First, we measured the period of simple pendulum with fixed releasing angle $\theta = 5^{\circ}$. According to eq2.5, we can obtain the theoretical period T by $T(l) = 2\pi\sqrt{\frac{l}{g}}$. It's easy to find out that $T \propto \sqrt{l}$. By using Fitting_Curve function with python and the experimental data, we can get the figure4.1:

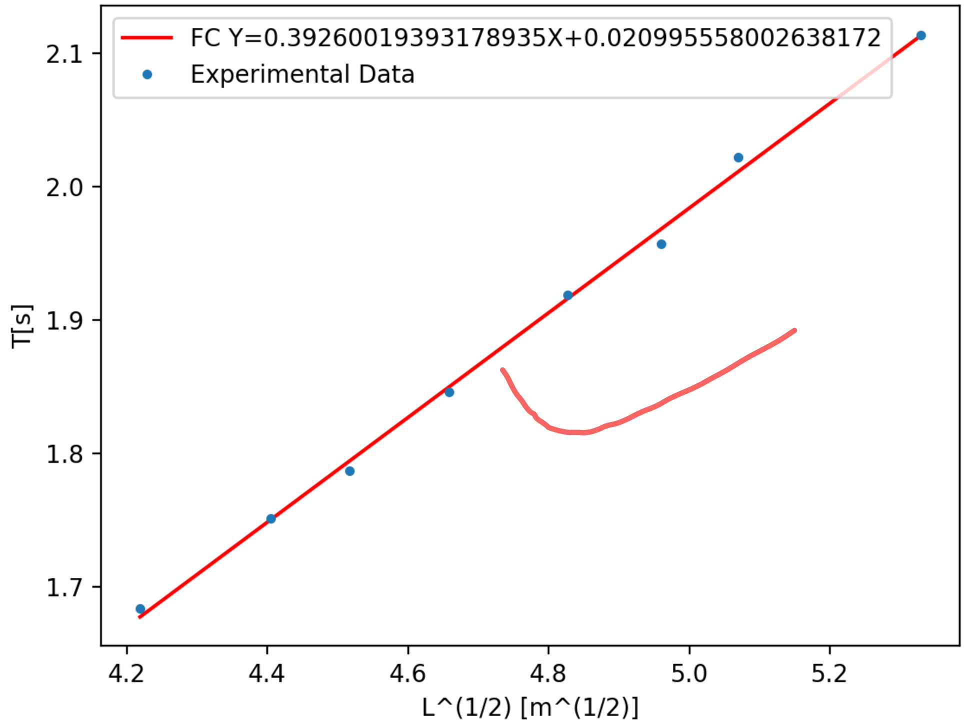

Fig4.1: Fitting curve of period with square root of the length of the pendulum.

This fitting result showed that period and square root of the length of pendulum have a linear relationship with a goodness of fit $R^2 = 0.9976880115920531 > 0.9$ which means that they are strongly linear. And we can get the experimental fitting coefficient $g_{exp} = 9.53391976217814 \mp 0.01 \, [\frac{m}{s^2}]$. Compare to the g constant in ShanTou $g = 9.79127 \mp 0.000005 [\frac{m}{s^2}]$, the difference [1] is about $2.6\% < 10\%$. So this measurement can be calculated in the final result.

4.2

This time we measured the period of simple pendulum with fixed releasing length $L = 24.6cm$. Figure4.2 shows the period of each experiment by changing the releasing angles to $(9°, 12°, 15°, 20°, 25°, 30°, 40°, 50°, 60°, 70°, 80°, 90°)$:

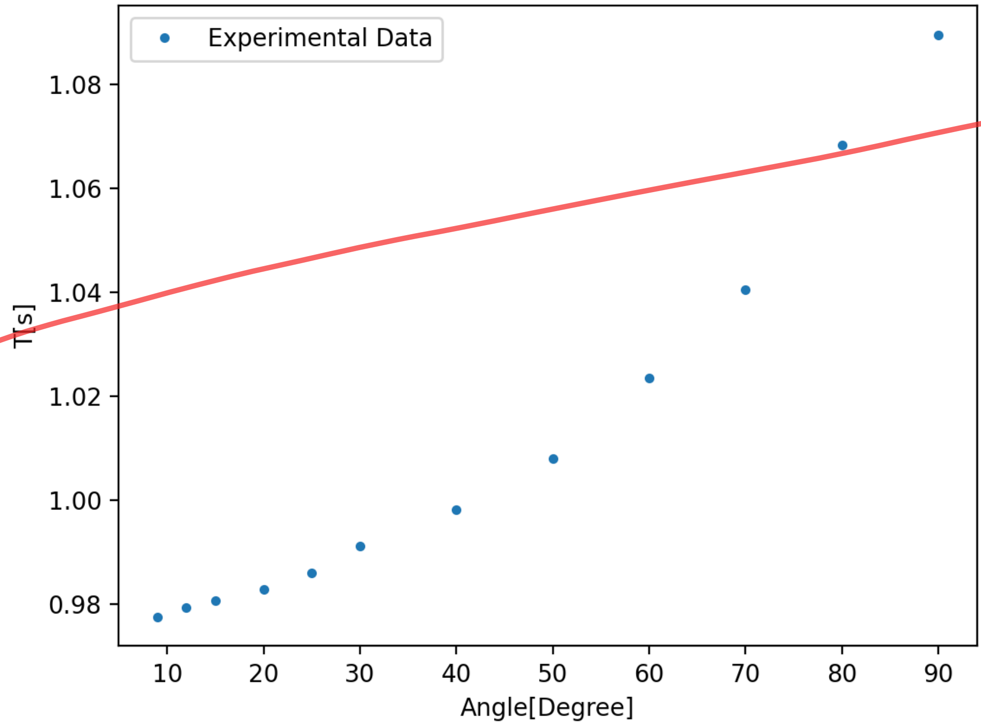

Figure4.2: period T with the released angles.

Due to the shape of the figure, we assumed that this figure has a high resemblance of a Quadratic function. By using python, we get the fitting curve figure4.3:

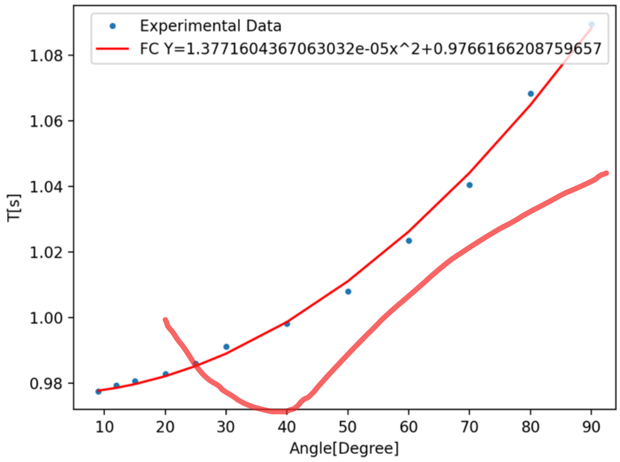

Figure4.3: fitting curve of the period T with releasing angles.

We can get from the figure4.3 the equation of period T with releasing angle:

$$T(\theta) = a * \ \theta^2 + b$$

$$\Rightarrow \begin{cases} a = 1.3771604 \\ b = 0.9766167 \end{cases}$$

This fitting result showed that period and releasing angle of pendulum have a quadratic relationship with a goodness of fit $R^2 = 0.9959279222010375 > 0.9$

Which means that they' re strong quadratic-relationship.

5.Conclusion

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In the first experiment, the result shows that we the releasing angle is small $(< 10°)$, Time period T will follow a linear relationship with the square root of the length of the simple pendulum, which proved the equation $T(l) = 2\pi\sqrt{\frac{l}{g}}$ is correct. And the experimental g coefficient is very close to the g constant in ShanTou.

However, when the releasing angle turn bigger$(> 10°)$, with a fixed length, we cannot find a clear equation between time period T and releasing angle(θ). The closest equation we can get so far is:

$$\cancel{T(\theta) = a * \ \theta^2} + b$$

For

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$$\Rightarrow \begin{cases} a = 1.3771604 \\ b = 0.9766167 \end{cases}$$

Reference

GTIIT moodle site
https://moodle.gtiit.edu.cn/moodle/pluginfile.php/36036/mod_resource/content/1/6%20Simpl

e%20Pendulum%20lab%20guide_v2.pdf citing the figure of the experimental system

https://moodle.gtiit.edu.cn/moodle/pluginfile.php/36037/mod_resource/content/0/%5B2006%5D%20An%20accurate%20formula%20for%20the%20period%20of%20a%20simple%20pendulum%20-%20Lima.pdf the derivation of the motion of oscillation

Appendix

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1 Calculate the error (in percentage) of a measurement:

$$error = \frac{|TheoreticalValue - ExperimentalValue|}{TheoreticalValue}$$

= ?

2 $\boldsymbol{R^2}$Method for testing goodness of the fitting curve:

$$R^2 = 1 - \frac{SSE}{SST}$$

$$SSE = \sum (y_{therotical} - y_{experimental})^2$$

$$SST = \sum (y_{therotical\_Average} - y_{experimental})^2$$

= ?