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Friction and drag in harmonic oscillator

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Abstract

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A harmonic oscillator is a system that, when displaced from its equilibrium position, experiences a restoring force F proportional to the displacement x. We recorded the displacement of the oscillator with respect to time. We are now interested in the system when damp force is exerted since this is what happens practically. There are two forces that we tested. One is friction, in which case the amplitude of the displacement is to be decreasing linearly with respect to time. The other one is the "drag" type, where amplitude is expected to decrease exponentially.

Theoretical Background

Harmonic oscillation

The equation for the motion of the oscillator on a frictionless track is defined by

$$m_{osc}\frac{d^2x_{osc}}{dt^2} = -(k_1 + k_2)x_{osc} \qquad (eq.3)$$

m_{osc} is the weight of the cart.

$k_1 k_2$ are the elastic constant of two springs.

x_{osc} is the displacement of the cart from the equilibrium point.

We can adapt from the upper equation for the displacement of harmonic oscillator

$$x_{osc} = A * \cos(\omega_o * t + \emptyset) \qquad (eq.7)$$

Spring and its energy

The force of a spring within its elastic limit is given by:

$$F_{spring} = -kx \qquad (eq.1)$$

k is the elastic constant of the spring.

x is the displacement from the equilibrium point. -3

The potential energy stored in a spring is described as:

$$U(x) = \frac{1}{2}kx^2 \qquad (eq.2)$$

$\omega_0 = \ldots$

$A_0 = \ldots$

$U(x)$ is the parabolic potential energy.

After plotting two types of the damp force to oscillation, we expect the amplitude to descend in two distinct ways. When the damp force is a constant force, we expect it to be a linear model. When a drag force which relates the oscillator's velocity, we expect it to be a exponential model. (the reason for our expectation is listed in appendix in detail)

Materials and methods

In this experiment, we need two springs with elastic constants k_1 and k_2, a magnetic track and a cart with sensor that is available for the recording of its displacement form the point where the cart was in equilibrium with respect to time. The recorded data are stored in the computer as $X_{osc}[m]$ and $t[s]$. Magnets are also required, to create drag force. We perform the experiment in the following procedure. Place the cart on the track. Connect the springs to the cart and the track in the way the graph (1) shows. After the system is set up properly, pull the cart for a adequate distance so that it can begin the harmonic oscillation. The computer will record the cart's displacement with respect to time. In this way we obtained the data of harmonic oscillation under the friction, which is a force that stays constant numerically. When the system returns to equilibrium again and the data is recorded properly, we add the magnet to the cart so that a force that will ascend with respect to velocity is asserted to the oscillator as a damp force. This time we obtain the data under a drag force. We will then use curve fitting to find the function of the displacement for verification, the goodness of the curve is also conducted.

Results and discussion

In the first experiment, we record the displacement of oscillator with respect to time. (figure 1)

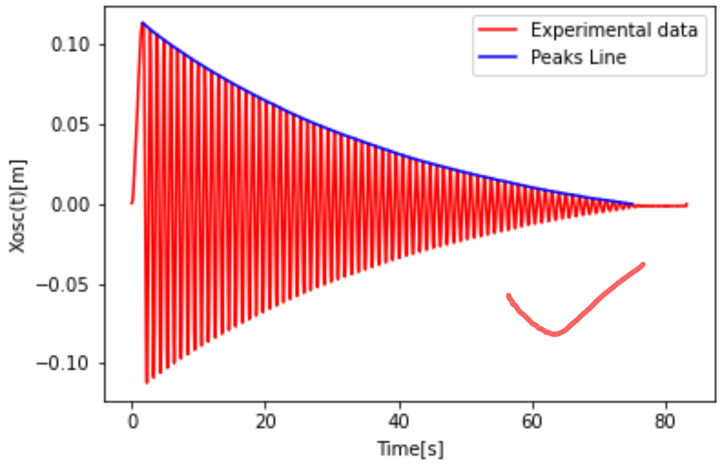

Figure 1

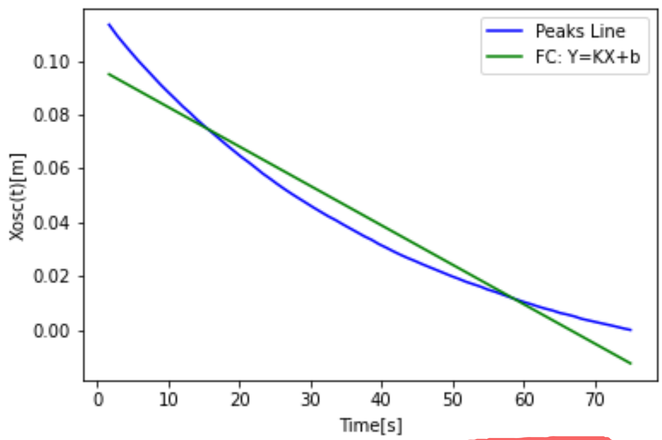

Figure 2

We expect the peaks line to be fitted in a linear model with goodness. Using the method of curve fitting, the peaks line can be described as (figure 2):

$$y = -0.00146751x + 0.09754138$$

Where y stands for the peaks

x stands for the time

Using R^2 method to test the goodness of the fit

$$R^2 = 1 - \frac{SSE}{SST} = 0.95089529702341 28$$

Since R^2 is larger than 0.9, we can say that the fit is quite good. In another word, the experimental results support our assumption.

After measuring the oscillator under friction, we then perform the experiment under drag force. The displacement of the oscillator with respect to time is shown in the *figure 3* below.

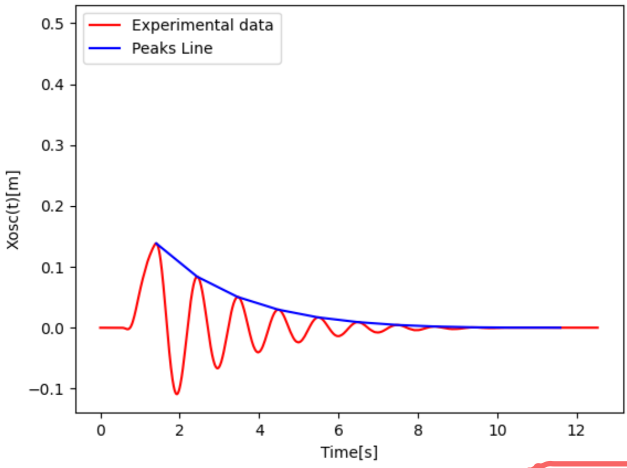

Figure 3

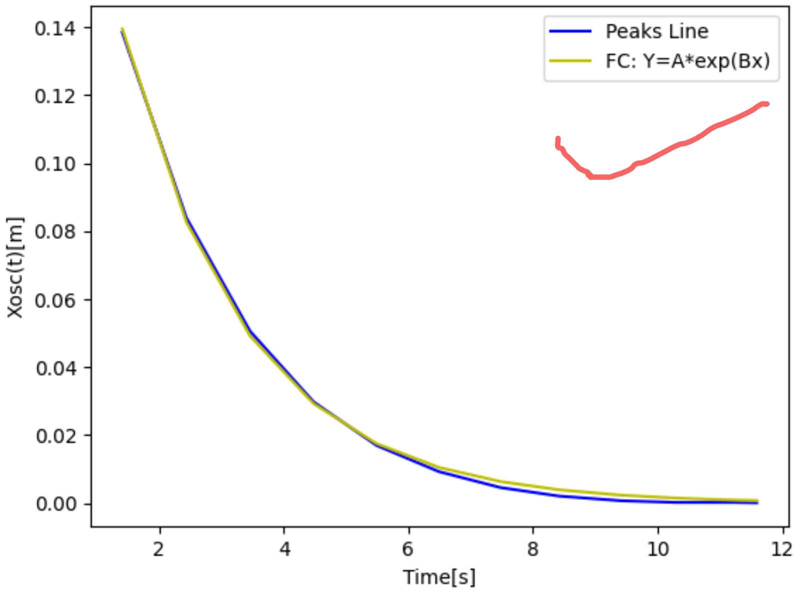

Figure 4

In the case of a drag force, we expect the decline of the peaks line to be an exponential model (figure 3). Repeat the procedure of last data analysis. We obtained following equations.

The decline of the peak lines can be fitted into (figure 4):

$$y = 0.60276104 * e^{-0.70884852x}$$

Where y stands for the peaks

x stands for the time

Applying the test of the goodness to the upper equation,

$$R^2 = 1 - \frac{SSE}{SST} = 0.98230864914137563$$

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R^2 in this case is also larger than 0.9, which represents that the fitting is very accurate. Therefore we can conclude that our assumption for the oscillator under drag force is also practical.

$\omega_0 = ?$ $\tau = ?$ $k = ?$

Conclusion

By using curve fitting and test for the goodness of curve, we can conclude from the experimental result we obtained that our assumption for the motion of oscillator under drag force and friction is correct.

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Appendix

The force of a spring within its elastic limit is given by:

$$F_{spring} = -kx \qquad (eq.1)$$

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k is the elastic constant of the spring.

x is the displacement from the equilibrium point.

The potential energy stored in a spring is described as:

$$U(x) = \frac{1}{2}kx^2 \qquad (eq.2)$$

$U(x)$ is the parabolic potential energy.

The equation for the motion of the oscillator on a frictionless track is defined by

$$m_{osc}\frac{d^2x_{osc}}{dt^2} = -(k_1 + k_2)x_{osc} \qquad (eq.3)$$

m_{osc} is the weight of the cart.

$k_1 k_2$ are the elastic constant of two springs.

x_{osc} is the displacement of the cart from the equilibrium point.

After calculations, we can transform $(eq.3)$ into another equation:

$$x_{osc}(t) = A\cos(\omega_0 t + \phi) \qquad (eq.4)$$

A is a constant can be found in experimental data.

ω_0 is the natural frequency of the oscillator.

Natural frequency of the oscillator is defined as

$$\omega_0 = \sqrt{\frac{k_1 + k_2}{m_{osc}}} \qquad (eq.5)$$

The equation of motion is transformed into this

$$\frac{d^2 x_{osc}}{dt^2} = -\omega_0 * x_{osc} \qquad (eq.6)$$

In another word this equation

$$x_{osc} = A * \cos(\omega_o * t + \emptyset) \qquad (eq.7)$$

The kinetic friction can be described as:

$$f = -u_k N \qquad (eq.8)$$

f is the friction.

u_k is the friction constant.

N is the normal force.

When the oscillator reaches the turning point, it has no velocity, so the energy is only stored in the spring. For the oscillator under kinetic friction, the work done by the friction fill in the energy difference between two turning point.

$$\text{F}*(A_1 + A_2) = \frac{1}{2} * (k_1 + k_2) * (A_1 - A_2)^2 \# (eq.9)$$

Solve the equation and we can obtain:

$$A_1 - A_2 = \frac{f}{k_1 + k_2} \qquad (eq.10)$$

We can see that difference of two adjacent amplitude is constant, therefore the function of the descending of amplitude is expected to be a linear function with respect to time.

As for cases that the damp force is a drag force which changes with respect to the velocity of the oscillator, which is, in our experiment, the magnetic force, we have following equation:

$$f_d = -\left(\frac{m_{osc}}{\tau}\right) \cdot \dot{x}_{osc} \qquad (eq.11)$$

So the equation of the motion should be :

$$m_{osc} \frac{d^2 x_{osc}}{dt^2} = -(k_1 + k_2) x_{osc} - \left(\frac{m_{osc}}{\tau}\right) \cdot \dot{x}_{osc} \qquad (eq.12)$$

After calculation, we can turn $(eq.12)$ into :

$$x_{osc} = A e^{-\frac{t}{2\tau}} \cos(\omega_1 t + \emptyset_1) \qquad (eq.13)$$

So, we can conclude from the upper equation that the x_{osc} will be reducing exponentially.

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Reference

*Fitting_Curve GTIIT Moodle Site
https://moodle.gtiit.edu.cn/moodle/pluginfile.php/36032/mod_resource/content/0/5.1%20Curve%20Fitting.pdf
*Lab_Guide GTIIT Moodle Site
https://moodle.gtiit.edu.cn/moodle/pluginfile.php/36033/mod_resource/content/1/5.2%20Harmonic%20Oscillations%20lab%20guide_v2.pdf