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Explore the best method to get the accurate acceleration

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1.Abstract

In this experiment, we recorded the position of a cart on an inclined plane with respect to time. We tried different method to process our data and get two sets of initial velocity and three different sets of results of friction constant. First method is to use curve fitting directly on the raw data. Second one is to apply center difference method once and then curve fitting. The third one is to simply use center difference method twice. We compare the results of three methods and compare the friction constant with previous data. We discover that the method 2 is the best one in our case. We concluded that the center difference method has great power in eliminating noise and oversampling error but also possess the shortcoming of losing data throughout the data processing.

2.Theoretical background

2.1 Types of Errors and Methods to reduce it.

Noise: noise in an experiment can refer to any random fluctuations of data that hinders perception of a signal.
Oversampling: oversampling happens when sampling rate is increased to a certain degree where the change in measured signal can not be obtained and signal is observed as same value.
Finite difference method: It is a class of numerical techniques for solving differential equations by approximating derivatives with finite differences. This method is applied to adjust the error cause by noise, discrete sampling and oversampling when we are dealing with problems that require differentiation. We have forward Euler difference, backward Euler difference, and centered difference. In this experiment, we used centered difference, which is the most accurate one.
Centered Difference:

$$v(t_j) = v'(t_j) \approx \frac{x(t_j + \Delta t) - x(t_j - \Delta t)}{2\Delta t} \qquad eq2.1.1$$

Among the three methods centered difference yields a more accurate approximation since it is twice differentiable.
To solve the problem of oversampling if $x_1 = x_2 = x_3 \ldots \ldots$we will improve the equation of centered difference to:

$$v(t_j) = v'(t_j) \approx \frac{x(t_j + n\Delta t) - x(t_j - n\Delta t)}{2n\Delta t} \qquad eq2.1.2$$

n is the factor depends on our choice.
Generally, the bigger the n, the smaller the errors.

2.2 Main equations:

The relationship between acceleration a, velocity v and displacement x:

$$x(t) = \frac{1}{2}at^2 \quad + x_0 \qquad eq2.2.1$$

$$v(t) = at = \dot{x} \qquad eq2.2.2$$

$$a = \dot{v} = \ddot{x} \qquad eq2.2.3$$

The magnitude of the acceleration when uphill or downhill:

$$Uphill: \ F = ma = mgsin\theta + \mu mgcos\theta$$

$$==> a = gsin\theta + \mu gcos\theta \qquad eq2.2.4$$

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$$Downhill: \ F = ma = mgsin\theta - \mu mgcos\theta$$

$$==> a = \cancel{gsin\theta - \mu gcos\theta} \qquad eq2.2.5$$

Where μ is the friction factor and θ is the angle of the inclin

3.Methods and material

$u = ?$

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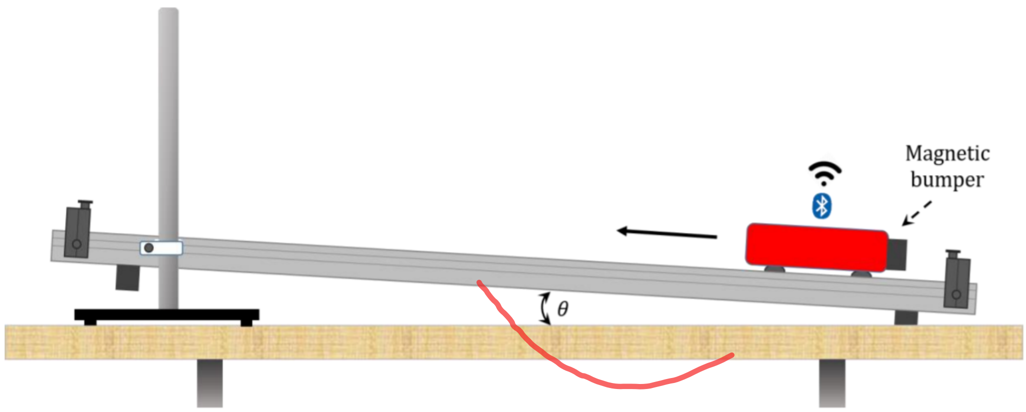

Figure 3.1 Acceleration experiment setup

We can see from the figure above the setup we will be using to perform this experiment. An inclined plane in placed on the horizontal surface. The inclined angle is measured by gradienter. The cart is released with an initial acceleration and move upwards along the plane with its velocity declining along the way. When it reaches the highest point and stops, it will accelerate backwards. A position detector is located inside the cart which transmits the location of the cart via Bluetooth to the computer. We will therefore proceed with data analysis with this data to derive velocity and acceleration of the cart, the friction factor is also calculated. Experimental data derived from different methods will be together compared with the theoretical value. Experimental friction factor is derived from different methods by calculating separately from acceleration uphill and downhill, the average is taken into the consideration of the reliability of methods by comparison with the data we got in Harmonic Oscillator experiment. We're going to compare with the errors that get from

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each experimental friction factors to deicide which method is more accurated.

4 Results and discussion

4.1 X-T Curve

We first draw the original data into the x-t curve. We separated the uphill part into "left" part and the downhill part into "right" part in order to simplify the analysis.

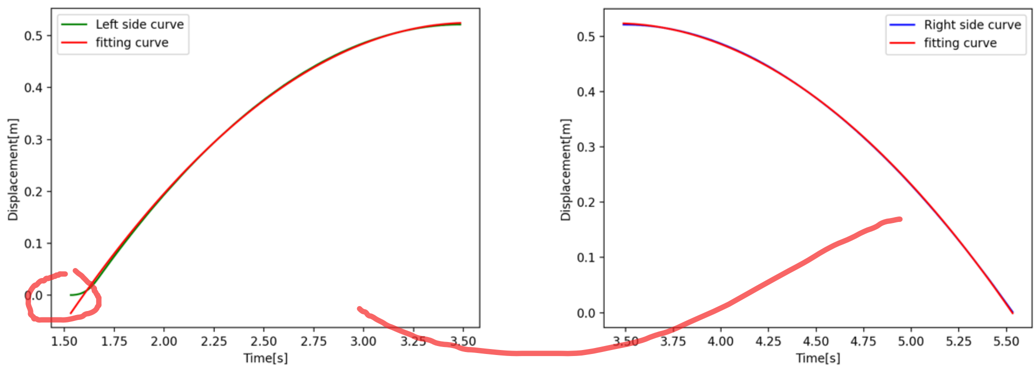

Figure4.1 x-t curve of the original data.

By using the curve-fitting method, we got the asymptotic curves of each parts:

$$Uphill: \ x(t) = 0.98433\text{t} - 0.1391496t^2 - 1.21599 \qquad eq4.1$$

$$Downhill: \ x(t) = 0.8219292\text{t} - 0.1195789t^2 - 0.888419 \qquad eq4.1.2$$

From the **Theoretical Background** we've already known that the theoretical relationship between the displacement x and acceleration a is described as:

$$x(t) = \frac{1}{2}at^2 \qquad eq2.2.1$$

As a result, we can get the practical acceleration a :

$$Uphill: \ a_{left} = -0.2782990043490084 \ \frac{m}{s^2} \qquad eq4.1.3$$

$$Downhill: \ a_{right} = -0.23915790961775105 \ \frac{m}{s^2} \qquad eq4.1.4$$

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By using the equation from the theoretical background:

$$Uphill: \ a = gsin\theta + \mu gcos\theta \qquad eq2.2.4$$

$$Downhill: \ a = gsin\theta - \mu gcos\theta \qquad eq2.2.5$$

With the g constant in Shantou:

$$g = 9.79127 \mp 0.000005[\frac{m}{s^2}]$$

We can calculate the μ :

$$\mu_{left} = 0.05636$$

$$\mu_{right} = 0.05236$$

$$\mu_{mean} = 0.05436$$

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Compare to the fiction constant we measured in the previous lab(Appendix7.3):

$$\mu_{theoretical} = 0.0613$$

The error is about(Appendix 7.1):

$$\%error = 0.1132137$$

4.2 V-T curve

In this part, we first use the centered difference approximation method with different factors to get the velocity of each point in the origin data. Following are the figures that plotted with different factors(1,5,10,15):

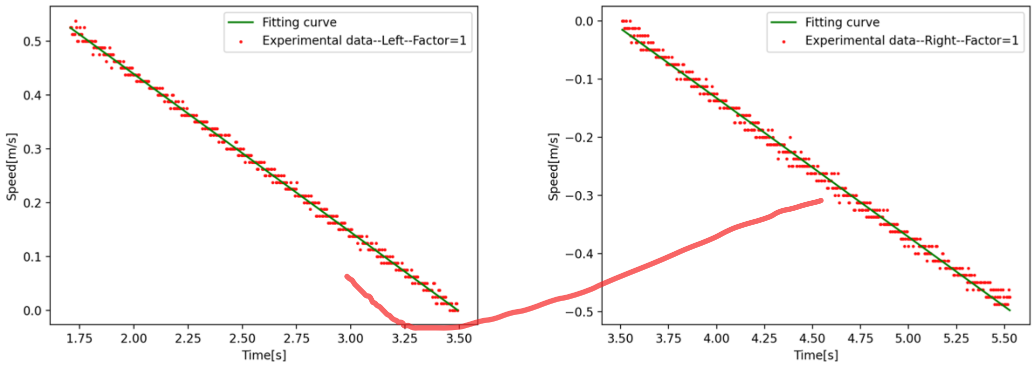

Figure4.2.1 v-t curve of the original data with factor = 1.

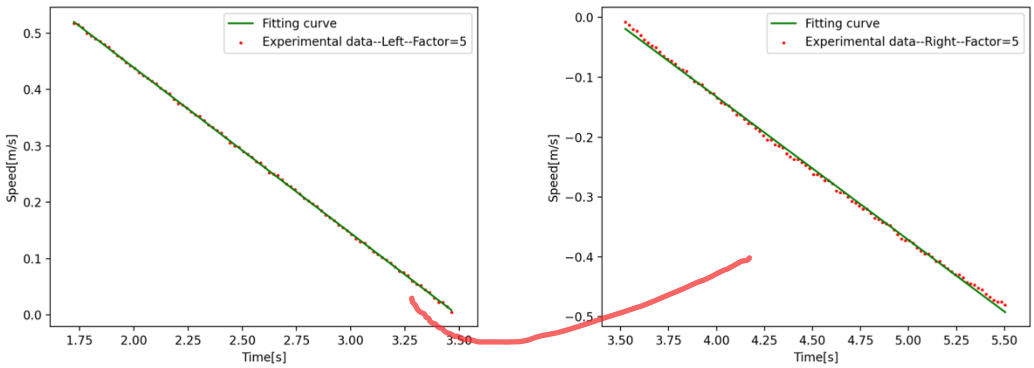

Figure4.2.2 v-t curve of the original data with factor = 5.

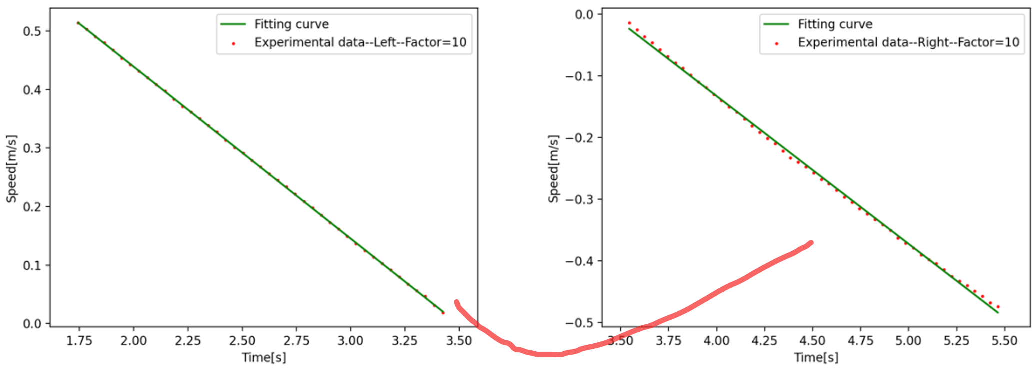

Figure4.2.3 v-t curve of the original data with factor = 10.

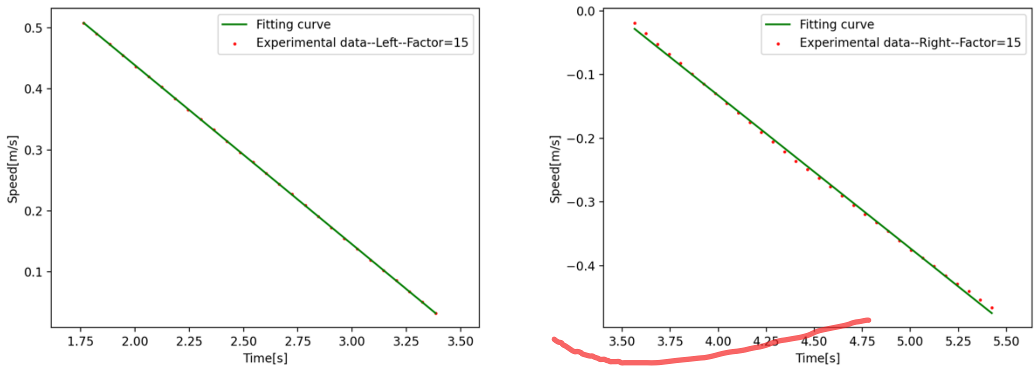

Figure4.2.4 v-t curve of the original data with factor = 15.

After comparing with each figure, we finally decided to choose to use figure4.2.3 to do the analysis as it shows the data's properties most clearly.

As it in 4.1, we get the asymptotic curves of each parts: m/s

$$Uphill: \ v(t) = -0.29354056t + 1.025782 \quad \text{eq4.2.1}$$

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$$Downhill: \ v(t) = -0.23976407t + 0.826119 \quad \text{eq4.2.2}$$

We test its R-squared value:

$$Uphill: \ R^2 = 0.99997305274 77673 > 0.9 \quad \text{eq4.2.3}$$

$$Downhill: \ R^2 = 0.99902981 0118037 > 0.9 \quad \text{eq4.2.4}$$

All shows a strong goodness of fit.

From:

$$v(t) = at \quad eq2.2.2$$

We can obtain:

$$Uphill: \ a_{left} = -0.2935405643676701 \ \frac{m}{s^2} \pm \quad eq4.2.5$$

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$$Downhill: \ a_{right} = -0.0.23976407226691096 \ \frac{m}{s^2} \pm \quad eq4.2.6$$

Same as 4.1, we can get:

$$\mu_{left} = 0.05792$$

$$\mu_{right} = 0.05242$$

$$\mu_{mean} = 0.05517$$

Compare to the fiction constant we measured in the previous lab:

$$\mu_{theoretical} = 0.0613$$

The error is about:

$$\%error = 0.10000$$

4.3 A-T graph

By applying again center difference method to the V-T curve above, we obtained 2 graphs which represent the acceleration of the cart with respect to time.

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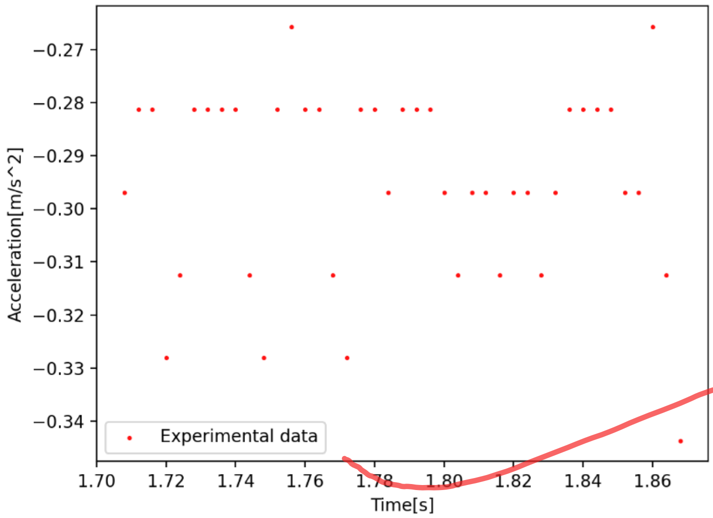


Figure4.3.1 A-T of cart moving up

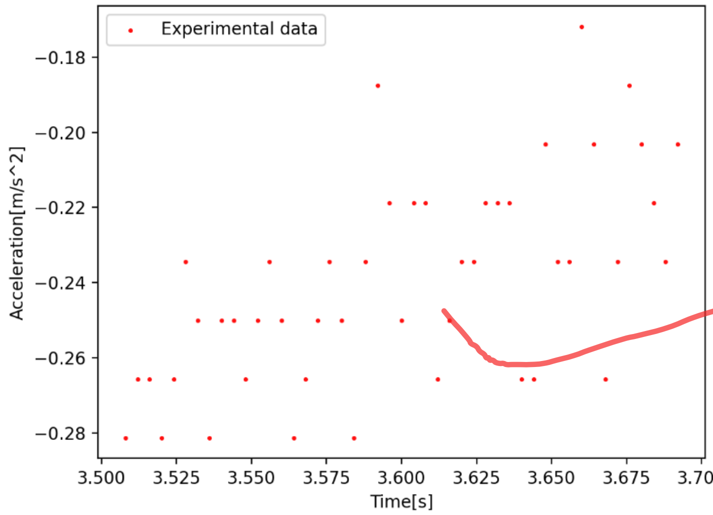


Figure4.3.2 A-T graph of cart moving down

Instead of using curve fit to find acceleration, we can just calculate the mean value of these points and will reach the same result as curve fit.

$$a_{up} = -0.29458841463414647\frac{m}{s^2} \quad \pm$$

$$a_{down} = -0.24002659574468094\frac{m}{s^2} \quad \pm$$

$$\mu_{left} = 0.05803$$

$$\mu_{right} = 0.05245$$

$$\mu_{mean} = 0.0524$$

Compare to the fiction constant we measured in the previous lab:

$$\mu_{theoretical} = 0.0613$$

The error is about:

$$\%error = 0.1451876$$

4.4 Comparison between the errors

only get best

As the result of three methods what we used in previous, we get a table with errors and initial velocities(Appendix7.2) in each method:

Methods	Initial velocity[m/s]	Experimental μ	%error
X-T Curve	0.98433	0.0613	0.1132137
V-T Curve	1.025782	0.05517	0.1000000
A-T Graph	None	0.0524	0.1451876

Table4.4.1 %error in each method

It is clear that V-T Curve has a larger initial velocity than X-T Curve. We presume that the result of X-T Curve is influenced by noise and oversampling and is less accurate and reliable because normally the application of center difference method will make the slope gentler, which means that the initial velocity of X-T Curve should be larger than that of V-T Curve if the data is good. (without affecting factor) So for the initial velocity, data of V-T Curve is more reliable.

We recalled the data we calculated in the harmonic oscillator and get the friction constant we measured. We compare it with the data in this experiment. We can see that the error percentage of V-T Curve is the lowest, with A-T Graph the highest and X-T Curve in the middle. This largely meets our expectation, because V-T Curve, compared with X-T Curve, solve the problem of noise, oversampling and obtain a "smoother" result, which is more reliable. While A-T Graph, compared with V-T Curve, applies one more time of center difference method, this results in loss of data, making results less reliable. We believe the aim of these process of data is to approximate the "true" value to the greatest extent, but A-T Graph overdo it therefore less reliable

5.Conclusion

We try three method to find initial velocity and friction constant and make comparison. We find that V-T Curve, which applies one time of center difference method is the best one. The center difference method helps us to efficiently eliminate experimental error like noise and oversampling. However, it also has some shortcomings. The application of it results in the loss of data, which we can see in the results of A-T Graph, making the result even less reliable. Our conclusion is that the usage of center difference method is about controlling the "balance". Too much using it can cause counter-action to our goal. In our case, the best way is to use it once, this depends on the quality of the data measured. The more noises and oversampling in the raw data, the more need we have to use center difference method to eliminate these errors, hence we apply more times of it. If we use a cruder machine to record data, we might need to apply center difference method twice and the results will be the most reliable one.

6.Reference

Statistic equations: https://www.statisticshowto.com/percent-error-difference/
Lab Guide:
https://moodle.gtiit.edu.cn/moodle/pluginfile.php/36043/mod_resource/content/0/7%20Acceleration%20Lab%20Guide.pdf

7.Appendix

7.1 Calculate the %error

$$\%error = \frac{|TheoreticalValue - ExperimentalValue|}{TheoreticalValue}$$

7.2 Get the initial velocities in X-T Curve and V-T Curve

We used curve-fitting and find the equation of X-T Curve and V-T Curve:
For X-T Curve:

$$Uphill: \ x(t) = 0.98433\text{t} - 0.1391496t^2 - 1.21599 \qquad eq4.1$$

From $eq2.2.1$ we can know that, if there exist an initial velocity, the relationship between displacement x and acceleration a should be:

$$x(t) = v_0 t + \frac{1}{2}at^2$$

v_0 is the initial velocity and in this case, it should be $0.98433\frac{m}{s}$

For V-T Curve:

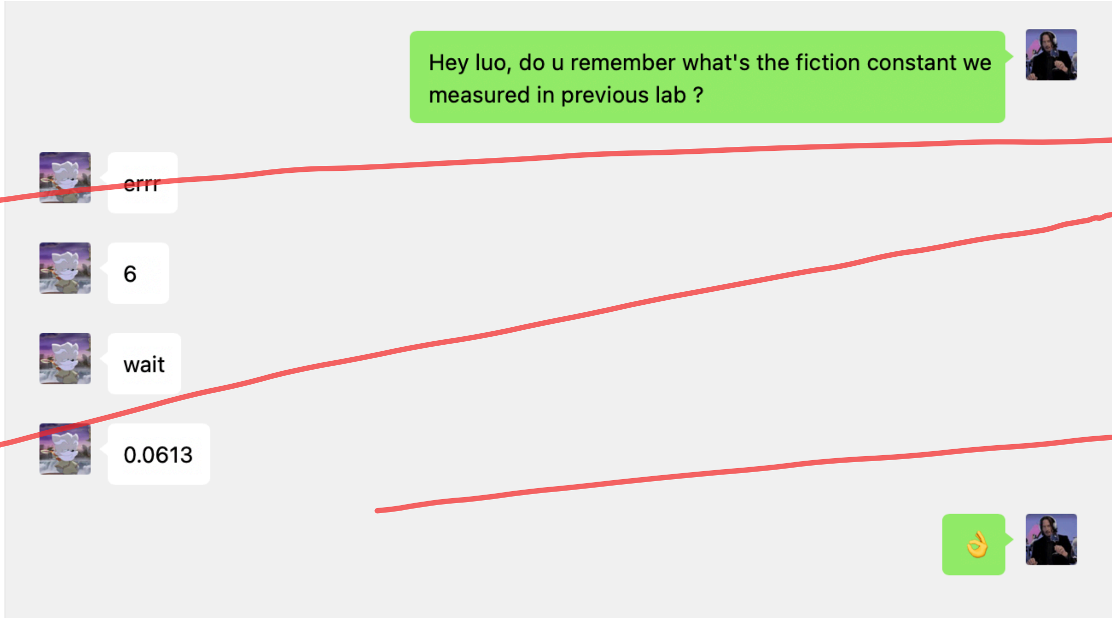
$$Uphill: \ v(t) = -0.29354056\text{t} + 1.025782 \qquad \text{eq4.2.1}$$

From $eq2.2.2$ we can know that, if there exist an initial velocity, the relationship between velocity v and acceleration a should be:

$$v(t) = v_0 + at$$

v_0 is the initial velocity and in this case, it should be $1.025782\frac{m}{s}$

7.3 The theoretical fiction constant



Just kidding

$$(A_0 + A_1) f_k = \frac{1}{2} \times k(\sqrt{A0^2 - A1^2})$$

$$-f_k = -\mu mg$$

$$A_0 = 0.0817 \qquad A_1 = 0.0783$$

$$\mu = \frac{1}{2} * \frac{k * (A_0 - A_1)}{mg} = 0.0613$$

Measured in Damped Harmonic Oscillator Lab