SPSS TUTORIALS

BASICS DATA ANALYSIS T-TEST ANOVA CHI-SQUARE TEST

n Correlations – Quick Introduction

You are here: H → Basics → Must-Know Statistics → arson Correlations Quick Introduction

A Pearson correlation is a number between -1 and 1 that indicates the extent to which two variables are linearly related.

The Pearson correlation is also known as the "product moment correlation coefficient" (**PMCC**) or simply "**correlation**".

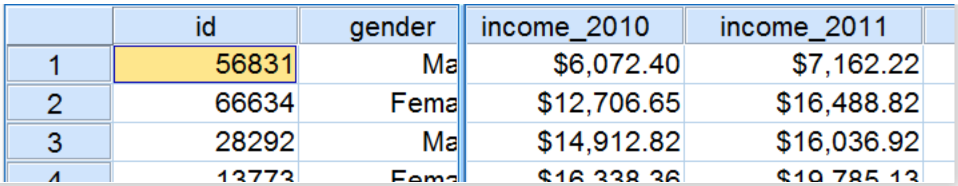
Pearson correlations are suitable only for metric variables (which include dichotomous variables).

- For **ordinal variables**, use the Spearman correlation or Kendall's tau and
- for **nominal variables**, use Cramér's V.

Correlation Coefficient - Example

We asked 40 freelancers for their yearly incomes over 2010 through 2014. Part of the raw data are shown below.

Ne
Di

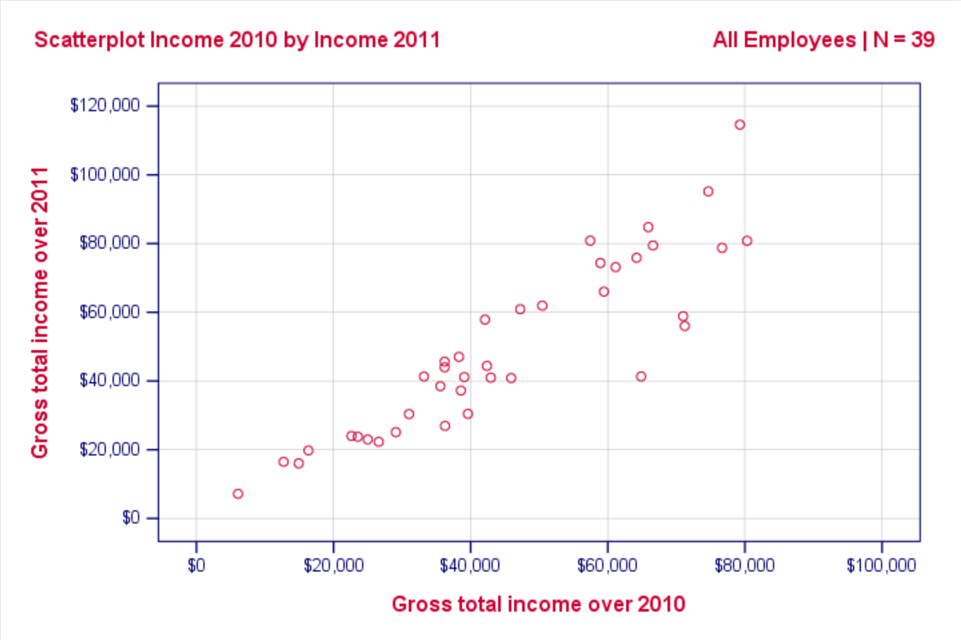


	id	gender	income_2010	income_2011
1	56831	Ma	$6,072.40	$7,162.22
2	66634	Fema	$12,706.65	$16,488.82
3	28292	Ma	$14,912.82	$16,036.92
4	13773	Fema	$16,338.36	$19,785.13

Today's question is:

is there any relation between income over 2010 and income over 2011?

Well, a splendid way for finding out is inspecting a scatterplot for these two variables: we'll represent each freelancer by a dot. The horizontal and vertical positions of each dot indicate a freelancer's income over 2010 and 2011. The result is shown below.



Our **scatterplot shows a strong relation** between income over 2010 and 2011: freelancers who had a low income over 2010 (leftmost dots) typically had a low income over 2011 as well (lower dots) and vice versa. Furthermore, **this relation is roughly linear**; the main pattern in the dots is a straight line.

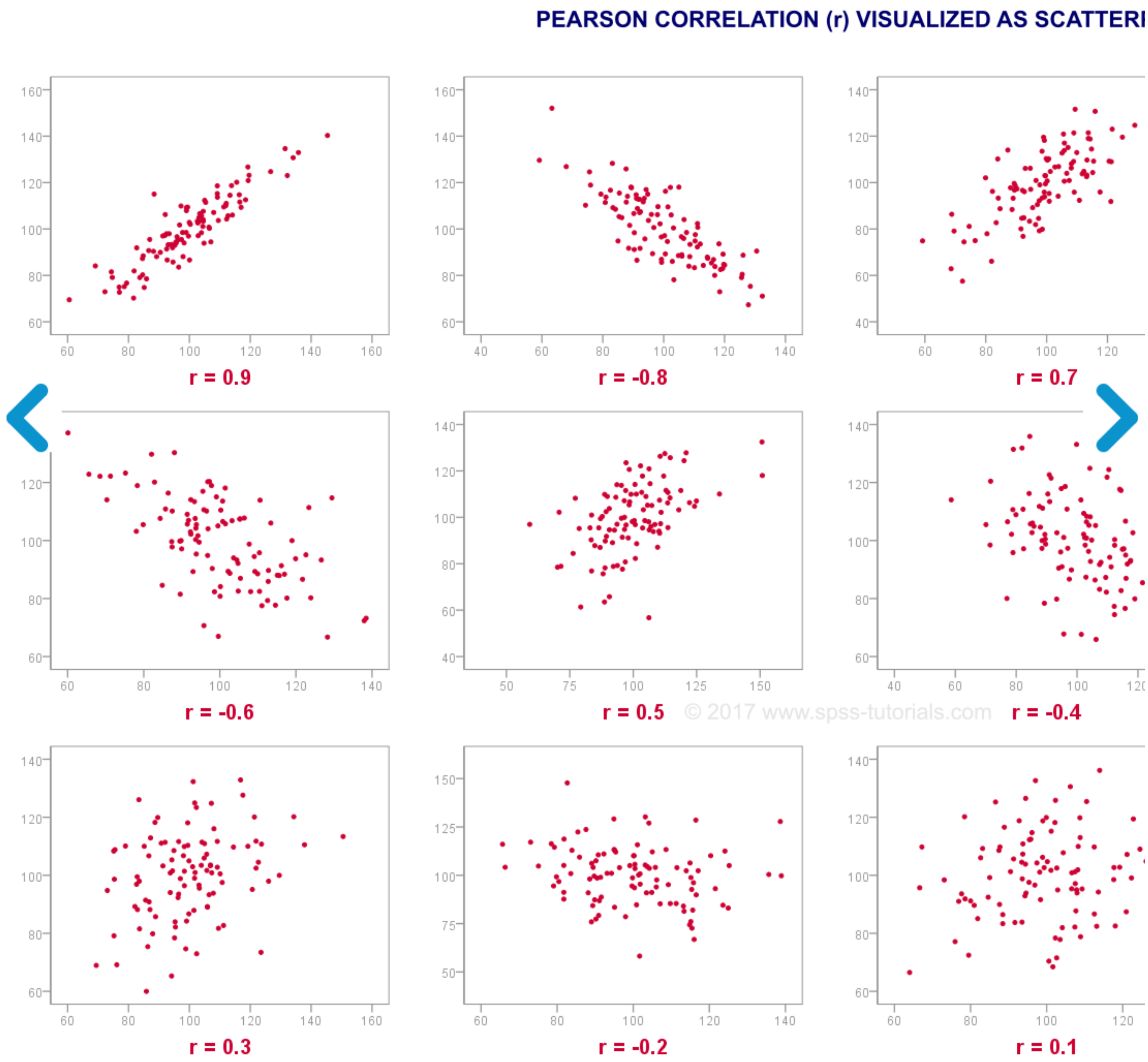
The extent to which our dots lie on a straight line indicates the strength of the relation. The Pearson correlation is a number that indicates the exact strength of this relation.

Ne
Di

Correlation Coefficients and Scatterplots

A correlation coefficient indicates the extent to which dots in a scatterplot lie on a straight line. This implies that we can usually estimate correlations pretty accurately from nothing more than scatterplots. The figure below, in which the **correlation coefficient is denoted by "r"** nicely illustrates this point.

Ne
Di



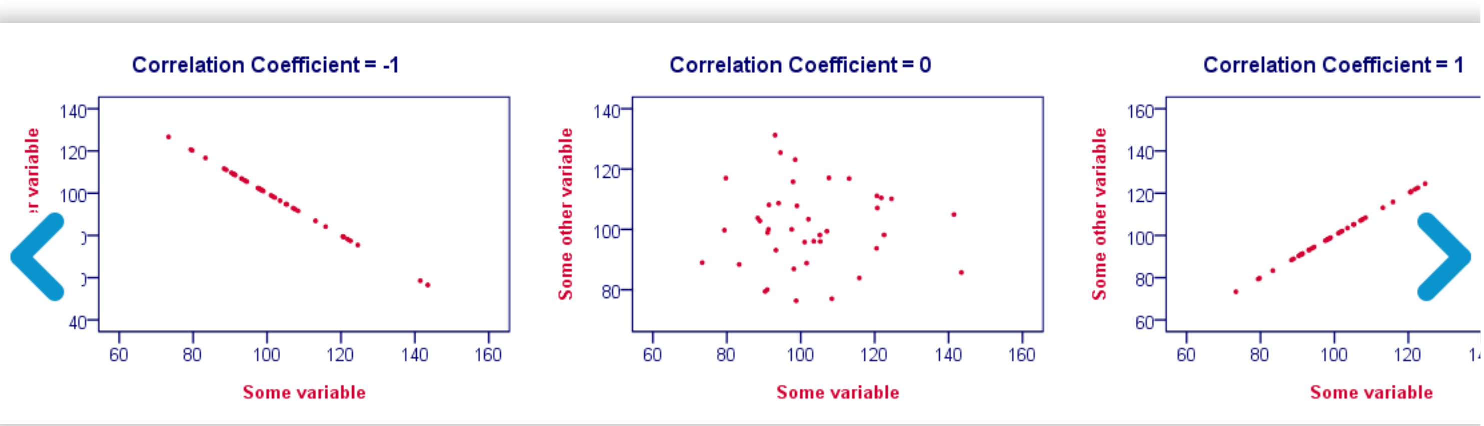
Correlation Coefficient - Basics

Some basic points regarding correlation coefficients are nicely illustrated by the previous figure. The least you should know is that

- **Correlations are never lower than -1.** A correlation of -1 indicates that the data points in a scatter plot lie exactly on a straight descending line; the two variables are perfectly negatively linearly related.

Ne
Di

- A **correlation of 0** means that two variables don't have any linear relation whatsoever. However, some non linear relation may exist between the two variables.
- **Correlation coefficients are never higher than 1.** A correlation coefficient of 1 means that two variables are *perfectly* positively linearly related; the dots in a scatter plot lie exactly on a straight ascending line.



Correlation Coefficient - Interpretation Caveats

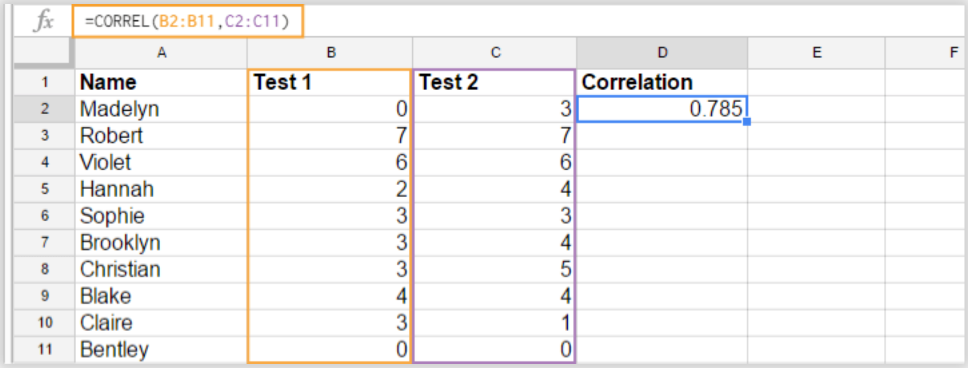
When interpreting correlations, you should keep some things in mind. An elaborate discussion deserves a separate tutorial but we'll briefly mention two main points.

- Correlations may or may not indicate **causal relations**. Reversely, causal relations from some variable to another variable may or may not result in a correlation between the two variables.
- Correlations are very sensitive to **outliers**; a single unusual observation may have a huge impact on a correlation. Such outliers are easily detected by a quick inspection a scatterplot.

Ne
Di

Correlation Coefficient - Software

Most spreadsheet editors such as Excel, Google sheets and OpenOffice can compute correlations for you. The illustration below shows an example in Googlesheets.

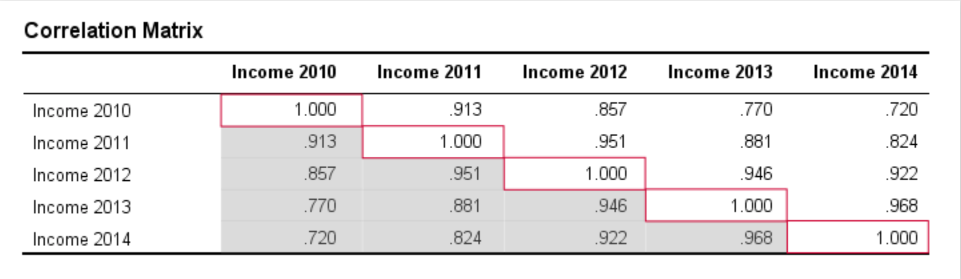


fx =CORREL(B2:B11,C2:C11)

	A	B	C	D	E	F
1	**Name**	**Test 1**	**Test 2**	**Correlation**		
2	Madelyn	0	3	0.785		
3	Robert	7	7			
4	Violet	6	6			
5	Hannah	2	4			
6	Sophie	3	3			
7	Brooklyn	3	4			
8	Christian	3	5			
9	Blake	4	4			
10	Claire	3	1			
11	Bentley	0	0			

Correlation Coefficient - Correlation Matrix

Keep in mind that correlations apply to pairs of variables. If you're interested in more than 2 variables, you'll probably want to take a look at the correlations between all different variable pairs. These correlations are usually shown in a square table known as a **correlation matrix**. Statistical software packages such as SPSS create correlations matrices before you can blink your eyes. An example is shown below.



Correlation Matrix

	Income 2010	Income 2011	Income 2012	Income 2013	Income 2014
Income 2010	1.000	.913	.857	.770	.720
Income 2011	.913	1.000	.951	.881	.824
Income 2012	.857	.951	1.000	.946	.922
Income 2013	.770	.881	.946	1.000	.968
Income 2014	.720	.824	.922	.968	1.000

Ne
Di

Note that the **diagonal elements** (in red) are the correlations between each variable and itself. This is why they are always 1.
Also note that the **correlations beneath the diagonal** (in grey) are redundant because they're identical to the correlations above the

diagonal. Technically, we say that this is a symmetrical matrix. Finally, note that the pattern of correlations makes perfect sense: correlations between yearly incomes become lower insofar as these years lie further apart.

Pearson Correlation - Formula

If we want to inspect correlations, we'll have a computer calculate them for us. You'll rarely (probably never) need the actual formula. However, for the sake of completeness, a Pearson correlation between variables X and Y is calculated by

$$r_{XY} = \frac{\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \overline{X})^2}\sqrt{\sum_{i=1}^{n}(Y_i - \overline{Y})^2}}$$

The formula basically comes down to dividing the covariance by the product of the standard deviations. Since a coefficient is a number divided by some other number our formula shows why we speak of a correlation *coefficient*.

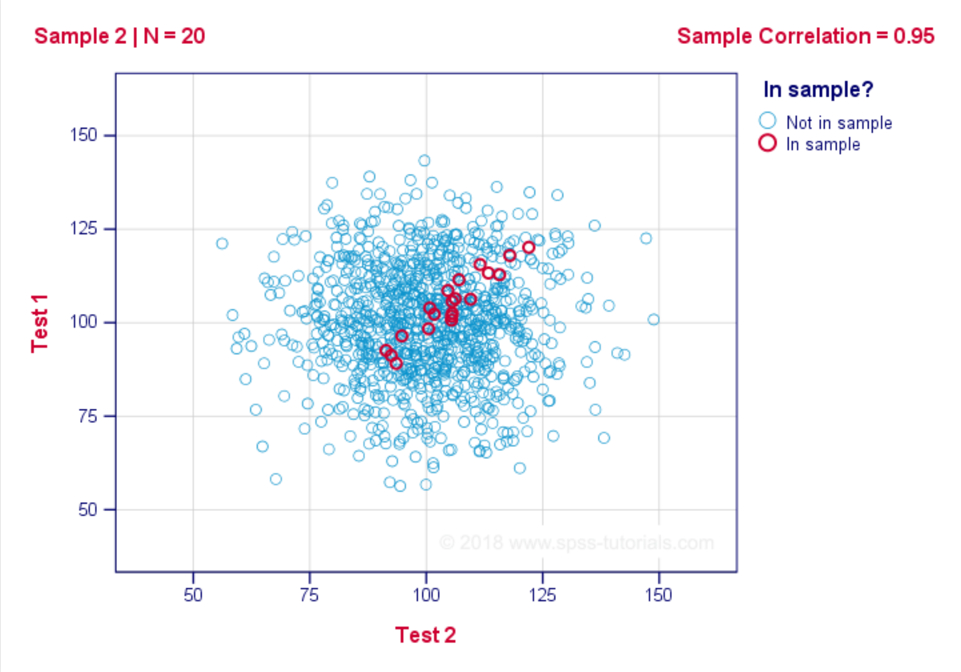
Correlation - Statistical Significance

The data we've available are often -but not always- a small sample from a much larger population. If so,

we may find a non zero correlation in our sample even if it's zero in the population.

The figure below illustrates how this could happen.

Ne
Di



If we ignore the colors for a second, all 1,000 dots in this scatterplot visualize some population. The **population correlation -denoted by ρ- is zero** between test 1 and test 2.

Now, we *could* draw a sample of N = 20 from this population for which the correlation **r = 0.95**. Reversely, this means that a sample correlation of 0.95 doesn't prove with certainty that there's a non zero correlation in the entire population. However, finding r = 0.95 with N = 20 is **extremely unlikely** if ρ = 0. But precisely how unlikely? And how do we know?

Correlation - Test Statistic

If ρ -a population correlation- is zero, then the probability for a given sample correlation -its statistical significance- depends on the sample size. We therefore combine the sample size and r into a single number, our test statistic t:

$$T = R\sqrt{\frac{(n-2)}{(1-R^2)}}$$

Now, T itself is not interesting. However, we need it for finding the significance level for some correlation. T follows a t distribution with ν = n - 2 degrees of freedom but only if some assumptions are met.

Ne
Di

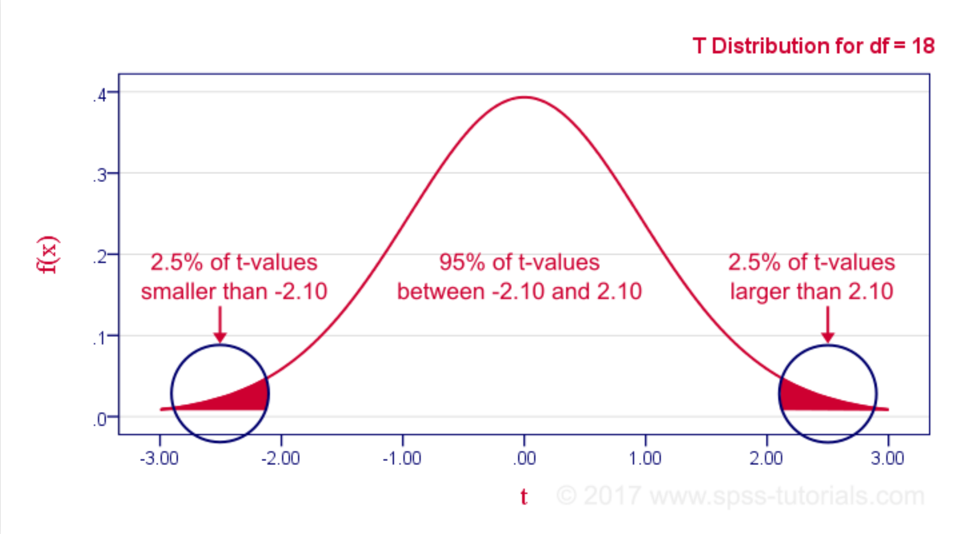
Correlation Test - Assumptions

The statistical significance test for a Pearson correlation requires 3 assumptions:

- **independent observations**;
- the population correlation, **ρ = 0**;
- **normality**: the 2 variables involved are bivariately normally distributed in the population. However, this is not needed for a reasonable sample size -say, N ≥ 20 or so.*

Pearson Correlation - Sampling Distribution

In our example, the sample size N was 20. So if we meet our assumptions, T follows a t-distribution with df = 18 as shown below.



This distribution tells us that there's a 95% probability that -2.1 < t < 2.1, corresponding to -0.44 < r < 0.44. Conclusion:

if N = 20, there's a 95% probability of finding -0.44 < r < 0.44.

Ne
Di

There's only a 5% probability of finding a correlation outside this range. That is, such correlations are statistically significant at α = 0.05 or lower: they are (highly) unlikely and thus refute the null hypothesis of a zero population correlation.

Last, our sample correlation of 0.95 has a p-value of $1.55e^{-10}$ -one to 6,467,334,654. We can safely conclude there's a non zero correlation in our entire population.

Thanks for reading!

Let me know what you think!

Ne
Di

Your name*

Your email address*

Your website

Your comment*

Done!

**Required field. Your comment will show up after approval from a moderator.*

This tutorial has 23 comments

By Ruben Geert van den Berg on September 24th, 2019

Hi Anna!

Expand comment | all comments

By Anna K on September 24th, 2019

Hi! Thank you for excellent tutorial. I have 1 question: why if n=20, df=18 there is 95% probability that -2.1 < t < 2.1??? (I look into t-distribution tables and do not understand why |2.1| - maybe |1.7| instead?) Thanx!

By Sandor on September 2nd, 2019

Beside of SPSS this correlation coefficient calculator can help you in quick r, p-value, and visualization. https://www.answerminer.com/calculators/correlation-test

Ne
Di

1 … 5

Get In Touch!

Ruben Geert van den Berg

LinkedIn

Facebook

SPSS Help (Netherlands)

Sigma Plus Statistiek

www.sigma-plus-statistiek.nl

info@sigma-plus-statistiek.nl

SPSS Help (International)

SPSS tutorials

www.spss-tutorials.com

info@spss-tutorials.com

Ne
Di