olmogorov-Smirnov Test for Normality

An alternative normality test is the Shapiro-Wilk test.

One-Sample Kolmogorov-Smirnov Test

		Most Extreme Differences				
	N	Absolute	Positive	Negative	Test Statistic	Asymp. Sig. (2-tailed)
Reaction time trial 1	233	.073	.073	-.031	.073	.0047[c]
Reaction time trial 2	233	.090	.090	-.086	.090	.0001[c]
Reaction time trial 3	235	.385	.385	-.219	.385	.0000[c]
Reaction time trial 4	226	.045	.045	-.027	.045	.2000[c,d]
Reaction time trial 5	235	.120	.120	-.067	.120	.0000[c]

a. Test distribution is Normal.
b. Calculated from data.
c. Lilliefors Significance Correction.
d. This is a lower bound of the true significance.

P < 0.05? REJECT NULL HYPOTHESIS OF NORMAL POPULATION DISTRIBUTION

What is a Kolmogorov-Smirnov normality test?

The Kolmogorov-Smirnov test examines if scores are likely to follow some distribution in some population.

For avoiding confusion, there's 2 Kolmogorov-Smirnov tests:

- there's the **one sample Kolmogorov-Smirnov test** for testing if a variable follows a given distribution in a population. This “given distribution” is usually -not always- the normal distribution, hence “Kolmogorov-Smirnov normality test”.
- there's also the (much less common) **independent samples Kolmogorov-Smirnov test** for testing if a variable has identical distributions in 2 populations.

In theory, “Kolmogorov-Smirnov test” could refer to either test (but usually refers to the one-sample Kolmogorov-Smirnov test) and had better be avoided. By the way, both Kolmogorov-Smirnov tests are present in SPSS.

Kolmogorov-Smirnov Test - Simple Example

So say I've a population of 1,000,000 people. I think their reaction times on some task are perfectly normally distributed. I sample 233 of these people and measure their reaction times.
Now the observed frequency distribution of these will probably differ a bit -but not too much- from a normal distribution. So I run a histogram over observed reaction times and superimpose a normal distribution with the same mean and standard deviation. The result is shown below.

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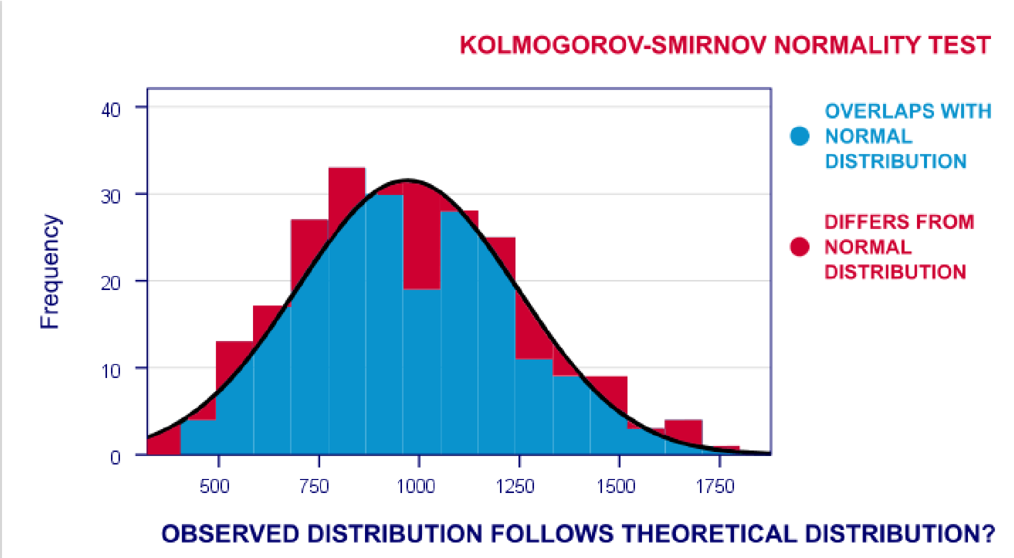

The frequency distribution of my scores doesn't entirely overlap with my normal curve. Now, I could calculate the **percentage of cases that deviate from the normal curve** -the percentage of red areas in the chart. This percentage is a test statistic: it expresses in a single number how much my data differ from my null hypothesis. So it indicates to what extent the observed scores deviate from a normal distribution.

Now, if my null hypothesis is true, then this deviation percentage should probably be quite small. That is, a small deviation has a high probability value or p-value.

Reversely, a huge deviation percentage is very unlikely and suggests that my reaction times don't follow a normal distribution in the entire population. So a **large deviation has a *low* p-value**. As a rule of thumb, we

reject the null hypothesis if $p < 0.05$.

So if $p < 0.05$, we *don't* believe that our variable follows a normal distribution in our population.

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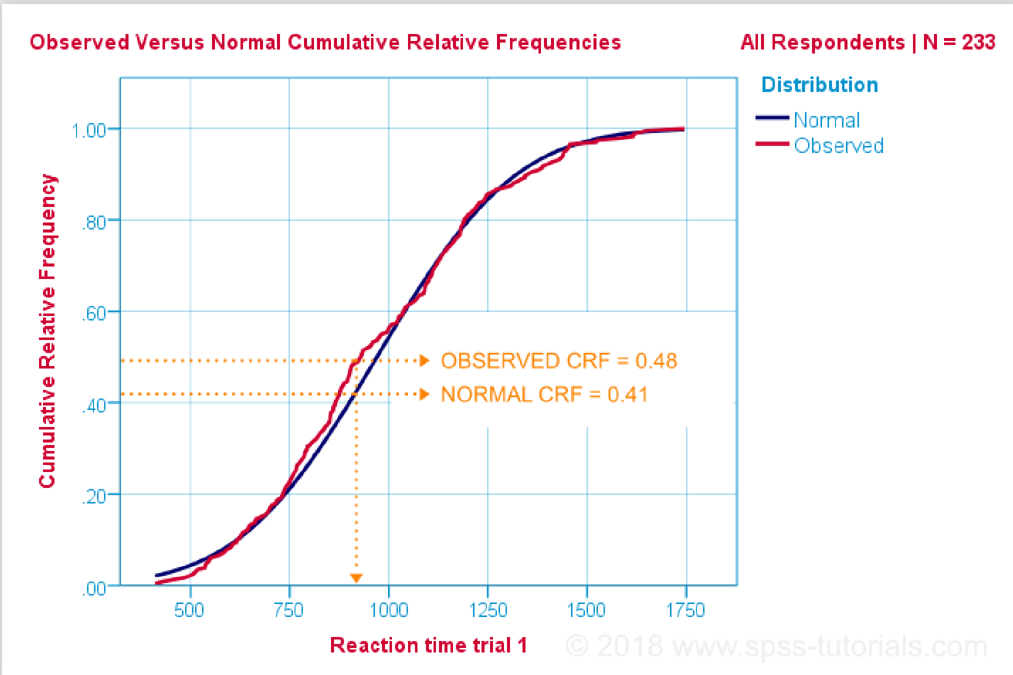
Kolmogorov-Smirnov Test - Test Statistic

So that's the easiest way to understand how the Kolmogorov-Smirnov normality test works. Computationally, however, it works differently: it compares the observed versus the expected cumulative relative frequencies as shown below.



The Kolmogorov-Smirnov test uses the **maximal absolute difference** between these curves as its test statistic denoted by D. In this chart, the maximal absolute difference D is (0.48 - 0.41 =) 0.07 and it occurs at a reaction time of 960 milliseconds. Keep in mind that **D = 0.07** as we'll encounter it in our SPSS output in a minute.

The Kolmogorov-Smirnov test in SPSS

There's 2 ways to run the test in SPSS:

- NPAR TESTS as found under Analyze ▸ Nonparametric Tests ▸ Legacy Dialogs ▸ 1-Sample K-S... is our method of choice because it creates nicely detailed output.

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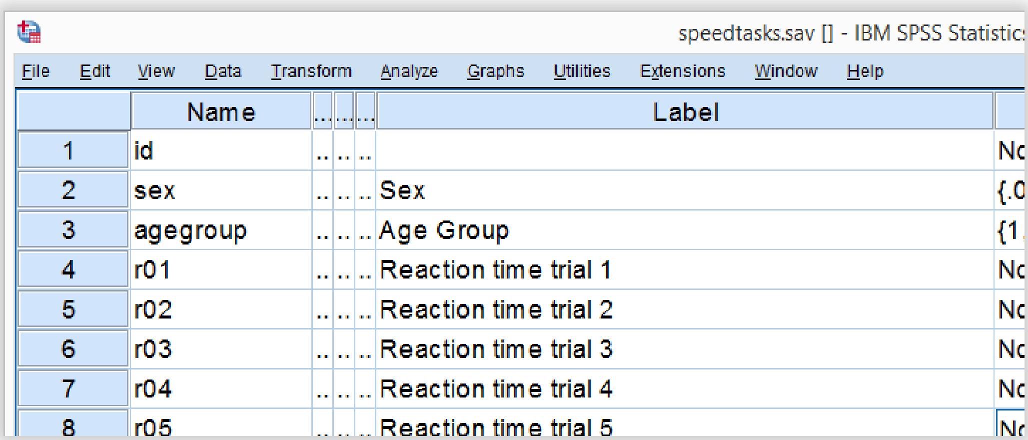
- EXAMINE VARIABLES from Analyze ▸ Descriptive Statistics ▸ Explore is an alternative. This command runs both the Kolmogorov-Smirnov test and the Shapiro-Wilk normality test.
  Note that EXAMINE VARIABLES uses **listwise exclusion of missing values** by default. So if I test 5 variables, my 5 tests only use cases which don't have any missings on any of these 5 variables. This is usually not what you want but we'll show how to avoid this.

We'll demonstrate both methods using speedtasks.sav throughout, part of which is shown below.


speedtasks.sav [] - IBM SPSS Statistics

File Edit View Data Transform Analyze Graphs Utilities Extensions Window Help

	Name	...	...	...	Label	
1	id	..	..	..		No
2	sex	..	..	..	Sex	{.0
3	agegroup	..	..	..	Age Group	{1
4	r01	..	..	..	Reaction time trial 1	No
5	r02	..	..	..	Reaction time trial 2	No
6	r03	..	..	..	Reaction time trial 3	No
7	r04	..	..	..	Reaction time trial 4	No
8	r05	..	..	..	Reaction time trial 5	No

Our main research question is

which of the reaction time variables is likely to be normally distributed in our population?

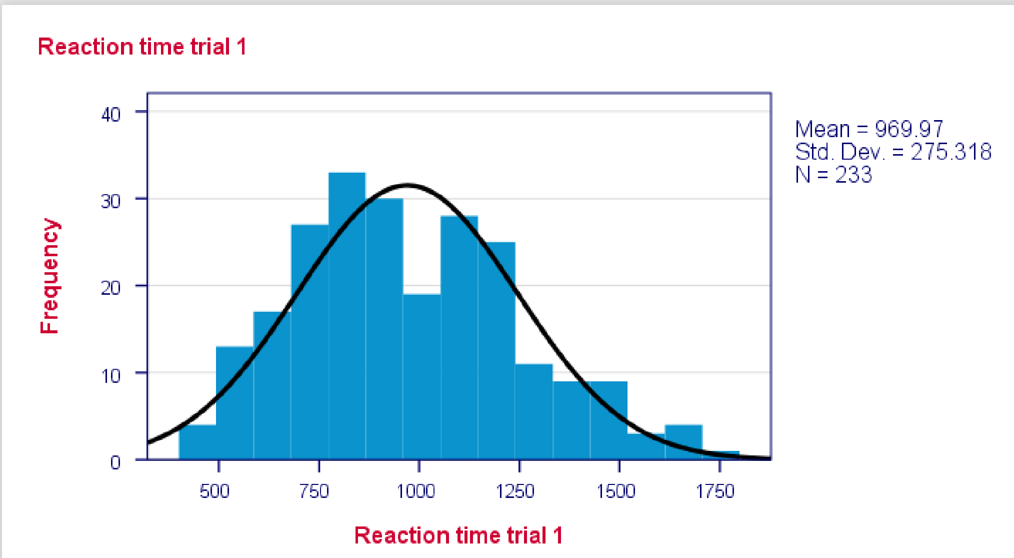
These data are a textbook example of why you should thoroughly inspect your data before you start editing or analyzing them. Let's do just that and run some histograms from the syntax below.

```
*Run basic histograms for inspecting if distributions

frequencies r01 to r05
/format notable
/histogram normal.

*Note that some distributions do not look plausible a
```

Result



Note that some distributions do not look plausible at all. But which ones are likely to be normally distributed?

SPSS Kolmogorov-Smirnov test from NPAR TESTS

Our preferred option for running the Kolmogorov-Smirnov test is under Analyze ▸ Nonparametric Tests ▸ Legacy Dialogs ▸ 1-Sample K-S... as shown below.

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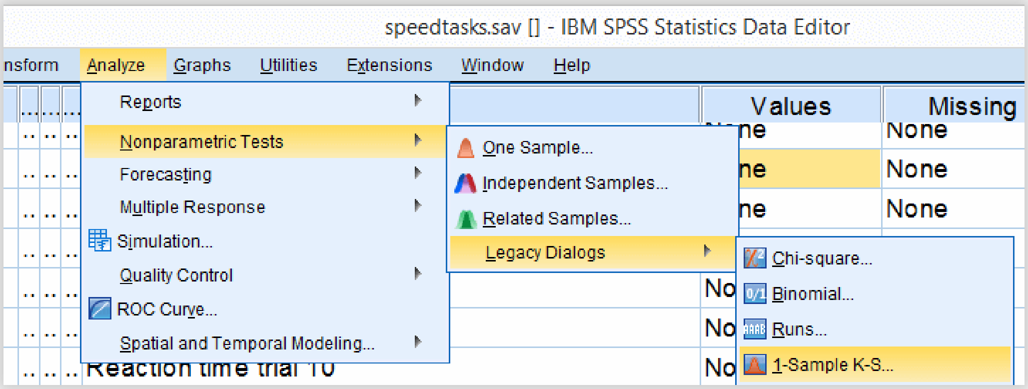
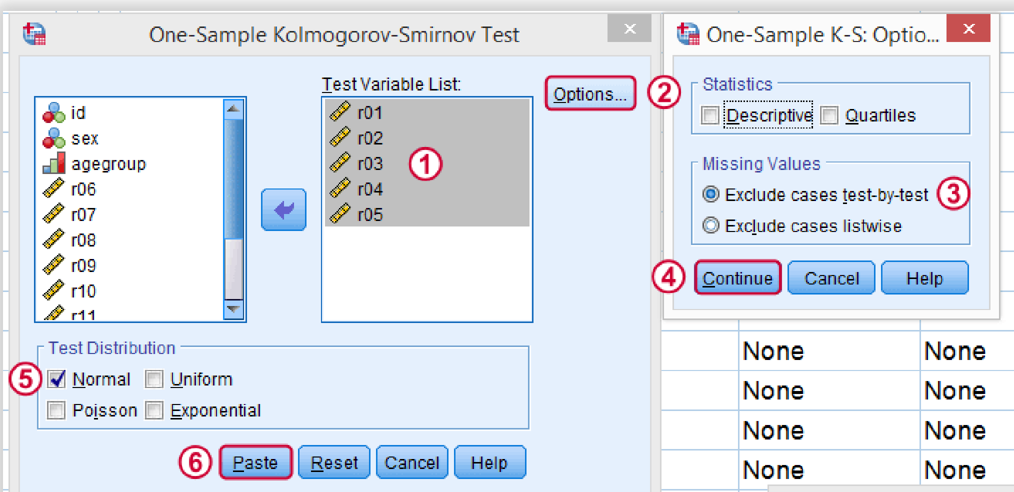


Next, we just fill out the dialog as shown below.



Clicking Paste results in the syntax below. Let's run it.

Kolmogorov-Smirnov Test Syntax from Nonparametric Tests

```
*One-sample Kolmogorov-Smirnov test from analyze - no

NPAR TESTS
```

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```
/K-S(NORMAL)=r01 r02 r03 r04 r05
/MISSING ANALYSIS.

*Only reaction time 4 has p > 0.05 and thus seems nor
```

Results

One-Sample Kolmogorov-Smirnov Test

	N	Most Extreme Differences Absolute	Positive	Negative	Test Statistic	Asymp. Sig. (2-tailed)
Reaction time trial 1	233	.073	.073	-.031	.073	.0047[c]
Reaction time trial 2	233	.090	.090	-.086	.090	.0001[c]
Reaction time trial 3	235	.385	.385	-.219	.385	.0000[c]
Reaction time trial 4	226	.045	.045	-.027	.045	.2000[c,d]
Reaction time trial 5	235	.120	.120	-.067	.120	.0000[c]

a. Test distribution is Normal.
b. Calculated from data.
c. Lilliefors Significance Correction.
d. This is a lower bound of the true significance.

P < 0.05? REJECT NULL HYPOTHESIS OF NORMAL POPULATION DISTRIBUTION

First off, note that the test statistic for our first variable is 0.073 -just like we saw in our cumulative relative frequencies chart a bit earlier on. The chart holds the exact same data we just ran our test on so these results nicely converge.
Regarding our research question: only the reaction times for trial 4 seem to be normally distributed.

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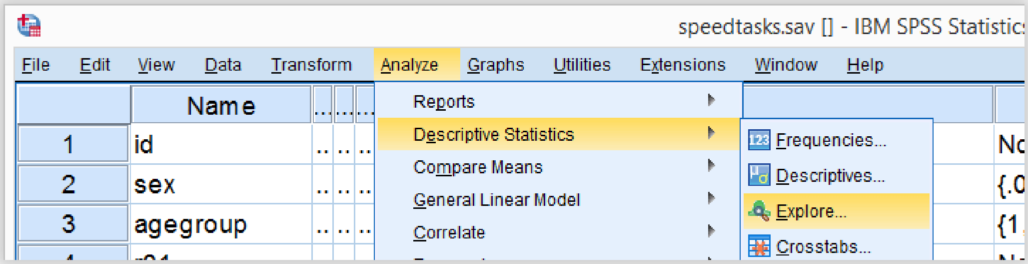
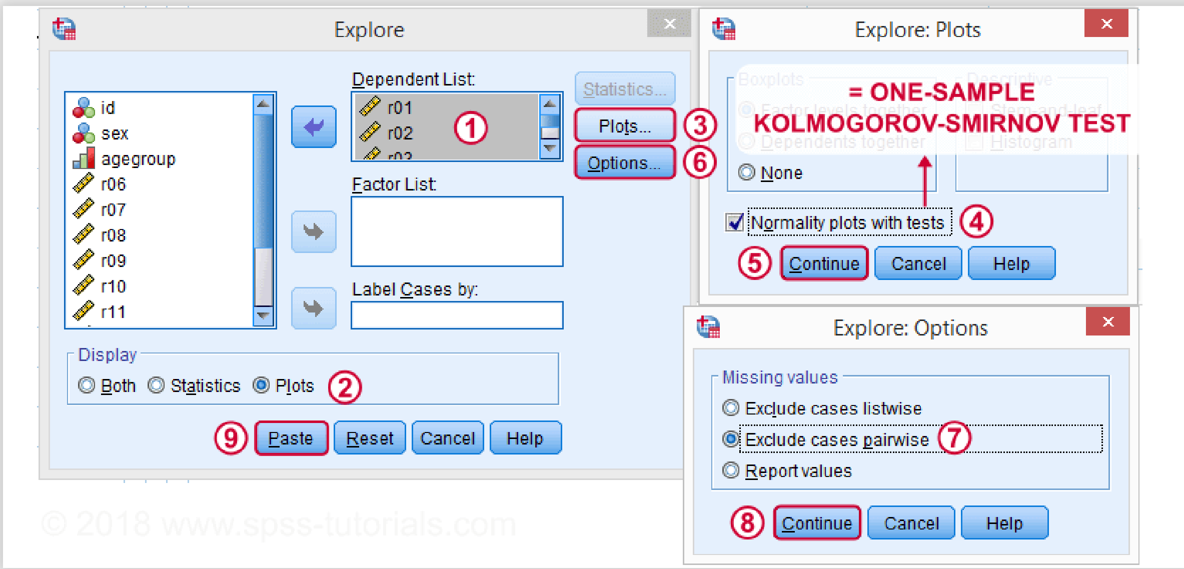
SPSS Kolmogorov-Smirnov test from EXAMINE VARIA

An alternative way to run the Kolmogorov-Smirnov test starts from Analyze ▸ Descriptive Statistics ▸ Explore as shown below.





Kolmogorov-Smirnov Test Syntax from Nonparametric Tests

```
*One-sample Kolmogorov-Smirnov test from analyze - de

EXAMINE VARIABLES=r01 r02 r03 r04 r05
/PLOT BOXPLOT NPPLOT
/COMPARE GROUPS
/STATISTICS NONE
/CINTERVAL 95
/MISSING PAIRWISE /*IMPORTANT!*/
/NOTOTAL.

*Shorter version.

EXAMINE VARIABLES r01 r02 r03 r04 r05
```

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```
/PLOT NPPLOT
/missing pairwise /*IMPORTANT!*/.
```

Results

Tests of Normality

	Kolmogorov-Smirnov[a]			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Reaction time trial 1	.073	233	.0047	.984	233	.0099
Reaction time trial 2	.090	233	.0001	.840	233	.0000
Reaction time trial 3	.385	235	.0000	.697	235	.0000
Reaction time trial 4	.045	226	.2000*	.989	226	.0752
Reaction time trial 5	.120	235	.0000	.801	235	.0000

*. This is a lower bound of the true significance.

a. Lilliefors Significance Correction

As a rule of thumb, we conclude that

a variable is *not* normally distributed if "Sig." < 0.05.

So both the Kolmogorov-Smirnov test as well as the Shapiro-Wilk test results suggest that only Reaction time trial 4 follows a normal distribution in the entire population.
Further, note that the Kolmogorov-Smirnov test results are identical to those obtained from NPAR TESTS.

Reporting a Kolmogorov-Smirnov Test

For reporting our test results following APA guidelines, we'll write something like

“a Kolmogorov-Smirnov test indicates that the reaction times on trial 1 do not follow a normal distribution, D(233) = 0.07, p = 0.005.”

For additional variables, try and shorten this but make sure you include

- **D** (for “difference”), the Kolmogorov-Smirnov test statistic,
- **df**, the degrees of freedom (which is equal to N) and
- **p**, the statistical significance.

One-Sample Kolmogorov-Smirnov Test

	N (= df)	Normal Parameters[a,b]			
			Deviation	Test Statistic	Asymp. Sig. (2-tailed)
Reaction time trial 1	233	969.97	275.318	.073	.0047[c]
Reaction time trial 2	233	1001.57	341.493	.090	.0001[c]
Reaction time trial 3	235	476.77	365.437	.385	.0000[c]
Reaction time trial 4	226	980.96	267.895	.045	.2000[c,d]
Reaction time trial 5	235	1009.52	348.779	.120	.0000[c]

“...is *not* normally distributed, D(233) = 0.07, p = 0.005.”

Wrong Results in SPSS?

If you're a student who just wants to pass a test, you can **stop reading now**. Just follow the steps we discussed so far and you'll be good.

Right, now let's run the exact same tests again in SPSS version 18 and take a look at the output.

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One-Sample Kolmogorov-Smirnov Test

	N	Most Extreme Differences Absolute	Most Extreme Differences Positive	Most Extreme Differences Negative	Kolmogorov-Smirnov Z	Asymp. Sig. (2-tailed)	Exact Sig. (2-tailed)	Point Probability
Reaction time trial 1	233	.073	.073	-.031	1.108	.172	.164	.000
Reaction time trial 2	233	.090	.090	-.086	1.377	.045	.042	.000
Reaction time trial 3	235	.385	.385	-.219	5.895	.000	.c	.000
Reaction time trial 4	226	.045	.045	-.027	.676	.750	.733	.000
Reaction time trial 5	235	.120	.120	-.067	1.840	.002	.002	.000

a. Test distribution is Normal.

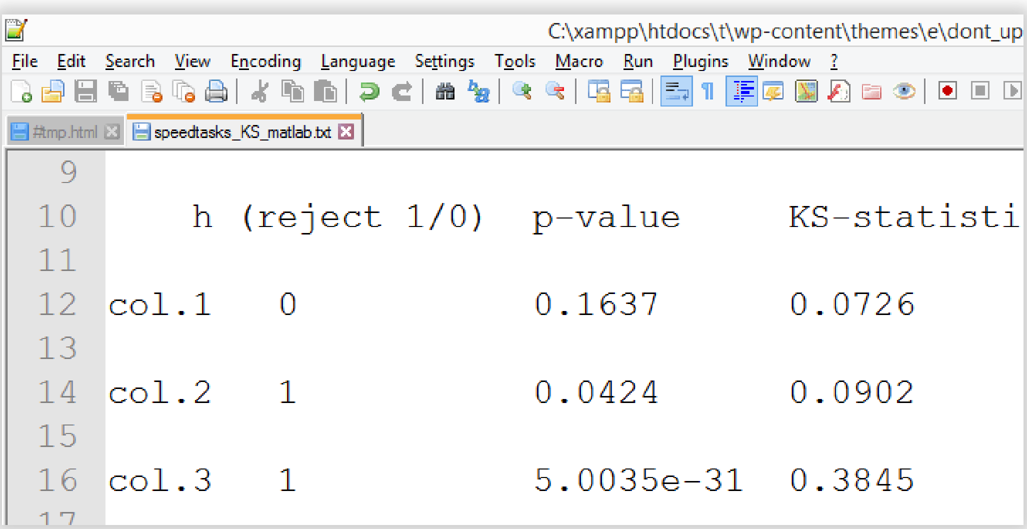
b. Calculated from data.

c. Numerical difficulties prevente

In this output, the **exact p-values** are included and -fortunately- they are very close to the asymptotic p-values. Less fortunately, though,

the SPSS version 18 results are wildly different from the SPSS version 24 results

we reported thus far.
The reason seems to be the Lilliefors significance correction which is applied in newer SPSS versions. The result seems to be that the asymptotic significance levels differ much more from the exact significance than they did when the correction is not implied. This raises **serious doubts regarding the correctness** of the “Lilliefors results” -the default in newer SPSS versions.
Converging evidence for this suggestion was gathered by my colleague Alwin Stegeman who reran all tests in Matlab. The Matlab results agree with the SPSS 18 results and -hence- not with the newer results.



Kolmogorov-Smirnov normality test - Limited Usefulness

The Kolmogorov-Smirnov test is often to test the normality assumption required by many statistical tests such as ANOVA, the t-test and many others. However, it is almost routinely overlooked that such tests are robust against a violation of this assumption if sample sizes are reasonable, say N ≥ 25.* Therefore,

normality tests are only needed for small sample sizes

if the aim is to satisfy the normality assumption.
Unfortunately, small sample sizes result in low statistical power for normality tests. This means that substantial deviations from normality will *not* result in statistical significance. The test says there's no deviation from normality while it's actually huge. In short, the situation in which normality tests are needed -small sample sizes- is also the situation in which they perform poorly.

Thanks for reading.

Let me know what you think!

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Your name*

Your email address*

Your website

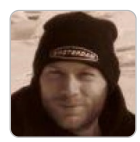
Your comment*

Done!

Required field. Your comment will show up after approval from a moderator.

This tutorial has 10 comments

By Ruben Geert van den Berg on October 9th, 2019



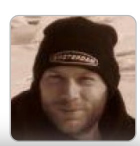
Hi Saman!

Please throw the syntax you pasted into another comment.

By Saman Brown on October 9th, 2019

hi there,
When I click on the paste, the syntax shown for me is different from what is shown in the pictures!

By Ruben Geert van den Berg on August 29th, 2019



The KS-test involves just 1 quantitative variable. No such thing as "(in)dependent variable" in this case.

For testing the distribution of 1 cat

Expand comment | all comments

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By K.Chitra Ravishankaran on August 28th, 2019

Hello,
Greetings. Thank you for your helpful resource in statistics.

Expand comment | all comments

By Tengku Arif on July 15th, 2019

your website is very useful for me to do a research. Can i know the reference that you use

1 2

Get In Touch!

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