

# Introduction To Problem Solving

21st March, 2022

## AGENDA:

A.

- Need for Problem Solving
- Some simple and fun problems  
Quizzes
- Real World Examples

B.

Next 2 months topic and schedule

C.

Doubts

## About Me:

- Graduated from IIT Kharagpur  
Studied Mathematics and Computing
- Software Developer @ Cohesity
- Instructor @ Scaler
- Plays Badminton, Basketball. Loves reading books, watching TV series (Loves The Office :))



## Prime Number

15  
12  
9  
23

N.B. :-

- Answers in Private chats.
- Questions in Question Tab.; Discussion in public chat.
- Acknowledge with Yes/No



What is a prime number?

$N :$

- a) 1 and itself as factors  $\cancel{N \geq 2}$   $\cancel{1}$
  - b) cnt factors  $\geq 2$
  - c) cnt unique factors = 2  $\cancel{N \geq 1}$  -  $\frac{1}{1} : 1$   
 $\frac{2}{2} : 1, 2$
- Not be applicable for 1...  $\times$

1  $\rightarrow$  1, 1  $\leftarrow$  1 is not prime

Q.

Check if the given input number  $N$  is prime or not.

if Count of factors = 2 ✓

def isPrime(N):

$N \rightarrow [1, N]$

cnt = 0

for i in range(1, N+1):  $\rightarrow$  [1, N]  $\leftarrow$  N numbers

if  $N \% i == 0$ : Modulo

(5)

||       $\equiv$       cnt += 1  
 ||      if    cnt == 2 :      }  
 ||           return True      // No. is prime.  
 ||      else  
 ||           return False. // No. is not prime.  
 1, 2, 3, 4, 5 ... N  
 ✓ X ✓ X ✓ ✓  
 cnt = 1      cnt = 2

Q. Time taken by above code?

Assumption -

$10^8$  iterations in 1 sec

<u>Input</u>	<u>Iterations</u>	<u>Time.</u>
$N = \underbrace{10^9}$	$10^9$	$10^9 / 10^8$ $= \underline{10 \text{ sec.}}$
{ $N = \underbrace{10^{18}}$	$10^{18}$	$\rightarrow \frac{10^{18}}{10^8} / \underline{10 \text{ sec}}$ $\rightarrow a^m / a^n = a^{m-n}$

$$10^{10} \text{ sec} \approx \rightarrow \text{min} \rightarrow \text{days} \rightarrow \text{years} \rightarrow \approx 317 \text{ years}$$

Q: How can we optimise?



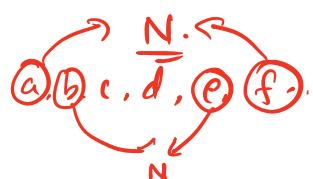
If  $A \leq 57$ , what is the max value  
of  $A$ ?  $A \leq 57$ .  
Max. value  $\rightarrow 57$ .



$$N = 16$$

Factors:

$$\begin{array}{c} 1 \\ 2 \\ 4 * 4 = 16 \\ 8 \\ 16 \end{array}$$



$$N = 24$$

1st half	1	*	24	=	24	$a * b = N$
	2	*	12	=	24	
	3	*	8	=	24	
	4	*	6	=	24	
	5	*	4	=	24	
	6	*	2	=	24	

\* Factors come in pairs.

$$\sqrt{24} = 4 \dots$$

$\begin{array}{ c c c c } \hline & 6 & * > 4 & \\ \hline 2nd half: & 8 & * & 3 \\ & 12 & * & 2 \\ \hline & 24 & * & 1 \\ \hline \end{array}$	$\stackrel{\text{etc}}{\dots}$ In 2nd half, factors start repeating itself. $a \leq b$ in 1st half $a > b$ in 2nd half
--	---

1, 2, 3, 4, ... 23, 24

$$\begin{array}{l} 1, \quad 24/1 \quad \checkmark \\ 2, \quad 24/2 = 12 \quad \checkmark \\ 3, \quad 24/3 = 8 \quad \checkmark \\ 4, \quad 24/4 = 6 \quad \checkmark \\ \hline (6) \\ \downarrow 8 \end{array}$$

1st half

$$\left\{ \begin{array}{l} a \leq b \\ a * b = N \end{array} \right\}$$

Max value of  $a$ ? =  $b$

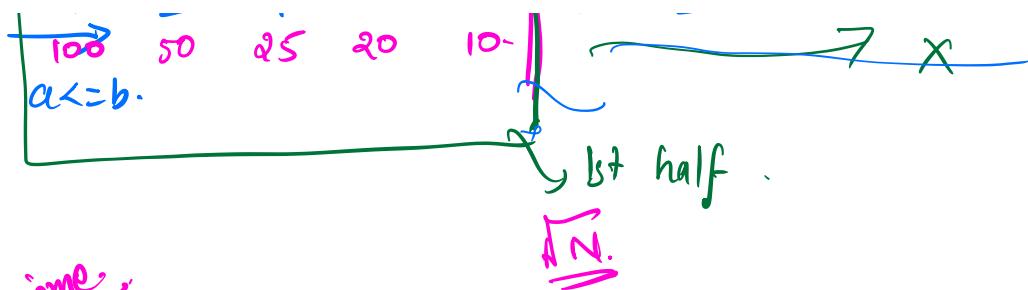
Eliminate  $b, a$  -  
Ans. in terms of  $N$ .

$$\begin{aligned} & a \leq b. \\ & \underline{\text{Max. value of } a = b}. \\ & a * b = N \\ & \Rightarrow b * b = N \\ & \Rightarrow b = \sqrt{N} = a \end{aligned}$$

$a \leq b$ ,  
Max. value of  $a$  can be  $\sqrt{N}$ .

$$\begin{array}{c} \overbrace{\quad \quad \quad \quad \quad}^{\sqrt{N}=10.} \\ \overbrace{\quad \quad \quad \quad \quad}^{N=100} \end{array}$$

1	2	4	5	10	20	25	50	100
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For prime.

$$\begin{array}{l} [1, N] \xrightarrow{N} \text{cnt factors} = 2 \\ - [1, \sqrt{N}] \xrightarrow{x} \text{cnt factors} = 1 \end{array}$$

$$\begin{array}{l} [1, 51] \\ \approx [1, \sqrt{5}] \\ \approx [1, 2 \cdot 2] \end{array} \leftarrow \textcircled{1} \checkmark$$

$$\begin{array}{l} N \geq 2 \\ 1 \times [2, \sqrt{N}] \rightarrow \text{cnt factors} = \underline{\underline{0}} \end{array} \checkmark$$

Code!

```
def isPrime(N):  
    if (N==1) return False  
    for i in range(2, int(sqrt(N))+1):  
        if N % i == 0:  
            return False  
    return True
```

$N=3$ :  
 $i \in \text{range}(2, \lceil \sqrt{3} \rceil + 1)$   
 $\lceil \sqrt{3} \rceil + 1 = 2$

$N=5$ :  
 $i \in \text{range}(2, \lceil \sqrt{5} \rceil + 1)$   
 $\lceil \sqrt{5} \rceil + 1 = 3$

$N=1$ ?  
 $\text{range}(2, 2)$  → X

range(2,  $\lceil \sqrt{2} \rceil + 1$ )  
 $\lceil \sqrt{2} \rceil + 1 = 2$

$\lceil \sqrt{2} \rceil + 1 = 2$  → Empty.  
 $N=2$

$\lceil \sqrt{2} \rceil + 1 = 2$  → 2.

$\lceil \sqrt{2} \rceil + 1 = 2$  → 2, 414.

$\lceil \sqrt{2} \rceil + 1 = 2$  → 2, 414.

Q. Time taken by above approach?  $10^8$  iter → 1 sec.

<u>Input.</u>	<u>Iterations.</u>	<u>Time taken</u>
$10^{10}$	$\sqrt{10^0}$ $= 10^5$	$10^5 / 10^8$ <u><math>= 1 \text{ ms.}</math></u>

$10^{18}$	$\sqrt{10^{18}}$ $= 10^9$	$10^9 / 10^8$ <u><math>= 10 \text{ sec.}</math></u>
<u>317 years.</u>	<u>10 seconds.</u>	



BREAK.

10:15 pm.

Sum of first N Natural numbers?

$$1+2+3+\dots+N = \frac{N(N+1)}{2}$$

Gauss:

$$\rightarrow 1+2+3+\dots+1000 = S$$

$$\underbrace{(+) 1000 + 999 + 998 + \dots + 1}_{(+)} = S$$

$$\times \underbrace{(1001 + 1001 + 1001 + \dots + 1001)}_{=} = 2S$$

$$1000 * 1001 = 2S$$

$$\Rightarrow S = \frac{1001 * 1000}{2} \quad \checkmark$$

$$1+2+\dots+N = \frac{N(N+1)}{2}$$



Given  $N$ , how many times we need to divide by 2 to make it 1?

$\log_2 N$

<u><math>N</math></u>	<u>Num Steps</u>
1	0
2 $\rightarrow 2/2 = 1$	1
3 $\rightarrow 3/2 = 1$	1
4 $\rightarrow 4/2 = 2 / 2 = 1$	2
5 $\rightarrow 5/2 = 2$	2
7 $\rightarrow$	2
8 $\rightarrow$	3
12 $\rightarrow 12/2 = 6 \quad 6/2 = 3 \quad 3/2 = 1$	3
15 $\rightarrow$	3
16 $\rightarrow$	4
27 $\rightarrow$	4
32 $64^{33-63} \rightarrow 5$	5
	6

$$\begin{aligned}\log_2 4 &= \\ \log_2 8 &= \\ (\log_2 12 = (3 \cdot 58)) &= 3.\end{aligned}$$

$$2^{\frac{3.61}{2}} \approx 12$$

$$\left. \begin{array}{l} \text{if } \rightarrow a^x = n \\ \Rightarrow \rightarrow x = \underline{\log_a n} \end{array} \right\} \text{ logarithms.}$$

$$\underbrace{a * a * \dots}_{?} = n.$$

amazon.

Q.

Given a number (which is a perfect square),  
find the square root.

$$\begin{array}{r} \xrightarrow{?} \\ 2 \times \\ 10 \times \\ \textcircled{4} \rightarrow \end{array}$$



Which of these is a perfect square?

24

35

49

50

$$7 \times 7 = 49. \quad \sqrt{49} = 7. \quad \checkmark$$



Search for the square root!

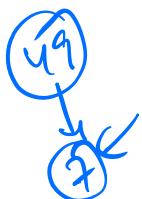
$$\begin{array}{r} \xrightarrow{\text{sqrt}(49)} \checkmark \\ \textcircled{(49)^{1/2}} \quad \checkmark \quad \times \end{array}$$

$$\log_2 \textcircled{49} = ?$$

$[1, N]$

$[1, 49]$

$$\begin{array}{l} \downarrow \text{number} \\ \text{num} * \text{num} = 49. \quad \checkmark \end{array}$$



def my\_sqrt(N):

{

    for i in range(1, N+1):

        if  $i * i == N$ :

            return i

    \textcircled{49}

$$1 \times 1 = 49 \quad \times$$

$$2 \times 2 = 49 \quad \times$$

$$\vdots$$

$$7 \times 7 = 49. \quad \checkmark$$

48, 49.

$\sqrt{N}$



No. of iterations in above code?

\* N

$\log(N)$

$\text{sqrt}(N)$

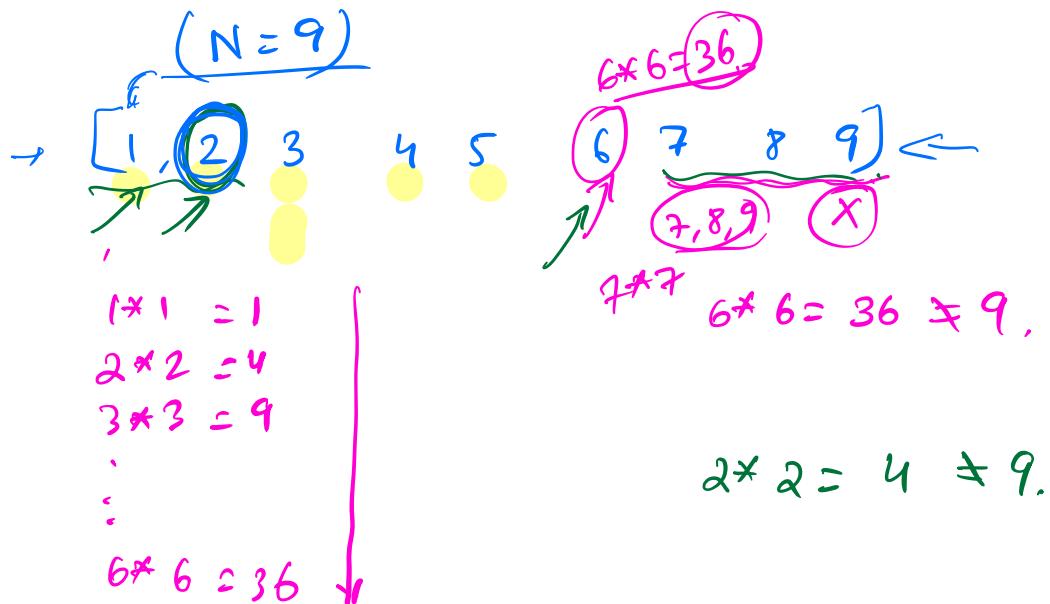
N/2

Q.

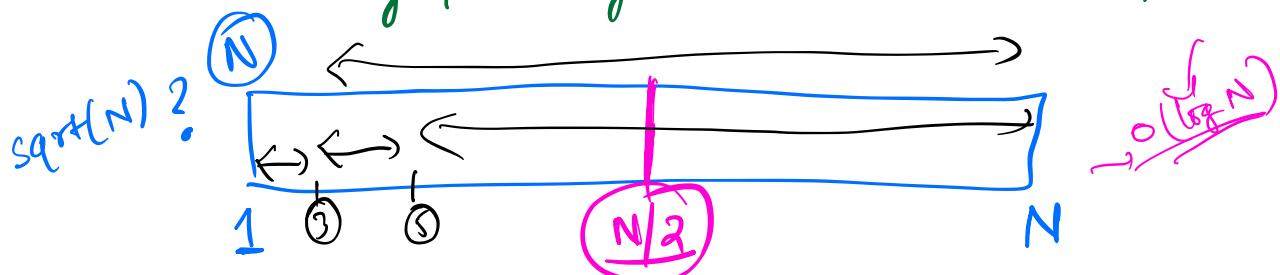
How can we optimise?



Observe how you can cut the search space.



Eliminating / Cutting down our search space.



```

mid = N/2  $[(\text{start} + \text{end})/2]$ 
if mid * mid == N :
    return mid
else if mid * mid < N :
    // eliminate Left half  $\leftarrow \text{start} = \text{mid} + 1$ 

```

else :  $(\text{mid} * \text{mid} > N)$  ;  
 // eliminate Right half .  $\text{end} = \text{mid} - 1$

$$\sqrt{100} = ?$$

$$\frac{100+1}{2} \rightarrow [1, 100]$$

$\downarrow$   
 $50$

$$\frac{100+1}{2} \rightarrow [1, 49]$$

$$\frac{100-1+1}{2} = 50$$

$$50 * 50 > 100$$

Right half X

$$\frac{49-1+1}{2} = 24 \quad \text{Righ } X$$

$$24 * 24 > 100$$

$$[1, 23] \rightarrow [1, 11]$$

$\xrightarrow{a+b}$   
 $b-a+1$

$$\times [1, 5] \rightarrow [6, 11]$$

$$\frac{23-1+1}{2} = 12 \quad \text{Right } X.$$

$$12 * 12 > 100$$

$$\frac{11-1+1}{2} = 5 \quad \text{Right } X.$$

$$6 * 6 = 36 < 100$$

$$5 * 5 < 100$$

$$[1, 3] \rightarrow 3-1+1 = 3$$

$\xrightarrow{a+b}$   
 $b-a+1$

$$[10, 11] \rightarrow [10, 5] \rightarrow [10, 10]$$

⑥/5

$$[6, 11] \rightarrow ⑧/9$$

$$\begin{aligned} [9, 11] &\rightarrow \frac{11-6+1}{2} = 3 \\ 8 * 8 &< 100 \\ 9 * 9 &< 100 \end{aligned}$$

$$\overline{10 * 10 = 100}$$

Eureka!

→ Binary Search . ←

Pseudo-code :

Q

① Find mid of range

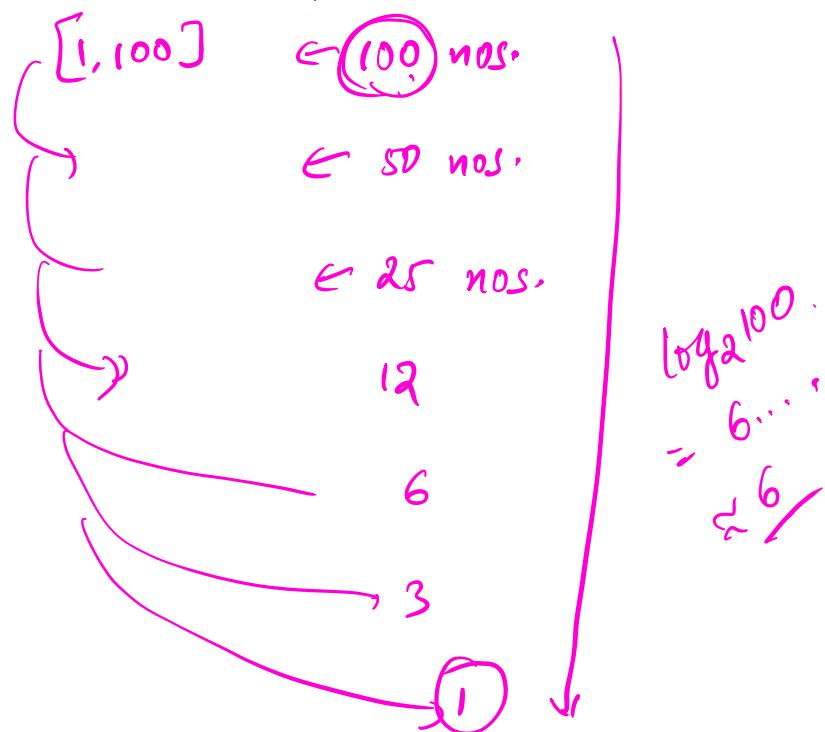
② Check if mid is sqrt(N)

- If  $\text{mid} * \text{mid} < N$ , ignore left half.
- If  $\text{mid} * \text{mid} > N$ , ignore right half.
- If  $\text{mid} * \text{mid} = N$ , EUREKA!

Q. No. of iterations/steps using above approach?

For general N?

No. of iterations/steps =  $\log_2 N$ .



## CONCLUSIONS !

\*\* **OBSERVATION is the key!**

Observations :  
Importance  
of  
optimization!

\*\* Real world examples -

amazon.com

filtering - Search  
Sorting - Sort

facebook / linkedin

Graphs

uber / swiggy

Shortest path b/w 2 nodes

google autocomplete

Trie

Trie.

Linux / Windows file systems

Trees

## Intermediate Content

**Goal :** Learn how to be better at observations and problem solving!

**Class Details :**

- M/W/F
- 9 PM

9-11 → Content

11-11:30 → Doubts & Discussions

**Curriculum :**

* Time Complexity	-	2
* <b>Arrays / Lists</b>	-	6/7 sessions
· Intro		
· Prefix sum		
· Carry forward		
· Subarrays		
· 2D Matrices		
· Interview problems		
* Bit Manipulation	-	2
* <b>Maths</b>	-	2
* Sorting	-	1
* Strings	-	1
* Hashing	-	1
* <b>Recursion</b>	-	2/3
* Linked Lists Basics	-	1
* Stacks & Queues Basics	-	1
* Trees Basics	-	2
* Problem solving	-	2/3

~2 months

Advance DS M.V.

$N=100$



Advanced DS content:

- Advanced searching, sorting
- Trees
- BST, Trie
- Greedy, DP
- Graphs
- etc.

### Some more points :

- \* Notes will be uploaded.
- \* 6-7 questions in HW+ Assignment (Python 3.x)

≈ 20 problems weekly

≈ 160 ~ 170 problems in the session

### Doubt Resolution:

#### ① Self Debug

Try it yourself!  
~ 30-40 mins

#### ② Help from TA

1:1 call with TA  
Option available  
on dashboard.

#### ③ Ask Me

Conceptual  
doubts or  
session doubts.

- i. Ask during class.
- ii. Doubt session at end
- iii. Ping on slack!

\* Python Primer material available in Dashboard!

GOOD LUCK !