

Hashing - Part I

29th April, 2022

Agenda:

- What/Why Hash Map / Dictionary ?
- Dictionary in Python
 - Operations defined on them
- Sneak peek into Implementation of Dictionary
- Some interesting problems

* Why Hash Map / Dictionary ?

Hotel.

1 - 20 rooms.
Checking each room's availability.

Room no.	
1	→ Yes.
2	→ No
3	→ Yes
:	:
20	↓

0	0	0	1	0	0	0	0	0	1
0	1	2	3		...	19			

→ Iterate., whether I have an entry in this array with '0'.

K*

Room nos. 1, 2, 3, ..., 20, ↙

Room nos. 1 10 100 10^3 10^4 ... 10^{20}

101 , 102 , 104

201 ,

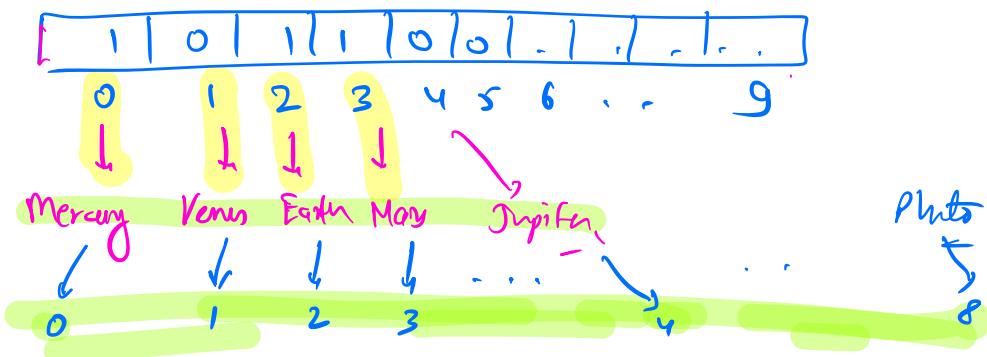
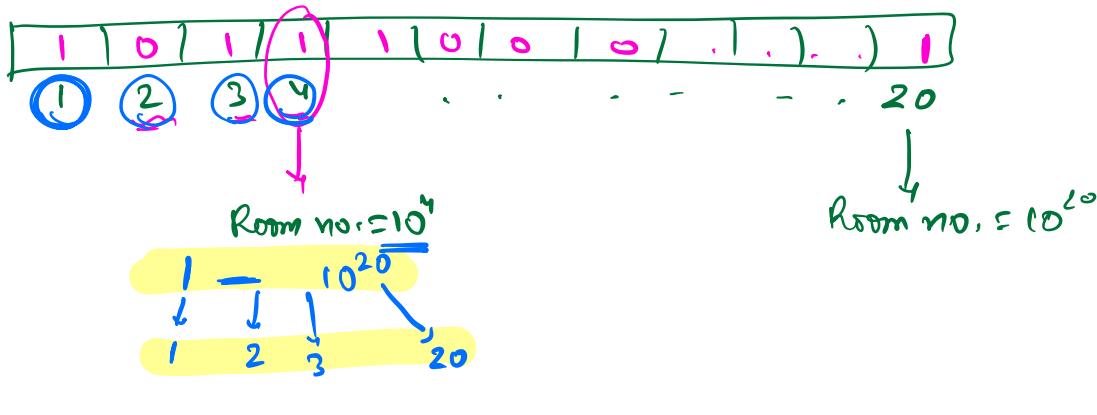
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K*

0/1	-1	-1	-1		1	-1	-1	-1	0
1	2	3	4		...	10^20			

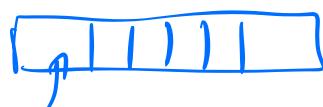
Waste of space. | X

Vie:
** Index as the power of 10



Hash Maps

(Key - value pair)



Element of list +
integer

Entry in a Hash Map.

(key-value pair)

Room name	Occupancy
Mercury	0
Venus	1
Earth	1
.	=
Uranus	0
Neptune	1

key

4 entries.

Direct access.

$O(1)$ time

* 2D array / 2D list

$2 \times N$

Mercury	Venus	0	0	Uranus
0	0			b

9 entries
 9×2 entries.

$O(N)$

Iterate.

* Dictionary.

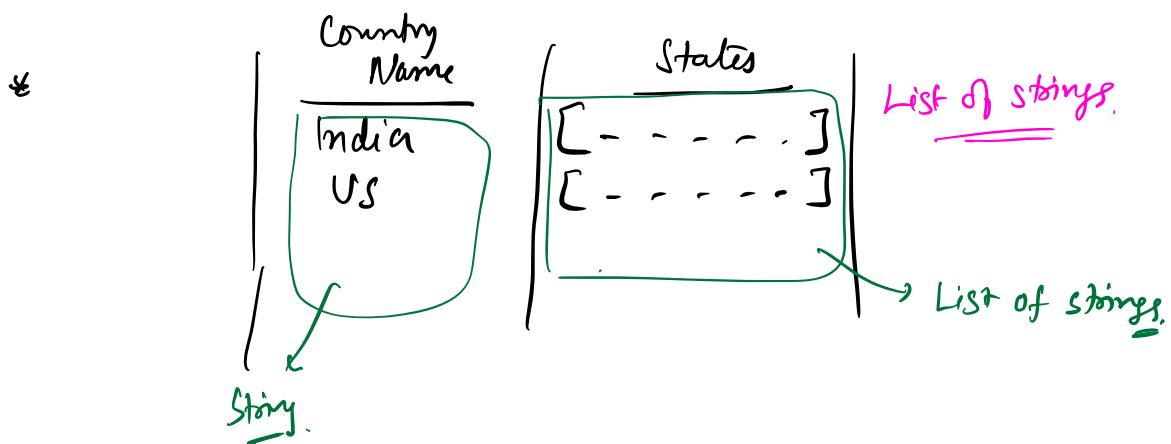
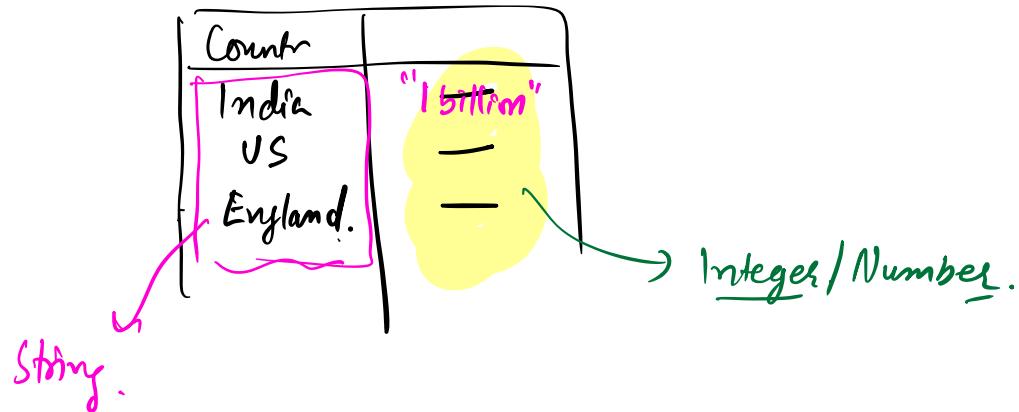
Lookup.

(Given a key, return value) $\rightarrow O(1)$ time.

Insertion.

(Insert a K,V pair) $\rightarrow O(1)$ time

* Population of every country.



Dictionary in Python

li = []

my-dict = {} // Dictionary

my-dict = { "Name": "Gautam", "Age": 25 }

Name	Gautam
Age	25

2 important properties:

→ Keys should be immutable.

↳ String, Tuple, Number

Mutable.

↳ List, Dictionary

* Can values be mutable?

Yes.

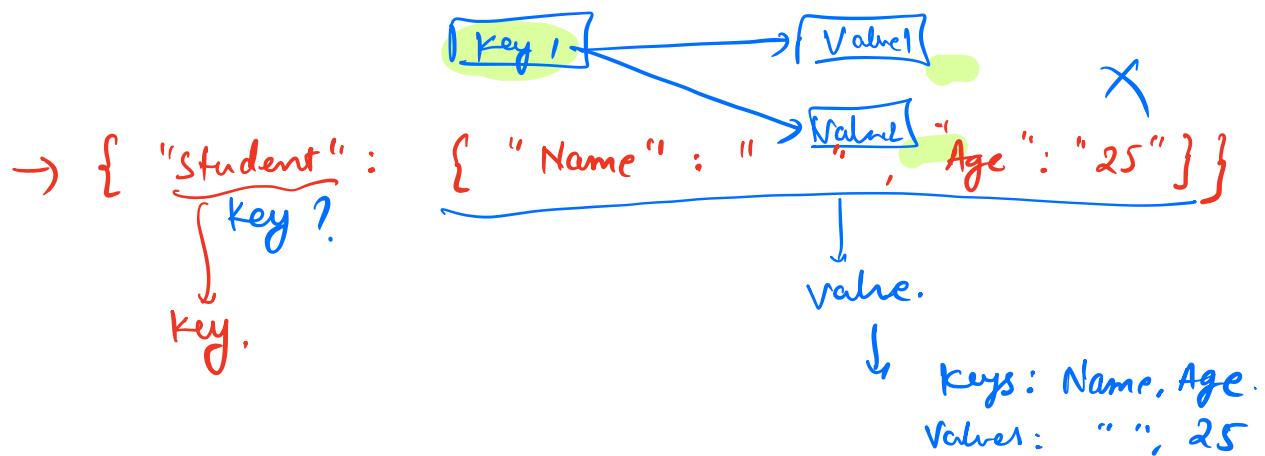
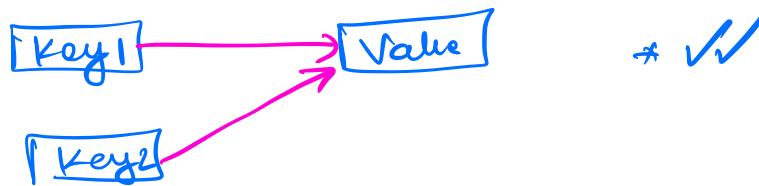
↳ Number, String, List, Tuple,
Dictionary.

→ Keys are unique.

(Each key is mapped to only one value).

India → 1 billion
India → 2 billion X

India → 1 billion
China → 1 billion } ✓



Functions on Dictionary.

* length

`len(my_dict)`



No. of entries present.

* `my_dict = {}`

Add a new k.v pair:

$\rightarrow \text{my_dict}[\text{"Govind"}] = 27$
 $= \{ \text{"Govind": 27} \}$

$\rightarrow \text{my_dict}.\underline{\text{update}}(\{ \text{"Govind": 27}, \text{"Deepak": 29} \})$

↓ Add multiple k.v pairs at once.

* `del my_dict["Govind"]`

↓ $\{ \text{"Deepak": 29} \}$

* Iterate over a dictionary.

`for ele in arr:
 for key, value in my_dict.items():
 print(key)
 print(value)`

`my_dict: { "Govind": 27, "Deepak": 29 }`

Govind
27
Deepak
29

* Iterate over the keys.

for k in my_dict.keys():

↳ List of keys,
iterator

* Iterate over the values.

range(0, 10)
↓
iterator

for v in my_dict.values():

* Get a value given a key.

* my_dict[key] → value. None if key is not present.

* my_dict.get(key) default → value.

* Check if a key is present in dictionary.

① if my_dict.get(key) == None:

O(1) time

// Key is not present.

② if key in my_dict.keys():

$\in O(N)$ time

// Key is present.

~~**~~

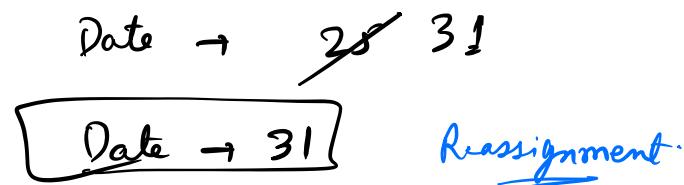
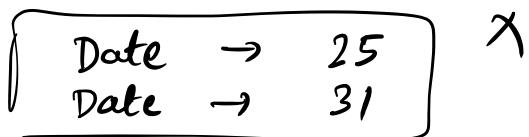
③ if key in my_dict:

$O(1)$ time

// Key is present.

- * $["Bar", "Mumbai"] : "cities"$
- * X Invalid dictionary

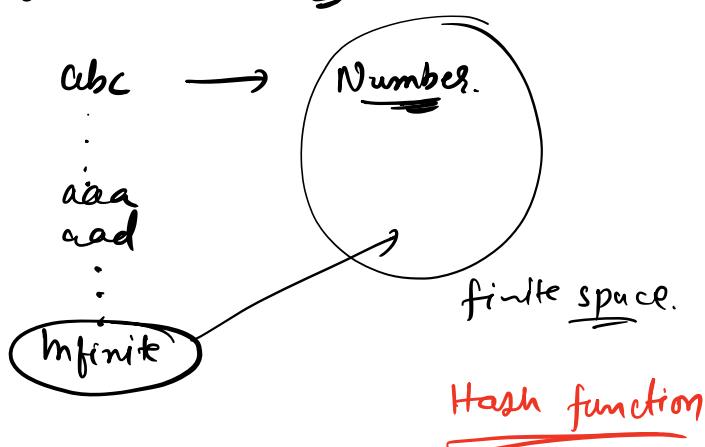
~~xx~~



~~xx~~ How hashmaps are implemented?

<u>Key</u>	<u>Value</u>
abc	"Mumbai"
aaa	"Cvrgan"
aad	"Kolkata"

Keys are hashed.



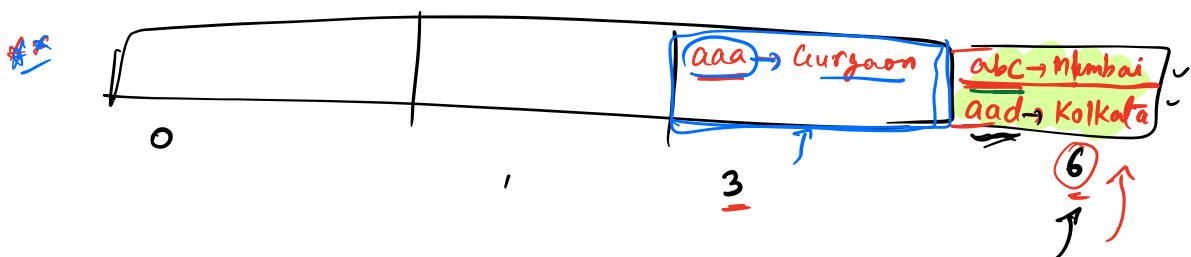
$$\begin{array}{c}
 \text{abc} \quad \xrightarrow{\text{Hash}} \quad a+b+c \\
 \downarrow \downarrow \downarrow \\
 \text{1 2 3} \quad \xrightarrow{\text{Hash}} \quad 6 \\
 \text{3 bits}
 \end{array}$$

$$\begin{array}{c}
 \text{aaa} \quad \xrightarrow{\text{Hash}} \quad 3 \\
 \text{aad} \quad \xrightarrow{\text{Hash}} \quad 6 \\
 \text{26} \times \text{26} \times \text{26} \quad \xrightarrow{\text{Hash}} \quad 1 - \frac{26 \times 3}{26 \times 26 \times 26} \\
 \text{zzz.}
 \end{array}$$

$$\left(\underbrace{-\infty \dots +\infty}_{\text{of integers}} \right) \xrightarrow{\% M} \underbrace{0 \dots M-1}_{\text{M}}$$

map

<u>Key</u>	<u>Value</u>
abc	"Mumbai"
aaa	"Curgaon"
aad	"Kolkata"



$$\begin{array}{l}
 \text{aaa} \rightarrow 3 \\
 \text{aad} \rightarrow
 \end{array}$$

$$\begin{array}{c}
 \text{aad} \\
 \text{abc} \\
 \text{ada} \\
 \text{cba}
 \end{array}
 \left. \right\} 10$$

Worst case TC of lookup: $O(N)$.

** In real life, Hash functions are very well designed.

a "collision."

A good hash function \rightarrow collisions should be very less.

+ Avg time complexity \rightarrow $O(1)$

SHA-256

$$\left\{ \begin{array}{l} \text{aaa} \rightarrow a \times 26^2 + a \times 26^1 + a \times 26^0 \\ \text{zzz} \rightarrow z \times 26^2 + . \end{array} \right.$$

$$\left\{ \begin{array}{l} "a" : \text{None} \end{array} \right\} \quad \checkmark$$

* * Break till 10:18

$$\left[\begin{array}{l} \underline{10}, \underline{20}, \underline{40} \end{array} \right] \quad \rightarrow \quad \underline{\underline{2}}$$

Problems.

Q. You are given an array of N elements.
Return Frequency of value 'X'.

Q queries

arr = [2, 6, 3, 8, 2, 8, 2, 3, 8]

Q queries } Freq of 2 → 3 } 2 : X < 3
 } Freq of 3 → 2 } 6 : 1
 } Freq of 8 → 2 } 3 : > 2
 } } 8 : X > 3

Brute force.

```
for i in Q:  

    // ita Query.  

    for ele in arr:  

        cnt // Count.
```

TC: $\boxed{O(N*Q)}$

Optimise?

** Create a dictionary // $O(N)$
 Key → elements in array
 value → Count of each element

** For Q queries, lookup from dictionary & return. // $O(Q)$

TC: $O(N+Q)$

```
my_dict = {}  
for ele in arr:  
    if ele not in my_dict:  
        my_dict[ele] = 1  
    else:  
        my_dict[ele] += 1  
  
for q in Q:  
    // return value of val.  
    print my_dict[val]
```

$\rightarrow O(N)$

$\rightarrow O(Q)$

$O(N+Q)$

Q. Given an array , count no. of distinct elements .

Create a dictionary.

return len(my_dict)

** Sets.

Keys values
len(set(array)) **

Imp
~~* *~~

Order in a dictionary

→ "Deepak" : 25,
→ "Alok" : 20

< Python 3.7

~~Unordered~~

≥ Python 3.7

~~↑ * Insertion order is maintained.~~

- Q Given an array, return the first non-repeating element in the array.

e.g.

[
 8, 2, 8, 3, 1, 2, 6, 5
 * * * * ↓
 ans = 3

~~*~~ Create a dictionary

{
 → 8 : 2
 → 2 : 2
 → 3 : 1
 | : 1
 6 : 1
 5 : 1 }
 |
 |

Q. Given an array, check if there exists a subarray with $\text{sum} = 0$.

2 2 1 -3 4 3 1 -2 -3

Brute force

→ Consider all subarrays.
Get their sum.

```
for i in range(len(A)):  
    for j in range(i, len(A)):  
        sum = 0  
        for k in range(i, j+1):  
            sum += A[k]  
        if sum == 0:  
            return True  
return False
```

$O(N^2)$

$O(N)$

$O(N^3)$

prefix sum

II optimisation

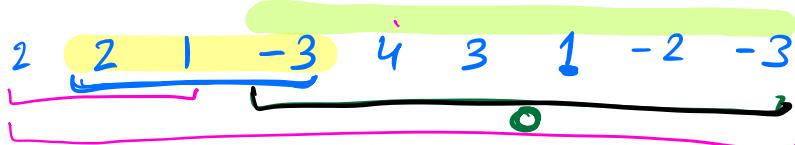
```

// Build prefix sum.
// Pre[i] → Gives prefix sum till i;
for i in range(len(A)):
    for j in range(i, len(A)):
        //sum of A[i:j]
        if Pre[j] - Pre[i-1] == 0
            return True
    return False.

```

$O(n^2)$

Optimisation



Prefix-sum → 2 4 5 2 6 9 10 8 5

$\text{Pre}[i] \rightarrow$ sum from 0 to i

$\text{Pre}[j] \rightarrow$ sum from 0 to j

* If for some $i, j \geq 0$ $\text{Pre}[i] = \text{Pre}[j]$,

** Build the prefix sum array. $\hookleftarrow O(N)$
Check if there is a repeating element.

$O(N)$ ✓

Dictionary.

**

$$[5, 3, 0, 4]$$

$$\downarrow \begin{matrix} 5 & 8 & 8 & 12 \end{matrix} \quad \checkmark$$

$$[0 \ 0 \ 0 \ 0]$$

$$\downarrow \begin{matrix} 0 & 0 & 0 & 0 & 0 \end{matrix} \quad \checkmark$$

$$[5 \ 5 \ 5 \ 5]$$

$$\downarrow [5 \ 10 \ 15 \ 20] \quad \checkmark$$

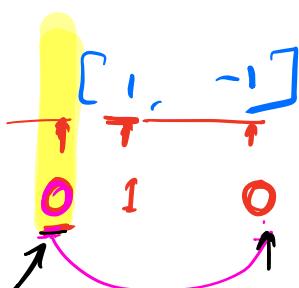
**

$$[-1, 1]$$

$$[-1, 0]$$

↗ No repeating element

Edge case . \rightarrow Subarray with sum 0 is starting from index 0.



$$\begin{bmatrix} -1, 1, 3, 4 \\ -1, 0, 3, 7 \end{bmatrix}$$

$$\boxed{\text{pre}[i] = \text{arr}[i] + \text{pre}[i-1]}$$

Pseudo code.

```
dict [0] = 1    // For edge case.  
sum = 0  
for ele in A :  
    sum += ele  
    if sum in dict :  
        return True  
    else  
        dict [sum] = 1  
return False
```

Can also do
using sets ↗

Doubt session

$s = "cbobcdbobc"$
prev=0 ← s.find("bob", 0) prev-1
-1

C bobcd bobc
X ↑
obcdbobc
X ↑
b \ obc