

Recursion - Part 1

May 4, 2022

AGENDA :

- How to write recursive code ?
- How does recursion work internally ?
- Some simple problems

Why Recursion ?

**

what ^{is} Recursion ?

→ a function calls itself

Why Recursion ?

- Merge sort / Quicksort
- Binary Tree / BST
- Dynamic programming.
- Graphs
- backtracking

Recursion

A function calling itself.

```
def sum( )
```

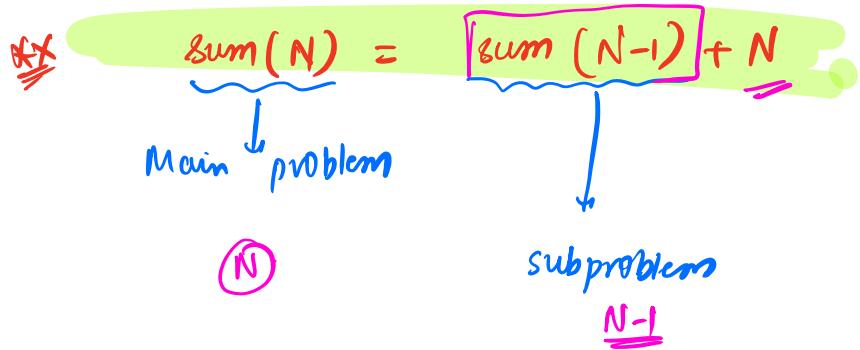
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- - -

- * When you have to solve a large problem, use the function itself to solve a sub-problem.
- * Calculate sum of nos. from 1 to N.

```
| def sum( N ):  
|     s = 0  
|     for i in range(1,N+1):  
|         s = s+i  
|  
|     return s
```

Iterative code (use for/while)

problem \Rightarrow $\boxed{\text{sum}(N)}$ = $\underbrace{1 + 2 + 3 + 4 + 5 + 6 + \dots + N-1 + N}_{\text{sum}(N-1) + N}$



* How to write Recursive code?

1. Assumption : a) Define what your function is supposed to do.
b) Believe in it;
Believe that your function returns what you expect.
2. Main Logic : How do you solve the main problem in terms of sub-problem?
3. Base condition : What is the smallest sub-problem that your function can solve?

Q. Sum of nos from 1 to N.

1. Assumption:

def sum(N):



Believe that it returns sum
of $1+2+3+\dots+N$

$$\begin{cases} \text{sum}(3) \rightarrow 6 \\ \text{sum}(2) \rightarrow 3 \end{cases}$$

2. Main Logic

$$\text{sum}(N) = \text{sum}(N-1) + N$$

Code:

return sum(N-1) + N

3. Base condition

if $N=1$:
return 1

Code:

```
def sum(N):  
    if N==1:  
        return 1  
    return N + sum(N-1)
```

Q. $\text{fact}(N) = 1 * 2 * 3 * 4 * 5 \dots N-1 * N$

Write a code to compute factorial of N.

Recursive code:

1. Assumption:

def fact(N) :
 ↓
 Return $1 * 2 * 3 \dots N$

$$\begin{aligned}\text{fact}(3) &\rightarrow 6 \\ \text{fact}(4) &\rightarrow 24\end{aligned}$$

2. Main Logic

$$\text{fact}(N) = \text{fact}(N-1) * N$$

$$\underbrace{1 * 2 * 3 * 4 \dots N-1 * N}_{\text{fact}(N-1) * N}$$

return $N * \text{fact}(N-1)$

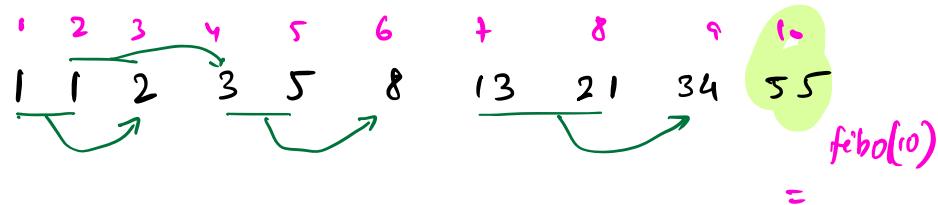
3. Base condition

```
if N==1 :  
    return 1
```

Code:

```
def fact(N):  
    if N==1:  
        return 1  
    return N * fact(N-1)
```

Q. Fibonacci series.



Golden ratio

$$\frac{55}{34}$$

$$\frac{34}{21}$$

$$\frac{21}{13}$$

11
1.618. ϕ

Recursive code

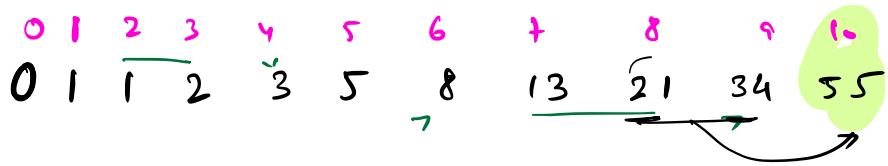
Y To return Nth Fibonacci number.

Assumption:

```
def fibo(N):
```

↓
Return Nth fibonacci number

Main logic:



$$\text{fib}(10) = \underline{34} + \underline{21} \\ = \underline{\underline{\text{fib}(9)}} + \underline{\underline{\text{fib}(8)}}$$

$$\text{fib}(n) = \text{fib}(n-1) + \text{fib}(n-2)$$

$$\text{return } \underline{\underline{\text{fib}(n-1)}} + \underline{\underline{\text{fib}(n-2)}}$$

* Base Condition:

```
if N==1 :  
    return 1  
if N==2 :  
    return 1
```

$$\text{fib}(3) = \text{fib}(\underline{\underline{2}}) + \text{fib}(\underline{\underline{1}})$$

↑ ↑
valid inputs

* fib(1) = fib(0) + fib(-1)

↑ ↑
invalid invalid

* fib(2) = fib(1) + fib(0)

↑ ↑
valid invalid

def fib(n):

```
{= if N==1 OR N==2 :  
    return 1
```

$$\text{return } \underline{\underline{\text{fib}(n-1)}} + \underline{\underline{\text{fib}(n-2)}}$$

** Internals of Recursion

e.g.

```
def add(a, b):  
    return a+b
```

```
def double(a):  
    return a*2
```

```
def square(a):  
    return a*a
```

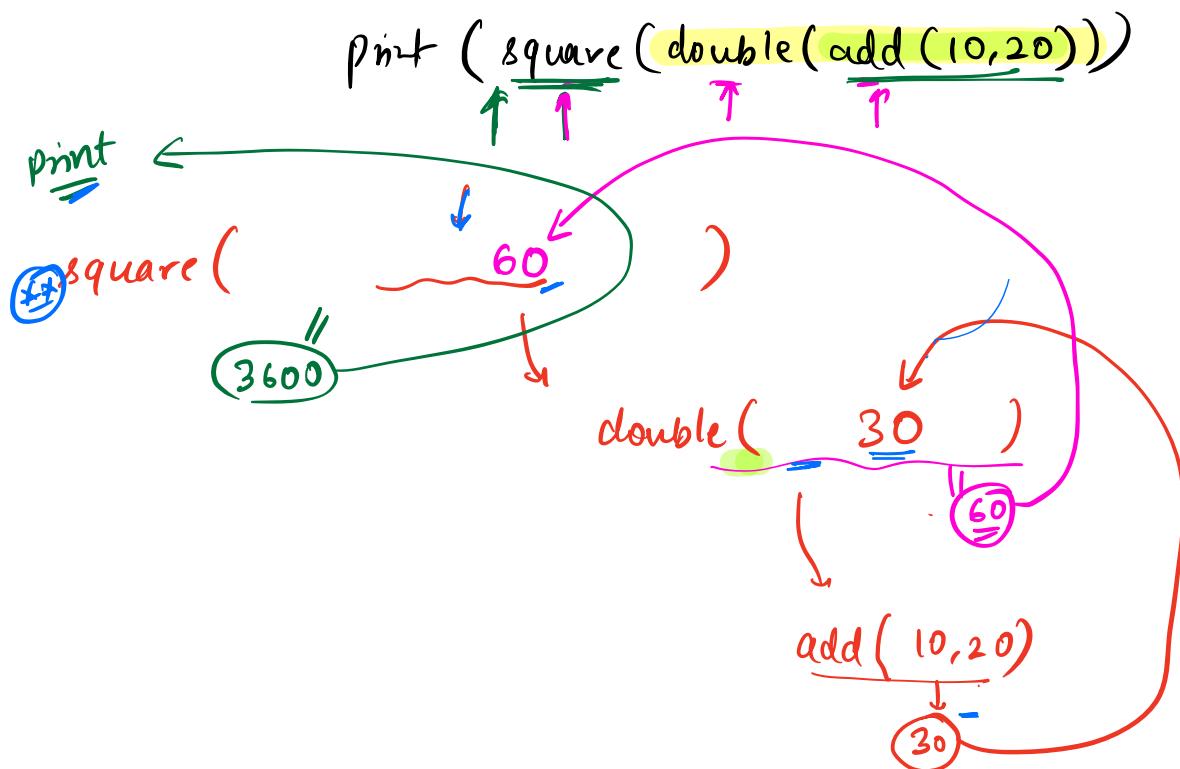
```
def main():
```

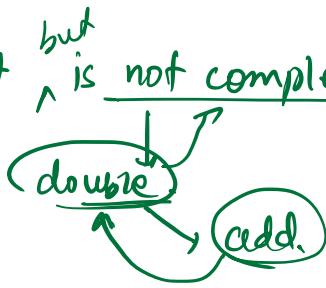
```
    → x = add(10, 20)  
    → y = double(x)  
    z = square(y)  
    print(z)
```

* first add. → double → square.
→ print

Function calls are happen sequentially

```
def main():
```

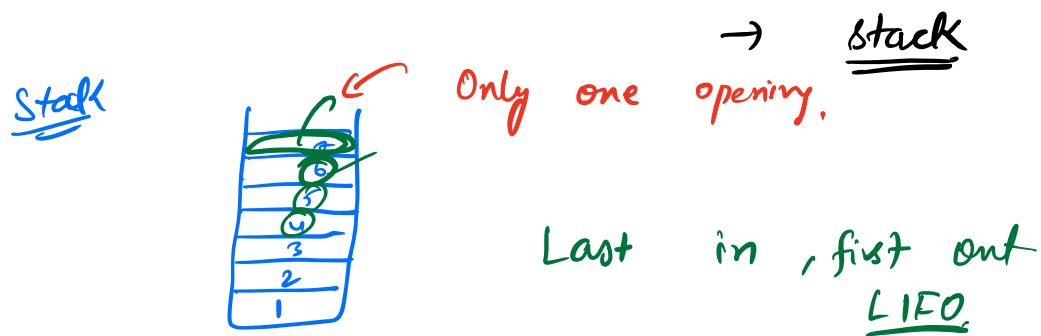


- * Square gets executed first ^{but} is not completed.


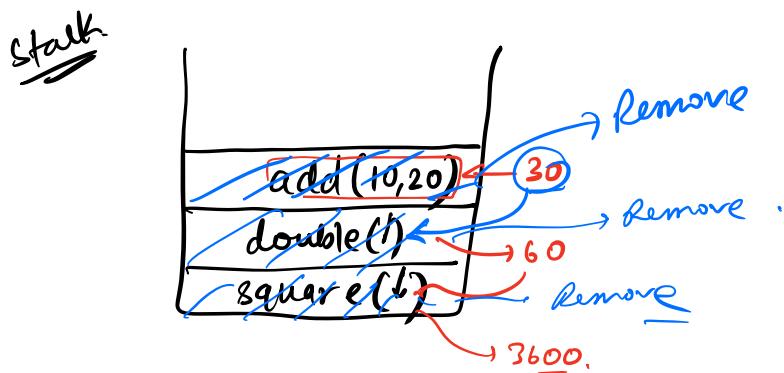
$f_1(f_2(f_3(f_4(f_5(f_6(f_7(\text{imp}))))))))$



- * Data structure to save function calls:

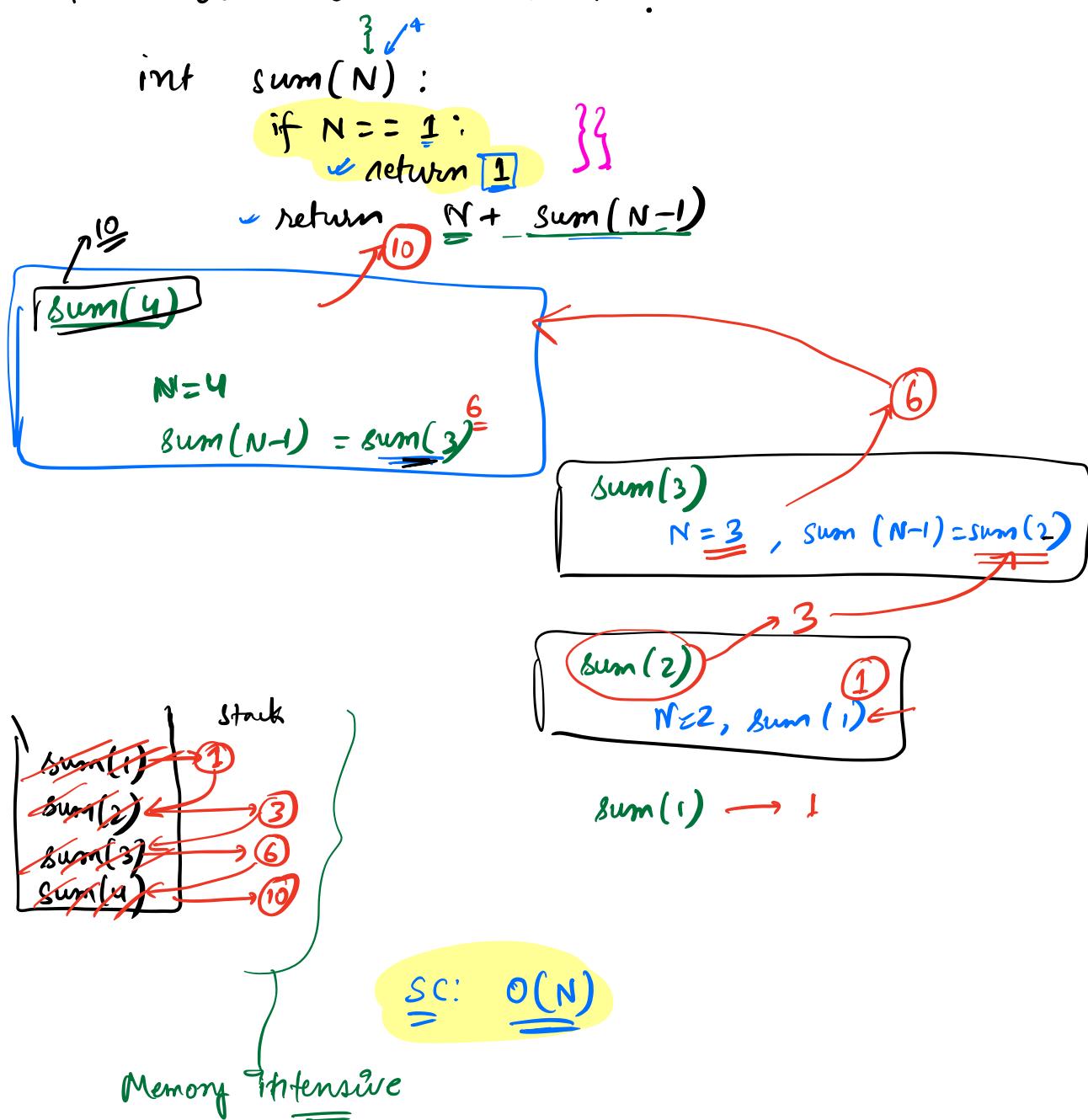


Push.
Pop



$\text{sum}(N)$:
 $\text{sum of } 1+2+3+\dots+N$

How does recursion work?



* what will happen if there is no base condition?

→ <Stack overflow> ←

Break till 10:15

Q. Given N , print all nos. from 1 to N .

Assumption:

* def printInc(N):

// print all nos. from 1 to N .

Main logic:

** { Print all nos. from 1 to $N-1$
 + Print Nth no.

* { printInc($N-1$)
 + print(N)

1
2
3
4
5
6
.
.
.
 N

Base condition:

if $N == 1$:
 print(1)

Code

def printInc(N):

 if $N == 1$:

 print(1)
 return

 + printInc($N-1$)
 - print(N)

$N=1$
printInc(0) X

1
2
3
4
5
.
.
.
 $N-1$
 N

H.W. Try seeing the output by building up the stack.

Q. Print all nos. from N to 1.
N, N-1, N-2, ... 3, 2, 1

Assumption

def printDec(N):
 ↓
 N, N-1, N-2, ... 1

Main Logic.

print(N)
printDec(N-1)



Code

def printDec(N):
 if N == 1
 print(1)
 return
 {
 print(N)
 printDec(N-1)
 }

Dry Run is a little difficult in recursive code.

N, N-1, ..., 3, 2, 1
print(N).
printDec(N-1)

Q: Given a string, check whether it is palindrome or not.

→

2 pointer approach.

Recursive approach:

Assumption:

def isPalindrome (string, start, end)

Main logic:

if $st[0] == st[-1]$:
isPalin($st[1:-1]$) \leftarrow // extra space.

if $string[\underline{start}] == string[\underline{end}]$:
return isPalindrome($string,$
 $=$ $start + 1,$
 $end - 1$)

else

return false

Base condition

if $\text{start} \geq \text{end}$:
return True.

$\text{start} == \text{end}$ ✓

Code :

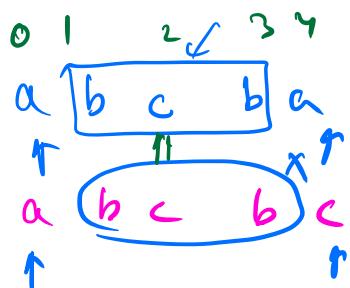
= def isPalin(start, end, string) <

if $\text{start} \geq \text{end}$:
return True

↑ return $\text{string}[\text{start}] == \text{string}[\text{end}]$
and $\text{isPalin}(\text{start}+1, \text{end}-1, \text{string})$

def checkalindrome(string) :

start end .
 isPalin($\underbrace{0}_{\text{↑}}, \underbrace{\text{len(string)} - 1}_{\text{l, len} - 2}, \text{string})$



("abcba", ②, ②)

Q. Given a directory structure, search for a file in it.

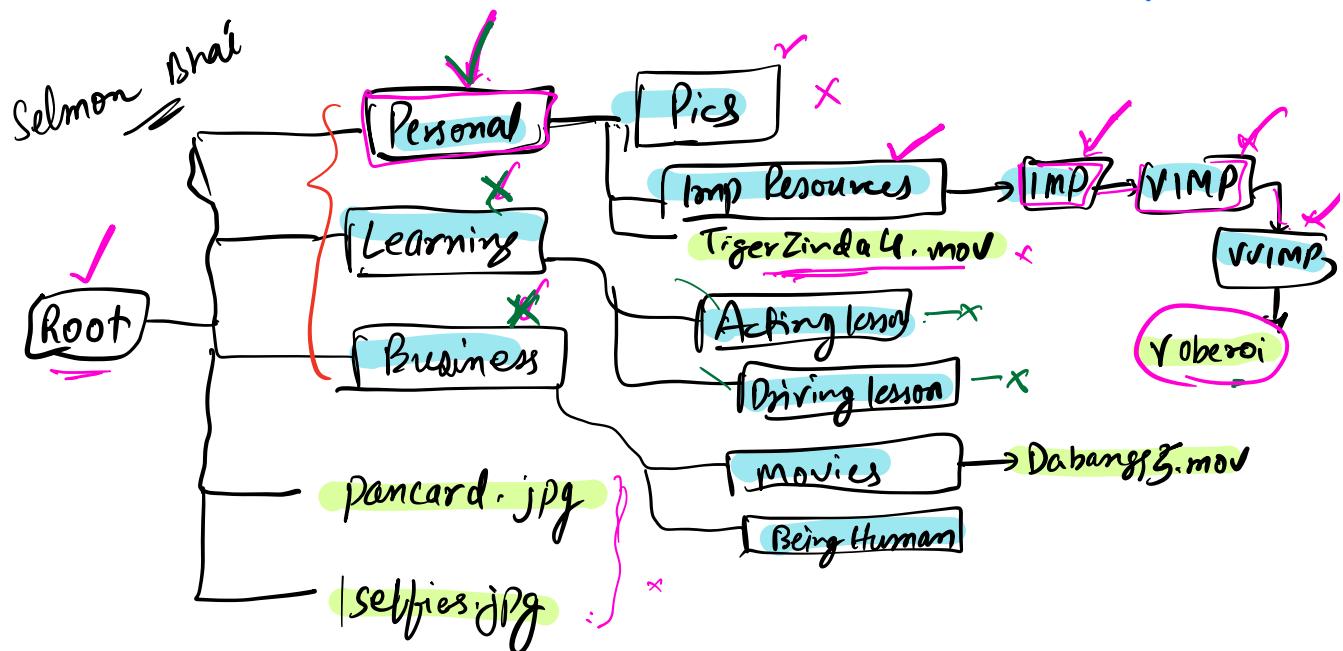
Inbuilt functions:

.. GetAllDirectories(Directory Name)

↳ Returns all immediate directory names

GetAllFiles(Directory Name)

↳ Returns all immediate files in this directory.



* GetAllDirectories(Root) → Personal, Learning, Business,

* GetAllFiles(Personal) → TigerZinda4.mov

Search File (root, Target File)

return whether a file is present or not.

{ Search File (root, "Aish") → No.

Search File (root, "TigerZindaH") → Yes.

Assumption:

Search File (DirectoryName, Target File)



return True / False.

bool search(DirectoryName, TargetFile):

dir_files = GetAllFiles(DirectoryName)

if TargetFile in dir_files:].
return True].

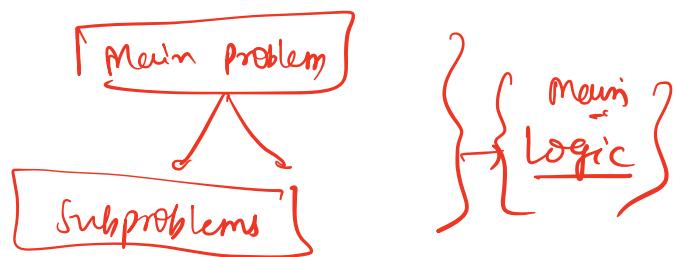
directories = GetAllDirectories(DirectoryName)

for dir in directories:
res = search(dir, TargetFile)
if res == True:
 return True

return False.

Personal
Business
Learn

O(N)

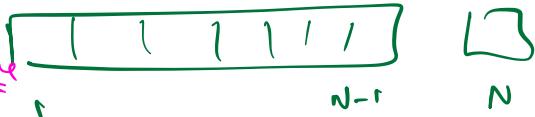


Palindrome



Sum of $A + B$.

this is
also a palindrome



xx

② "hello"

h - ello

ello

hello

{ hello
hell
hel
he
h.