

Arithmetic Coding: Probability to Bits

Model Output
Logits [batch, 512, 256]
Raw scores

Shannon's Theorem:
Optimal bits = $-\log_2 p(x)$
Arithmetic coding achieves
near-optimal compression
with < 2 bits overhead

Softmax
Convert to probabilities
 $\sum p(x) = 1$

Example for byte 147:
 $p(147 \mid \text{context}) = 0.089$

Cumulative Distribution
 $\text{CDF}[i] = \sum_{j=0 \text{ to } i} p(j)$
 $\text{CDF}[147] = 0.412$
 $\text{CDF}[148] = 0.501$

Interval Encoding
Byte 147 $\rightarrow [0.412, 0.501]$
Interval width = 0.089

Bits Required
 $-\log_2(0.089) \approx 3.49$ bits
vs 8 bits (fixed)
Savings: 4.51 bits!

torchac Library
Range coding implementation
16-bit precision
Near-optimal encoding

Compressed Bitstream
Average: 3.04 bits/byte
Theoretical: 2.63× compression