I. DBDACB

II.

1. P(X=2)=0.2*0.2*0.9+2*0.2*0.1*0.9=0.0680

2. $1-(1-F_X(z))(1-F_Y(z))$

3. 1/2

4. 5.6

5. 1/2

6. 5/6

III.

Solution：

a. It is known that

$P(\text{no virus}) = 0.98, P(\text{virus}) = 0.02, P(\text{positive}|\text{virus}) = 0.98,$

and $P(\text{positive}|\text{no virus}) = 1 - P(\text{negative}|\text{no virus}) = 1 - 0.95 = 0.05$ 2 分

$$P(\text{virus}|\text{positive}) = \frac{P(\text{positive}|\text{virus})P(virus)}{P(\text{positive}|\text{virus})P(virus)+P(\text{positive}|\text{no virus})P(\text{no virus})}$$

$$= \frac{0.98\times 0.02}{0.98\times 0.02+0.05\times 0.98} = 0.286$$ 4 分

b. $P(\text{virus}|\text{second positive}) = \frac{0.98\times 0.286}{0.98\times 0.286+0.05\times 0.714} = 0.887.$ 4分

IV.

Solution:

a. $P(AB) = P(B|A)(1-P(A')) = \frac{1}{8},$ 1 分

$P(AB') = (1-P(B|A))P(A) = \frac{3}{8},$ 1 分

$P(B) = \frac{P(AB)}{1-P(A'|B)} = \frac{3}{8},$

$P(A'B) = P(B) - P(AB) = \frac{1}{4},$ 1 分

$P(A'B') = P(B') - P(AB') = \frac{1}{4},$ 1 分

The joint probability table of X and Y is given by 2 分

P(x,y)		-1	1
x	-1	0.25	0.25
	1	0.375	0.125

$b. V[X] = E[X^2] - E[X]^2 = 1 - 0 = 1$ 1 分

$V[Y] = E[Y^2] - E[Y]^2 = 1 - 0.25^2 = \frac{15}{16}$ (0.9375) 1 分

$Cov(X,Y) = E[XY] - E[X]E[Y] = -0.25$ 1 分

$\rho = \frac{-1/4}{\sqrt{15}/4} = -\frac{\sqrt{15}}{15} = -0.2582$ 1 分

V.

Solution:

a. The joint pdf of X and Y is $f(x,y)=\begin{cases}4e^{-(x+4y)}, & x>0,\ y>0\\ 0, & \text{otherwise}\end{cases}$ 2 分

$$P(X<Y)=\int_0^{+\infty}\int_0^{y}4e^{-(x+4y)}\,dx\,dy=\frac{1}{5}.$$ 3 分

$b.\ P(Z\le 1)=\binom{3}{1}\cdot\frac{1}{5}\cdot\frac{16}{25}+\binom{3}{0}\frac{4^3}{5^3}=\frac{112}{125}=0.896.$ 3 分

c. $\mathrm{E}(Y-X+1)=E(Y)-E(X)+1=1/4-1+1=1/4$ 2 分

$\mathrm{V}(Y-X+1)=V(Y)+V(X)=\frac{1}{4^2}+1=\frac{17}{16}$ 1 分

$\sigma_{Y-X+1}=\frac{\sqrt{17}}{4}=1.0308$ 1 分

VI.

Solution：

a. Since $\int_0^{\theta}\frac{ax}{\theta^3}(\theta-x)dx=\frac{a}{\theta^3}\int_0^{\theta}x(\theta-x)\,dx=\frac{a}{6}=1,\ a=6.$ 3 分

b. $\mathrm{E(X)}=\int_0^{\theta}\frac{6x^2}{\theta^3}(\theta-x)dx=\frac{6}{\theta^3}\int_0^{\theta}x^2(\theta-x)\,dx=\frac{\theta}{2}.$ 2 分

Let $\frac{\theta}{2}=\bar{X}$ to find that $\hat{\theta}=2\bar{X}$. 2 分

c. $\mathrm{E(X^2)}=\int_0^{\theta}\frac{6x^3}{\theta^3}(\theta-x)dx=\frac{3}{10}\theta^2$ 2 分

$\mathrm{V(X)}=\mathrm{E(X^2)}-E(X)^2=\frac{\theta^2}{20}$ 1 分

$\mathrm{V}(\hat{\theta})=\mathrm{V}(2\bar{X})=4\mathrm{V}(\bar{X})=4\frac{V(X)}{n}=4\frac{\theta^2}{20n}=\frac{\theta^2}{5n}$ 2 分

VII.

Solution:

$a.\ \bar{X}\sim N(\mu,4).$ $\mathrm{P}(|\bar{X}-\mu|<2)=P\left(-1<\frac{\bar{X}-\mu}{2}<1\right)=0.6526.$ 4 分

b. The 95% CI has a width $2Z_{0.025}(\sigma/\sqrt{n})=2\times 1.96\left(\frac{10}{\sqrt{n}}\right)=\frac{39.2}{\sqrt{n}}.$ 4 分

Solve $\frac{39.2}{\sqrt{n}}\le 2$ to obtain $n\ge 19.6^2=384.16.$

Since n must be an integer, a sample size of 358 is required.

VIII

Solution:

a. $\bar{x} \pm t_{0.05,8} \cdot \frac{S}{\sqrt{9}} = 31 \pm 1.86 \cdot \frac{2.208}{3} = 31 \pm 1.369 = (29.631, 32.369)$ 4 分

b. Null hypothesis: $H_0: \mu = 30$

Alternative hypothesis: H_α: $\mu > 30$

The testing statistic value $z = \frac{\bar{x}-30}{2/3}$.

Rejection region: $z \geq z_{0.05} = 1.64$. 4 分

Substituting $\bar{x} = 31, z = 1.5$

The computed value does not fall in the rejection region. So we cannot reject H_0 and think that $\mu > 30$. 1 分

c. $\beta(31.0933) = \Phi\left(z_{0.05} + \frac{30-31.0933}{2/3}\right) = \Phi(0) = 0.5.$ 3 分