Упр. 3 $C_1 \|x\|_2 \le \|x\|_1 \le C_2 \|x\|_2$

$$\sum_{i=1}^{n} |x_i|^2 \le \left(\sum_{i=1}^{n} |x_i|\right)^2 \Rightarrow C_1 = 1.$$

Коши-Буняковский: $$\left(\sum_{i=1}^{n} |x_i|\right)^2 \le \left(\sum_{i=1}^{n} 1\right) \sum_{i=1}^{n} |x_i|^2 = n \sum_{i=1}^{n} |x_i|^2$$

$$\Rightarrow C_2 = \sqrt{n}.$$

Упр 4. а) $\sqrt{\sum_{i=1}^{m} x_i^2} \le \sqrt{\sum_{i=1}^{m} x_{max}^2} = \sqrt{m\, x_{max}^2} = \sqrt{m}\, \|x\|_\infty$; $x = \begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}$ $\|x\|_2 = \sqrt{m} \|x\|_\infty = \sqrt{m}$

б) $\|Ax\|_2 \ge \|Ax\|_\infty$; $\sqrt{a_{max}^2 + \dots} \ge |a_{max}|$

$$\|A\|_\infty = \sup_{x \ne 0} \frac{\|Ax\|_\infty}{\|x\|_\infty} \le \sup_{x \ne 0} \frac{\|Ax\|_\infty \sqrt{n}}{\|x\|_2} \le \sqrt{n} \sup_{x \ne 0} \frac{\|Ax\|_2}{\|x\|_2} = \sqrt{n} \|A\|_2$$

$$A = \begin{pmatrix} 1 & \cdots & 1 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{pmatrix} n \qquad \|A\|_\infty = \sqrt{n}\,\|A\|_2 = n.$$

$$A \overset{m}{\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix}} = \begin{pmatrix} n \\ 0 \\ \vdots \\ 0 \end{pmatrix}; \qquad A \begin{pmatrix} \frac{1}{\sqrt{n}} \\ \vdots \\ \frac{1}{\sqrt{n}} \end{pmatrix} = \begin{pmatrix} \sqrt{n} \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

Упр 5.

$$\|A\|_F^2 = \operatorname{tr} A^T A = \operatorname{tr} A A^T. \qquad \|UA\|_F^2 = \operatorname{tr} A^T U^T U A = \|A\|_F^2$$

$$\|AU\|_F^2 = \operatorname{tr} A U U^T A^T = \operatorname{tr} A A^T = \|A\|_F^2.$$