Упр. 3

1) $ay^3 + d = 0$ $\quad a = 1 \pm 10^{-3}$ $\quad d = 8 \pm 10^{-3}$

$y = \sqrt[3]{-\frac{d}{a}}$ $\quad y \approx -2$; $\quad \Delta y = \left| \frac{1}{3}\left(\frac{d}{a}\right)^{-\frac{2}{3}} \frac{\Delta d}{a} + \frac{1}{3}\left(\frac{d}{a}\right)^{-\frac{2}{3}} \cdot \frac{d}{a^2} \Delta a \right| \approx$

$\approx \frac{1}{12}\left(\frac{10^{-3}}{1} + \frac{8 \cdot 10^{-3}}{1}\right) \approx \frac{3}{4} 10^{-3} = 7{,}5 \cdot 10^{-4}$.

2) $u'(x) \approx \frac{u(x-2h) - 8u(x-h) + 8u(x+h) - u(x+2h)}{12h}$

$\varepsilon_{\text{округл}} = \frac{\Delta u + 8\Delta u + 8\Delta u + \Delta u}{12h} = \frac{3}{2}\frac{\Delta u}{h}$;

$\varepsilon_{\text{метода}} = \left| u'(x) - \frac{u(x-2h) - 8u(x-h) + 8u(x+h) - u(x+2h)}{12h} \right| =$

$$= \left| \frac{1}{12h} \cdot \frac{48}{120} h^5 \cdot M_5 \right| = \frac{M_5 \cdot h^4}{30}$$

$$\varepsilon = \frac{3}{2} \frac{\Delta u}{h} + \frac{M_5 h^4}{30}$$

$$\varepsilon' = \frac{4}{30} h^3 M_5 - \frac{3}{2} \frac{\Delta u}{h^2} = 0$$

$$h_{опт} = \sqrt[5]{\frac{45}{4} \frac{\Delta u}{M_5}}$$

4-й порядок аппроксимации.

Упр 4.

$$x^2 - 2x + 0{,}999999\,3751 = 0 \to x^2 - 2x + k = 0$$

$$x = \frac{2 \pm \sqrt{4 - 4k}}{2} = 1 \pm \sqrt{1-k} \;;\; k = 1 - \varepsilon \;;\; \sqrt{1-k} = 10^{-4}$$

$$\sqrt{\varepsilon} = 10^{-4}$$

$$\varepsilon = 10^{-8}$$

Ответ: 8 знаков

Упр 5

$5X_{n+1} - X_n = 4$ ; $X_{n+1} = \frac{1}{5}X_n + \frac{4}{5}$ ; $X_n = y_n + a$

$y_{n+1} + a = \frac{1}{5}(y_n + a) + \frac{4}{5}$ ; $y_{n+1} = \frac{1}{5}y_n + \left(-\frac{4}{5}a + \frac{4}{5}\right)$ $a = +1$

$y_n = \left(\frac{1}{5}\right)^n y_0$

$X_n = \left(\frac{1}{5}\right)^n (X_0 - 1) + 1$ $\Delta X_n = \left(\frac{1}{5}\right)^n \Delta X_0$

$\frac{\Delta X_n}{X_n} = \frac{\Delta X_0}{(X_0 - 1) + 5^n} \to 0$ при $n \to +\infty$.

Не при каких $X_0$ погрешность не растёт.