Task Group Allocation for Solving Multi-Load Agent Pickup and Delivery problem

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ABSTRACT

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KEYWORDS

Multi-load agent, Pickup and delivery problem, Task allocation

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1 INTRODUCTION

The multi-load agent pickup and delivery problem (MLAPD) is a variant of the pickup and delivery problem which is a well-known combinatorial optimization problem in the field of logistics and transportation. In the MLAPD problem, there are multiple agents, each of which can carry multiple tasks at the same time. The goal is to allocate tasks to agents to optimize the completion time of all tasks. The MLAPD problem is a NP-hard problem, and it is difficult to solve it optimally in polynomial time [1].

2 RELATED WORK

3 PROBLEM DEFINITION

In this section, we introduce the multi-load agent pickup and delivery problem, which we address in this paper. Before this, we first give the definitions of map, task, agent, comparative metrics and MLAPD problem.

Definition 3.1 (Map). A map $G = (V, E)$ is the , whose vertices V correspond to locations and edges E represent edges between adjoining locations. The shortest distance of the path between two locations v_i and v_j is denoted as $dis(v_i, v_j)$.

Definition 3.2 (Task). A task $\tau \in \Gamma$ is a tuple $< v_\tau^s, v_\tau^g, t_\tau^r, t_\tau^d, t_\tau^c >$, where v_τ^s and v_τ^g are the pickup location and delivery location, respectively. t_τ^r, t_τ^d and t_τ^c are the release time, deadline and completed time, respectively.

Definition 3.3 (Agent). An agent $a \in A$ is represented as a tuple $< v_a, c_a, \Gamma_a, S_a >$, where v_a is location, c_a represents the maximum capacity, Γ_a is the set of tasks, and S_a is the schedule of Γ_a. The $S_{\Gamma_a} =< v_1, v_2, ..., v_n >$, where v_i is a pickup or delivery location of task $\tau \in \Gamma_a$, and the cost of schedule S_{Γ_a} is denoted as $Cost(S_{\Gamma_a}) = \sum_{i=1}^{n-1} dis(v_i, v_{i+1})$.

A task $\tau \in \Gamma$ is released at t_τ^r. When system assigns τ to an agent a, a needs to reach v_τ^s before t_τ^d, and then deliver τ to v_τ^g while recording the completed time t_τ^c. If τ could not be completed before t_τ^d, then the task is considered as failed.

Definition 3.4 (Comparative metrics). In this paper, we demonstrate algorithmic superiority through three comparative metrics, service time (ST), makespan (MS) and completion ratio (CR). $ST = \sum_{\tau_i \in \Gamma} (t_{\tau_i}^c - t_{\tau_i}^r)$, is defined as the sum of differences between the completed times and the release times of all tasks. $MS = max\{t_\tau^c\}$ $(\forall \tau \in \Gamma)$ is time when last task has completed. $CR = \sum_{\tau_i \in \Gamma} X_{\tau_i} / |\Gamma|$, where X_{τ_i} is a decision variable, is completion ratio of Γ. If task τ_i is completed, then X_{τ_i} is 1; otherwise, it is 0.

Definition 3.5 (MLAPD). The multi-load agent pickup and delivery problem (MLAPD) is a tuple $P =< G, A, \Gamma >$, defined by a map G, a set of agents A, and a set of tasks Γ. Moreover, P amounts to finding the allocation of tasks to agents, which optimize three comparative indicators, (ST, MS, CR). Time is discretized into timesteps, and agents can only move to adjacent locations or wait at its current location in one timestep. Howerver, a conflict occurs when two agents are at the same location or pass through the same edge at the same time. The time required to avoid conflicts needs to be considered, since it will affect the comparative metrics. Therefore, we assue that agents can take θ time to avoid a conflict.

4 SOLUTION

In this study, Task-Group Allocation Algorithm (TGA) is developed to solve MLAPD problem efficiently. The TGA method first analyzes the likelihood of carpooling between each task, and then divide the set of tasks into groups via K-Capacity Hierarchical Clustering Algorithm (KCHC). Then, we allocate task groups to agents.

4.1 Carpooling Chance

The biggest difference between multi-load agent and single-load agent is that multi-load agent can load multiple items at the same time. This means that multi-load agent performing a task affects the completion time of the "co-passenger" tasks which are also executed to the same agent at same time.

Example 4.1 (Co-passenger tasks). As shown as in Figure 1, there is an agent, a, and three tasks, τ_1, τ_2 and τ_3. The agent a is located at pickup location of τ_1. The blue lines are the best schedule for agent a to complete τ_1 and τ_2. The red lines are the best schedule for agent a to complete τ_1 and τ_3. When we allocate τ_1 and τ_2 to the agent a, the agent a needs to take the blue paths to complete

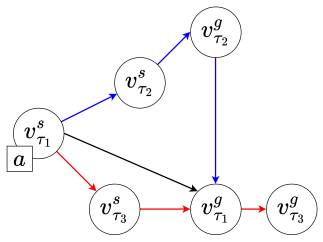


Figure 1: The impact of co-passenger tasks

τ_1 and τ_2 by greedy strategy, and it will not impact the completion time of τ_2 but lengthen the completed time of τ_1. Howerver, if we allocate τ_1 and τ_3 to the agent a, a needs to take the red paths to complete τ_1 and τ_3. While it also lengthens the completed time of τ_1, it is obvious that τ_3 has less of an effect on τ_1 as compared to τ_2. Therefore, we need to consider the impact of "co-passenger" tasks when allocating tasks to agents.

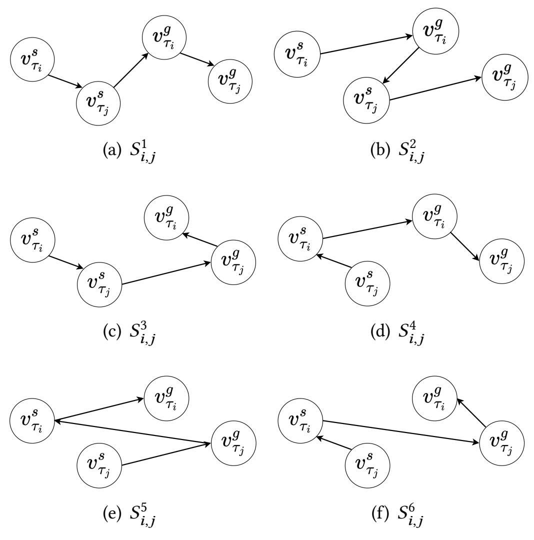


Figure 2: All possible schedules of two tasks

We first analyze the likelihood of carpooling between two tasks. As shown in Figure 2, there are 6 schedules to the completion of τ_i and τ_j, ignoring the location of agent. The schedules are $S^1_{i,j}$, $S^2_{i,j}$, $S^3_{i,j}$, $S^4_{i,j}$, $S^5_{i,j}$ and $S^6_{i,j}$. Therefore, we define the carpooling chance between two tasks τ_i and τ_j as follows.

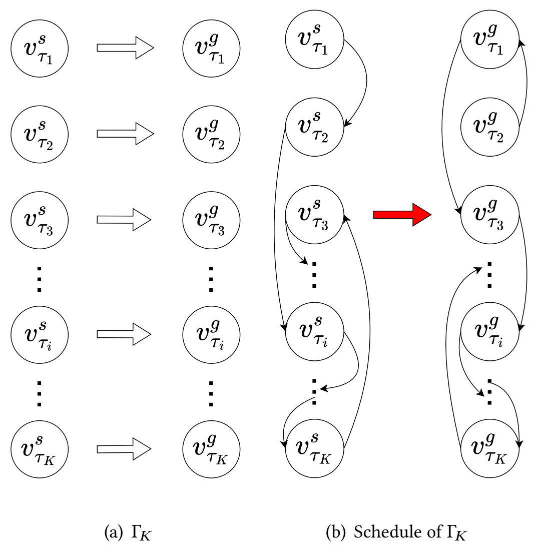
Definition 4.2 (Carpooling chance). The carpooling chance between two tasks τ_i and τ_j is defined as

$$\delta_{\tau_i,\tau_j} = min\{Cost(S^q_{i,j})\} - dis(v^g_{\tau_i}, v^s_{\tau_i}) - dis(v^g_{\tau_j}, v^s_{\tau_j}) \quad (1)$$

where $q \in [1, 6] \wedge q \in \mathbb{N}_+$. $\delta_{i,j}$ is difference between the best schedule of two tasks and the sum of the distances between the pickup and delivery locations of the two tasks. The small $\delta_{i,j}$ indicates that τ_i and τ_j being executed by the same agent has less impact on each other's completed time, which can efficiently optimize ST.

Howerver, in this paper, we focus on multi-load agents, which can carry multiple tasks at the same time, not just two agents. Therefore, we need to consider the carpooling chance between multiple tasks. In the beginning, we encounter an issue that we cannot find the best schedule to complete multiple tasks, which is a NP-hard problem. Therefore, we firtly find a novel schedule to complete there tasks. Then we evaluate cost of the schedule and proof it. Finally, we quantify the carpooling chance between multiple tasks.

As shown in Figure 3(a), there is a set of task Γ_K, which contains K tasks, τ_1, τ_2, ..., τ_K. The schedule $\dot{S}_{\Gamma_K}$ of K tasks is shown in Figure 3(b). This schedule starts at the pickup location of a task in Γ_K, then goes to the pickup location of other tasks in Γ_K, and finally to the delivery location of all tasks in Γ_K. More significantly, in $\dot{S}_{\Gamma_K}$, the path between two locations about two tasks in Γ_K is is part of paths of the best schedules of two tasks. Therefore, Theorem 4.3 is proposed to evaluate the cost of the schedule of K tasks, and prove it.



(a) Γ_K (b) Schedule of Γ_K

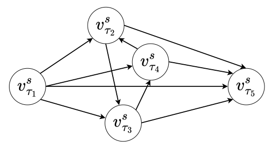
Figure 3: Schedule of K Tasks

THEOREM 4.3. *Given a set of tasks Γ_K, there exists a path shown in Figure 3(b) such that the following bound holds.*

$$Cost(\dot{S}_{\Gamma_K}) \leq 2 * \sum_{\tau_i \in \Gamma_K} dis(v^s_{\tau_i}, v^g_{\tau_i}) + 2*(K-1)\delta \quad (2)$$

where δ is the carpooling chance between two tasks.

PROOF. Firstly, we need to prove that there is a path to complete Γ_K. As shown in Figure 3(b), the schedule first connects the pickup locations of all tasks in Γ_K, then connects the delivery locations of all tasks in Γ_K. Focusing on the pickup locations set of Γ_K and


(a) Tournament

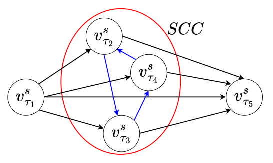

(b) SCC

Figure 4: The tournament and SCC

connecting the pickup locations, we can get a tournament [3], which is a directed graph with exactly one edge between each two vertices, in one of the two possible directions. As shown in Figure 4(a), there are 5 pickup locations, and a path between each two locations. Accroding to the definition of tournament, there is a Hamiltonian path in the tournament [3], that means there exists a path to connect all pickup locations of Γ_K. Similarly, there is a path to connect all delivery locations of Γ_K. Then, we need to find a path to connect the pickup locations and delivery locations. Therefore, we introduce the Strongly Connected Component (SCC) [2], which is a subgraph that is strongly connected. As shown in Figure 4(b), $v^s_{\tau_2}$, $v^s_{\tau_3}$ and $v^s_{\tau_4}$ combine to form a SCC where there is a path between each two locations. When we shrink the SCC to a single vertex and focus on the last SCC of the pickup locations and first SCC of the delivery locations, we can find two locations, $v^s_{\tau_i}$ and $v^g_{\tau_j}$, in two SCCs respectively. Therefore, there is a path between $v^s_{\tau_i}$ and $v^g_{\tau_j}$ to connect the pickup locations and delivery locations.

Secondly, we need to evaluate the cost of the path. In the beginning, we evaluate the cost of the path between the pickup locations. $dis(v^s_{\tau_j}, v^s_{\tau_i})$ is the distance between the pickup locations of τ_i and τ_j, and all possible schedules of two tasks are shown in Figure 2. If the best schedule of two tasks is $S^1_{i,j}$, we can get Equation 3.

$$\begin{aligned} Cost(S^1_{i,j}) &= dis(v^s_{\tau_j}, v^s_{\tau_i}) + dis(v^g_{\tau_i}, v^s_{\tau_j}) + dis(v^g_{\tau_j}, v^g_{\tau_i}) \\ &\geq dis(v^s_{\tau_j}, v^s_{\tau_i}) + dis(v^g_{\tau_j}, v^s_{\tau_j}) \qquad (3) \end{aligned}$$

$$dis(v^g_{\tau_i}, v^s_{\tau_i}) + \delta_{\tau_i,\tau_j} \geq dis(v^s_{\tau_j}, v^s_{\tau_i}) \qquad (4)$$

Then, based on this Definition 4.2, we can get Equation 4. Finally, we check all the possible schedules of two tasks in Figure 2, and we ensure the Equation 4 holds. Similarly, we can evaluate the cost of the path between the delivery locations by the same method and get Equation 5.

$$dis(v^g_{\tau_j}, v^s_{\tau_j}) + \delta_{i,j} \geq dis(v^g_{\tau_j}, v^g_{\tau_i}) \qquad (5)$$

Therefore, we can evaluate the cost of the path between the pickup locations and delivery locations. The cost of paths in pickup locations less than or equal to $\sum_{\tau_i \in \Gamma_K \setminus \tau_i} dis(v^s_{\tau_i}, v^g_{\tau_i}) + (K-1)\delta$, which τ_i is the last pickup location in $S_{\dot{\Gamma}_K}$. The cost of paths in delivery locations less than or equal to $\sum_{\tau_i \in \Gamma_K \setminus \tau_j} dis(v^s_{\tau_i}, v^g_{\tau_i}) + (K-1)\delta$, which τ_j is the first delivery location in $S_{\dot{\Gamma}_K}$. The cost of the path $dis(v^g_{\tau_j}, v^s_{\tau_i})$, which connects the pickup locations and delivery locations in $S_{\dot{\Gamma}_K}$, is less than or equal to $dis(v^g_{\tau_j}, v^s_{\tau_j}) + dis(v^g_{\tau_i}, v^s_{\tau_i})$. Therefore, the cost of the schedule is less than or equal to $2 * \sum_{\tau_i \in \Gamma_K} dis(v^s_{\tau_i}, v^g_{\tau_i}) + 2*(K-1)\delta$.

□

Therefore, we propose the Get Carpooling Chance (GCC) Algorithm between multiple tasks.

Algorithm 1 Get Carpooling Chance Algorithm

```
Input: Set of tasks $\Gamma_p$ and $\Gamma_q$
Output: Carpooling chance $\Delta_{\Gamma_p,\Gamma_q}$
1: Initialize $\Delta_{\Gamma_p,\Gamma_q} = 0$
2: $\Gamma_{p+q} = \Gamma_p \cup \Gamma_q$
3: Call Tarjan algorithm to get $SCC_s$ and $SCC_g$ of $\Gamma_{p+q}$
4: for $SCC^i_s \in SCC_s$ do
5:     for $\tau_i \in SCC^i_s$ do
6:         for $\tau_j \in SCC^i_s$ do
7:             $\Delta_{\Gamma_p,\Gamma_q} += \delta_{\tau_i,\tau_j}$
8:         end for
9:     end for
10: end for
```

4.2 K-Capacity Hierarchical Clustering

Based on Theorem 4.3,

The K-Capacity Hierarchical Clustering Algorithm (KCHC) is proposed to divide the set of tasks into groups. The KCHC algorithm is shown in Algorithm 2.

Algorithm 2 K-Capacity Hierarchical Clustering Algorithm

```
Input: Set of tasks $\Gamma$, the max capacity of agents $K$
Output: Set of task groups $\Pi$
1: Initialize $\Pi = \emptyset$
2: for $\tau_i \in \Gamma$ do
3:     Treat $\tau_i$ as a task group $\Gamma_i$, and add $\Gamma_i$ into $\Pi$
4:     for $\Gamma_j \in \Pi$ do
5:         GetCarpoolingChance($\Gamma_i$, $\Gamma_j$)
6:     end for
7: end for
8: while $\Pi$ has changed do
9:     Select minimum carpooling chance $\Delta_{\Gamma_p,\Gamma_q}$ from $\Pi$
10:    Merge $\Gamma_p$ and $\Gamma_q$ into $\Gamma_{p+q}$
11:    Remove $\Gamma_p$ and $\Gamma_q$ from $\Pi$ and add $\Gamma_{p+q}$ into $\Pi$
12:    for $\Gamma_j \in \Pi$ do
13:        if $|\Gamma_j| + |\Gamma_{p+q}| \leq K$ then
14:            GetCarpoolingChance($\Gamma_{p+q}$, $\Gamma_j$)
15:        end if
16:    end for
17: end while
18: return $\Pi$
```

The pseduo-code of K-Capacity Hierarchical Clustering Algorithm is shown in Algorithm 2, where the input is a set of tasks Γ and the max capacity of agents K, and the optput is a set of task groups Π.

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