Appendix

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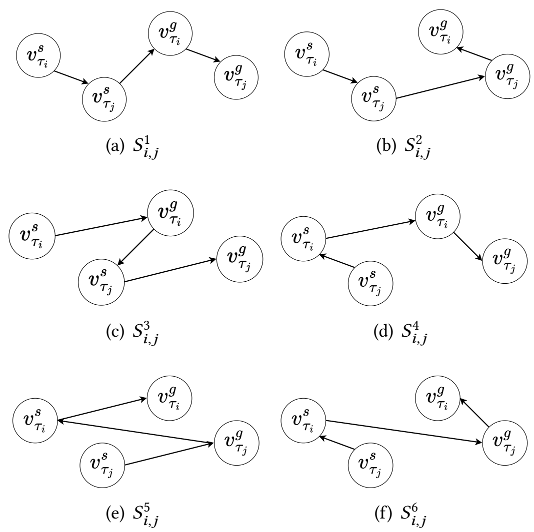
1 COST OF SCHEDULE

In this section, we will proof the cost of the schedule of multiple tasks in detail.



(a) $S^1_{i,j}$ (b) $S^2_{i,j}$ (c) $S^3_{i,j}$ (d) $S^4_{i,j}$ (e) $S^5_{i,j}$ (f) $S^6_{i,j}$

Figure 1: All possible schedules of two tasks

As shown in Figure 1, there are six possible schedules for two tasks. It means the shortest path between two tasks is the minimum cost of these six schedules. Then, we introduce the carpooling scores between two tasks.

Definition 1.1 (Carpooling Scores). The carpooling scores between two tasks τ_i and τ_j is defined as

$$\delta_{\tau_i,\tau_j} = min\{Cost(S^q_{i,j})\} - dis(v^g_{\tau_i}, v^s_{\tau_i}) - dis(v^g_{\tau_j}, v^s_{\tau_j}) \tag{1}$$

where $q \in [1,6] \wedge q \in \mathbb{N}_+$. $\delta_{i,j}$ is difference between the best schedule of two tasks and the sum of the distances between the pickup and delivery locations of the two tasks.

There exits a pickup-to-pickup path and a delivery-to-delivery path in all possible schedules of two tasks.

Proc. of the 24th International Conference on Autonomous Agents and Multiagent Systems (AAMAS 2025), A. El Fallah Seghrouchni, Y. Vorobeychik, S. Das, A. Nowe (eds.), May 19 – 23, 2025, Detroit, Michigan, USA.

If the shortest schedule is $S^1_{i,j}$, in figure 1(a), the cost of the pickup-to-pickup path is $dis(v^s_{\tau_j}, v^s_{\tau_i})$, and the cost of the delivery-to-delivery path is $dis(v^g_{\tau_j}, v^g_{\tau_i})$. The cost of the schedule $S^1_{i,j}$ is $dis(v^s_{\tau_j}, v^s_{\tau_i}) + dis(v^g_{\tau_i}, v^s_{\tau_j}) + dis(v^g_{\tau_j}, v^g_{\tau_i})$.

$$\begin{aligned}
Cost(S^1_{i,j}) &= dis(v^s_{\tau_j}, v^s_{\tau_i}) + dis(v^g_{\tau_i}, v^s_{\tau_j}) + dis(v^g_{\tau_j}, v^g_{\tau_i}) \\
&\geq dis(v^s_{\tau_j}, v^s_{\tau_i}) + dis(v^g_{\tau_j}, v^s_{\tau_j}) \\
dis(v^g_{\tau_i}, v^s_{\tau_i}) + \delta_{\tau_i,\tau_j} &\geq dis(v^s_{\tau_j}, v^s_{\tau_i}) \qquad (2) \\
Cost(S^1_{i,j}) &= dis(v^s_{\tau_j}, v^s_{\tau_i}) + dis(v^g_{\tau_i}, v^s_{\tau_j}) + dis(v^g_{\tau_j}, v^g_{\tau_i}) \\
&\geq dis(v^g_{\tau_j}, v^g_{\tau_i}) + dis(v^g_{\tau_i}, v^s_{\tau_i}) \\
dis(v^g_{\tau_j}, v^s_{\tau_j}) + \delta_{\tau_i,\tau_j} &\geq dis(v^g_{\tau_j}, v^g_{\tau_i}) \qquad (3)
\end{aligned}$$

If the shortest schedule is $S^2_{i,j}$, in figure 1(b), the cost of the pickup-to-pickup path is $dis(v^s_{\tau_j}, v^s_{\tau_i})$, and the cost of the delivery-to-delivery path is $dis(v^g_{\tau_i}, v^g_{\tau_j})$. The cost of the schedule $S^2_{i,j}$ is $dis(v^s_{\tau_j}, v^s_{\tau_i}) + dis(v^g_{\tau_j}, v^s_{\tau_j}) + dis(v^g_{\tau_i}, v^g_{\tau_j})$.

$$\begin{aligned}
Cost(S^2_{i,j}) &= dis(v^s_{\tau_j}, v^s_{\tau_i}) + dis(v^g_{\tau_j}, v^s_{\tau_j}) + dis(v^g_{\tau_i}, v^g_{\tau_j}) \\
&\geq dis(v^s_{\tau_j}, v^s_{\tau_i}) + dis(v^g_{\tau_j}, v^s_{\tau_j}) \\
dis(v^g_{\tau_i}, v^s_{\tau_i}) + \delta_{\tau_i,\tau_j} &\geq dis(v^s_{\tau_j}, v^s_{\tau_i}) \qquad (4) \\
Cost(S^2_{i,j}) &= dis(v^s_{\tau_j}, v^s_{\tau_i}) + dis(v^g_{\tau_j}, v^s_{\tau_j}) + dis(v^g_{\tau_i}, v^g_{\tau_j}) \\
&\geq dis(v^g_{\tau_i}, v^g_{\tau_j}) + dis(v^g_{\tau_j}, v^s_{\tau_j}) \\
dis(v^g_{\tau_i}, v^s_{\tau_i}) + \delta_{\tau_i,\tau_j} &\geq dis(v^g_{\tau_i}, v^g_{\tau_j}) \qquad (5)
\end{aligned}$$

If the shortest schedule is $S^3_{i,j}$, in figure 1(c), the cost of the pickup-to-pickup path is $dis(v^s_{\tau_i}, v^g_{\tau_j}) = dis(v^g_{\tau_i}, v^s_{\tau_i}) + dis(v^s_{\tau_j}, v^g_{\tau_i})$, and the cost of the delivery-to-delivery path is $dis(v^g_{\tau_j}, v^s_{\tau_i}) = dis(v^s_{\tau_j}, v^g_{\tau_i}) + dis(v^g_{\tau_j}, v^s_{\tau_j})$.

$$\begin{aligned}
Cost(S^3_{i,j}) &= dis(v^g_{\tau_i}, v^s_{\tau_i}) + dis(v^s_{\tau_j}, v^g_{\tau_i}) + dis(v^g_{\tau_j}, v^s_{\tau_j}) \\
&= dis(v^s_{\tau_j}, v^s_{\tau_i}) + dis(v^g_{\tau_j}, v^s_{\tau_j}) \\
dis(v^g_{\tau_i}, v^s_{\tau_i}) + \delta_{\tau_i,\tau_j} &= dis(v^s_{\tau_j}, v^s_{\tau_i}) \qquad (6) \\
Cost(S^3_{i,j}) &= dis(v^g_{\tau_i}, v^s_{\tau_i}) + dis(v^s_{\tau_j}, v^g_{\tau_i}) + dis(v^g_{\tau_j}, v^s_{\tau_j}) \\
&= dis(v^g_{\tau_j}, v^g_{\tau_i}) + dis(v^g_{\tau_i}, v^s_{\tau_i}) \\
dis(v^g_{\tau_j}, v^s_{\tau_j}) + \delta_{\tau_i,\tau_j} &= dis(v^g_{\tau_j}, v^g_{\tau_i}) \qquad (7)
\end{aligned}$$

$S^4_{i,j}$, $S^5_{i,j}$, and $S^6_{i,j}$ are similar to $S^2_{i,j}$, $S^3_{i,j}$, and $S^1_{i,j}$, respectively. Because these schedules are symmetric, we can get the same results as above.

In summary, when we observe these equations 2, 3, 4, 5, 6, and 7, we can draw the following two conclusions:

Figure 2: The schedule of multiple tasks

$$dis(v^g_{\tau_i}, v^s_{\tau_i}) + \delta_{\tau_i,\tau_j} \geq dis(v^s_{\tau_j}, v^s_{\tau_i}) \tag{8}$$

$$dis(v^g_{\tau_j}, v^s_{\tau_j}) + \delta_{\tau_i,\tau_j} \geq dis(v^g_{\tau_j}, v^g_{\tau_i}) \tag{9}$$

where δ_{τ_i,τ_j} is the carpooling scores between two tasks τ_i and τ_j. Equations 8 presents the upper bound on the cost of pickup-to-pickup path of two tasks, and equations 9 presents the upper bound on the cost of delivery-to-delivery path of two tasks.

Then, we can get the cost of the schedule of multiple tasks. In this paper, we introduce a novel method to schedule multiple tasks.