SLOŽITOST

$f, g : \mathbb{N} \to \mathbb{R}^+$

$f \in O(g) \iff \exists c > 0 \quad \exists n_0 \quad \forall n > n_0 : f(n) \leq c \cdot g(n)$

$f \in \Omega(g) \iff \exists c > 0 \quad \exists n_0 \quad \forall n > n_0 : f(n) \geq c \cdot g(n)$

$f \in \Theta(g) \iff f \in O(g) \wedge f \in \Omega(g)$

1. $n^2 \in O(n^3)$

   ANO: $c = 1 \quad \forall n \geq 1 \quad n^2 \leq n^3 \iff 1 \leq n$

2. $n^3 \in O(n^2)$

   NE: $\forall c \quad \forall n_0 \quad \exists n > n_0 : f(n) > c \cdot g(n) \to n^3 > c \cdot n^2$

   $n > c$

   $\Rightarrow$ vezmu lib. $n > \max(c, n_0)$

3. $f \in O(g) \Rightarrow g \in O(f)$

   NE: $f(n) = n^2$, $g(n) = n^3$

4. $f \in O(g) \Rightarrow g \in \Omega(f)$

   víme: $f \in O(g) \iff \exists c \quad \exists n_0 \quad \forall n > n_0 \quad f(n) \leq c\, g(n)$

   chceme: $g \in \Omega(f) \iff \exists d \quad \exists n_0 \quad \forall n > n_0 \quad g(n) \geq d\, f(n)$

   $f(n) \leq \frac{1}{d} g(n)$

   ANO: $d = \frac{1}{c}$

5. $f \in \Theta(g) \Rightarrow g \in O(f)$

   ANO: $f \in \Omega(g) \iff \exists c > 0 \quad \exists n_0 \quad \forall n > n_0 : f(n) \geq c\, g(n)$

   $g(n) \leq \frac{1}{c} f(n)$

   $d = \frac{1}{c}$: $\exists d > 0 \quad \exists n_0 \quad \forall n > n_0 : g(n) \leq d\, f(n) \iff g \in O(f)$

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6. $f \in O(g) \Rightarrow \frac{1}{f} \in O(g)$

NE: $f(n) = \frac{1}{n}$, $g(n) = \frac{1}{n}$ zjevně $f \in O(g)$

$\frac{1}{f}(n) = n$, ukážeme $n \notin O(\frac{1}{n}) \Leftrightarrow \forall c, n_0 \; \exists n > n_0 : n > c \cdot \frac{1}{n}$

$n > \frac{c}{n} \Leftrightarrow n^2 > c \Leftrightarrow n > \sqrt{c}$ → volím $n > \max(\sqrt{c}, n_0)$

7. $f \in O(g) \Rightarrow \frac{1}{f} \in O(\frac{1}{g})$

NE: $f(n) = n$, $g(n) = n^2$ zjevně $f \in O(g)$

$\frac{1}{f}(n) = \frac{1}{n}$, $\frac{1}{g}(n) = \frac{1}{n^2}$, ukážeme $\frac{1}{n} \notin O(\frac{1}{n^2}) \Leftrightarrow \forall c, n_0 \; \exists n > n_0 : \frac{1}{n} > c \cdot \frac{1}{n^2}$

$\frac{1}{n} > c \frac{1}{n^2} \Leftrightarrow n > c$ → volím $n > \max(c, n_0)$

8. $f \in O(g) \vee g \in O(f)$

NE: $f(n) = \begin{cases} 1 & n \text{ sudé} \\ n & n \text{ liché} \end{cases}$ $g(n) = \begin{cases} n & n \text{ sudé} \\ 1 & n \text{ liché} \end{cases}$

ukážeme $f \notin O(g) \Leftrightarrow \forall c, n_0 \; \exists n > n_0 : f(n) > c \cdot g(n)$

liché n: $f(n) = n$, $g(n) = 1$ $n > c \cdot 1$

→ vezmu liché $n > \max(c, n_0)$

9. $f_1 \in O(g), f_2 \in O(g) \Rightarrow f_1 + f_2 \in O(g)$

ANO: $f_1 \in O(g) \Leftrightarrow \exists c_1 > 0 \; \forall n > n_1 : f_1(n) \leq c_1 g(n)$

$f_2 \in O(g) \Leftrightarrow \exists c_2 > 0 \; \forall n > n_2 : f_2(n) \leq c_2 g(n)$

pro $c = c_1 + c_2$, $n_0 = \max(n_1, n_2)$

$\forall n > n_0 : f_1(n) + f_2(n) \leq c_1 g(n) + c_2 g(n) = c \cdot g(n)$