

National University of Computer & Emerging Sciences

Homework # 18: Tensor Unfolding, Mode-n Product, and Inverses

Section 1: Tensor Unfolding

Question 1: Given the tensor $\mathcal{A} \in \mathbb{R}^{2 \times 3 \times 2}$:

$$\mathcal{A}(:, :, 1) = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, \quad \mathcal{A}(:, :, 2) = \begin{bmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}.$$

1. Unfold \mathcal{A} along **mode-1**, and write the resulting matrix $A_{(1)}$.
2. For the same tensor \mathcal{A} , unfold it along **mode-2**, and write the resulting matrix $A_{(2)}$.
3. Unfold the tensor \mathcal{A} along **mode-3**, and write the resulting matrix $A_{(3)}$.

Question 2: For a tensor $\mathcal{B} \in \mathbb{R}^{4 \times 3 \times 5}$, write the dimensions of $B_{(1)}$, $B_{(2)}$, and $B_{(3)}$.

Section 2: Mode-n Product

Question 3: Mode-1 Product Given $\mathcal{A} \in \mathbb{R}^{2 \times 3 \times 2}$ as in Question # 1 and a matrix $M \in \mathbb{R}^{4 \times 2}$:

$$M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

Compute $\mathcal{B} = \mathcal{A} \times_1 M$, and provide the slices $\mathcal{B}(:, :, 1)$ and $\mathcal{B}(:, :, 2)$.

Question 4: Mode-2 Product For the same tensor \mathcal{A} , compute $\mathcal{B} = \mathcal{A} \times_2 N$, where:

$$N = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix}.$$

Question 5: Mode-3 Product For the tensor \mathcal{A} and matrix $P \in \mathbb{R}^{3 \times 2}$:

$$P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix},$$

compute $\mathcal{B} = \mathcal{A} \times_3 P$, and describe the resulting tensor's dimensions.

Section 3: Inverse of Mode-n Product

Question 6: Inverse of Mode-2 Product Let $\mathcal{A} \in \mathbb{R}^{2 \times 3 \times 2}$ as in Question # 1 and $N \in \mathbb{R}^{3 \times 3}$ as in Question # 4. Show that:

$$(\mathcal{A} \times_2 N) \times_2 N^{-1} = \mathcal{A}.$$