

National University of Computer & Emerging Sciences

Homework # 17: Affine & Convex Combinations

1. Let

$$\mathbf{v}_1 = \begin{pmatrix} -1 \\ -3 \\ 4 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 0 \\ -3 \\ 1 \end{pmatrix}.$$

Find

- (i) $\text{span}\{\mathbf{v}_1\}$, $\text{aff}\{\mathbf{v}_1\}$ and $\text{conv}\{\mathbf{v}_1\}$
- (ii) $\text{span}\{\mathbf{v}_1, \mathbf{v}_2\}$, $\text{aff}\{\mathbf{v}_1, \mathbf{v}_2\}$ and $\text{conv}\{\mathbf{v}_1, \mathbf{v}_2\}$

Geometrically explain all these spans, affine hull and convex hull. Also construct the geometric relationship between these.

2. Check affine independence, if so find barycentric coordinates of \mathbf{p} with respect to given affinely independent set, if set is affinely dependent give dependence relation

$$(i) \quad \mathbf{v}_1 = \begin{pmatrix} 1 \\ -1 \\ 2 \\ 1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 2 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 1 \\ 2 \\ -2 \\ 0 \end{pmatrix}, \quad \mathbf{p} = \begin{pmatrix} 5 \\ 4 \\ -2 \\ 2 \end{pmatrix}$$

$$(ii) \quad \mathbf{v}_1 = \begin{pmatrix} 0 \\ 1 \\ -2 \\ 1 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 2 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 1 \\ 4 \\ -6 \\ 3 \end{pmatrix}, \quad \mathbf{p} = \begin{pmatrix} -1 \\ 1 \\ -4 \\ 0 \end{pmatrix}$$

$$(iii) \quad \mathbf{v}_1 = \begin{pmatrix} 3 \\ -3 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} 0 \\ 6 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \quad \mathbf{p} = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$