Day: Muneeb Lone DS-B Date: / /
23i-2623

Assignment #8

Q1: $B = \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} \right\}$

1. $v = \begin{pmatrix} 4 \\ -3 \\ 7 \end{pmatrix}$

$\boxed{\text{Coordinate Vector} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1/2 \\ -3/2 \\ -5 \end{pmatrix}}$ ← Ans

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 4 \\ 1 & -1 & 1 & -3 \\ 1 & -1 & -1 & 7 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 4 \\ 0 & -2 & 2 & -7 \\ 0 & -2 & 0 & 3 \end{array}\right] \begin{array}{l} \\ R2-R1 \\ R3-R1 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 4 \\ 0 & -2 & 2 & -7 \\ 0 & 0 & -2 & 10 \end{array}\right] \begin{array}{l} \\ \\ R3-R2 \end{array}$$

$$\begin{array}{c} \begin{array}{ccc} x_1 & x_2 & x_3 \end{array} \\ \left[\begin{array}{ccc|c} 1 & 1 & -1 & 4 \\ 0 & -2 & 2 & -7 \\ 0 & 0 & -2 & 10 \end{array}\right] \end{array}$$

$-2x_3 = 10$ —(1)

$-2x_2 + 2x_3 = -7$ —(2)

$x_1 + x_2 - x_3 = 4$ —(3)

$x_3 = -5$

$-2x_2 + 2(-5) = -7$

$-2x_2 - 10 = -7$

$x_2 = -\frac{3}{2}$

$x_1 - \frac{3}{2} + 5 = 4$

$x_1 = 4 - 5 + \frac{3}{2} = \frac{1}{2}$

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2. $[u]_\beta = \begin{pmatrix} 5 \\ -4 \\ 1 \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$

$u = (\beta)([u]_\beta) = 5\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + -4\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} + 1\begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix}$

$u = \begin{bmatrix} 5 \\ 5 \\ 5 \end{bmatrix} + \begin{bmatrix} -4 \\ 4 \\ 4 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 0 \\ 10 \\ 8 \end{bmatrix}$ ← Ans

Q2: Let $C = \{1 + 2t + t^2, 3 - 3t, -t + 5t^2\}$

1. $x^0 = c_1 + c_2 = 0$
$x^1 = c_1 + c_2 + c_3 = 0$
$x^2 = c_1 + c_3 = 0$

$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 3 & -3 & 0 & 0 \\ 0 & -1 & 5 & 0 \end{array}\right]$

$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & -9 & -3 & 0 \\ 0 & -1 & 5 & 0 \end{array}\right]$ R2 − 3R1

$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 3 & 1 & 0 \\ 0 & 0 & 16 & 0 \end{array}\right]$ −R2/3, 3R3 + R2

As the system is consistent, we can deduce it is linearly independent and hence is a valid basis.
Dim C = n+1 = 3
Dim C = highest degree +1.

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2. $[U]_C = \begin{pmatrix} 5 \\ -4 \\ 1 \end{pmatrix}$

$U = \begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix} \begin{bmatrix} 1 + 2t + t^2 \\ 3 - 3t \\ -t + 5t^2 \end{bmatrix}$

$U = (5 + 10t + 5t^2) + (-12 + 12t) + (-t + 5t^2)$

$U = -7 + 21t + 10t^2$

Q3: $W = \left\{ \begin{pmatrix} a & b \\ b & a \end{pmatrix} \middle| \; a, b \in \mathbb{R} \right\}$ of $M_{2\times2}$

1. $D = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right\}$

- D is linearly independent as M_1 and M_2 can clearly not be written as scalar multiples of each other.
- $a = 1$, $b = 0$ → This satisfies the condition $a, b \in \mathbb{R}$
- $$c_1 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + c_2 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$c_1 + c_2 = 0$ and $c_1 = c_2$

D is a basis of W

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2. $E = \left\{ \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}, \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \right\}$ ~~is~~ ~~is~~ Muneeb

$$c_1 \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} + c_2 \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$\left[\begin{array}{cc|c} 1 & -2 & 0 \\ 2 & 1 & 0 \\ 2 & 1 & 0 \\ 1 & -2 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{array}\right] \begin{array}{l} \\ R2 - R3 \\ R3 - 2R1 \\ R4 - R1 \end{array}$$

$$\begin{array}{c} \begin{array}{cc} c_1 & c_2 \end{array} \\ \left[\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \end{array} \quad \text{Swap } R2, R3$$

$5c_2 = 0$ — (1)

$c_1 - 2c_2 = 0$ — (2)

$c_2 = 0$

$c_1 = 0$

E is linearly independent and hence a basis for W

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3. $V = \begin{pmatrix} 5 & 3 \\ 3 & 5 \end{pmatrix}$ w.r.t $\mathbb{D}$ and $\mathbb{E}$

W.R.T D: $\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \\ 1 & 0 & 5 \end{array}\right]$

$\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \end{array}\right] R4-R1$

$\begin{array}{ccc} c_1 & c_2 & \\ \end{array}$
$\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] R3-R2$

~~Coordinate Vector~~

$c_1 = 5, \quad c_2 = 3$

Coordinate Vector $= \begin{bmatrix} 5 \\ 3 \end{bmatrix} = [V]_D$

w.r.t E:

$\left[\begin{array}{cc|c} 1 & -2 & 5 \\ 2 & 1 & 3 \\ 2 & 1 & 3 \\ 1 & -2 & 5 \end{array}\right]$

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$$\left[\begin{array}{cc|c} 1 & -2 & 5 \\ 0 & 0 & 0 \\ 0 & 5 & -7 \\ 0 & 0 & 0 \end{array}\right] \begin{array}{l} \\ R2-R3 \\ R2-2R1 \\ R4-R1 \end{array}$$

$$\left[\begin{array}{cc|c} 1 & -2 & 5 \\ 0 & 5 & -7 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right] \text{ Swap } R2, R3$$

$5c_2 = -7$ — (1)

$c_1 - 2c_2 = 5$ — (2)

$c_2 = \frac{-7}{5}$

$c_1 = 5 + \frac{14}{5} = \frac{39}{5}$

Coordinate Vector $= \begin{bmatrix} \frac{39}{5} \\ \frac{-7}{5} \end{bmatrix}$

4. $[u]_E = \begin{pmatrix} 5 \\ -4 \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix}$

$u = c_1 \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} + c_2 \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}$

$u = 5 \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix} - 4 \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix}$

$u = \begin{pmatrix} 5 & 10 \\ 10 & 5 \end{pmatrix} + \begin{pmatrix} 8 & -4 \\ -4 & 8 \end{pmatrix} = \begin{pmatrix} 13 & 6 \\ 6 & 13 \end{pmatrix}$ Ans.

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