

Muneeb Lone  
231-2623  
DS-B

Date: 

HW #18



$$Q_1: A(:, :, 1) = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}, A(:, :, 2) = \begin{bmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}$$

• Along mode 1:

$$A_{(1)} = \begin{bmatrix} 1 & 2 & 3 & 7 & 8 & 9 \\ 4 & 5 & 6 & 10 & 11 & 12 \end{bmatrix}$$

$$A_{(1)} \in \mathbb{R}_{l \times m \times n}^{l \times m \times n}$$

• Along mode 2:

$$A_{(2)} = \begin{bmatrix} 1 & 4 & 7 & 10 \\ 2 & 5 & 8 & 11 \\ 3 & 6 & 9 & 12 \end{bmatrix}$$

$$A_{(2)} \in \mathbb{R}^{m \times l \times n}$$

Along mode 3:

$$A_{(3)} = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 7 & 8 & 9 & 10 & 11 & 12 \end{bmatrix}$$

$$A_{(3)} \in \mathbb{R}^{n \times l \times m}$$

Date: \_\_\_\_\_

Q2:  $B \in \mathbb{R}^{4 \times 3 \times 5}$

Murad  
2

$$B_{(1)} \in \mathbb{R}^{4 \times 15}$$

$$B_{(2)} \in \mathbb{R}^{3 \times 20}$$

$$B_{(3)} \in \mathbb{R}^{5 \times 12}$$

Q3:  $M = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \quad A \in \mathbb{R}^{2 \times 3 \times 2}$

$$B = A \times_1 M$$

$$A_{(1)} = \begin{bmatrix} 1 & 2 & 3 & 7 & 8 & 9 \\ 4 & 5 & 6 & 10 & 11 & 12 \end{bmatrix}$$

$$B = M A_{(1)} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 7 & 8 & 9 \\ 4 & 5 & 6 & 10 & 11 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 & 7 & 8 & 9 \\ 4 & 5 & 6 & 6 & 10 & 12 \\ 5 & 7 & 9 & 17 & 19 & 21 \\ 4 & 5 & 6 & 6 & 10 & 12 \end{bmatrix}$$

Date: \_\_\_\_/\_\_\_\_/\_\_\_\_

$$B(:, :, 1) = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 5 & 7 & 9 \\ 4 & 5 & 6 \end{bmatrix} \quad B(:, :, 2) = \begin{bmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \\ 17 & 19 & 21 \\ 10 & 11 & 12 \end{bmatrix}$$

Mixed

$$B(:, :, 2) = \begin{bmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \\ 17 & 19 & 21 \\ 10 & 11 & 12 \end{bmatrix}$$

$$Q4: N = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix}$$

$$B = A \times_2 N$$

$$B = N \cdot A_{(2)}$$

$$A_{(2)} = B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 7 & 10 \\ 2 & 5 & 8 & 11 \\ 3 & 6 & 9 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 10 & 16 & 22 \\ 2 & 5 & 8 & 11 \\ 4 & 7 & 10 & 13 \end{bmatrix}$$



Date: \_\_\_\_\_

$$B(:, :, 1) = \begin{bmatrix} 4 & 2 & 4 \\ 10 & 5 & 7 \end{bmatrix}$$

*Handwritten signature*

$$B(:, :, 2) = \begin{bmatrix} 16 & 8 & 10 \\ 22 & 11 & 13 \end{bmatrix}$$

$$Q5: P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$$

$$B = A \times P$$

$$B = PA_{(3)}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 7 & 8 & 9 & 10 & 11 & 12 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 7 & 8 & 9 & 10 & 11 & 12 \\ 8 & 10 & 12 & 14 & 16 & 18 \end{bmatrix}$$

$$B(:, :, 1) = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$B(:, :, 2) = \begin{bmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix}$$

$$B(:, :, 3) = \begin{bmatrix} 8 & 10 & 12 \\ 14 & 16 & 18 \end{bmatrix}$$

Date: \_\_\_\_\_

Dimensions of resulting tensors are 2 x 3

$$\Phi_6: (A \times_2 N) \times_2 N^{-1} = A$$

$$\underbrace{B}_{(N A_{(2)})} \times_2 N^{-1} = A$$

$$B = N A_{(2)}$$

$$B \times_2 N^{-1} = A$$

~~$$B = N A_{(2)} N^{-1} \quad B_{(2)} = (N A_{(2)}) \times_2$$~~

$$\boxed{B B^{-1} = A}$$

~~$$A = A$$~~

$$\rightarrow N^{-1} B = \begin{bmatrix} 1/2 & 1/2 & -1/2 \\ 0 & 1 & 0 \\ 1/2 & -1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 4 & 10 & 16 & 22 \\ 2 & 5 & 8 & 11 \\ 4 & 7 & 10 & 13 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 4 & 7 & 10 \\ 2 & 5 & 8 & 11 \\ 3 & 6 & 9 & 12 \end{bmatrix}$$

$$(:, :, 1) = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$(:, :, 2) = \begin{bmatrix} 7 & 8 & 9 \\ 10 & 11 & 12 \end{bmatrix} = A \quad \boxed{\text{Prove d}}$$

$\leftarrow A$