Date

ASSIGNMENT #09

Q1: $\int_2^3 \frac{1}{x^2-1}\,dx$

$= -\int_2^3 \frac{1}{1-x^2}\,dx$

$= -\tan^{-1}(x)\Big]_2^3$

$= \left|-\tan^{-1}(3) + \tan^{-1}(2)\right| = |-0.142| = 0.142$ Ans.

Q2: $\int_0^\infty \frac{1}{\sqrt{x}(1+x)}\,dx$

Improper because — upper bound infinity
— Degree of denominator is greater than numerator

$\infty = p$

$\int_0^p \frac{1}{\sqrt{x}(1+x)}\,dx$

$\lim_{p\to\infty} \int_0^p f(x)\,dx = \lim_{p\to\infty} \int_0^p \frac{1}{\sqrt{x}(1+x)}\,dx \qquad u = \sqrt{x} \qquad du = \frac{1}{2\sqrt{x}}\,dx$

$\cancel{= \lim_{p\to\infty} \int_0^p \frac{1}{\sqrt{x}+1} \cdot \frac{1}{\sqrt{x}} \cdot \frac{1}{1+x}\,dx}$

$= \lim_{p\to\infty} \int_0^p \frac{2\,du}{u^2+1}$

$= \lim_{p\to\infty} \left[2\tan^{-1}(u)\right]_0^p$

$= \lim_{p\to\infty} \left[2\tan^{-1}(p) - 2\tan^{-1}(0)\right] = \pi$ Ans.

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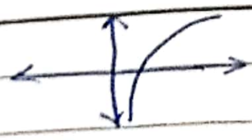
Q3: $\int_e^{\infty} \frac{1}{x[\ln(x)]^p} \, dx$ $\qquad t = \ln(x)$



$dt = \frac{1}{x} \, dx$

$\int_e^{\infty} \frac{1}{t^p} \, dt$

$\ln(e) = 1 \quad , \ln(\infty) = \infty$

$\int_1^{\infty} \frac{1}{t^p} \, dt$

$\infty = a$

$\lim_{a \to \infty^+} \int_1^{a} \frac{1}{t^p} \, dt$

$\lim_{a \to \infty^+} \left[\frac{1}{-p+1} t^{1-p} \right]_1^a$

$\lim_{a \to \infty^+} \left[\frac{1}{-p+1} a^{1-p} + \frac{1}{p+1} b^{1-p} \right]$

$\cancel{\lim_{a \to \infty^+} \left[\frac{1}{-p+1} \infty^{1-p} + 0 \right]}$

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$\lim \left[\frac{1}{1-p} \infty^{1-p} + \frac{1}{1-p} (1)^{1-p} \right]$

$\lim_{a \to \infty^+} \left[\frac{1}{1-p} a^{1-p} + \frac{1}{1-p} \right]$

Converges for $p > 1$, diverges for $p < 1$