

Muneeb Lone
DS-B
23i-2623

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Homework #14

Muneeb

$$1. \quad U^T = \begin{bmatrix} 1/2 & 1/2 & -1/2 & -1/2 \end{bmatrix}, \quad u = \begin{bmatrix} 1/2 \\ 1/2 \\ -1/2 \\ -1/2 \end{bmatrix}$$

$$(a) \quad P = UU^T = \begin{bmatrix} 1/2 \\ 1/2 \\ -1/2 \\ -1/2 \end{bmatrix} \begin{bmatrix} 1/2 & 1/2 & -1/2 & -1/2 \end{bmatrix}$$

$$P = \begin{bmatrix} 0.25 & 0.25 & -0.25 & -0.25 \\ 0.25 & 0.25 & -0.25 & -0.25 \\ -0.25 & -0.25 & 0.25 & 0.25 \\ -0.25 & -0.25 & 0.25 & 0.25 \end{bmatrix}$$

$$(b) \quad Q = I - 2P$$

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - 2 \begin{bmatrix} 0.25 & 0.25 & -0.25 & -0.25 \\ 0.25 & 0.25 & -0.25 & -0.25 \\ -0.25 & -0.25 & 0.25 & 0.25 \\ -0.25 & -0.25 & 0.25 & 0.25 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & 1.5 & 0.5 & 0.5 \\ 0.5 & 0.5 & 1.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & 1.5 \end{bmatrix}$$

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(a) ~~$P = P$~~

Answer

~~$$P = \begin{bmatrix} 0.25 & 0.25 & -0.25 & -0.25 \\ 0.25 & 0.25 & -0.25 & -0.25 \\ -0.25 & -0.25 & 0.25 & 0.25 \\ -0.25 & -0.25 & 0.25 & 0.25 \end{bmatrix}$$

Idempotent~~

~~$$Q = \begin{bmatrix} 3 & -1 & 1 & 1 \\ -1 & 3 & 1 & 1 \\ 1 & 1 & 3 & -1 \\ 1 & 1 & -1 & 3 \end{bmatrix}$$~~

(c) $P^2 = (UU^T)(UU^T)$
 $P^2 = U(U^T U)U^T$
 $P^2 = UU^T$
 $P^2 = P$

$Q^2 = (I - 2UU^T)^2$
 $Q^2 = I - 4UU^T + 4(UU^T)(UU^T)$
 $Q^2 = I - 4UU^T + 4UU^T$
 $Q^2 = I$

(d) Eigenvalues of P:

$\lambda = 1$

$\lambda = 0$

Eigenvectors:

$\{ u \}$

$\{ u \}^\perp$

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Eigenvalues of Q :

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$$\lambda = 1$$

$$\lambda = -1$$

Eigenvectors:



$$\{u\}^\perp$$

$$\{u\}$$

$$Q_2: A = \begin{bmatrix} 3 & -2 & 4 \\ -2 & 6 & 2 \\ 4 & 2 & 3 \end{bmatrix} = A^T \rightarrow \text{Symmetric}$$

$$\det(A - \lambda I) = 0$$

$$A - \lambda I = \begin{bmatrix} 3-\lambda & -2 & 4 \\ -2 & 6-\lambda & 2 \\ 4 & 2 & 3-\lambda \end{bmatrix} = 0$$

$$\det = \begin{vmatrix} 3-\lambda & 6-\lambda & 2 \\ 2 & 3-\lambda & -2 \\ -2 & 2 & 3-\lambda \end{vmatrix} + 2 \begin{vmatrix} -2 & 2 \\ 4 & 3-\lambda \end{vmatrix} + 4 \begin{vmatrix} -2 & 6-\lambda \\ 4 & 2 \end{vmatrix}$$

$$\det = -\lambda^3 + 12\lambda^2 - 21\lambda - 98 = 0$$

$$\lambda_1 = -2 \quad \lambda_2 = 7 \quad \lambda_3 = 7$$

Repeated Roots

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$$\lambda_1 = -2:$$

$$\lambda_2 = -7$$

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$$x_1 = \begin{bmatrix} -1 \\ -1/2 \\ 1 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} -1/2 \\ 1 \\ 0 \end{bmatrix}$$

$$x_3 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 7 \end{bmatrix}$$

$$P = \begin{bmatrix} -1 & -1/2 & 1 \\ -1/2 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$Q = \begin{bmatrix} -0.67 & -0.45 & 1/\sqrt{2} \\ -0.33 & 0.89 & 0 \\ 0.67 & 0 & 1/\sqrt{2} \end{bmatrix}$$

$$A = QDQ^T$$