Assignment #5

$$C = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \qquad \begin{bmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix}$$

$$2 \times 2 \qquad\qquad 3 \times 2$$

(i)
$$\begin{bmatrix} ① & 2 \\ 0 & -2 \end{bmatrix} \quad R2 - 3R1$$

$$\begin{bmatrix} ① & 0 \\ 0 & ⊖ \end{bmatrix} \quad R1 + R2$$

Basis of Col C $= Span \left\{ \begin{matrix} 1 \\ 3 \end{matrix} , \begin{matrix} 2 \\ 4 \end{matrix} \right\}$

Dimension = No. of pivot columns = 2

Rank Col C = Dim (Col C) = 2

Col D

Rank = 1

$$\begin{bmatrix} ① & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix}.$$

$$\begin{bmatrix} ① & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{matrix} R2 - 2R1 \\ R3 - 3R1 \end{matrix}$$

Basis of Col D $= Span \left\{ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} \right\}$

Col D

Dim = 1

(ii) $\begin{bmatrix} ① & 2 \\ 0 & -2 \end{bmatrix}$ R2 - 3R1   Mueeb

$\begin{bmatrix} ① & 0 \\ 0 & ⊖ \end{bmatrix}$ R1 + R2

Basis of Row C = Span $\left\{ (1 \quad 0), (0 \quad -2) \right\}$

Dim(Row C) = 2 = No. of ~~non-zero~~ Pivot rows
Rank (Row C) = 2

$\begin{bmatrix} ① & 2 \\ 2 & 4 \\ 3 & 6 \end{bmatrix}$

$\begin{bmatrix} ① & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ R2 - 2R1
               R3 - 3R1

Basis of Row D = Span $\left\{ (1, 2) \right\}$

Dim (Row D) = 1
Rank (Row D) = 1

(iii) $\begin{bmatrix} ① & 2 & \vdots & 0 \\ 3 & 4 & \vdots & 0 \end{bmatrix}$

Basis of Nul C = $\left\{ \vec{\cancel{\emptyset}} \right\}$
Empty because the $\vec{0}$
cannot form the basis
for a null space

$x_1 \quad x_2$

$\begin{bmatrix} ① & 0 \\ 0 & ②̵ \end{bmatrix} \begin{matrix} R1+R2 \\ R2-3R1 \end{matrix}$

$-2x_2 = 0 \quad ①̵$

$x_1 = 0 \quad ②̵$

Muneet

No free variables.
Dim Nul C = 0 = No non-pivot column.

$\begin{bmatrix} 1 & 2 & \vdots & 0 \\ 2 & 4 & \vdots & 0 \\ 3 & 6 & \vdots & 0 \end{bmatrix}$

$x_1 \quad x_2$

$\begin{bmatrix} ① & 2 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{matrix} \\ R2-2R1 \\ R3-3R1 \end{matrix}$

$x_1 + 2x_2 = 0 \quad ①̵$
x_2 free variable; $x_2 = t$

$x_1 = -2x_2 \qquad \text{or} \qquad x_1 = -2t$

$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -2t \\ t \end{bmatrix} = t \begin{bmatrix} -2 \\ 1 \end{bmatrix}$

Basis of Nul D = $\left\{ \text{Span} \begin{Bmatrix} -2 \\ 1 \end{Bmatrix} \right.$
Dim(Nul D) = 1

Yousaf

CamScanner

(iv) $C^T = \begin{bmatrix} ① & 3 \\ 2 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Muneeb

$\begin{array}{cc} x_1 & x_2 \end{array}$

$\begin{bmatrix} ① & 3 \\ 0 & ⊘② \end{bmatrix}$ R2-2R1

$x_1 + 3x_2 = 0$ Dim Nul $C^T = 0$

$-2x_2 = 0$

$x_1 = 0 , \quad x_2 = 0$

No free variables

No basis of Null space in this matrix

$D^T = \begin{bmatrix} ① & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$\begin{bmatrix} ① & 2 & 3 \\ 0 & 0 & 0 \end{bmatrix}$ R2-2R1 $x = t_1 \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix} + t_2 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$

$x_1 + 2x_2 + 3x_3 = 0$

x_2, x_3 free variables

$x_2 = t_1 , \quad x_3 = t_2$ Basis of Nul $D^T =$

Span $\left\{ \begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix} , \begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix} \right\}$

$x_1 = -2x_2 - 3x_3$

$x_1 = -2t_1 - 3t_2$

$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -2t_1 - 3t_2 \\ t_1 \\ t_2 \end{bmatrix}$ Dim Nul $D^T = 2$

Yousaf