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DS-B

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Assignment 2

Q1: A: 7/10 years of education
B: Is convicted within 2 years ....

(i) $P(A) = 0.4$

(ii) $P(B) = 0.37$

(iii) $P(A \cap B) = 0.1$

(iv) $P(A \cup B) = 0.4 + 0.37 - 0.1 = 0.67$

(v) $P(A') = 1 - 0.4 = 0.6$

(vi) $P((A \cup B)') = 1 - P(A \cup B) = 1 - 0.67 = 0.33$

(vii) $P((A \cap B)') = 1 - P(A \cap B) = 1 - 0.1 = 0.9$

(viii) $P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.1}{0.37} = 0.270$

(ix) $P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{0.1}{0.4} = 0.25$

Q2: TOMORROW

(a) Arrangements $= \frac{8!}{3!\,2!} = 3360$

(b) R [O O O] _ _ _ R (4!, 3!; 1, 1)

O's together $= 4! \times 1 \times 1 = 24$

Not together = Total − together
$= \frac{6!}{3!} - 4!$
$= 120 - 24 = 96$

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(c) Total = $^8C_4 = 70$

Case 1: 1R, 1B

Case 2: 2R, 1B

3: 1R, 2B

4: 1R, 3B

5: 2R, 2B

$= {}^3C_1 + {}^2C_1 + {}^3C_2 + {}^2C_2 + {}^3C_1 + {}^3C_1 + {}^3C_2 + {}^2C_1 + {}^3C_1 + {}^3C_3 + {}^2C_1 + {}^3C_0 + {}^3C_2 + {}^3C_2 = 50$

Probability: $\frac{50}{70} = \frac{5}{7}$

Q3: G: College Graduate

T: At least 3 years experience

(a) $P(G) = \frac{18+36}{90} = \frac{54}{90} = 0.6$

(b) $P(T') = 1 - \underbrace{P(T)}_{0.3} = 1 - \frac{(18+9)}{90} = 1 - \frac{27}{90} = 0.7$

(c) $P(G \cap T) = P(G) \times P(T)$

$= 0.6 \times 0.3$

$= 0.18$

~~0.2~~

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(d) $P(G' \cap T') = P((G \cup T)')$

$= 1 - P(G \cup T)$

$= 1 - [P(G) + P(T) - P(G \cap T)]$

$= 1 - 0.7$

$P(G' \cap T') = 0.3$

(e) $P(T|G) = \frac{P(T \cap G)}{P(G)} = \frac{0.2}{0.6} = 0.333$

(f) $P(G'|T') = \frac{P(G' \cap T')}{P(T')} = \frac{1 - P(G \cup T)}{1 - P(T)}$

$= \frac{0.3}{1-0.3} = \frac{0.3}{0.7} = 0.43$

Q4: Red = 2, Green = 3, Black = 4

3 drawn

(a) Diff color: $\frac{{}^2C_1 \times {}^3C_1 \times {}^4C_1}{{}^9C_3} = \frac{2}{7}$

(b) 2 same and :-

1 different

Case 1: $(2R + 1G) \Rightarrow {}^2C_2 \times {}^3C_1 = 3$

Case 2: $(2R, 1B) \Rightarrow {}^2C_2 \times {}^4C_1 = 4$

Case 3: $(2G, 1R) \Rightarrow {}^3C_2 \times {}^2C_1 = 6$

Case 4: $(2G, 1B) \Rightarrow {}^3C_2 \times {}^4C_1 = 12$

Case 5: $(2B, 1R \Rightarrow {}^4C_2 \times {}^2C_1 = 12$

Case 6: $(2B, 1G) \Rightarrow {}^4C_2 \times {}^3C_1 = 18$

Prob $= \frac{55}{{}^9C_3} = \frac{55}{84}$

$= 0.654$

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(c) Case 1 : 3B : ${}^3C_3 = 1$

Case 2 : 3G : ${}^4C_3 = 4$

Probability $= \frac{5}{84} = 0.059$

Q5: ~~(a) $P(A \cap B^c) = P(A) \cdot P(B^c)$~~

$P(A) = 0.7$, $P(B \cup A) = 0.9$, $P(A \cap B) = 0.3$

(a) $P(A \cap B^c) = P(A) \cdot P(B^c)$

$= [P(A)](1 - P(B))$

$P(A \cap B) = P(A) \cdot P(B)$

$0.3 = 0.7 \cdot P(B)$

$P(B) = 0.428$

$P(A \cap B^c) = (P(A))(1 - P(B))$

$= (0.7)(1 - 0.428)$

$= 0.7 - 0.57$

$= 0.4004$

(b) $P(B \cap A^c) = P(B)(1 - P(A))$

$= 0.42 \cdot (1 - 0.7)$

$= (0.42)(0.3)$

$= 0.126$

(c) $P(A^c \cap B^c) = 1 - P(A \cup B)$

$= 1 - 0.9 = 0.1$

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(d) $P(A^c \cup B) = P(A^c) + P(B) - P(A^c \cap B)$

$= 0.3 + 0.42 - 0.126$

$= 0.594$

(e) $P(A^c \cup B^c) = P(A^c) + P(B^c) - P(A^c \cap B^c)$

$= 0.3 + 0.57 - 0.1$

$= 0.77$

Q6: $P(M_1) = 0.4$ $P(M_1^c) = 0.6$

$P(M_2) = 0.5$ $P(M_2^c) = 0.5$

$P(M_3) = 0.6$ $P(M_3^c) = 0.4$

(a) All missiles hit the target: $0.4 \times 0.5 \times 0.6 = 0.12$

(b) At least one hits: $1 - P(\text{None hit})$

$= 1 - (P(M_1)^c \times P(M_2)^c \times P(M_3)^c)$

$= 1 - (0.6 \times 0.5 \times 0.4)$

$= 1 - 0.12$

$= 0.88$

(c) At most one: $(A^c \times B^c \times C^c) + (A \times B^c \times C^c) + (A^c \times B \times C^c) + (A^c \times B^c \times C)$

$= (0.12) + (0.4 \times 0.5 \times 0.4) + (0.6 \times 0.5 \times 0.4) + (0.6 \times 0.5 \times 0.6)$

$= 0.12 + 0.08 + 0.12 + 0.18 = 0.5$

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(d) Exactly one:

$= P(A \times B^c \times C^c) + P(A^c \times B \times C^c) + P(A^c \times B^c \times C)$

$= (0.4 \times 0.5 \times 0.4) + (0.6 \times 0.5 \times 0.4) + (0.6 \times 0.5 \times 0.6)$

$= 0.08 + 0.12 + 0.18$

$= 0.38$

(e) Exactly two missiles: $P(A \times B \times C^c) + P(A \times B^c \times C) + P(A^c \times B \times C)$

$= (0.4 \times 0.5 \times 0.4) + (0.4 \times 0.5 \times 0.6) + (0.6 \times 0.5 \times 0.6)$

$= 0.08 + 0.12 + 0.18$

$= 0.38$

Q7: $P(C) = 0.44$ C: Cleaned

$P(F) = 0.24$ F: Cavity Filled

$P(E) = 0.21$ E: Tooth extracted

$P(C \cap E) = 0.08$

$P(C \cap E) = 0.11$

$P(F \cap E) = 0.07$

$P(C \cap E \cap F) = 0.03$

$P(C \cap F \cup E) = P(C) + P(F) + P(E) - (P(C \cap F) + P(C \cap E) + P(F \cap E)) + P(C \cap E \cap F)$

$= 0.44 + 0.24 + 0.21 - 0.08 - 0.11 - 0.07 + 0.03$

$= 0.66$

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Q8: $P(H) = 0.21$ $\quad P(H^c) = 0.79$

$P(W) = 0.28$ $\quad P(W^c) = 0.72$

$P(H \cap W) = 0.15$ $\quad P(H^c \cup W^c) = 0.85$

(a) At least one:

$= (P(H) \times P(W^c)) + (P(H^c) \times P(W)) + (P(H) \times P(W))$

$= (0.21 \times 0.72) + (0.79 \times 0.28) + (0.21 \times 0.28)$

$= 0.4312$

(b) $P(W|H) = \frac{P(W \cap H)}{P(H)} = \frac{0.15}{0.21} = 0.714$

(c) $P(H|W^c) = \frac{P(H \cap W^c)}{P(W^c)} = \frac{P(H) - P(H \cap W)}{P(W^c)}$

$= \frac{0.21 - 0.15}{0.72} = 0.0833$

Q9: $P(b) = 0.4$ $\quad P(L|B) = 0.55$ $\quad P(L^c) = 0.48$

$P(T) = 0.35$ $\quad P(L|T) = 0.7$ $\quad P(L) = 1 - 0.48 = 0.52$

$P(C) = 0.25$ $\quad P(L|C) = x$

(a) $x = ?$

$P(L|C) = \frac{P(L \cap C)}{P(C)}$ ~~$= P(L) = 1 - P(L|B)(P(B)$~~

$P(L|C) = [1 - P(L|B)(P(B))] + [1 - P(L|T)(P(T))] + [1 - P(L|C)(P(C))]$

$0.48 = (0.45 \times 0.4) + (0.3 \times 0.35) + (1 - x)(0.25)$

$0.48 = 0.18 + 0.105 + 0.25 - 0.25x \rightarrow x = 0.22$

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(b) $P(T|L) = \frac{P(L|T)\,P(T)}{P(L)}$

$P(L) = 0.52$

$P(T|L) = \frac{0.7 \times 0.35}{0.52}$

$P(T|L) = \frac{0.245}{0.52} = 0.4711$

Q10: $P(J) = 0.2$ J: John

$P(T) = 0.6$ T: [redacted] Tom

$P(E) = 0.15$ F: Jeff

$P(P) = 0.05$ P: Pat

$P(F|J) = 0.005$, $P(F|T) = 0.01$

$P(F|E) = 0.011$, $P(F|P) = 0.005$

$P(J|F) = ?$

$P(F) = (0.005 \times 0.2) + (0.01 \times 0.6) + (0.0111 \times 0.15) + (0.005 \times 0.05)$

$P(F) = 0.008915$

$P(J|F) = \frac{P(F|J) \cdot P(J)}{P(F)} = \frac{0.005 \times 0.2}{0.008915} = 0.1122$