National University of Computer & Emerging Sciences
Islamabad

FAST School of Computing | Fall-2024 | Islamabad Campus

MT1004 – Linear Algebra

Homework # 2

Question # 1

Determine if the vector $\boldsymbol{b} = \begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix}$ is in the span of the columns of the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}.$$

Question # 2

Let $\boldsymbol{u} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, $\boldsymbol{v} = \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}$ and $\boldsymbol{w} = \begin{bmatrix} h \\ -3 \\ -5 \end{bmatrix}$. For what value(s) of h is $\boldsymbol{w}$ in the plane generated by $\boldsymbol{u}$ and $\boldsymbol{v}$?

Question # 3

Consider the matrix

$$A = \begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 3 & -4 \\ 0 & 2 & 6 & 7 \\ 2 & 9 & 5 & -7 \end{bmatrix}$$

(i) Do the columns of A span $\mathbb{R}^3$?
(ii) Do the columns of A span $\mathbb{R}^4$?
(iii) Does the equation $A\boldsymbol{x} = \boldsymbol{b}$ have a solution for each $\boldsymbol{b}$ in $\mathbb{R}^4$?

Question # 4

Let

$$A = \begin{bmatrix} 2 & 2 & 4 \\ -4 & -4 & -8 \\ 0 & -3 & -3 \end{bmatrix} \text{ and } \boldsymbol{b} = \begin{bmatrix} 6 \\ -12 \\ 0 \end{bmatrix}$$

(i) Describe all solutions of $A\boldsymbol{x} = \boldsymbol{b}$. Express the solutions in parametric form.
(ii) Describe all solutions of $A\boldsymbol{x} = \boldsymbol{0}$. Express the solutions in parametric form.
(iii) Describe the geometric interpretation of the solution sets obtained in part **(i)** and **(ii)**.