Mureeb Lone
23i-2623
DS-B

Date: ___/___/___

Homework 12

Q1: $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}_{2\times 3}$ $\quad A^T = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix}_{3\times 2}$

(a) Linear Independence: ① $A_{m\times n}$ has dimensions 2×3. Since $m < n$, the columns cannot be linearly independent. It is also obvious that Col 2 = Col 1 + Col 3 so its in their span. The rows however are linearly independent as they cannot be written as a linear combination of each other.

② AA^T is a 2×2 matrix and is invertible as the rows of A are linearly independent

③ A^TA is a 3×3 matrix but since the columns of A are not linearly independent, A^TA is not invertible.

(b) $A_R^{-1} = A^T(AA^T)^{-1}$

$$AA^T = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$(AA^T)^{-1} = \frac{1}{3}\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

Date: ___/___/______

$A_R^{-1} = A^T(AA^T)^{-1}$

$$= \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \left(\frac{1}{3}\right)$$

$$A_R^{-1} = \frac{1}{3} \begin{bmatrix} 2 & -1 \\ 1 & 1 \\ -1 & 2 \end{bmatrix}$$

(c) Std Matrix $= A^T(AA^T)^{-1}A$

$$P = \begin{bmatrix} 1 & 0 \\ 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \left(\frac{1}{3}\right)$$

$$P = \frac{1}{3} \begin{bmatrix} 2 & 1 & -1 \\ 1 & 2 & 1 \\ -1 & 1 & 2 \end{bmatrix}$$

Q2: $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 1 & 2 \end{bmatrix}$, $b = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}$

$$\text{proj}_{A_1} b = \overbrace{\frac{a_1 \cdot b}{a_1 \cdot a_1}}^{u_1} a_1 + \overbrace{\frac{a_2 \cdot b}{a_2 \cdot a_2}}^{u_2} . a_2$$

$$u_1 = \frac{3+1+5}{1+1+1} = 3 \qquad u_2 = \frac{6 + -4 + 10}{4+16+4} = \frac{1}{2}$$

$$\hat{u} = \hat{x} = \begin{bmatrix} 3 \\ 1/2 \end{bmatrix}$$

Date: ___/___/___

(b) P1(2,3) , P2(-4,1) , P3(2,5)

$y = \alpha + \beta x$

$3 = \alpha + 2x \qquad 1 = \alpha - 4x \qquad 5 = \alpha + 2\beta$

$$A = \begin{bmatrix} 2 & 1 \\ -4 & 1 \\ 2 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}$$

$$x = (A^T A)^{-1} A^T b$$

$$x = \left(\begin{bmatrix} 2 & -4 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ -4 & 1 \\ 2 & 1 \end{bmatrix} \right)^{-1} \begin{bmatrix} 2 & -4 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}$$

$$x = \begin{bmatrix} 1/2 \\ 3 \end{bmatrix}$$

Least square line: $y = \frac{1}{2} x + 3$

Errors :

P1 : $y = (\frac{1}{2})(2) + 3 = 4$

P2 : $y = (\frac{1}{2})(-4) + 3 = 1$

P3 : $y = (\frac{1}{2})(2) + 3 = 4$

$$(y - \hat{y})^2 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Total error = $1 + 1 + 0 = 2$

Date: ___/___/___

Q3 (a) (1,1),(2,-2),(3,3),(4,4)

Mmt

$y = ax^2 + bx + c$

$$A = \begin{bmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \\ x_4^2 & x_4 & 1 \end{bmatrix} \quad b = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}, \quad x = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \\ 16 & 4 & 1 \end{bmatrix} \quad b = \begin{bmatrix} 1 \\ -2 \\ 3 \\ 4 \end{bmatrix}$$

$$A^TA = \begin{bmatrix} 354 & 90 & 30 \\ 90 & 30 & 10 \\ 30 & 10 & 4 \end{bmatrix} \quad A^Tb = \begin{bmatrix} 54 \\ 14 \\ 6 \end{bmatrix}$$

$A\hat{x} = \hat{b}$

$$\begin{bmatrix} 354 & 90 & 30 \\ 90 & 30 & 10 \\ 30 & 10 & 4 \end{bmatrix} \hat{x} = \begin{bmatrix} 54 \\ 14 \\ 6 \end{bmatrix}$$

Date: ____

(b) $(1,1,1), (2,-2,2), (3,3,3), (4,4,4)$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & -2 & 1 \\ 3 & 3 & 1 \\ 4 & 4 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}$$

$$A^T A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & -2 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 2 & -2 & 1 \\ 3 & 3 & 1 \\ 4 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 30 & 14 & 10 \\ 14 & 30 & 10 \\ 10 & 10 & 4 \end{bmatrix}$$

$$A^T b = \begin{bmatrix} 30 \\ 20 \\ 10 \end{bmatrix}$$

$$\begin{bmatrix} 30 & 14 & 10 \\ 14 & 30 & 10 \\ 10 & 10 & 4 \end{bmatrix} \hat{x} = \begin{bmatrix} 30 \\ 20 \\ 10 \end{bmatrix}$$