ans(1) (a) (i) span $\{v_1\}$

• $\text{span}\{v_1\} = \{cv_1 \mid c \in \mathbb{R}\} = \left\{ \begin{pmatrix} c(-1) \\ c(-3) \\ c(4) \end{pmatrix} \mid c \in \mathbb{R} \right\}$

→ Geometrically it is a line passing through origin in direction of v_1.

• $\text{aff}\{v_1\} = \{cv_1 \mid c = 1\}$

→ Geometrically it would be point v_1 (flat)

(ii) $\text{conv}\{v_1, v_2\} = \{c_1v_1 + c_2v_2 \mid c_1 + c_2 = 1, c_1, c_2 \geq 0\}$

→ Geometrically it would be a line segment joining v_1 & v_2

(ii) → $\text{span}\{v_1, v_2\} = \{c_1v_1 + c_2v_2 \mid c_1, c_2 \in \mathbb{R}\}$

$= \left\{ c_1 \begin{bmatrix} -1 \\ -3 \\ 4 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix} \mid c_1, c_2 \in \mathbb{R} \right\}$

→ it is plane in $\mathbb{R}^3$ (2 vectors) passing through origin

→ $\text{aff}\{v_1, v_2\} = \{c_1v_1 + c_2v_2 \mid c_1 + c_2 = 1, (c_1 + c_2) \in \mathbb{R}\}$

→ it would be a translated line passing though v_1 & v_2 but not extending beyond them.

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$$\text{conv}\{v_1\} = \{v_1\} = \begin{bmatrix} -1 \\ -3 \\ 4 \end{bmatrix} = \text{aff}\{v_1\} = \text{conv}\{v_1\}$$

→ it would be a point in space

★ the convex hull is a subset of affine hull, which is a subset of span.

$\text{conv}\{v_1, v_2\}$ (line segment) $\subset \text{aff}\{v_1, v_2\}$ (line) $\subset \underbrace{\text{span}\{v_1, v_2\}}_{\text{(plane)}}$

→ Geometric Relationships :-

(1) for $\text{span}\{v_1, v_2\}$ imagine a plane in $\mathbb{R}^3$ going through both vectors & origin

(2) for $\text{aff}\{v_1, v_2\}$, it is essentially the same plane but translated. So, it includes all points in the plane formed by v_1 & v_2.

(3) for $\text{conv}\{v_1, v_2\}$, it is just the line segment btw v_1 & v_2, which is the shortest path connecting them.

Ans 2 :-

→ for affine independence

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ -1 & 1 & 2 & 0 \\ 2 & 0 & -2 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \end{array}\right] \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 2 & 3 & 0 \\ 0 & -2 & -4 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \end{array}\right] \begin{array}{l} R_2+R_1 \\ R_3-R_2 \\ R_4-R_1 \\ R_5-R_1 \end{array} \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -2 & -4 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 2 & 3 & 0 \end{array}\right] R_2 \leftrightarrow R_5$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 3 & 0 \end{array}\right] \begin{array}{l} R_3+2R_2 \\ R_5-2R_2 \end{array} \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 3 & 0 \end{array}\right] R_3/-4$$

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→ so, as the rank = 3, which is no. of vectors, so vectors are affine independent.

· for Barycentric coordinates:-

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 5 \\ -1 & 1 & 2 & 4 \\ 2 & 0 & -2 & -2 \\ 1 & 1 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{array}\right] \underset{R_1 \leftrightarrow R_5}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ -1 & 1 & 2 & 4 \\ 2 & 0 & -2 & 2 \\ 1 & 1 & 0 & 2 \\ 1 & 2 & 1 & 5 \end{array}\right] \underset{\substack{R_2+R_1 \\ R_3-2R_1 \\ R_4-R_1 \\ R_5-R_1}}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 2 & 3 & 5 \\ 0 & -2 & -4 & 0 \\ 0 & 0 & -1 & -3 \\ 0 & 1 & 0 & 4 \end{array}\right]$$

$$\underset{\substack{R_2 \leftrightarrow R_5 \\ R_3+2R_2 \\ R_5-2R_2}}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & -1 & -3 \\ 0 & 0 & 3 & -3 \end{array}\right] \underset{R_3 \leftrightarrow R_4}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & -1 & -3 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 3 & -3 \end{array}\right] \underset{\substack{R_1-R_2 \\ R_3 \to -R_3 \\ R_1-R_3}}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & -4 & 0 \\ 0 & 0 & 3 & -3 \end{array}\right]$$

$$\underset{\substack{R_4+4R_3 \\ R_5-3R_3}}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 12 \\ 0 & 0 & 0 & -12 \end{array}\right] \rightarrow -6+4+3+12-12 = 1$$

· So, barycentric coordinates are $\{-6, 4, 3, 12, -12\}$

(ii) → for affine independence:-

$$\left[\begin{array}{ccc|c} 0 & 1 & 1 & 0 \\ 1 & 1 & 4 & 0 \\ -2 & 0 & -6 & 0 \\ 1 & 2 & 3 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right] \underset{\substack{R_1 \leftrightarrow R_5 \\ R_2-R_1 \\ R_3+2R_1 \\ R_4-R_1}}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 2 & -4 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \underset{\substack{R_2 \leftrightarrow R_5 \\ R_3-2R_2 \\ R_4-R_2}}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -6 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 \end{array}\right]$$

$$\underset{R_3 \leftrightarrow R_4}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -6 & 0 \\ 0 & 0 & 3 & 0 \end{array}\right]$$

as the rank is 3, which is equal to no. of vectors. so the vectors are affine independent.

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• for barycentric coordinates :-

$$\left[\begin{array}{ccc|c} 0 & 1 & 1 & -1 \\ 1 & 1 & 4 & 1 \\ -2 & 0 & -6 & -4 \\ 1 & 2 & 3 & 0 \\ 1 & 1 & 1 & 1 \end{array}\right] \underset{\substack{R_1 \leftrightarrow R_5 \\ R_2 - R_1 \\ R_3 + 2R_1 \\ R_4 - R_1}}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & 3 & 0 \\ 0 & 2 & -4 & -2 \\ 0 & 1 & 2 & -1 \\ 0 & 1 & 1 & -1 \end{array}\right] \underset{\substack{R_2 \leftrightarrow R_5 \\ R_3 - 2R_2 \\ R_4 - R_2}}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -6 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 3 & 0 \end{array}\right]$$

$$\underset{\substack{R_3 \leftrightarrow R_4 \\ R_4 + 6R_3 \\ R_5 - 3R_3}}{\sim} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \underset{\substack{R_1 - R_2 \\ R_2 - R_3}}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \rightarrow 2 \cdot -1 \cdot 1$$

→ so, Barycentric coordinates are $\{2, -1\}$

(iii) for affine independence :-

(alternative method)

$$v_2 - v_1 = \begin{bmatrix} 0 \\ 6 \end{bmatrix} - \begin{bmatrix} 3 \\ -3 \end{bmatrix} = \begin{bmatrix} -3 \\ 9 \end{bmatrix}$$

$$v_3 - v_1 = \begin{bmatrix} 2 \\ 0 \end{bmatrix} - \begin{bmatrix} 3 \\ -3 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$$

• As both vectors are scalar multiples to each other, vectors are linearly independent

so, As $(v_2 - v_1) = 3(v_3 - v_1)$

$\cancel{(v_2 - v_1) - 3(}$ $(v_2 - v_1) - 3(v_3 - 1) = 0$

$\boxed{2v_1 + v_2 - 3v_3 = 0}$ → Affine dependence relation