MT1003 Calculus and Analytical Geometry
Homework 03

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Due Date: September 20, 2023

Problem 1

Consider the function $f(x) = \frac{5x^3 - 3x^2 + 2x - 1}{2x^3 - x^2 + 4}$.

    a. Find the horizontal asymptote, if any, and justify your answer using limits.

    b. Determine the vertical asymptote(s), if any, and explain your reasoning.

    c. Determine if there is an oblique (slant) asymptote for the function $f(x)$. If there is, find the equation of the oblique asymptote and explain your approach.

Problem 2

Given the function $g(x) = \frac{\sin(x)}{x}$, investigate its asymptotic behavior as x approaches both positive and negative infinity. Determine whether there are horizontal asymptotes and vertical asymptotes, and explain your findings using limits.

Problem 3

Analyze the rational function $h(x) = \frac{3x^2 - 2x + 1}{x^2 + 4x + 4}$.

    a. Find the horizontal asymptote(s), if any, and explain your reasoning.

    b. Determine the vertical asymptote(s), if any, and provide a detailed explanation of your approach.

Problem 4

Evaluate the following limits using the Sandwich Theorem (Squeeze Theorem):

    a. $\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right)$

    b. $\lim_{x \to \infty} \frac{\ln(x)}{x}$

    c. $\lim_{x \to 0} x^2 \cos\left(\frac{1}{x}\right)$