

# National University of Computer & Emerging Sciences

## Homework # 15

1. Let

$$Q(x_1, x_2, x_3) = 3x_1^2 + 4x_2^2 + 5x_3^2 + 4x_1x_2 - 4x_2x_3$$

- (a) Express the quadratic form in the matrix notation  $x^T Ax$ , where  $A$  is a symmetric matrix.
- (b) Find an orthogonal change of variables that eliminates the cross product terms in the quadratic form  $Q(x_1, x_2, x_3)$ , and express  $Q(x_1, x_2, x_3)$  in terms of the new variables.
- (c) Identify the quadric surface represented by the equation  $Q(x_1, x_2, x_3) = 1$ .
- (d) Classify the matrix as positive definite, negative definite, or indefinite.
- (e) Find the maximum and minimum values of the given quadratic form subject to the constraint  $x_1^2 + x_2^2 + x_3^2 = 1$ , and determine the values of  $x_1$ ,  $x_2$  and  $x_3$  at which the maximum and minimum occur.
- (f) Explain the role of Positive/negative definiteness in ensuring the existence of a unique minimum/maximum for  $Q(x_1, x_2, x_3)$ .