Homework # 9

Case-I: Distinct Eigenvalues Always Diagonalizable

1. Find eigenvalues and eigenvectors of the matrix A, where

$$A = \begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}$$

Also

(a) Compute $\begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}^{10} \begin{pmatrix} 5 \\ 1 \end{pmatrix}$

(b) Diagonalize the matrix A

(c) Compute $\begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}^{10}$

Hint: Follow these steps to provide solution

(i) **For Eigenvalues:** Solve the Characteristic polynomial $\lambda^2 - tr(A)\lambda + det(A) = 0$ to obtain eigenvalues

(ii) **For Eigenvectors v_1 and v_2:** Find λ-eigenspace$=Nul(A - \lambda I)$

(iii) **For $A^n v$:** First write $\mathbf{v} = \mathbf{x_1 v_1 + x_2 v_2}$ where $\mathbf{v_1}$ and $\mathbf{v_2}$ are eigenvectors, then use the fact $A^n\mathbf{v_1} = \lambda_{\mathbf{1}}^{\mathbf{n}}\mathbf{v_1}$ and $A^n\mathbf{v_2} = \lambda_{\mathbf{2}}^{\mathbf{n}}\mathbf{v_2}$

(iv) **For Diagonalization:** Write $A = PDP^{-1}$, where D is 2×2 matrix with eigenvalues in its diagonal and P is 2×2 matrix with eigenvectors as its columns.

(v) **For A^n:** Use $A = PDP^{-1}$, which gives $A = PD^nP^{-1}$.