Homework # 5

Fundamental Subspaces of a Matrix

September 9, 2024

Introduction

In this homework, we will explore the four fundamental subspaces associated with a matrix. Understanding these subspaces is crucial for various applications, including dimensionality reduction, data representation, and linear regression. For each question, perform the necessary calculations and provide detailed explanations.

Fundamental Subspaces

Consider the matrix

$$C = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \qquad D = \begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{pmatrix}.$$

Answer the following:

1. Compute the column space of matrix $C \& D$. Provide its dimension and basis.

2. Compute the row space of matrix $C \& D$. Provide its dimension and basis.

3. Find the null space of matrix $C \& D$. Determine its dimension and provide a basis.

4. Determine the left null space of matrix $C \& D$ i.e., Null space of $C^T \& D^T$. Provide its dimension and a basis.