National University of Computer & Emerging Sciences

Multi-Variable Calculus

Homework#05

Instructions:

1. Must write your *Roll Number, Name, Section and Homework no.*
2. Submit your Homework on *GCR* as well as *hard form* in class.
3. Homework should be neat.

Question#01-

Lagrange Multipliers Method: (50 Marks)

I. Use the Lagrange Multipliers Method to find the minimum of $f(x, y) = x^2 + 4y^2 - 2x + 8y$ subject to the constraint $x + 2y = 7$.

II. Find the minimum distance of the parabola $y = x^2$ from the point (0,3).

III. A rectangular solid is contained within a tetrahedron with vertices at $(1,0,0)$, $(0,1,0)$, $(0,0,1)$ and the origin. The base of the box has dimensions x and y, and the height is z. If $x + y + z = 1$, find the volume that maximizes the volume of the rectangular solid.

IV. Show that the equilateral triangle has the largest parameter among all the triangles inscribed in a circle of radius r.

V. Write a Python Code for the first two functions and their Geometries. **(5+5 Marks)**

Question#02-

Double Integrals Method: (50 Marks)

I. Finding the Average Value of a Function
$f(x, y) = \frac{1}{2} xy$ over the plane region where is a rectangle with vertices $(0, 0), (4, 0), (4, 3), (0, 3)$.

II. Evaluate the integral by reversing the order of integration.

$$\int_0^2 \int_{x^2}^4 x \cos y^2 \, dy dx$$

III.   Evaluate the integral by reversing the order of integration.

$$\int_0^2 \int_{\frac{y}{2}}^1 e^{\frac{y}{x}} \, dx \, dy$$

IV.   Sketch the region of integration and write an equivalent double integral with the order of integration reversed.

$$\int_1^e \int_0^{\ln x} xy \, dy \, dx$$

V.   Evaluate the integral by reversing the order of integration.

$$\int_0^4 \int_{\sqrt{x}}^2 \sin y^3 \, dy \, dx$$