Muneeb Lone, 23i-2623

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Question # 1:

SVD AGTR $1\times2\times3$

$A(:,:,1) = [1\ 0]$

$A(:,:,2) = [+1\ -1]$

$A(:,:,3) = [0\ 1]$

$A = S \times_1 u_1 \times_2 u_2 \times_3 u_3$

S → core tensor driven :- $1\times2\times3$

$u_1 \rightarrow 1\times1$, $u_2 \rightarrow 2\times2$, $u_3 \Rightarrow 3\times3$

Compute u_1 :- unfolding by mode 1 :-

$A_{(1)} = [1\ 0\ \ 1\ -1\ \ 0\ 1]$.

We need to compute u_1 =

$A_{(1)}\ A_{(1)}^T = [1\ 0\ 1\ \ -1\ \ 0\ 1] \begin{bmatrix} 1 \\ 0 \\ +1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

$= [4]$

Eigan values = 4

Vectors = [1]

$u_1 = [1]$

compute u_2 :-

$A_{(2)} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

Eigan values : $\lambda^2 - 4\lambda + 3 = 0$

$\lambda = 3, 1$

$\lambda = 3 \rightarrow EV :- \begin{pmatrix} -1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} -1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$

$\lambda = 1 \rightarrow EV :- \begin{pmatrix} 1 \\ 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1/\sqrt{2} \\ 1/\sqrt{2} \end{pmatrix}$

$u_2 = \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$

compute $u_3 = A_{(3)} = \begin{bmatrix} 1 & 0 \\ 1 & -1 \\ 0 & 1 \end{bmatrix}$

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$A_{(3)} \; A_{(3)}^T = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

eigen values :- 3, 1, 0

vectors :- $\begin{pmatrix} -1 \\ -2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$

$U_3 = \begin{bmatrix} -1/\sqrt{6} & 1/\sqrt{2} & -1/\sqrt{3} \\ -2/\sqrt{6} & 0 & 1/\sqrt{3} \\ 1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{3} \end{bmatrix}$

$S = A \times_1 u_1^T \times_2 u_2^T \times_3 u_3^T$

$A \times_1 u_1^T = u_1^T \times A_{(1)}$

$= -[1] \times [1 \; 0 \; 1 \; -1 \; 0 \; 1]$

$= [1 \; 0 \; 1 \; -1 \; 0 \; 1]$

folding $= [\{+1, 0\}, \{1, -1\} \{0 \; 1\}]$

$1 \times 2 \times 3$

$\in \mathbb{R}$

$= B$

$B \times_2 u_2^T = u_2^T \times B_{(2)}$

$= \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}^T \times \begin{bmatrix} 1 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$

$= \begin{bmatrix} -1/\sqrt{2} & -2/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 0 & 1/\sqrt{2} \end{bmatrix}$

$= \left\{ \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}, \begin{bmatrix} -1/\sqrt{2} & 0 \end{bmatrix}, \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \right\}$

$C \times_3 u_3^T = u_3^T \times C_{(3)}$

$= \begin{bmatrix} -1/\sqrt{6} & 1/\sqrt{2} & -1/\sqrt{3} \\ -2/\sqrt{6} & 0 & 1/\sqrt{3} \\ 1/\sqrt{6} & 1/\sqrt{2} & 1/\sqrt{3} \end{bmatrix}$

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$$= \begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$= \begin{bmatrix} -\sqrt{3} & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$S_1 = \begin{bmatrix} -\sqrt{3} & 0 \end{bmatrix}$$

$$S_2 = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$S_3 = \begin{bmatrix} 0 & 0 \end{bmatrix}$$