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DS-B
23i-2623

Date 14/10/2023

ASSIGNMENT #4

Q1: CITY REVENUE

$R(n) = \frac{8000n}{n+2}$, $n(t) = 30 - 6t$ → 30hrs daily decreasing by 6 hrs per day.

$\frac{dR}{dn} = \frac{d}{dn}\frac{8000n}{n+2} = \frac{u'v - uv'}{v^2}$

$\frac{dR}{dn} = \frac{(8000)(n+2) - (8000n)(1)}{(n+2)^2}$

$n(t) = 30 - 6t$

$\frac{dn}{dt} = 0 - 6$

$\frac{dn}{dt} = -6$

CHAIN RULE:

$\frac{dR}{dt} = \frac{dR}{dn} \times \frac{dn}{dt}$

$\frac{dR}{dt} = \left[\frac{(8000)(n+2) - (8000n)}{(n+2)^2}\right]\left[-6\right]$

$n(0) = 30$

$\frac{dR}{dt} = \left[\frac{(8000)(32) - (8000 \times 30)}{(32)^2}\right]\left[-6\right]$

$\frac{dR}{dt} = -93.75$

Due to the pandemic, revenue is decreasing at a rate of \$93.75 everyday.

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Q2: Compound Interest

$A = 500\left(1 + \frac{r}{1200}\right)^{120}$, years = 10, initial value = \$500

$\frac{dA}{dt} = (500)(120)\left(1 + \frac{r}{1200}\right)^{+119} \left(\frac{1}{1200}\right)$

when $r = 5$

$\frac{dA}{dr} = (500)(120)\left(1 + \frac{5}{1200}\right)^{119}\left(\frac{1}{1200}\right) = 82.01$

when $r = 7$

$\frac{dA}{dr} = (500)(120)\left(1 + \frac{7}{1200}\right)^{119}\left(\frac{1}{1200}\right) = 99.9$

Q3: $u(t) = 4\cos t$ → measures displacement in inches.

For velocity:	For acceleration:
$\frac{du}{dt} = \frac{d}{dt} 4\cos t$	$\frac{dv}{dt} = \frac{d}{dt} -4\sin(t)$
$\frac{du}{dt} = (4)(-\sin(t))(1)$	$\frac{dv}{dt} = -4\cos(t) = a(t)$
$\frac{du}{dt} = -4\sin(t) = v(t)$	$a'(0) = -4\cos(t)$
	$0 = \cos(t)$
~~Max velocity when~~	$t = \cos^{-1}(0)$
~~$0 = -4\sin(t)$~~	$t = \frac{\pi}{2}, \frac{3\pi}{2}$ → Max velocity time
~~$\sin^{-1}(0) = t$~~	

~~$t = 0$ s → Max velocity time.~~

Max $v = v\left(\frac{\pi}{2}\right) = -4\sin\left(\frac{\pi}{2}\right) = -4$ inches per second is max velocity

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Q4: TANGENT LINE

(a) $x^3 + y^3 = 3xy$ - (1)

$\cancel{3}x^2 + (\cancel{3}y^2)\left(\frac{dy}{dx}\right) = (\cancel{3}x)\left(\frac{dy}{dx}\right) + \cancel{3}y$

$x^2 + y^2\frac{dy}{dx} = x\frac{dy}{dx} + y$

$x^2 - y = \frac{dy}{dx}(x - y^2)$

$\frac{dy}{dx} = \frac{x^2 - y}{x - y^2}$

(b) $\frac{dy}{dx} = 0$, First quad → $D\,(0, +\infty)$, $R\,(0, +\infty)$

$0 = \frac{x^2 - y}{x - y^2}$

$x^2 = y$ - (2)

$x^3 + (x^2)^3 = 3x(x^2)$

$x^3 + x^6 = 3x^3$

$x^6 = 2x^3$

$-2x^3 + x^6 = 0$

$x^3(x^3 - 2) = 0$

$x^3 = 0$ $\qquad$ $x^3 = +2$

$x = 0$ $\qquad$ $x = \sqrt[3]{+2}$

$y = 0$ $\qquad$ $y = (\sqrt[3]{+2})^2 = 2^{\frac{2}{3}}$

$(0, 0)$ $\qquad$ $(\sqrt[3]{2}, 2^{2/3})$

Horizontal tangent lines at above two points

P5: IMPLICIT DIFFERENTIATION

$x = x(t)$, $t \ln x = xe^t - 1$

$$\frac{d}{dt}(t \ln x) = \frac{d}{dt}(xe^t - 1)$$

$$u'v + vu' = u'v + vu' - 0$$

$$(1)(\ln x) + (t)\left(\frac{1}{x}\right)\left(\frac{dx}{dt}\right) = \left(\frac{dx}{dt}\right)e^t + (e^t)(x)$$

$$\frac{1}{x} t \frac{dy}{dx} + \ln x = e^t \frac{dx}{dt} + xe^t$$

$$\left(\frac{1}{x} t - e^t\right)\frac{dx}{dt} = xe^t - \ln(x)$$

$$\frac{dx}{dt} = \frac{xe^t - \ln x}{\cancel{x}t - e^t}$$

Point $(0, 1)$ (t, x)

$$\frac{dx}{dt} = \frac{(1)(e^0) - \ln 1}{0 - e^0} \quad , \ln 1 = 0 \quad , e^0 = 1$$

$$\frac{dx}{dt} = \frac{1 - 0}{0 - 1}$$

$\frac{dx}{dt} = -1$ at $(0, 1)$

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Q6: LOGARITHMIC DIFFERENTIATION

a) $y = \dfrac{x^{\frac{3}{4}}\sqrt{x^2+1}}{(3x+2)^5}$

$\ln y = \ln\left(\dfrac{x^{\frac{3}{4}}\sqrt{x^2+1}}{(3x+2)^5}\right)$

$\ln y = \ln(x^{\frac{3}{4}}) + \ln(\sqrt{x^2+1}) - \ln(3x+2)^5$

$\ln y = \frac{3}{4}\ln(x) + \frac{1}{2}\ln(x^2+1) - 5\ln(3x+2)$

$\frac{1}{y}\frac{dy}{dx} = \left(\frac{3}{4}\right)\left(\frac{1}{x}\right) + \left(\frac{1}{\cancel{2}}\right)\left(\frac{1}{x^2+1}\right)(\cancel{2}x) - (5)\left(\frac{1}{3x+2}\right)(3)$

$\frac{dy}{dx} = y\left(\frac{3}{4x} + \frac{x}{x^2+1} - \frac{15}{3x+2}\right)$

$\frac{dy}{dx} = \left(\frac{x^{\frac{3}{4}}\sqrt{x^2+1}}{(3x+2)^5}\right)\left(\frac{3}{4x} + \frac{x}{x^2+1} - \frac{15}{3x+2}\right)$

b) $y = x^{\sqrt{x}}$

$\ln y = \ln(x^{\sqrt{x}})$

$\frac{1}{y}\frac{dy}{dx} = \frac{\sqrt{x}}{x}$

$\frac{dy}{dx} = \frac{y}{\sqrt{x}}$

$\frac{dy}{dx} = \frac{x^{\sqrt{x}}}{\sqrt{x}}$

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Q7: $x = 2t^3 - 15t^2 + 24t$

a) $v(t) = \frac{dx}{dt} = 6t^2 - 30t + 24$

$a(t) = \frac{dv}{dt} = 12t - 30$

b) $v(2) = 6(2)^2 - 30(2) + 24$

$v(2) = +36$

$a(2) = 12(2) - 30$

$a(2) = -6$

At $t = 2s$, the direction is along positive x-axis while slowing down.

Q8: SAWTOOTH CURVE

$f(x) = \sin^{-1}(\sin(x))$

a) $\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\sqrt{1-x^2}}$, $\frac{d}{dx}\sin(x) = \cos(x)$

$f'(x) = \frac{d}{dx}\left(\sin^{-1}(\sin(x))\right) = \frac{1}{\sqrt{1-\sin^2 x}} \cdot \cos x$

$f'(x) = \frac{\cos(x)}{\pm\sqrt{\cos^2 x}} = \frac{\cos(x)}{|\cos(x)|} = \pm 1$

b) The function is not differentiable at points where $\cos(x) = 0$ due to jump discontinuity.
$\cos(x) = 0$ at $(2n+1)\left(\frac{\pi}{2}\right) = x$ so $f(x)$ is continuous and differentiable everywhere except odd multiples of $\frac{\pi}{2}$.

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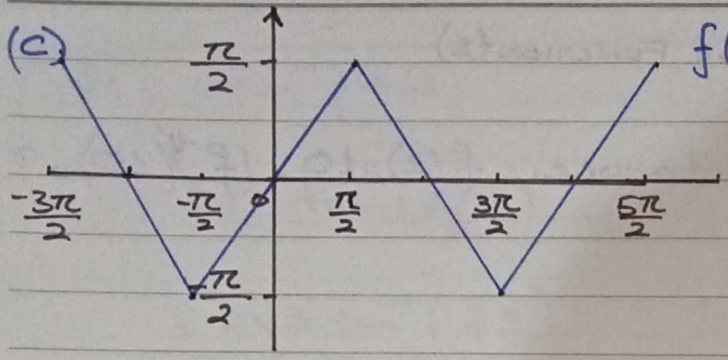
(c) $f(x) = \sin^{-1}(\sin(x))$



(d) PICTURE ADDED TO END OF PDF ON GCR

COMMANDS:

```
x= pi:0.01:pi;
f_x= asin(sin(x));
f_prime_x = cos(x)./abs(cos(x));

figure;
subplot(2,1,1);
plot(x,f_x,'b','LineWidth',2);
title('f(x)= sin^{-1}(sin(x))');
xlabel('x');
ylabel('f(x)');
grid on;

subplot(2,1,2)
plot(x,f_prime_x,'r','LineWidth',2);
title('f''(x)');
xlabel('x');
ylabel('f''(x)');
grid on;

subplot(2,1,1);
axis tight;
subplot(2,1,2);
axis tight;
sgtitle('plot of f(x) and its derivative f''(x)');
```

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Q9: DIFFERENTIATION OF INVERSE FUNCTION(S)

(a) $f(x) = x^3 + x$ is one-to-one, $f(2) = 10$, $(f^{-1})'(10) = ?$

$f(x) = x^3 + x$
$f'(x) = 3x^2 + 1$

$f'(x)$ is positive for any input on the real number line

$f'(x)$ is ~~hence~~ always increasing and hence is a one-to-one function

$(f^{-1})'(a) = \frac{1}{f(b)}$

$f(2) = 10$
$(f^{-1})(10) = 2$

$f'(x) = 3x^2 + 1$
$f'(2) = 3(2)^2 + 1$
$f'(2) = 13$

$(f^{-1})'(10) = \frac{1}{f'(2)}$

$(f^{-1})'(10) = \frac{1}{13}$ Ans.

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(b) $f'(2\sqrt{3}) = x\tan^{-1}(x/2)$

$= 2\sqrt{3}\tan^{-1}\left(\frac{\cancel{2}\sqrt{3}}{\cancel{2}}\right)$

$= 2\sqrt{3}\tan^{-1}(\sqrt{3})$

$= (2\sqrt{3})\left(\frac{\pi}{3}\right)$

$= 2\pi \times \frac{\sqrt{3}}{3}$

$f'(2\sqrt{3}) = \frac{2\pi}{\sqrt{3}}$ Ans.

(c) $\sin^{-1}\left(\frac{x}{a}\right) = y$

$\cancel{\frac{dy}{dx} = \frac{d}{dx}\sin^{-1}\left(\frac{x}{a}\right)}$

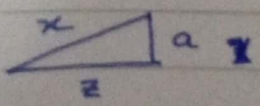
$\sin(y) = \frac{x}{a}$

$\frac{d}{dx}\sin(y) = \frac{d}{dx}\frac{x}{a}$

$(\cos y)\left(\frac{dy}{dx}\right) = \frac{1}{a}$

$\frac{dy}{dx} = \frac{1}{a\cos(y)}$

$\sin(y) = \frac{x}{a}$



$\cos^2 y + \sin^2 y = 1$

$\cos(y) = \sqrt{1 - \left(\frac{x}{a}\right)^2}$

$\cancel{\sqrt{x^2}}$ ~~/-/d~~ E/Z

$\frac{dy}{dx} = \frac{1}{(a)(\sqrt{1-(x/a)^2})}$

→ We can see format of

$\int \frac{dx}{\sqrt{a^2-x^2}}$ $\frac{d}{dx}\sin^{-1}(x)$ where $a > 0$

$a=1$ ~~We can see identity~~

$\cancel{x = a\sin(\theta),\ dx = a\cos(\theta)d\theta}$

$\cancel{\int \frac{a\cos(\theta)\,d\theta}{\sqrt{a^2 - a^2\sin^2\theta}}}$

$x = a\sin(\theta)$

$dx = a\cos(\theta)d\theta$

$\sqrt{a^2 - x^2} = a\cos(\theta)$

$\int \frac{\cancel{a\cos(\theta)}}{\cancel{a\cos(\theta)}}\,d\theta$

$\int d\theta = \theta + C$

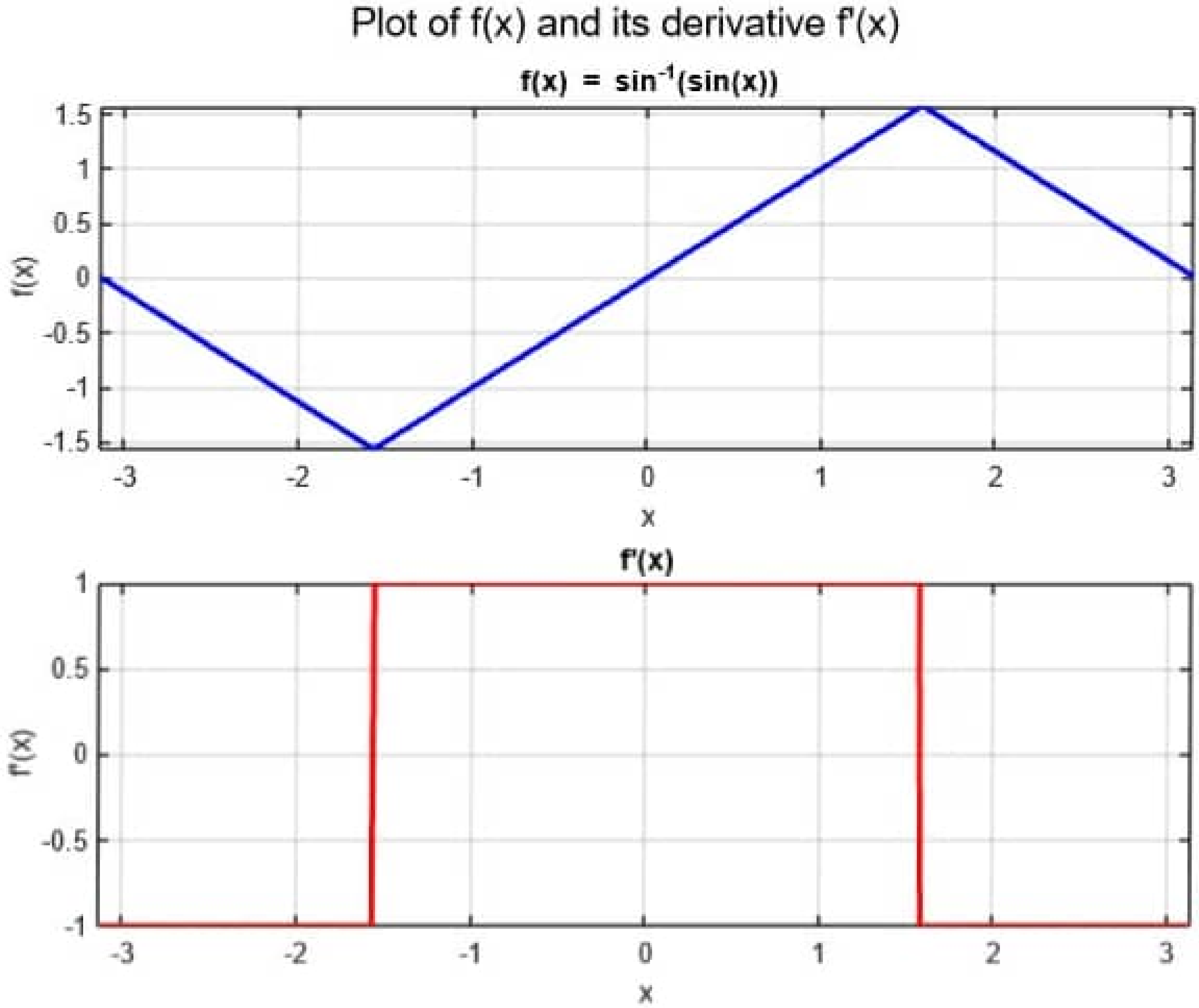
$x = a\sin(\theta)$

$\theta = \sin^{-1}\left(\frac{x}{a}\right)$

$\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$

where c is a constant

```
x = -pi:0.01:pi;


f_x = asin(sin(x));
f_prime_x = cos(x) ./ abs(cos(x));


figure;

subplot(2, 1, 1);
plot(x, f_x, 'b', 'LineWidth', 2);
title('f(x) = sin^{-1}(sin(x))');
xlabel('x');
ylabel('f(x)');
grid on;

subplot(2, 1, 2);
plot(x, f_prime_x, 'r', 'LineWidth', 2);
title('f''(x)');
xlabel('x');
ylabel('f''(x)');
grid on;


subplot(2, 1, 1);
axis tight;
subplot(2, 1, 2);
axis tight;




sgtitle('Plot of f(x) and its derivative f''(x)');
```



Plot of f(x) and its derivative f'(x)
f(x) = sin^-1(sin(x))
1.5
1
0.5
0
-0.5
-1
-1.5
f(x)
-3
-2
-1
0
1
2
3
x
f'(x)
1
0.5
0
-0.5
-1
f'(x)
-3
-2
-1
0
1
2
3
x