



National University of Computer & Emerging Sciences Islamabad

FAST School of Computing

Fall-2024

Islamabad Campus

MT1004 – Linear Algebra

Homework # 3

Question # 1

Determine whether the following statements are true (give an explanation) or false (give a counterexample.)

- i) If S is a linearly dependent set, then each vector is a linear combination of the other vectors in S .
- ii) If a set in \mathbb{R}^n is linearly dependent, then the set contains more than n vectors.
- iii) There are 3 pivot columns in a 5×3 matrix if its columns are linearly independent.
- iv) If \mathbf{u} and \mathbf{v} are linearly independent, and if $(\mathbf{u}, \mathbf{v}, \mathbf{w})$ is linearly dependent then \mathbf{w} is in $\text{Span}(\mathbf{u}, \mathbf{v})$.
- v) If A is an $m \times n$ matrix and if the equation $A\mathbf{x} = \mathbf{b}$ has at most one solution for each \mathbf{b} in \mathbb{R}^m , then the columns of A are linearly independent.

Question # 2

Let $\mathbf{u} = \begin{bmatrix} h \\ -1/2 \\ -1/2 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -1/2 \\ h \\ -1/2 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} -1/2 \\ -1/2 \\ h \end{bmatrix}$.

- i) For what real values of h is \mathbf{w} in $\text{Span}(\mathbf{u}, \mathbf{v})$.
- ii) For what real values of h is $(\mathbf{u}, \mathbf{v}, \mathbf{w})$ linearly dependent? Justify each answer.

Question # 3

Show that the three vectors $\mathbf{u} = \begin{bmatrix} 0 \\ 3 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 6 \\ 0 \\ 5 \\ 1 \end{bmatrix}$ and $\mathbf{w} = \begin{bmatrix} 4 \\ -7 \\ 1 \\ 3 \end{bmatrix}$ form a linearly

dependent set in \mathbb{R}^4 . Find the linear dependence relation among the vectors.