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DS-B

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Homework #15

Q1: $Q(x_1, x_2, x_3) = 3x_1^2 + 4x_2^2 + 5x_3^2 + 4x_1x_2 - 4x_2x_3$

(a) $Q(x) = x^T A x$ where $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$

$$A = \begin{bmatrix} 3 & 2 & 0 \\ 2 & 4 & -2 \\ 0 & -2 & 5 \end{bmatrix}$$

$$Q(x) = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} \begin{bmatrix} 3 & 2 & 0 \\ 2 & 4 & -2 \\ 0 & -2 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

(b) $\det(A - \lambda I) = 0$

$$(3-\lambda)((4-\lambda)(5-\lambda) - (-2)^2) - 2(2(5-\lambda) - 0) + 0$$

$$-\lambda^3 + 12\lambda^2 - 39\lambda + 28 = 0$$

$$\lambda_1 = 2, \lambda_2 = 4, \lambda_3 = 8$$

λ_1 Eigenvector: $\quad$ λ_2 Eigenvector: $\quad$ λ_3 Eigenvector:

$$V1 = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} \quad V2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad V3 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

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$$P = \begin{bmatrix} 2/\sqrt{5} & 1/\sqrt{3} & 0 \\ -1/\sqrt{5} & 1/\sqrt{3} & 1/\sqrt{2} \\ 0 & 1/\sqrt{3} & -1/\sqrt{2} \end{bmatrix}$$

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

$$Q(x) = y^T D y = 2y_1^2 + 4y_2^2 + 8y_3^2$$

(c) $2y_1^2 + 4y_2^2 + 8y_3^2 = 1$

All eigenvalues are positive hence ellipsoid.

(d) All eigenvalues are positive so positive definite.

(e) $R(x) = x^T A x$

$\lambda_1 = 2, \lambda_2 = 4, \lambda_3 = 8$

Maximum Value $= 8$
Minimum Value $= 2$

At max $= v_3 = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$ At min $= v_1 = \frac{1}{\sqrt{5}} \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$

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(f) • Positive definiteness implies all eigenvalues are positive, $Q(x)$ has a unique global minimum.

• Negative definiteness implies all eigenvalues are negative, $Q(x)$ has a unique global maximum.

• Indefiniteness means $Q(x)$ can have saddle points