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23i-2623 DS-B

ASSIGNMENT #4

Muneeb

(i) $H = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a+b+c = 0 \right\}$

This set is a vector space as this set is closed under addition, scalar multiplication and has a zero vector.

$k\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} ka \\ kb \\ kc \end{bmatrix}$ = For any scalar k;

$ka + kb + kc$
$= k(a+b+c)$
$= k(0)$
$= 0$ - Multiplication

$\begin{bmatrix} a_1 \\ b_1 \\ c_1 \end{bmatrix} = \begin{bmatrix} a_2 \\ b_2 \\ c_2 \end{bmatrix}$

$a_1 + b_1 + c_1 = a_2 + b_2 + c_2$
$0 = 0$ - Addition

Given Vector: $\begin{bmatrix} a \\ b \\ c \end{bmatrix} = 0$

(ii) $H = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a+b+c = 1 \right\}$

This set is NOT a vector space as this set is not ~~closed under~~ containing a zero vector.

For $\begin{bmatrix} a=0 \\ b=0 \\ c=0 \end{bmatrix}$, ~~the vector does not pass through the origin~~ we get 0 which is $\neq 1$

PAPERWORK

(iii) $H = \left\{ \begin{bmatrix} a - 2b \\ c \end{bmatrix} \right\}$ Muneeb

This is a vector space as it is closed under addition, scalar multiplication and has a zero vector.

For $a = b = c = 0$ (Checking zero vector)

$0 - 2(0) + 0 = 0$ Hence zero vector present

$k \begin{bmatrix} a - 2b \\ c \end{bmatrix} = \begin{bmatrix} ka - 2kb \\ kc \end{bmatrix}$ • Scaling remains same with k

$\begin{bmatrix} a_1 - 2b_1 \\ c_1 \end{bmatrix} + \begin{bmatrix} a_2 - 2b_2 \\ c_2 \end{bmatrix} = \begin{bmatrix} a_1 + a_2 - 4(b_1 + b_2) \\ c_1 + c_2 \end{bmatrix}$

(iv) $H = \left\{ \begin{bmatrix} a - 2 \\ c \end{bmatrix} \right\}$

This is not a vector space as it is not closed under scalar multiplication and doesn't have zero vector:

For $a = 0, c = 2$

$\begin{bmatrix} 0 - 2 \\ 0 \end{bmatrix} = -2 \neq 0$

$k\begin{bmatrix} a-2 \\ c \end{bmatrix} = \begin{bmatrix} ka & -2k \\ kc \end{bmatrix}$. If $k=0$, $H \neq 0$

Q#2 (a) $A = \begin{bmatrix} 1 & -3 & 2 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix}$

(i) $\begin{bmatrix} 1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$ Cannot be in Nul A or Col A as this vector is in $\mathbb{R}^4$ and A is in $\mathbb{R}^2$

(ii) $\begin{bmatrix} 1 \\ -1 \end{bmatrix} \Rightarrow \begin{array}{ccccc} x1 & x2 & x3 & x4 & b \\ \end{array} \left[\begin{array}{cccc|c} 1 & -3 & 2 & 0 & 1 \\ 0 & 0 & 3 & 0 & -1 \end{array}\right]$

$x3 = -1$

$x_1 - 3x2 + 2x3 = 1 \rightarrow x_1 - 3x = 3$

~~x1~~

$Ax = \vec{b}$ is consistent (does not satisfy $\{0\ 0 .. = b\}$ where $b \neq 0$) hence $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ is in Col A

$$\cancel{\left[\begin{array}{cccc|c} 1 & 3 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \end{array}\right]}$$

$A = \begin{bmatrix} 1 & -3 & 2 & 0 \\ 0 & 0 & 3 & 0 \end{bmatrix}$

$2 \times \boxed{4}$

$$\cancel{\left[\begin{array}{cccc|c} 1 & 3 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right]} \quad \cancel{R2/3} \quad \vec{u} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$\cancel{x3 = x1 + 3x2}$ $\boxed{2} \times 1$

$\cancel{x3 = 0}$

$\cancel{x1 + 3x2 = 0}$ Cannot be multiplied hence not in Null A.

(iii) $$\left[\begin{array}{cccc|c} 1 & -3 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 & 0 \end{array}\right]$$ Muneeb

$$\left[\begin{array}{cccc|c} 1 & -3 & 2 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right] R2/3$$

$x_3 = 0 \quad (1)$

$x_1 - 3x_2 + 2x_3 = 0 \quad (2)$

$x_1 - 3x_2 = 0$

$x_1 = 3x_2$

$$\text{Span} = \begin{bmatrix} \cancel{3}\, 3x_2 \\ x_2 \\ 0 \\ 0 \end{bmatrix} = x_2 \begin{bmatrix} 3 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$