

MT1003 Calculus and Analytical Geometry

Homework 01

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Due Date: September 7, 2023

1. Given the function $f(x) = \frac{x^2-4}{x-2}$, find its natural domain and identify any holes in the graph.
2. Determine the symmetry properties of the function $g(x) = x^3 - 4x^2 + 3x + 6$. Is it odd, even, or neither? Provide a mathematical proof.
3. Express the floor function $f(x) = \lfloor x \rfloor$ as a piece-wise function for $x \in \mathbb{R}$.
4. Find the inverse of the function $f(x) = \sqrt{2x+3}$ and calculate the domain and range of the inverse.
5. Given the functions $f(x) = x^2 + 3x$ and $g(x) = \frac{1}{x}$, find $(f \circ g)(x)$ and determine its domain.
6. Sketch the graph of the exponential function $y = 2^x$ and $y = 2^{-x}$ and explain its growth behavior as x approaches negative and positive infinity.
7. Compute the composition $(g \circ f)(x)$ for $f(x) = \log_{10}(x)$ and $g(x) = \sin(x)$ and specify its domain.
8. Determine the domain and range of the inverse trigonometric function $\arctan(x)$, and sketch its graph.
9. Prove whether the function $g(x) = x^4 - 3x^2 + 2$ is even, odd, or neither using mathematical reasoning and symmetry properties.
10. Find the inverse of the function $f(x) = \ln(2x-1)$ and determine its domain and range.
11. Given the functions $f(x) = \frac{x}{x^2+1}$ and $g(x) = \sqrt{3x+2}$, compute $(g \circ f)(x)$ and specify its domain.
12. For each of the following functions, describe how they are obtained by shifting and/or scaling the basic function $y = f(x)$:
 - (a) $y = 2(x-3)^2 + 4$
 - (b) $y = \sin(2x)$
 - (c) $y = e^{x/2}$