MT1004 – Linear Algebra

Homework # 11

Question # 1

Let

$$A = \begin{bmatrix} 3 & 1 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

(a) Find a least-squares solution of the inconsistent system $A\boldsymbol{x} = \boldsymbol{b}$. (Hint: Use normal equations $A^T A\boldsymbol{x} = A^T\boldsymbol{b}$.)

(b) Using $A = QR$, find the least-squares solution of the inconsistent system $A\boldsymbol{x} = \boldsymbol{b}$. (Hint: Use $R\boldsymbol{x} = Q^T\boldsymbol{b}$.)

(c) Find the least-squares error vector and the least-squares error of the inconsistent system $A\boldsymbol{x} = \boldsymbol{b}$.

(d) Without calculations, comment on the linear independency of columns of A and invertibility of $A^T A$.

(e) Find the standard matrix for the orthogonal projection on the column space of matrix A. (Hint: Use $A(A^T A)^{-1}A^T$.)

(f) Use $A = QR$ to show that the projection matrix $A(A^T A)^{-1}A^T$ can be written as QQ^T.

(g) Find the projection of $\boldsymbol{b}$ on the column space of A.

(h) Find the left inverse of A. (Hint: Use $(A^T A)^{-1}A^T$.)