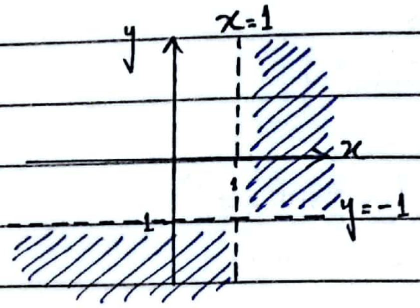
# Homework #1

1(a) $f(x,y) = \ln|xy + x - y - 1|$




$xy + x - y - 1 > 0$

$x(y+1) - 1(y+1) > 0$

$(x-1)(y+1) > 0$

At $(0,0) \to -1 > 0$ ×

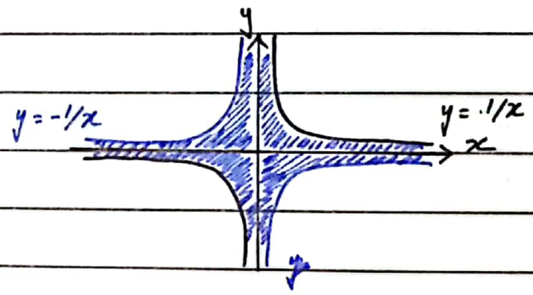
At $(0,-2) \to 1 > 0$ ✓

At $(2,0) \to 1 > 0$ ✓

$D: \{(x,y) \in R^2 \mid (x-1)(y+1) > 0\}$

$\cancel{\{(x,y) \in R^2 \mid y}$

1(b) $f(x,y) = \cos^{-1}(xy)$




$-1 \leq xy \leq 1$

$xy \leq 1$ $\qquad$ $xy \geq -1$

$y \leq \dfrac{1}{x}$ $\qquad$ $y \geq \dfrac{-1}{x}$

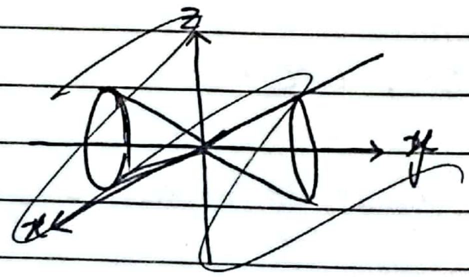
At $(0,0)$ $\quad 0 \leq 1/0$ ×

At $(0,1)$ $\quad 0 \leq 1/0$

At $(1,0)$ $\quad 0 \leq 1$

$D: \{(x,y) \in R^2 \mid -1 \leq xy \leq 1\}$

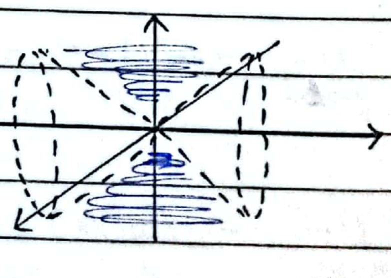
1(c) $f(x,y,z) = \ln(x^2 - y^2 + z^2)$



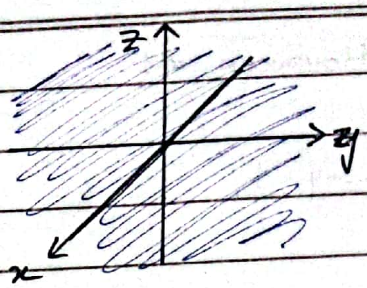

$x^2 - y^2 + z^2 > 0$

$x^2 + z^2 > y^2$

$D: \{(x,y,z) \in R^3 \mid x^2 + z^2 > y^2\}$



1(d) $f(x,y,z) = 1 - |y| - |z|$




• No restrictions

$D : \{ (x,y,z) \in R^3 \}$

1(e) $f(x,y,z) = \dfrac{\ln(x^2+y^2+z^2)}{\sqrt{z - \sin(xy)}}$

$x^2 + y^2 + z^2 > 0$

$z - \sin(xy) > 0$

$z > \sin(xy)$

$D : \left\{ \begin{array}{l} (x,y,z) \in R^3 \mid x^2+y^2+z^2 > 0 \\ (x,y,z) \in R^3 \mid \sin(xy) < z \end{array} \right\}$

3(a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{y}{x} = \dfrac{0}{0}$

Along $x = 0$:

$\lim\limits_{y\to 0} \dfrac{y}{0} = \infty$

Along $y = 0$:

$\lim\limits_{x\to 0} \dfrac{0}{x} = 0$

As $\infty \neq 0$, L.D.N.E

(b) $\lim\limits_{(x,y)\to(1,1)} \dfrac{\tan(y) - y\tan(x)}{y - x} = \dfrac{0}{0}$

Along $x = 0$:

$\lim\limits_{y\to 1} \dfrac{\tan(y) - 0}{y} = \dfrac{\tan(1)}{1}$

Along $y = 0$:

$\lim\limits_{x\to 1} \dfrac{-0\tan(x)}{x} = 0$

$\neq$ so L.D.N.E

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c. $f(x,y) = xy\left(\frac{x^2-y^2}{x^2+x^2}\right) = y\left(\frac{x^2-y^2}{2x}\right)$

$\lim_{(x,y)\to(0,0)} f(0,0) = \frac{0}{0}$

Along $x$-axis:

$$\lim_{y\to 0} \frac{y(-y^2)}{0} = \infty$$

Along $y$-axis:

$$\lim_{x\to 0} 0\left(\frac{x}{2}\right) = 0$$

As $\infty \neq 0$, L.D.N.E

d. $f(x,y) = \frac{xy^2\sin(x)}{x^2+y^2}$

$x = r\cos\theta$ , $y = r\sin\theta$

$f(r,\theta) = \frac{r\cos\theta\, r^2\sin^2\theta\,\{\sin(r\cos\theta)}{r^2\cos^2\theta + r^2\sin^2\theta} = r\cos(\theta)\sin^2(\theta)\sin(r\cos\theta)$

$\cancel{\lim_{(r,\theta)\to(0,\theta)} r(\cos(\theta)\sin^2(\theta)\sin(r\cos\theta)) = 0}$ $\lim_{r\to 0} r\cos(\theta)\sin^2(\theta)\sin(r\cos\theta) = 0$

$\cancel{\lim_{(r,\theta)\to(r,\theta)} r(\cos(\theta)\sin^2(\theta)\sin(r\cos\theta)) = 0}$

$L = 0$ , L.D.E

e. $f(x,y) = \frac{x^3\cos(y)}{x^2+2y^2}$

$x = r\cos\theta$ , $y = r\sin\theta$

$f(r,\theta) = \frac{r^3\cos^3\theta\cos(r\sin\theta)}{r^2\cos^2\theta + 2(r^2\sin^2\theta)} = \frac{r\cos^3\theta\cos(r\sin\theta)}{\cos^2\theta + 2\sin^2\theta}$

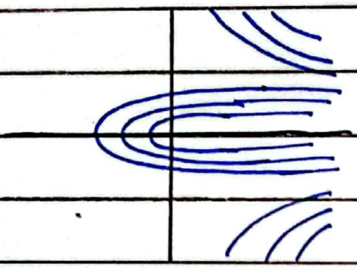
$$\lim_{(r,\theta)\to(0,\theta)} r\left[\frac{\cos^3\theta\cos(r\sin\theta)}{\cos^2\theta+2\sin^2\theta}\right]=0$$

$$\lim_{(r,\theta)\to(r,0)} \frac{r\cos^3\theta\cos(r\sin\theta)}{\cos^2\theta+2\sin^2\theta}=r\cos(r)$$

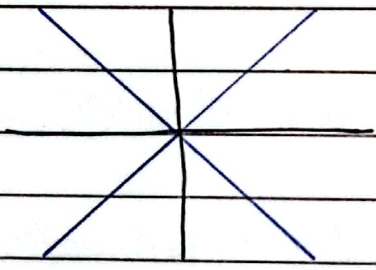
$$\lim_{(r,\theta)\to(0,0)} r\left[\frac{\cos^3\theta\cos(r\sin\theta)}{\cos^2\theta+2\sin^2\theta}\right]=0$$

$L=0$, L.D.E

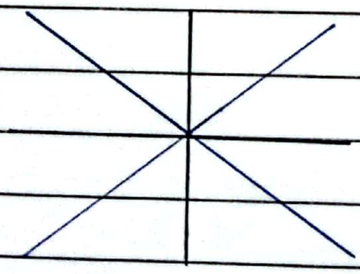
LEVEL CURVES:



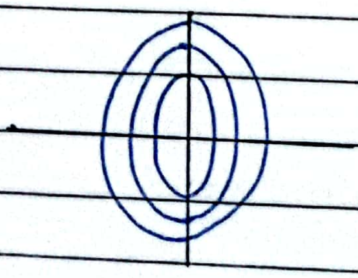
$f(x,y)=e^x\cos(y)$



$f(x,y)=xy^2-x^3$

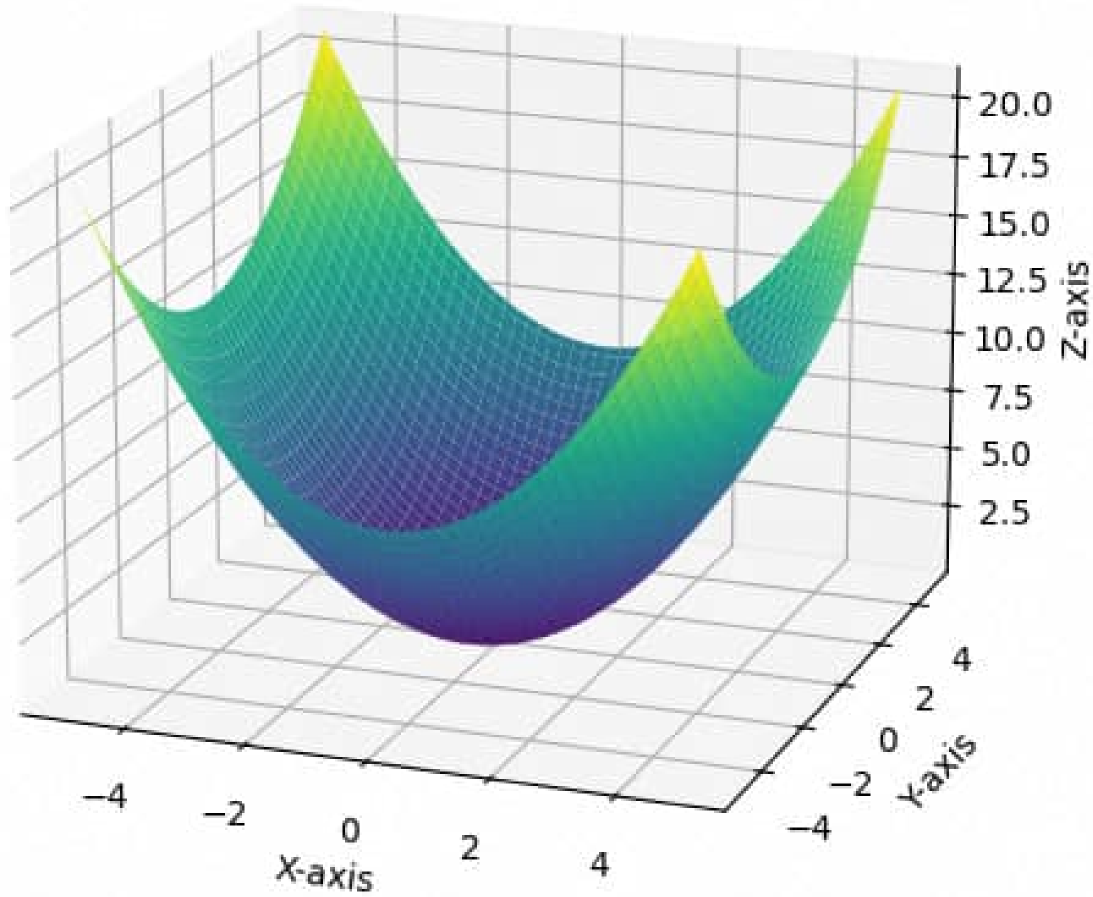


$f(x,y)=xy^3-yx^3$

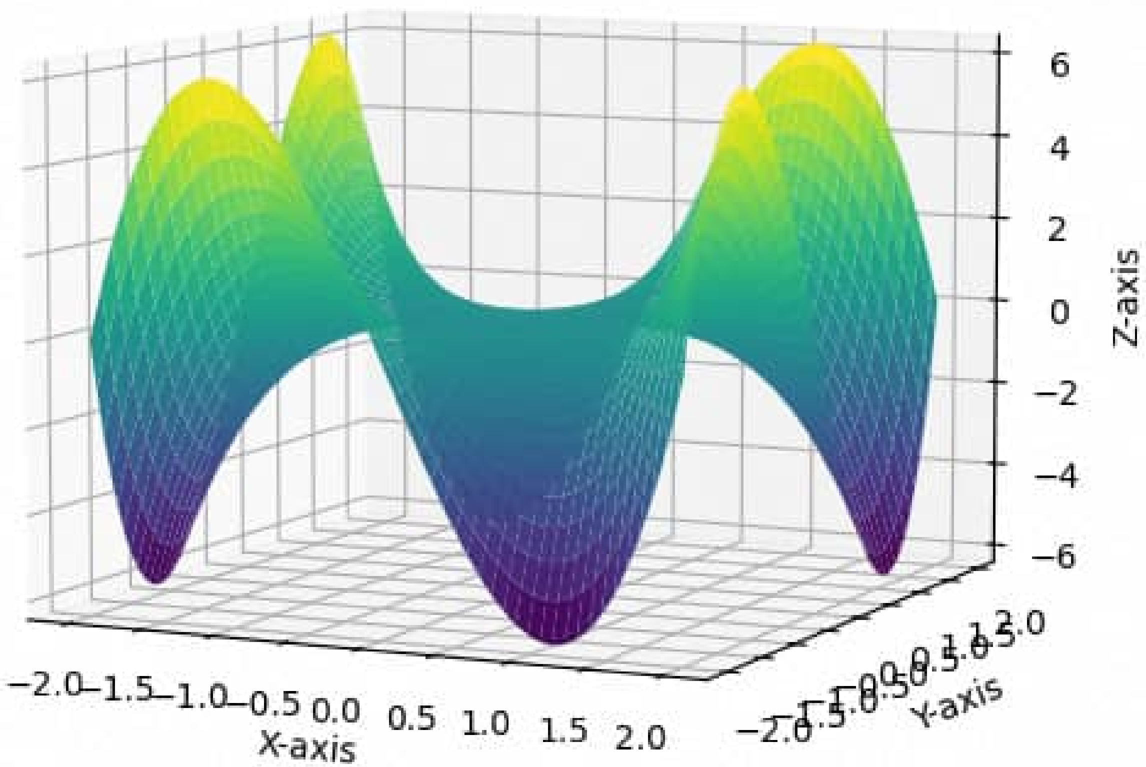


$f(x,y)=\frac{x^2}{2}+\frac{y^2}{3}$

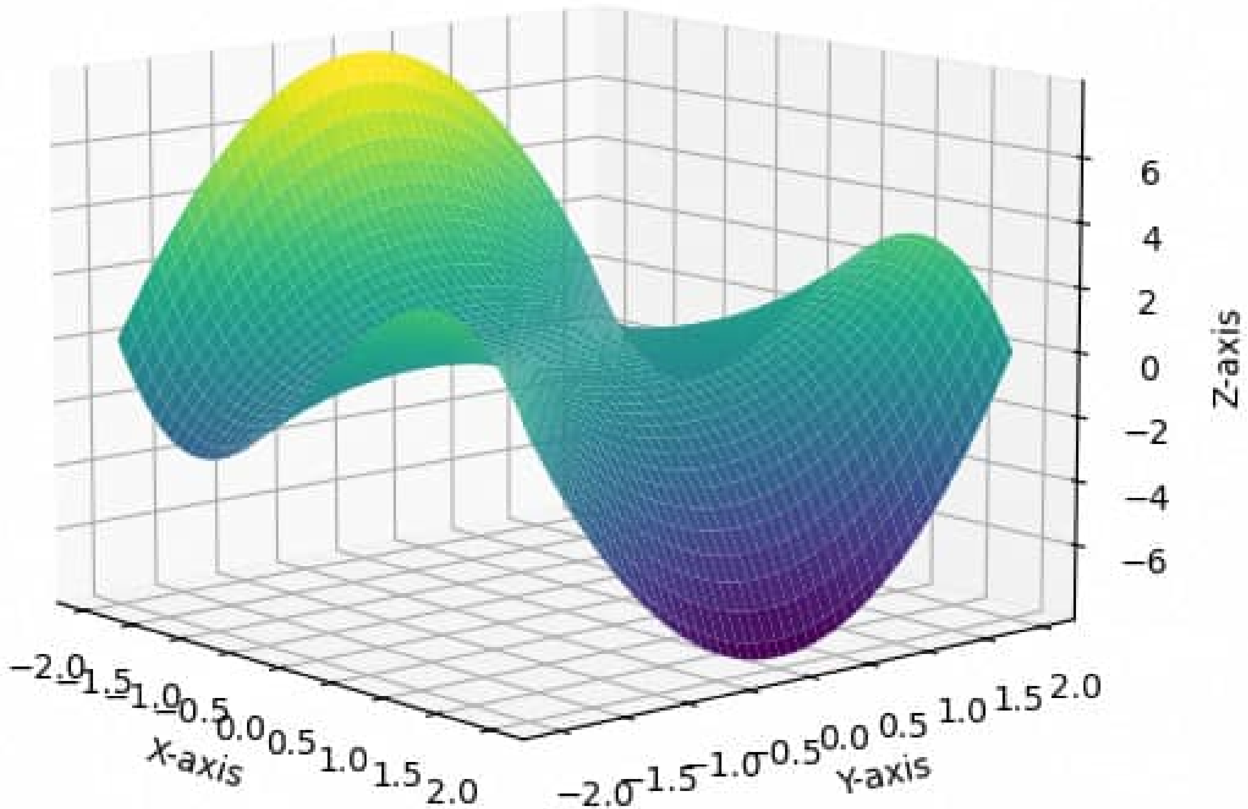
$$f(x,y) = \frac{x^2}{2} + \frac{y^2}{3}$$



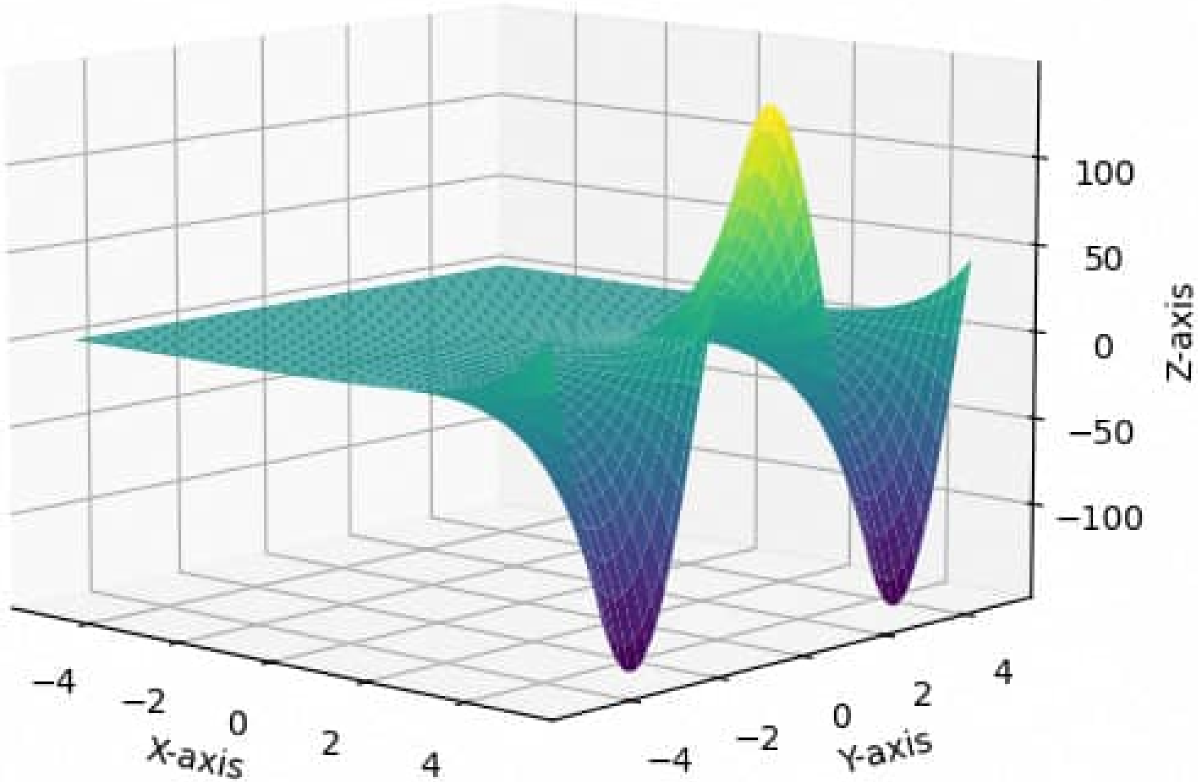
$f(x, y) = xy^3 - yx^3$ (Dog Saddle)



$f(x, y) = xy^2 - x^3$ (Monkey Saddle)



$$f(x, y) = e^x \cos(y)$$



$$f(x, y) = \frac{\sin(xy)}{x^2 + y^2}$$