Muneeb Lone
231-2623
DS-B
Day / Date

HOMEWORK #4

1(i) Let $\nabla f(1,2) = a\hat{i} + b\hat{j}$

$\underset{u_1}{\overrightarrow{P_0P_1}} = \langle 1,1 \rangle \qquad \underset{u_2}{\overrightarrow{P_0P_2}} = \langle 0,-2 \rangle$

$2\sqrt{2} = D_{u_1} f(1,2) = \nabla f(1,2) \cdot \frac{\langle 1,1 \rangle}{|\langle 1,1 \rangle|} = \langle a,b \rangle \cdot \frac{\langle 1,1 \rangle}{\sqrt{2}}$

$= \frac{a}{\sqrt{2}} + \frac{b}{\sqrt{2}}$

$-3 = D_{u_2} f(1,2) = \nabla f(1,2) \cdot \frac{\langle 0,-2 \rangle}{|\langle 0,-2 \rangle|} = \langle a,b \rangle \cdot \frac{\langle 0,-2 \rangle}{2}$

$\cancel{-}3 = \cancel{-}b$

$\frac{a}{\sqrt{2}} + \frac{3}{\sqrt{2}} = 2\sqrt{2}$

$a + 3 = 4$

$a = 1 \quad , b = 3 \Rightarrow \nabla f = \langle 1, 3 \rangle$

$P_3(0,0) \Rightarrow \overrightarrow{P_0P_3} = \langle -1, -2 \rangle$

$D_{u_3} f(1,2) = \nabla f(1,2) \cdot \frac{\langle -1,-2 \rangle}{|\langle -1,-2 \rangle|} = \langle a,b \rangle \cdot \frac{\langle -1,-2 \rangle}{\sqrt{5}}$

$= \frac{-7}{\sqrt{5}}$

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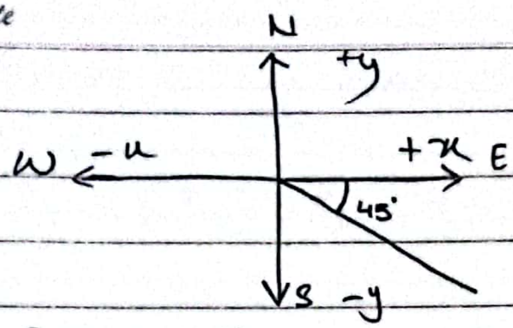
1 (ii)

~~$m_s = 4$ $m_e = \sqrt{2}$~~



$\nabla f(0,0) \cdot (-\hat{j}) = 4$ $\nabla f(0,0) \cdot \left(\frac{\hat{i} - \hat{j}}{\sqrt{2}}\right) = \sqrt{2}$

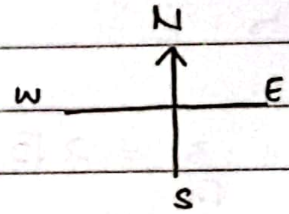
$f_y(0,0) = -4$ $\frac{f_x(0,0) - f_y(0,0)}{\sqrt{2}} = \sqrt{2}$

$f_x(0,0) + 4 = 2$

$f_x(0,0) = -2$

Slope in eastern direction $= -2$.

1 (iii) $f(x,y) = 6 - xy^2$, $P(2,1,4)$



a) $\nabla f(x,y) = -y^2\hat{i} - 2xy\hat{j} = -\hat{i} - 4\hat{j}$

$|\nabla f(x,y)| = \sqrt{1+16} = \sqrt{17}$ ← Slope

Path of steepest ascent : $\frac{-1}{\sqrt{17}}(-\hat{i} - 4\hat{j})$ or $\frac{-1}{\sqrt{17}} \langle -1, -4 \rangle$

b) $D_{\langle 0,1 \rangle} f(2,1) = D_N f(2,1) = \nabla f(2,1) \cdot \langle 0,1 \rangle = \langle -1, -4 \rangle \cdot \langle 0,1 \rangle = -4$

Slope $= 4$

Descending

c) To walk in same direction: $\nabla f \perp \hat{k}$

$\hat{k} = \frac{1}{\sqrt{17}} \langle 4, -1 \rangle$ ← Direction

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2(i) $P = xzy^2 \quad , S = x+y+z = 32$

$x = 32 - y - z \quad , P = 32y^2z - y^3z - y^2z^2$

$P_y = 64yz - 3y^2z - 2yz^2 \qquad P_z = 32y^2 - y^3 - 2y^2z$

$P_y = P_z = 0$

$0 = 64yz - 3y^2z - 2yz^2 \qquad 0 = 32y^2 - y^3 - 2y^2z$

$0 = yz(64 - 3y - 2z) \qquad 0 = y^2(32 - y - 2z)$

$yz = 0 \times \quad 64 - 3y - 2z = 0 \qquad y = 0 \times \quad 32 - y - 2z = 0$

$2z = 64 - 3y \qquad 2z = 32 - y$

$32 - y = 64 - 3y \qquad 2z = 16$

$2y = 32 \qquad z = 8$

$y = 16 \qquad (y, z)$

$(16, 8)$

$P_{yy} = 64z - 6yz - 2z^2 \qquad P_{zz} = -2y^2$

$P_{zy} = 64y - 3y^2 - 4yz$

@ $(16,8) \Rightarrow \quad P_{yy} = -384 \quad , P_{zz} = -512 \quad , P_{zy} = -256$

$P_{yy} \cdot P_{zz} - (P_{zy})^2 = (-384)(-512) - (-256)^2 = 131072 > 0$

$P_{yy} < 0$

Maximum

$x = 32 - 16 - 8$

$x = 8$

$(x, y, z) \Rightarrow (8, 16, 8)$

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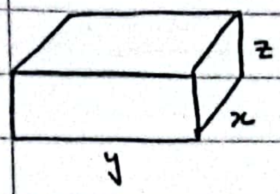
2 (ii)



$S = 2xz + 2yz + xy$

$V = xyz$

$S(x,y) = \frac{2V}{y} + \frac{2V}{x} + xy$

$S_x(x,y) = -\frac{2V}{x^2} + y$

$S_y(x,y) = -\frac{2V}{y^2} + x$

$x^2 y = 2V$ $\qquad$ $xy^2 = 2V$

$y = \frac{2V}{x^2}$ $\qquad$ $x\left(\frac{4V^2}{x^4}\right) = 2V$

$x^3 = 2V$

$y = \sqrt[3]{2V}$ $\qquad$ $x = \sqrt[3]{2V}$

$S_{xx} = \frac{4V}{x^3}$ $\qquad$ $S_{xx} = 2$

$S_{xy} = 1$ $\qquad$ @ (x,y) $\qquad$ $S_{xy} = 1$

$S_{yy} = \frac{4V}{y^3}$ $\qquad$ $S_{yy} = 2$

$S_{xx} S_{yy} - (S_{xy})^2 > 0$

$S_{xx} > 0$

$x = y = \sqrt[3]{2V}$ $\qquad$ $z = \sqrt[3]{\frac{V}{4}}$

Desired Dimensions: $\left(\sqrt[3]{2V}, \sqrt[3]{2V}, \sqrt[3]{\frac{V}{4}}\right)$

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2(iii) $\sqrt{x^2+y^2} = z$, $|x| \le 1$, $|y| \le 1$

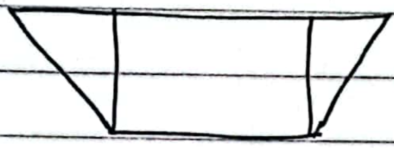
Cone shape with point at $(0,0)$

No derivates (z_x, z_y) at $(0,0)$

According to restriction: highest points $(\pm 1 \pm 1, \sqrt{2})$

Lowest points $(0, 0, 0)$

2(iv)



$$A(x,\theta) = (100-2x)x\sin\theta + 2.\frac{1}{2}x.\sin\theta.x\cos\theta$$

$$A(x,\theta) = (100x-2x^2)\sin\theta + x^2\sin\theta\cos\theta$$

$$A_x = (100-4x)\sin\theta + 2x\sin\theta\cos\theta = 0 \quad (1)$$

$$A_\theta = (100x-2x^2)\cos\theta + x^2(\cos^2\theta - \sin^2\theta) = 0 \quad (2)$$

$$\cos\theta = -\frac{100-4x}{2x} , \quad \sin^2\theta = 1-\cos^2\theta = 1-\frac{(100-4x)^2}{4x^2}$$

$$-(100-2x)\left(\frac{100-4x}{2x}\right) + x\left[\frac{(100-4x)^2}{2x^2} - 1\right] = 0$$

$$-(100-2x)(100-4x) + (100-4x)^2 - 2x^2 = 0$$

$$6x^2 = 200x$$

$$x = \frac{100}{3} , \quad \cos\theta = +\frac{1}{2} , \quad \theta = 60^\circ$$

$$\text{Max Area} = \frac{2500}{\sqrt{3}}$$

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2(v) $P_x = 2 - \frac{4x}{10^6} = 0$ (1), $\cancel{Q = 2y - \frac{4y^2+x^2}{2\times10^6}}$ $Q_y = 2 - \frac{8y}{2\times 10^6} = 0$ (2)

$x = \left(\frac{1}{2}\right)(10^6)$ $y = \left(\frac{1}{2}\right)(10^6)$

$P+Q = 2(x+y) - \frac{1}{10^6}\left(\frac{5}{2}x^2 + 3y^2\right)$

$= 10^6\left(1 + 1 - \frac{5}{8} - \frac{3}{4}\right) = \frac{5}{8}10^6$

$P+Q = f$

$f_x = 2 - \frac{5x}{10^6} = 0$ (1)

$x = \frac{2}{5}10^6$, $y = \frac{1}{3}10^6$

$f_y = 2 - \frac{6y}{10^6} = 0$ (2)

$f = 2(x+y) - \frac{1}{10^6}\left(\frac{5}{2}x^2 + 3y^2\right)$

$f = 10^6\left(\frac{4}{5} + \frac{2}{3} - \frac{2}{5} - \frac{1}{3}\right) = \frac{11}{15}10^6$