Sessional 1: Solution.

Q1: a) $m \to (e \vee p)$

b) $w \to (\neg d \vee s)$

c) $e \to a \wedge ((b \vee p) \wedge r)$

Q2:- $(\neg p \to (q \to r)), (q \to p \vee r)$ logically equiv.

p	q	r	$\neg p$	$q \to r$	$\neg p \to (q \to r)$	$p \vee r$	$q \to (p \vee r)$
T	T	T	F	T	T	T	T
T	T	F	F	F	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	F	F
F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	T

yes

(b) Prove: $p \to (q \vee r) \wedge \neg r \models p \to q$.

p	q	r	$q \vee r$	$\overset{a}{(q \vee r) \wedge \neg r}$	$p \to a$	$p \to q$
T	T	T	T	F	F	T
T	T	F	T	T	T	T
T	F	T	T	F	F	F
T	F	F	F	F	F	F
F	T	T	T	F	T	T
F	T	F	T	T	T	T
F	F	T	T	F	T	T
F	F	F	F	F	T	T

yes.

(c) $\neg p \wedge p \to q \to \neg q$

p	q	$\neg p \wedge p$	$a \to \neg a$	$a \to b$
T	T	F	F	T
T	F	F	T	T
F	T	F	F	T
F	F	F	T	T
		a	b	

tautology.

$s \leftrightarrow t \Longleftrightarrow (t \vee s) \leftrightarrow (s \wedge t)$

$\equiv (t \vee s) \longleftrightarrow (s \wedge t)$

$\equiv ((t \vee s) \rightarrow (s \wedge t)) \wedge ((s \wedge t) \rightarrow (t \vee s))$ ∴ Equivalance law

$\equiv (\neg(t \vee s) \vee (s \wedge t)) \wedge (\neg(s \wedge t) \vee (t \vee s))$ ∴ Implication law (2x)

$\equiv ((\neg t \wedge \neg s) \vee (s \wedge t)) \wedge ((\neg s \vee \neg t) \vee (t \vee s))$ ∴ DeMorgan's (2x)

$\equiv ((\neg t \wedge \neg s) \vee (s \wedge t)) \wedge ((\neg s \vee s) \vee (\neg t \vee t))$ ∴ Associative (2x)

$\equiv ((\neg t \wedge \neg s) \vee (s \wedge t)) \wedge (\text{true} \vee \text{true})$ ∴ Excluded Middle (2x)

$\equiv ((\neg t \wedge \neg s) \vee (s \wedge t)) \wedge (\text{true})$ ∴ Simplification I

$\equiv ((\neg t \wedge \neg s) \vee (s \wedge t))$ Simplification I

$\equiv (\neg s \wedge \neg t) \vee (s \wedge t)$ ∴ Commutativity

$\equiv ((\neg s \wedge \neg t) \vee s) \wedge ((\neg s \wedge \neg t) \vee t)$ ∴ Distributive

$\equiv ((\neg s \vee s) \wedge (\neg t \vee s)) \wedge ((\neg s \vee t) \wedge (\neg t \vee t))$ ∴ Distributive (2x)

$\equiv ((\text{true}) \wedge (\neg t \vee s)) \wedge ((\neg s \vee t) \wedge (\text{true}))$ ∴ Excluded Middle (2x)

$\equiv (\neg t \vee s) \wedge (\neg s \vee t)$ ∴ Simplification I (2x)

$\equiv (t \rightarrow s) \wedge (s \rightarrow t)$ ∴ Implication

$\equiv t \longleftrightarrow s$ ∴ Equivalance

$\equiv s \longleftrightarrow t$ ∴ Commutativity

Proved.

tion.

1. $p \land q \to r$
2. $r \to s$
3. $q \land \neg s$
4. $\neg s$ — 3, $\land E$
5. $\neg r$ — 2, 4, $\to E$
6. $\neg(p \land q)$ — 1, 5, $\to E$
    7. p — Assumption
    8. q — 3, $\land E$
    9. $p \land q$ — 7, 8, $\land I$
    10. false — 6, 9, $\neg E$
11. $\neg p$ — 7-10, $\neg I$