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23i-2623
DS-B

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ASSIGNMENT #1

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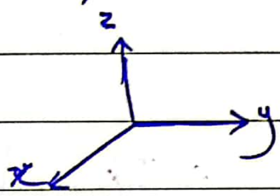
Q1: $P(2,3,1)$ across $z=0 \rightarrow$ (xy-plane)

$$\text{Standard Matrix} = \begin{array}{c} \begin{matrix} \hat{i} & \hat{j} & \hat{k} \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$



$$\hat{i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad T(\hat{i}) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \hat{j} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \quad T(\hat{j}) = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\hat{k} = \begin{bmatrix} 0 \\ 0 \\ +1 \end{bmatrix}, \quad T(\hat{k}) = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}$$

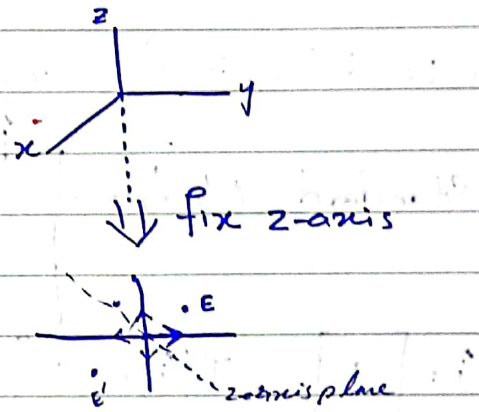
$$T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$ · Matrix of reflection across $z=0$.

$$T(\vec{P}) = T\left(\begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 2 \\ 3 \\ -1 \end{bmatrix}$$ Ans.

~~When viewed from below, the~~ This reflection basically mirrors the building along the ground thus allowing us to see how it would look when looking at it from both above and below the building.

Q2: $E = (4, 5, 2)$, Rotate 90° around the z-axis

$$\text{Std Matrix} = \begin{array}{c} \hat{i} \quad\ \hat{j} \quad\ \hat{k} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$



$$\hat{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \rightarrow T(\vec{k}) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\hat{i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \rightarrow T(\hat{i}) = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix} \qquad \hat{j} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \rightarrow T(\hat{j}) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$T = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{Transformation Matrix} = \begin{bmatrix} T(\hat{i}) \\ T(\hat{j}) \\ T(\hat{k}) \end{bmatrix} \text{ row wise}$$

$$\vec{E} = \begin{bmatrix} 4 \\ 5 \\ 2 \end{bmatrix}$$

$$T(\vec{E}) = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 2 \end{bmatrix}$$

$$T(\vec{E}) = \begin{bmatrix} -5 \\ 4 \\ 2 \end{bmatrix}$$

New Coordinates: $E' = (-5, 4, 2)$

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Q3: $A(1,2,3)$ translated by $(3,2,0)$

$\cancel{\hat{i} = \begin{bmatrix}1\\0\\0\end{bmatrix}}, \cancel{T(\hat{i}) = \begin{bmatrix}4\\0\\0\end{bmatrix}}, \cancel{\hat{j} = \begin{bmatrix}0\\1\\0\end{bmatrix}}, \cancel{T(\hat{j}) = \begin{bmatrix}0\\3\\0\end{bmatrix}}$

$\cancel{\hat{k} = \begin{bmatrix}0\\0\\1\end{bmatrix}}, \cancel{T(\hat{k}) = \begin{bmatrix}0\\0\\1\end{bmatrix}}$ $\quad A = \begin{bmatrix}1\\2\\3\end{bmatrix} \rightarrow T(A) = \begin{bmatrix}4\\4\\3\end{bmatrix}$

Not a linear transformation so we need homogenous coordinates

$$T = \begin{bmatrix}1 & 0 & 0 & d_x\\0 & 1 & 0 & d_y\\0 & 0 & 1 & d_z\\0 & 0 & 0 & 1\end{bmatrix} \qquad A_H = \begin{bmatrix}1\\2\\3\\1\end{bmatrix}$$

$$T(A_H) = \underbrace{\begin{bmatrix}1 & 0 & 0 & 3\\0 & 1 & 0 & 2\\0 & 0 & 1 & 0\\0 & 0 & 0 & 1\end{bmatrix}}_{\text{Translation Matrix.}} \begin{bmatrix}1\\2\\3\\1\end{bmatrix} = \begin{bmatrix}(1)(1)+(0)(2)+(0)(3)+(3)(1)\\(0)(1)+(1)(2)+(0)(3)+(2)(1)\\(0)(1)+(0)(2)+(1)(3)+(0)(3)\\(0)(1)+(0)(2)+(0)(3)+(1)(1)\end{bmatrix}$$

$$T(A_H) = \begin{bmatrix}4\\4\\3\\1\end{bmatrix}$$

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Q4: $B(5,7,8)$, project onto xy plane, then x-axis

T_1 : $\hat{i}$, $\hat{j}$ remain same. $\hat{k} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

T_2: $\hat{i}$, $\hat{k}$ unchanged

$\hat{j} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$B' = (5,0,0)$

Transformation Matrix: $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

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Q5: $C(2,3,4)$, First translated by $(1,0,0)$ then rotated 180° about the y-axis

Translation Matrix:
$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \overset{1}{\cancel{0}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C' = \begin{bmatrix} 1 & 0 & 0 & \overset{1}{\cancel{0}} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 4 \\ 1 \end{bmatrix}$$

Rotation Matrix:
$$\begin{bmatrix} \cos 180 & 0 & \sin 180 \\ 0 & 1 & 0 \\ -\sin 180 & 0 & \cos 180 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$C'' = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ -4 \\ 1 \end{bmatrix}$$

Final Coordinates: $(-3, 3, -4)$

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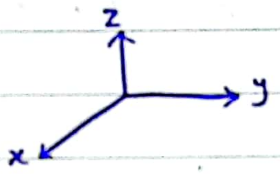
Q6: D(1, -3, 2), reflect across yz-plane



$\hat{i} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \rightarrow \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$

$\hat{j}, \hat{k}$ remain same

Transformation Matrix : $\begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$D' = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ -3 \\ 2 \end{bmatrix}$$

D(0,0,0)

Q7: Rotated 45° around x-axis then translated by (5,2,0)

After rotating at the origin, the model remains at the same place so no change.

$$D' = \begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \\ 0 \\ 1 \end{bmatrix}$$

D' = (5, 2, 0) final coordinates.

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Q8: E (3,4,5), reflected across xz-plane

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$$E' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ 5 \end{bmatrix}$$

Q9: F(2,2,2), translated by (1,1,1), rotated 90° about y-axis

$$F' = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 3 \\ 1 \end{bmatrix}$$

$$F'' = \begin{bmatrix} \cos 90 & 0 & \sin 90 \\ 0 & 1 & 0 \\ -\sin 90 & 0 & \cos 90 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 3 \\ 3 \end{bmatrix}$$

$$F'' = \begin{bmatrix} 3 \\ 3 \\ -3 \end{bmatrix}$$ Ans.

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Q10: G (0,0,0), rotated 30° about the z-axis, translated to (10,5,0) → final answer.

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Rotation at origin has no effect

$$G' = \begin{bmatrix} 1 & 0 & 0 & 10 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \\ 0 \\ 1 \end{bmatrix}$$

$G' = (10,5,0)$ Ans

Q11: H(1,1,1), rotate 90° around y-axis, then translates by (2,3,1)

$$H' = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix}$$

$$H'' = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 0 \\ 1 \end{bmatrix}$$

$H'' = (3,4,0)$ Ans.

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Q12: J(0,0,0), first translate to (4,5,0) then rotates 45° around z-axis, translates by (-2,-1,0)

We can combine transformation matrices for ease:

$$\begin{bmatrix} 4 \\ 5 \\ 0 \end{bmatrix} + \begin{bmatrix} -2 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix}$$

$$J' = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 0 \\ 1 \end{bmatrix}$$

$J' = (2,4,0)$ Ans.

Q13: K(3,4,5). Reflect across yz-plane, then rotate 180° about x-axis.

$$K' = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \\ 5 \end{bmatrix}$$

$\boxed{K'' = (-3,-4,-5) \text{ Ans}}$

$$K'' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} -3 \\ -4 \\ -5 \end{bmatrix}$$

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Q14: L(5,5,5), translated by (-3,-2,-1), then rotated 90° around x-axis, then projected onto xy-plane

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~~with rotation~~

$$L' = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 5 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 1 \end{bmatrix} = (2,3,4)$$

$$L'' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 3 \end{bmatrix}$$

$$L''' = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

L''' (2,4) final ans.

Q15: M(2,2,2), first rotate 90° around y-axis, then translated by (1,0,3), then reflected across xz-plane

$$M' = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 2 \\ -2 \end{bmatrix}$$

$$M'' = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 2 \\ -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -1 \\ 1 \end{bmatrix}$$

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$$M''' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ -1 \end{bmatrix}$$ Final as. Muneeb

Q16: P(4,5) in 2D. Translate by (3,-2)

1. Derive matrix of transformation.

$P' = (x + dx, y + dy)$
$P' = (4+3, 5-2) = (7,3)$

$$P = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \qquad T(dx, dy) = \begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{bmatrix}$$

$$T(3,-2) = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

2. Apply matrix

$$P'' = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 3 \\ 1 \end{bmatrix}$$

$P' = (7,3)$

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Q17: P(3,4) by 30° counterclockwise around the origin:

$$P' = \begin{bmatrix} \cos 30 & -\sin 30 \\ \sin 30 & \cos 30 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$P' = \begin{bmatrix} (3)(\sqrt{3}/2) + (-1/2)(4) \\ (4)(1/2) + (\sqrt{3}/2)(4) \end{bmatrix} = \begin{bmatrix} 3\sqrt{3}/2 - 2 \\ 2 + 2\sqrt{3} \end{bmatrix}$$ Ans.

Q18: P(1,0), rotate 90° counterclockwise, translate it by (2,3)

1. $R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

$R(90°) = \begin{bmatrix} \cos 90 & -\sin 90 \\ \sin 90 & \cos 90 \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$P' = (x + dx, y + dy)$

$T(dx, dy) = \begin{bmatrix} 1 & 0 & \cancel{0} & dx \\ 0 & 1 & \cancel{0} & dy \\ 0 & 0 & & 1 \end{bmatrix}$

$T(2,3) = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix}$

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2. Combined Matrix (A):

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$A = R(\theta) \cdot T(dx, dy)$

$A = R(90) \cdot T(2, 3)$

$$A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix}$$

3. $$P' = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix}$$

$P' = (2, 4)$ Ans