Day: Muneeb Lone 23i-2623 Date: / /

Homework #2

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Q1: A. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$ b= $\begin{bmatrix} 10 \\ 11 \\ 12 \end{bmatrix}$

$$\sim \left[\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 4 & 5 & 6 & 11 \\ 7 & 8 & 9 & 12 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 0 & -3 & -6 & -29 \\ 0 & -6 & -12 & -58 \end{array}\right] \begin{array}{l} \\ R2-4R1 \\ R3-7R1 \end{array}$$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ \end{array}$$
$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 10 \\ 0 & -3 & -6 & -29 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \\ \cancel{R2/3} \\ R3-2R2 \end{array}$$

$$x_1 + 2x_2 + 3x_3 = 10$$
$$-3x_2 - 6x_3 = -29$$

$\vec{b}$ is in the span of A as the system is consistent.

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Q2: $u = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$, $v = \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}$, $w = \begin{bmatrix} h \\ -3 \\ -5 \end{bmatrix}$ $w = a\vec{u} + b\vec{v}$

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$$\left[\begin{array}{cc|c} 1 & -2 & h \\ 0 & 1 & -3 \\ -2 & 7 & -5 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & -2 & h \\ 0 & 1 & -3 \\ 0 & 3 & -5+2h \end{array}\right] \quad R3 + 2R1$$

$$\left[\begin{array}{cc|c} 1 & -2 & h \\ 0 & 1 & -3 \\ 0 & 0 & -5+2h+9 \end{array}\right] \quad R3 - 3R2$$

$$\begin{array}{cc} x_1 & x_2 \end{array}$$
$$\left[\begin{array}{cc|c} 1 & -2 & h \\ 0 & 1 & -3 \\ 0 & 0 & 2h+4 \end{array}\right]$$

$x_1 - 2x_2 = h$ ①

$x_2 = -3$ ②

$\boxed{0 = 2h+4}$ ③

$x_1 + 6 = h$

$\boxed{h = \frac{-4}{2} = -2}$

$x_1 = -8$

System is only consistant when $2h+4$ evaluates to 0 which is only at $h=-2$ hence for $h=-2$ w lies in the plane generated by u and v

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Q3: $A = \begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 3 & -4 \\ 0 & 2 & 6 & 7 \\ 2 & 9 & 5 & -7 \end{bmatrix}$

(i) This matrix's span cannot be checked with $\mathbb{R}^3$ as this is ~~an $\mathbb{R}^4$ matrix~~ a matrix with each column being $\mathbb{R}^4$ vectors

(ii) $\begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 3 & -4 \\ 0 & 2 & 6 & 7 \\ 2 & 9 & 5 & -7 \end{bmatrix}$

$\begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 3 & -4 \\ 0 & 2 & 6 & 7 \\ 0 & 1 & 3 & -11 \end{bmatrix} R4 - 2R1$

$\begin{bmatrix} 1 & 4 & 1 & 2 \\ 0 & 1 & 3 & -4 \\ 0 & 0 & 0 & 15 \\ 0 & 0 & 0 & -7 \end{bmatrix} \begin{matrix} \\ \\ R3 - 2R2 \\ R4 - R2 \end{matrix}$

Not every row contains a pivot ~~and the system~~ & hence this matrix does not span all of $\mathbb{R}^4$

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(iii) As not all rows have a pivot, the equation $A\vec{x} = \vec{b}$ does not have a solution for each b in $\mathbb{R}^4$.

Q4. $A = \begin{bmatrix} 2 & 2 & 4 \\ -4 & -4 & -8 \\ 0 & -3 & -3 \end{bmatrix}$. $b = \begin{bmatrix} 6 \\ -12 \\ 0 \end{bmatrix}$

(i) $\begin{bmatrix} 2 & 2 & 4 \\ -4 & -4 & -8 \\ 0 & -3 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -12 \\ 0 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 6 \\ -4 & -4 & -8 & -12 \\ 0 & -3 & -3 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 1 & 1 & 2 & 3 \\ 0 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} R1/2 \\ -R2/4 \\ -R3/3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \quad R2 - R1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \text{Swap } R2, R3$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 3 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

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$x_1 + x_2 + 2x_3 = 3$ (i)

$x_2 + x_3 = 0$ (ii)

~~$x_2 = -x_3$~~

x_3 free variable

$x_3 = -x_2$

$x_1 - x_3 + 2x_3 = 3$

$\boxed{x_1 + x_3 = 3}$

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$$x = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

$x_3 \in \mathbb{R}$

(ii) $Ax = 0$

$$\left[\begin{array}{ccc|c} 2 & 2 & 4 & 0 \\ -4 & -4 & -8 & 0 \\ 0 & -3 & -3 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} R1/2 \\ -R2/4 \\ -R3/3 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \quad R2 - R1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \text{Swap } R2, R3$$

$x_1 + x_2 + 2x_3 = 0$

$x_2 + x_3 = 0$

x_3 free variable

$x_3 = -x_2$

$x_1 - x_3 + 2x_3 = 0$

$x_1 + x_3 = 0$

$\boxed{x_1 = -x_3}$

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$$x = x_3 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$$

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(iii) In part(i) the solution set represents a line in $\mathbb{R}^3$ which is ~~offset~~ translated away from the origin by the vector $\begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}$

In part(ii) the solution set represents a line in $\mathbb{R}3$ through the origin defined by the vector $\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

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