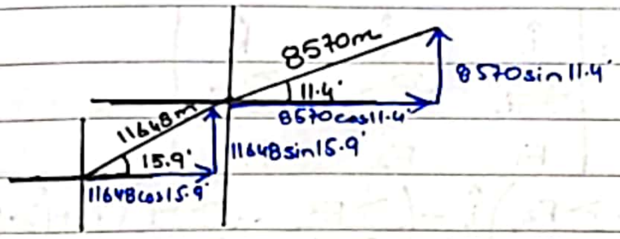
Date 3rd Sept. 2023

AP ASSIGNMENT

Q1:



↑ → : +ve
↓ ← : −v

Vertical Analysis

$R_y = 11648\sin15.9 + 8570\sin11.4$

$R_y = 4885.0m$

Horizontal Analysis

$R_x = 11648\cos15.9 + 8570\cos11.4$

$R_x = 19603.3m$

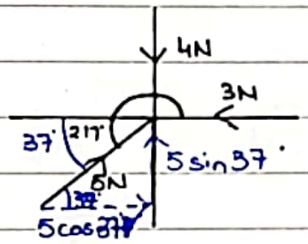
Resultant

$$R = \sqrt{(R_y)^2 + (R_x)^2}$$

$$R = \sqrt{(4885)^2 + (19603.3)^2}$$

$$R = 20202.8m$$

Q2:



↑ → : +ve
↓ ← : −ve

Vertical Analysis

$R_y = 5\sin37 - 4$

$R_y = -0.991N$

Horizontal Analysis

$R_x = 5\cos37 - 3$

$R_x = 0.993N$

Resultant

$$R = \sqrt{(R_y)^2 + (R_x)^2} = \sqrt{(0.993)^2 + (-0.991)^2}$$

$R = 1.403N$ net force

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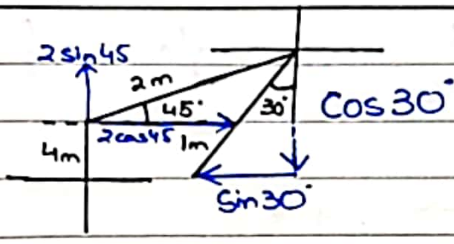
Q3:



↑ → : +ve

↓ ← : −ve

Vertical Analysis

$R_y = 4 + 2\sin45 - \cos30°$

$R_y = 4.55\,m$

Horizontal Analysis

$R_x = 2\cos45° - \sin30°$

$R_x = 0.91\,m$

Resultant

$R = \sqrt{R_y^2 + R_x^2}$

$R = \sqrt{(4.55)^2 + (0.91)^2}$

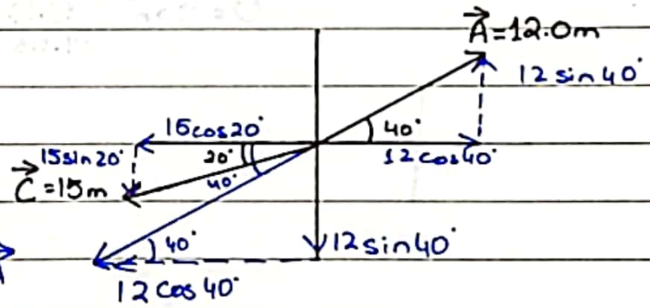
$R = 4.64\,m$

Starting at the same initial point, an expert golfer could make the hole in a single 4.64m displacement.

Q4: $\vec{A} + \vec{B} = \vec{C}$

$\vec{B} = \vec{C} - \vec{A}$

$\vec{B} = \vec{C} + (-\vec{A})$



↑ → : +ve

↓ ← : −ve

Vertical Analysis

$\vec{B}_y = -12\sin40 - 15\sin20$

$\vec{B}_y = -12.84$

Horizontal Analysis

$\vec{B}_x = -15\cos20° - 12\cos40°$

$\vec{B}_x = -23.29$

Resultant

$\vec{B} = \sqrt{B_y^2 + B_x^2}$

$\vec{B} = \sqrt{(-12.84)^2 + (-23.29)^2} = 26.59\,m$

$\tan\theta = B_y / B_x$

$\theta = \left(\frac{12.84}{23.29}\right) = 28.87°$ from −ve x-axis

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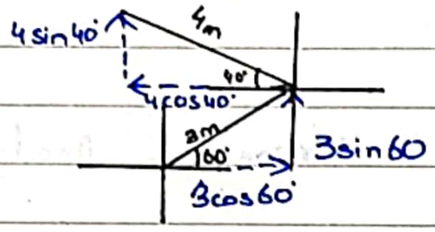
$\vec{B}$ has a magnitude of 26.59° at an angle of 28.87° from −ve x-axis.

Q5:



↑ → : +ve
↓ ← : −ve

Vertical Analysis

$R_y = 3\sin 60° + 4\sin 40°$

$R_y = 5.17$ km

Horizontal Analysis

$R_x = 3\cos 60° - 4\cos 40°$

$R_x = -1.56$ km

Resultant

$R = \sqrt{R_x^2 + R_y^2}$

$R = \sqrt{(5.17)^2 + (-1.56)^2}$

$R = 5.40$ km

$\tan\theta = \frac{R_y}{R_x}$

$\theta = \tan^{-1}\left(\frac{5.17}{-1.56}\right) = -73.21°$

~~Head~~.

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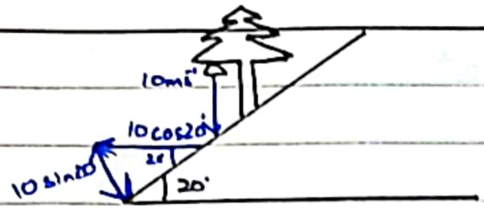
Q6:



$\uparrow \rightarrow$: $-ve$

$\downarrow \leftarrow$: $+ve$

Parallel

$10\cos 20 = P_A$

$P_A = 9.40 ms^{-1}$

Perpendicular

$10\sin 20 = P_E$

$P_E = 3.42 ms^{-1}$

Q7: $\vec{A}\cdot\vec{B} = |A||B|\cos\theta$

$14 = (6)(7)\cos\theta$

$\theta = \cos^{-1}\left(\frac{14}{(6)(7)}\right)$

$\theta = \cos^{-1}\left(\frac{1}{3}\right) = 70.53^\circ$

Q8: $\vec{F} = \cancel{q\vec{x}}\; q\vec{v} \times \vec{B}$

$4.0\hat{i} - 20.0\hat{j} + 12.0\hat{k} = (2)(2.0\hat{i} + 4.0\hat{j} + 6.0\hat{k}) \times \vec{B}$

$= (4.0\hat{i} + 8.0\hat{j} + 12.0\hat{k}) \times \vec{B}$

$$\begin{array}{ccc|ccc|ccc}
 & & & \hat{i} & \hat{j} & \hat{k} & & & \\
 & & & A1 & B1 & C1 & & & \\
V_x & V_y & V_z & V_x & V_x & V_z & V_x & V_y & V_z \\
 & & & A2 & B2 & C2 & & & \\
B_x & B_y & B_z & B_x & B_y & B_z & B_x & B_y & B_z
\end{array}$$

$= \hat{i}(B_1C_2 - B_2C_1) - \hat{j}(A_1C_2 - A_2C_1) + \hat{k}(A_1B_2 - A_2B_1)$

$= \hat{i}(8B_z - 12B_y) - \hat{j}(4B_z - 12B_x) + \hat{k}(4B_y - 8B_x)$

$\therefore$ Compare coefficients

$4.0\cancel{\hat{i}} = (8B_z - 12B_y)\cancel{\hat{i}}$

$B_z = -4$

$\cancel{-}20.0\cancel{\hat{j}} = \cancel{-}(4B_z - 12B_x)\cancel{\hat{j}}$

$B_x = B_y = B_{xy}$

$B_z = -4$

$12\cancel{\hat{k}} = (4B_y - 8B_x)\cancel{\hat{k}}$

$\cancel{B_{xy} = -3}$

$B_{xy} = -3$

$\vec{B} = -3\hat{i} - 3\hat{j} - 4\hat{k}$

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Q9 (a): $\phi(x,y,z) = 3x^2y - y^3z^2$

$\phi = 3x^2y - y^3z^2$

$\frac{\partial\phi}{\partial x} = 6xy$, $\frac{\partial\phi}{\partial y} = 3x^2 - 3y^2z^2$, $\frac{\partial\phi}{\partial z} = -2y^3z$

$\nabla.\phi = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$

$\nabla.\phi = (6xy)\hat{i} + (3x^2 - 3y^2z^2)\hat{j} - (2y^3z)\hat{k}$

(b) $F(x,y,z) = e^x \cos y \ln z$

$F = e^x \cos y \ln z = \phi$

$\frac{\partial\phi}{\partial x} = e^x \cos y \ln z$

$\frac{\partial\phi}{\partial y} = -e^x \sin y \ln z$

$\frac{\partial\phi}{\partial z} = \frac{e^x \cos y}{z}$

$\nabla.F = \frac{\partial\phi}{\partial x}\hat{i} + \frac{\partial\phi}{\partial y}\hat{j} + \frac{\partial\phi}{\partial z}\hat{k}$

$\nabla.F = (e^x \cos y \ln z)\hat{i} - (e^x \sin y \ln z)\hat{j} + \left(\frac{e^x \cos y}{z}\right)\hat{k}$

Q10(a): $\vec{v} = (xyz)\hat{i} + (3x^2y)\hat{j} + (xz^2 - y^2z)\hat{k}$

$\text{div } V = \nabla.V = \frac{\partial V_x}{\partial x}\hat{i} + \frac{\partial V_y}{\partial y}\hat{j} + \frac{\partial V_z}{\partial z}\hat{k}$

$= (yz)\hat{i} + (3x^2)\hat{j} + (2xz - y^2)\hat{k}$

(b) $\vec{u} = x^4\,\hat{i} + y^4\,\hat{j} + z^4\,\hat{k}$

$$\text{div } u = \nabla\cdot u = \frac{\partial U_x}{\partial x}\hat{i} + \frac{\partial U_y}{\partial y}\hat{j} + \frac{\partial U_z}{\partial z}\hat{k}$$

$$= (4x^3)\,\hat{i} + (4y^3)\,\hat{j} + (4z^3)\,\hat{k}$$

Q11(a): $\vec{v} = (xyz)\,\hat{i} + (3x^2y)\,\hat{j} + (xz^2 - y^2z)\,\hat{k}$

$$\text{curl } v = \nabla\times v = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_x & V_y & V_z \end{vmatrix}$$

$$\nabla\times V = \hat{i}\left(\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right) - \hat{j}\left(\frac{\partial V_z}{\partial x} - \frac{\partial V_x}{\partial z}\right) + \hat{k}\left(\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right)$$

$$= \hat{i}(-2yz - 0) - \hat{j}(z^2 - 0 - xy) + \hat{k}(6xy - xz)$$

$$= (-2yz)\,\hat{i} - (z^2 - xy)\,\hat{j} + (6xy - xz)\,\hat{k}$$

(b) $\vec{F} = (x^2 - y^2 + x)\,\hat{i} - (2xy + y)\,\hat{j} + 0\,\hat{k}$

$$\text{curl } F = \nabla\times F = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ F_x & F_y & 0 \end{vmatrix}$$

$$\frac{\partial}{\partial x} \quad \frac{\partial}{\partial y} \quad \frac{\partial}{\partial z}$$
$$F_x \quad F_y \quad 0$$

$$\nabla\times F = \hat{i}\left(\frac{\partial F_y}{\partial y} - \frac{\partial}{\partial y}\right) - \hat{j}\left(0 - \frac{\partial F_x}{\partial z}\right) + \hat{k}\left(\frac{\partial F_u}{\partial y} - \frac{\partial F_y}{\partial \cancel{Fx}}\right)$$

$$\cancel{\nabla\times F = \hat{i}(0-0) - \hat{j}(0-0) + \hat{k}(2y + 2y)}$$

$$\nabla\times F = \hat{k}\left(\frac{\partial(x^2 - y^2 + x)}{\partial y} - \frac{\partial(-2xy \mp y)}{\partial x}\right)$$

$$\cancel{\nabla\times F = 4y\,\hat{k}}$$

$$= \hat{k}(-2y + 2y)$$

$$= \hat{k}(0)$$

$$= 0 \quad \text{Ans.}$$