

Muneeb Lone  
23i-2623  
DS-B

Date: \_\_\_\_\_

Homework #7

Muneeb

$$Q1: b_1 = \begin{bmatrix} -5 \\ 1 \\ 0 \end{bmatrix} \quad b_2 = \begin{bmatrix} -2 \\ 5 \\ 0 \end{bmatrix} \quad b_3 = \begin{bmatrix} -2 \\ -5 \\ 4 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 2 & 0 \\ 1 & 1 & 3 \end{bmatrix}$$

volume of  $T(B) = (\det A)(\text{Volume of } B)$

$$\det A = 1 \times 2 \times 3 \quad [\text{Product of diagonal}] = 6$$

$$B = \begin{bmatrix} -5 & -2 & -2 \\ 1 & 5 & -5 \\ 0 & 0 & 4 \end{bmatrix}$$

$$V(B) = |\vec{b}_1 \cdot (\vec{b}_2 \times \vec{b}_3)|$$

$$= \vec{b}_1 \cdot \begin{vmatrix} i & j & k \\ -2 & 5 & 0 \\ -2 & -5 & 4 \end{vmatrix} = \vec{b}_1 \cdot \begin{bmatrix} -5 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} a_2 b_3 - a_3 b_2 \\ -a_3 b_1 - a_1 b_3 \\ a_1 b_2 - a_2 b_1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} (5)(4) - (0)(-5) \\ (0)(-2) - (-2)(4) \\ (-2)(-5) - (5)(-2) \end{bmatrix} = \begin{bmatrix} -5 \\ 1 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 20 \\ 8 \\ 0 \end{bmatrix}$$

$$V(B) = (-5)(20) + (1)(8) = |-92| = 92$$

$$\text{Volume of } T(B) = (92)(6) = 552$$

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$$Q_2: T \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x - y \\ 2x + y \end{bmatrix}$$

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$$A = \begin{bmatrix} 1 & -1 \\ 2 & 1 \end{bmatrix} \Rightarrow x \begin{bmatrix} 1 \\ 2 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix}$$

$\text{Det } A = (1)(1) - (-1)(2) = 3$   
Factor of 3: ...

$$Q_3: A = \begin{bmatrix} 1 & 2 & 3 \\ 7 & 5 & 4 \\ 8 & 7 & 10 \end{bmatrix}$$

1. Cofactor Expansion:

$$\text{Det } A = (1) \begin{vmatrix} 5 & 4 \\ 7 & 10 \end{vmatrix} - (2) \begin{vmatrix} 7 & 4 \\ 8 & 10 \end{vmatrix} + (3) \begin{vmatrix} 7 & 5 \\ 8 & 7 \end{vmatrix}$$

$$\text{Det } A = -27$$

$$\text{Cofactor Matrix } A^c = \begin{bmatrix} 22 & -38 & 9 \\ 1 & -14 & 9 \\ -7 & 17 & -9 \end{bmatrix}$$

$$\text{Adj}(A) = A^T = \begin{bmatrix} 22 & 1 & -7 \\ -38 & -14 & 17 \\ 9 & 9 & -9 \end{bmatrix}$$



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$$A^{-1} = \frac{\text{adj } A}{\det(A)} = \frac{-1}{27} \begin{bmatrix} 22 & 1 & -7 \\ -38 & -14 & 17 \\ 9 & 9 & -9 \end{bmatrix} \quad \text{Answer}$$

2. Row Operations:

$$\begin{bmatrix} \textcircled{1} & 2 & 3 & | & 1 & 0 & 0 \\ 7 & 5 & 4 & | & 0 & 1 & 0 \\ 8 & 7 & 10 & | & 0 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} \textcircled{1} & 2 & 3 & | & 1 & 0 & 0 \\ 0 & -9 & -17 & | & -7 & 1 & 0 \\ 8 & 7 & 10 & | & 0 & 0 & 1 \end{bmatrix} \quad R_2 - 7R_1$$

$$\sim \begin{bmatrix} \textcircled{1} & 2 & 3 & | & 1 & 0 & 0 \\ 0 & -9 & -17 & | & -7 & 1 & 0 \\ 0 & -9 & -14 & | & -8 & 0 & 1 \end{bmatrix} \quad R_3 - 8R_1$$

$$\sim \begin{bmatrix} \textcircled{1} & 2 & 3 & | & 1 & 0 & 0 \\ 0 & -9 & -17 & | & -7 & 1 & 0 \\ 0 & 0 & 3 & | & -1 & -1 & 1 \end{bmatrix} \quad R_3 - R_2$$

$$\sim \begin{bmatrix} \textcircled{1} & 2 & 3 & | & 1 & 0 & 0 \\ 0 & -9 & -17 & | & -7 & 1 & 0 \\ 0 & 0 & 1 & | & -1/3 & -1/3 & 1/3 \end{bmatrix} \quad R_3 \times 3$$

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$$\left[ \begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -9 & 0 & -16/3 & 2/3 & 17/3 \\ 0 & 0 & 1 & -1/3 & -1/3 & 1/3 \end{array} \right] \quad R_2 + 17R_3$$

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$$\left[ \begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 0 & 16/27 & -2/27 & -17/27 \\ 0 & 0 & 1 & -1/3 & -1/3 & 1/3 \end{array} \right] \quad -R_2/9$$

$$\left[ \begin{array}{ccc|ccc} 1 & 2 & 0 & 2 & 1 & -1 \\ 0 & 1 & 0 & 16/27 & -2/27 & -17/27 \\ 0 & 0 & 1 & -1/3 & -1/3 & 1/3 \end{array} \right] \quad R_1 - 3R_3$$

$$\left[ \begin{array}{ccc|ccc} \textcircled{1} & 0 & 0 & -22/27 & 1/9 & 7/27 \\ 0 & \textcircled{1} & 0 & 16/27 & -2/27 & -17/27 \\ 0 & 0 & \textcircled{1} & -1/3 & -1/3 & 1/3 \end{array} \right]$$

$$A^{-1} = \begin{bmatrix} -22/27 & 1/9 & 7/27 \\ 16/27 & -2/27 & -17/27 \\ -1/3 & -1/3 & 1/3 \end{bmatrix}$$

3. Write  $A$  as Elementary:

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -7 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -8 & 0 & 1 \end{bmatrix}, E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}$$



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$$E_4 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/3 \end{bmatrix}, E_5 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 17 & 1 \end{bmatrix}, E_6 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/a & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E_7 = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, E_8 = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = E_8 E_7 E_6 E_5 E_4 E_3 E_2 E_1$$

$$A = E_1^{-1} E_2^{-1} E_3^{-1} E_4^{-1} E_5^{-1} E_6^{-1} E_7^{-1} E_8^{-1}$$