National University of Computer and Emerging Sciences

Home Work
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Due Date: Sunday, October 15, 2023, 11:59PM.

MT1003 Calculus and Analytical Geometry
BS AI &BS DS

Differentiation

Q. 1 **City Revenue.** The revenue realized by a small city from the collection of fines from parking tickets is given by

$$R(n) = \frac{8000n}{n+2},$$

where n is the number of work-hours each day that can be devoted to parking patrol. At the outbreak of a flu epidemic, 30 work-hours are used daily in parking patrol, but during the epidemic that number is decreasing at the rate of 6 work-hours per day. How fast is revenue from parking fines decreasing at the outbreak of the epidemic?

Q. 2 **Compound Interest.** Suppose a sum of $\$500$ is deposited in an account with an interest rate of r percent per year compounded monthly. At the end of 10 years, the balance in the account (as illustrated in Figure 10) is given by

$$A = 500\left(1 + \frac{r}{1200}\right)^{120}.$$

Find the rate of change of A with respect to r if $r = 5$ or 7.*****

[***** Notice that r is given here as an integer percent, rather than as a decimal, which is why the formula for compound interest has 1200 where you would expect to see 12. This leads to a simpler interpretation of the derivative.]

Q. 3 Suppose that $u(t)$ measures the displacement (measured in inches) of a weight suspended from a spring t seconds after it is released and that $u(t) = 4\cos t$. Determine the maximum velocity.

Q. 4 **Tangent Line**

a. Use implicit differentiation to find dy/dx for the Folium of Descartes $x^3 + y^3 = 3xy$.

b. At what point(s) in the first quadrant is the tangent line to the Folium of Descartes horizontal?

Q. 5 **Implicit Differentiation**

a. Find x' for $x = x(t)$ defined implicitly by $t\ln x = xe^t - 1$ and evaluate x' at $(t, x) = (0,1)$.

Q. 6 **Logarithmic Differentiation**

Differentiate the following functions by logarithmic differentiation.

a.

$$y = \frac{x^{3/4}\sqrt{x^2+1}}{(3x+2)^5}.$$

b.

$$y = x^{\sqrt{x}}.$$

Q. 7 A point P moves along the x −axis in such a way that its position at time t s is given by

$$x = 2t^3 - 15t^2 + 24t\ ft.$$

a. Find the velocity and acceleration of P at time t.

b. In which direction and how fast is P moving at $t = 2\ s$? Is it speeding up or slowing down at that time?

Q. 8 A Sawtooth Curve. Let $f(x) = \sin^{-1}(\sin x)$ for all real numbers x.

a. Calculate and simplify $f'(x)$.

b. Where is f differentiable? Where is f continuous?

c. Use your results from (a) and (b) to sketch the graph of f.

d. Plot the graph of f and f' using computing tool.

Q. 9 Differentiation of Inverse Function(s)

a. Show that $f(x) = x^3 + x$ is one-to-one on the whole real line, and, noting that $f(2) = 10$, find $(f^{-1})'(10)$.

b. Evluate $f'\left(2\sqrt{3}\right) = x\tan^{-1}(x/2)$.

c. Find the derivative of $\sin^{-1}\left(\frac{x}{a}\right)$ and hence evaluate $\int \frac{dx}{\sqrt{a^2-x^2}}$ where $a > 0$.

Good Luck