

Assignment # 1

Application-Based Geometric Transformations in \mathbb{R}^3

Submission Guidelines

- Solve the following application-based questions related to geometric transformations in \mathbb{R}^3 . Provide detailed solutions and explanations for each question.
- Provide detailed solutions to each question, including any relevant mathematical derivations and transformation matrices used.
- Use diagrams where applicable to illustrate your answers, especially for reflection and rotation transformations.
- Submit your solutions in a PDF format by the due date.

Questions

Question # 1: Architectural Design

An architect is designing a building and needs to reflect the initial design point $P(2, 3, 1)$ across the plane defined by $z = 0$. Find the matrix of transformation and this reflection helps visualize how the building's design will appear when viewed from below.

Question # 2: Robotics Movement

A robotic arm's end effector is currently positioned at $E(4, 5, 2)$. The arm rotates 90° around the z -axis to reach a new position. Determine the new coordinates of the end effector after the rotation. This movement allows the robotic arm to access different areas, enhancing its operational range. Find the matrix for this transformation.

Question # 3: Animation Framework

In a 3D animation, a character is at position $A(1, 2, 3)$ and is moved to simulate walking by translating the character by the vector $(3, 2, 0)$. What will the new coordinates of the character

This translation helps animate the character's movement, making the scene more dynamic. Find the translation matrix.

Question # 4: Virtual Reality Rendering

A point in a virtual environment $B(5, 7, 8)$ needs to be projected onto the xy -plane and then on x -axis for a simplified rendering. Calculate the coordinates of the projected point B' . Find the matrix of transformation.

Question # 5: Game Development

In a game, a game object located at $C(2, 3, 4)$ is first translated by $(1, 0, 0)$ to simulate movement and then rotated 180° about the y -axis to face the opposite direction. Find the final coordinates of the object after these transformations.

Question # 6: Computer Graphics Design

A designer wants to create a mirror effect for an object at $D(1, -3, 2)$ by reflecting it across the yz -plane. Calculate the coordinates of the reflected object D' and this creates a mirror effect, improving visual fidelity. Find the matrix of transformation.

Question # 7: 3D Modeling Software

A 3D model of a car at the origin $(0, 0, 0)$ is rotated 45° around the x -axis to give it a tilt. After the rotation, the model is translated to a new position defined by $(5, 2, 0)$. What are the final coordinates of the car model?

Question # 8: Simulation of Light Reflection

In a simulation, a light source at $E(3, 4, 5)$ needs to be reflected across a mirror located in the xz -plane. Calculate the coordinates of the reflected light source E' .

Question # 9: Augmented Reality Application

In an augmented reality application, a point $F(2, 2, 2)$ representing an object is first translated by $(1, 1, 1)$ to simulate user interaction, then rotated 90° about the y -axis to change its orientation. What are the final coordinates of the object after these transformations.

Question # 10: Architectural Visualization

An architectural model at $G(0, 0, 0)$ is rotated 30° about the z -axis to visualize it from a different angle, and then translated to $(10, 5, 0)$ for presentation purposes. Calculate the final coordinates of the model.

Question # 11: Composite Movement in Robotics

A robotic arm is at position $H(1, 1, 1)$. First, it rotates 90° around the y -axis, and then it translates by the vector $(2, 3, 1)$. Determine the final coordinates of the end effector after both transformations.

Question # 12: Animation Path

An animated character starts at position $J(0, 0, 0)$. The character first translates to $(4, 5, 0)$, then rotates 45° around the z -axis, and finally translates by $(-2, -1, 0)$. What are the final coordinates of the character?

Question # 13: Camera View Adjustment

A camera in a 3D scene is positioned at $K(3, 4, 5)$. It first reflects across the yz -plane and then rotates 180° about the x -axis. Find the new position of the camera after these transformations.

Question # 14: Building Model Transformation

A building model is originally located at $L(5, 5, 5)$. It is first translated by $(-3, -2, -1)$, then rotated 90° around the x -axis, and finally projected onto the xy -plane. Determine the final position of the model.

Question # 15: Object Manipulation in VR

An object at position $M(2, 2, 2)$ undergoes the following transformations: first, it is rotated 90° around the y -axis, then it is translated by $(1, 0, 3)$, and finally, it is reflected across the xz -plane. What are the final coordinates of the object?

Question # 16: Image Translation

Consider the point $P(4, 5)$ in 2D space. Translate this point by the vector $(3, -2)$.

1. Derive the transformation matrix for this translation.
2. Apply the matrix to find the new coordinates of the point after translation.

Question # 17: Image Rotation

Rotate the point $P(3, 4)$ by 30° counterclockwise around the origin.

1. Derive the transformation matrix for a 30° rotation.
2. Use the matrix to compute the new coordinates of the point after rotation.

Question # 18: Composite Transformation (Rotation + Translation)

First, rotate the point $P(1, 0)$ by 90° counterclockwise, then translate it by $(2, 3)$.

1. Derive the rotation matrix for 90° and the translation matrix.
 2. Combine these matrices to create a composite transformation matrix.
 3. Apply this composite matrix to find the final coordinates of the point.
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