National University of Computer & Emerging Sciences

HW#3

Q.1) (9 points) Find and classify the relative extrema and saddle points of the following:

(i) $f(x, y) = x^2 - 6x - x\sqrt{y} + y$

(ii) $f(x, y) = -4y/(x^2 + y^2 + 1)$

(iii) $f(x, y) = \sin x + \sin y \quad 0 \le x \le 2\pi,\ 0 \le y \le 2\pi$

Q.2) (10 points)

(i) Find the points on the surface $z^2 = xy - x + 4y + 21$ that are closest to the origin. What is the shortest distance from the origin to the surface?

(ii) Find the dimensions of an open rectangular box of maximum volume that can be constructed from 48 ft^2 of cardboard?

Q.3) (10 points) Find the directional derivative of the function f at the point in the direction of the vector.

(i)) $f(x, y, z) = \sqrt{(xy^2 + 6y^2z^2)}$; $P(2, 3, -1)$, $v = 2i - k$

(ii) $f(x, y, z) = e^x(2\cos y + 3\sin z)$; $P(1, \pi/6, \pi/6)$ $v = 2i - j + 3k$

Q.4) (10 points) find a vector giving the direction in which the function increases and decreases most rapidly at the point. What is the maximum rate of increase and decrease?

(i) $f(x, y, z) = \ln(x^2 + 2y^2 + 3z^2)$; $P(1, 2, -1)$

(ii) $f(x, y, z) = \sqrt{xy}\cos z$; $P(4, 1, \pi/4)$

Q.5) (5 points) find the absolute extrema of the function over the region.

(i) $f(x, y) = 12 - 3x - 2y$ R: The triangular region in the plane with vertices (2,0),(0,1) and (1,2).