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MT1003 Calculus and Analytical Geometry

1. $f(x) = \dfrac{5x^3 - 3x^2 + 2x - 1}{2x^3 - x^2 + 4}$

Problem 1:

(a) Taking ∞ as Limit:

$$\lim_{x \to \infty} f(x) = \frac{5(\infty)^3 - 3(\infty)^2 + 2(\infty) - 1}{2(\infty)^3 - (\infty)^2 + 4} = \frac{\infty}{\infty}$$

$$\lim_{x \to \infty} f(x) = \frac{5(\infty)^3(\infty)^{-3} - 3(\infty)^2(\infty)^{-3} + 2(\infty)(\infty)^{-3} - 1(\infty)^{-3}}{2(\infty)^3(\infty)^{-3} - (\infty)^2(\infty)^{-3} + 4(\infty)^{-3}}$$

$$\lim_{x \to \infty} f(x) = \frac{5}{2}$$

Horizontal asymptote exists at $y = \frac{5}{2}$ as it approaches that value as $x = \infty$ but never reaches it.

(b) $f(x) = \dfrac{5x^3 - 2x^2 + 2x - 1}{2x^3 - x^2 + 4}$

$2x^3 - x^2 + 4 = 0$

$x \approx -1.113$ and x = Two imaginary solutions

Function is undefined at approximately $x \approx -1.113$ so verticle asymptote exists there.

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(c) $f(x) = \dfrac{5x^3 - 3x^2 + 2x - 1}{2x^3 - x^2 + 4}$ $5/2$

$$2x^3 - x^2 + 4 \,\overline{)\, \cancel{5x^3} - 3x^2 + 2x - 1} \qquad Q_1 = \frac{5\cancel{x^3}}{2\cancel{x^3}}$$

$$- \cancel{5x^3} + \frac{5x^2}{2} \mp 10$$

Long division does not produce linear equation so no diagonal asymptote

Problem 2:

$g(x) = \dfrac{\sin x}{x}$

Horizontal Asymptote

$\lim_{x \to \infty} f(x) = \dfrac{\sin(\infty)}{\infty} = \infty$

There is no horizontal asymptote due to oscillations and also due to it being continuous

Verticale Asymptote:

$x \neq 0$

$g(x) = \dfrac{\sin x}{x}$

$g(0) = \dfrac{\sin 0}{0} = \infty$

~~So~~ The ~~verticle~~ vertical asymptote exists at $x = 0$ or at the y-axis as the graph never touches the axis.

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Problem 3:

$$h(x) = \frac{3x^2 - 2x + 1}{x^2 + 4x + 4}$$

(a) $\lim_{x \to \infty} h(x) = \frac{3(\infty)^2 - 2(\infty) + 1}{(\infty)^2 + 4(\infty) + 4} = \frac{\infty}{\infty}$

$$\lim_{x \to \infty} h(x) = \frac{3\cancel{(\infty)^2}\cancel{(\infty)^{-2}} - 2(\infty)(\infty)^{-2} + (1)(\infty)^{-2}}{(\infty)^2(\infty)^{-2} + 4(\infty)(\infty)^{-2} + 4(\infty)^{-2}}$$

$$\lim_{x \to \infty} h(x) = \frac{3 - 2x^{-1} + 1x^{-2}}{1x^0 + 4x^{-1} + 4x^{-2}}$$

$$\lim_{x \to \infty} h(x) = 3$$

Horizontal asymptote exists at $h(x) = 3$ as the graph approaches this value at infinity and never reaches it.

(b) $h(x) = \frac{3x^2 - 2x + 1}{x^2 + 4x + 4}$

$x^2 + 4x + 4 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = -2$

At $x = -2$, $h(x)$ is undefined hence vertical asymptote exists at $x = -2$.

Problem 4:

(a) $\lim\limits_{x \to 0} x^2 \sin\left(\frac{1}{x}\right)$

$$-1 \leq \sin \frac{1}{x} \leq 1$$

$$-x^2 \leq x^2 \sin \frac{1}{x} \leq x^2$$

$$\lim_{x \to \cancel{-\infty}} -x^2 \leq x^2 \sin \frac{1}{x} \leq \lim_{x \to 0} x^2$$

$$\lim_{x \to 0} 0 \leq x^2 \sin \frac{1}{x} \leq 0$$

As $\lim\limits_{x \to 0} -x^2 = \lim\limits_{x \to 0} x^2 = 0$;

$$\lim_{x \to 0} x^2 \sin \frac{1}{x} = 0$$

(b) $\lim\limits_{x \to \infty} \frac{\ln(x)}{x}$

$$-\infty \leq \frac{1}{x} \leq \infty$$

$$-(\ln(x))(+\infty) \leq \frac{\ln(x)}{x} \leq \cancel{\infty} \ln(x)\infty$$

$$\lim_{x \to \infty} -(\ln x)(\infty) \leq \frac{\ln(x)}{x} \leq \lim_{x \to \infty} \ln(x)\infty$$

$$-\infty \leq \frac{\ln x}{x} \leq \infty$$

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) $\lim_{x\to 0} x^2\cos\left(\frac{1}{x}\right)$

$$-1 \leq \cos\left(\frac{1}{x}\right) \leq 1$$

$$-x^2 \leq x^2\cos\left(\frac{1}{x}\right) \leq x^2$$

$$\lim_{x\to 0} -x^2 \leq \lim_{x\to 0} x^2\cos\left(\frac{1}{x}\right) \leq \cancel{x} \lim_{x\to 0} x^2$$

$$0 \leq \lim_{x\to 0} x^2\cos\left(\frac{1}{x}\right) \leq 0$$

As $\lim_{x\to 0} -x^2 = \lim_{x\to 0} x^2 = 0$;

$$\lim_{x\to 0} x^2\cos\left(\frac{1}{x}\right) = 0$$