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23i-2623
DS-B

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HOMEWORK #10 Muneeb

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & -5 & 4 \end{pmatrix} \qquad A-\lambda I = \begin{pmatrix} -\lambda & 1 & 0 \\ 0 & -\lambda & 1 \\ 2 & -5 & 4-\lambda \end{pmatrix}$$

$$\det(A-\lambda I) = 0 = -\lambda \begin{vmatrix} -\lambda & 1 \\ -5 & 4-\lambda \end{vmatrix} - 1 \begin{vmatrix} 0 & 1 \\ 2 & 4-\lambda \end{vmatrix} + 0 \begin{vmatrix} 0 & -\lambda \\ 2 & -5 \end{vmatrix}$$

$$-\lambda^3 + 4\lambda^2 - 5\lambda + 2 = 0$$

$$\lambda_1 = 2, \quad \lambda_2 = \lambda_3 = 1$$

For $\lambda = 2$: $(A-2I)\vec{v} = \vec{0}$

$$\left[\begin{array}{ccc|c} -2 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 2 & -5 & \cancel{4-\lambda}\ 2 & 0 \end{array}\right]$$

$-2x_1 + x_2 = 0$ — ①

$-2x_2 + x_3 = 0$ — ②

$\boxed{x_3 = x_3}$ — ③

$$\left[\begin{array}{ccc|c} -2 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & -4 & 2 & 0 \end{array}\right] R3+R1$$

$\boxed{x_2 = x_3/2}$

$-2x_1 + \frac{x_3}{2} = 0$

$\frac{-4x_1 + x_3}{2} = 0$

$$\begin{array}{cccc} x_1 & x_2 & x_3 & \\ \end{array}$$
$$\sim \left[\begin{array}{ccc|c} -2 & 1 & 0 & 0 \\ 0 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] R3-2R1$$

$\boxed{x_1 = \frac{x_3}{4}}$

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$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} x_3/4 \\ x_3/2 \\ x_3 \end{pmatrix} = x_3 \begin{pmatrix} 1/4 \\ 1/2 \\ 1 \end{pmatrix}$$

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Eigenvector : $\begin{pmatrix} 1/4 \\ 1/2 \\ 1 \end{pmatrix}$

For $\lambda = 1$:

$(A - \lambda)\vec{v} = \vec{0}$

$$\left[\begin{array}{ccc|c} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 2 & -5 & 3 & 0 \end{array}\right]$$

1. $-x_1 + x_2 = 0$
2. $-x_2 + x_3 = 0$
3. $x_3 = x_3$

$$\left[\begin{array}{ccc|c} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & -3 & 3 & 0 \end{array}\right] R3 + 2R1$$

$\boxed{x_2 = +x_3}$

$-x_1 + x_2 = 0$

$\boxed{x_1 = x_3}$

$$\left[\begin{array}{ccc|c} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] R3 - 3R1$$

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$$\vec{x} = \begin{pmatrix} \vec{x_1} \\ \vec{x_2} \\ \vec{x_3} \end{pmatrix} = \begin{pmatrix} x_3 \\ x_3 \\ x_3 \end{pmatrix} = x_3 \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\text{Eigenvector} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

	AM	G.M
$\lambda = 1$	2	1
$\lambda = 2$	1	1
Σ	3	2

- The matrix is not diagonalizable as $\Sigma\, A.M \neq \Sigma\, G.M$

Q2: $A = \begin{pmatrix} -1 & 0 & 1 \\ 3 & 0 & -3 \\ 1 & 0 & -1 \end{pmatrix}$ $\quad A - \lambda I = \begin{pmatrix} -1-\lambda & 0 & 1 \\ 3 & -\lambda & -3 \\ 1 & 0 & -1-\lambda \end{pmatrix}$

$$\det(A - \lambda I) : 0 = -1-\lambda \begin{vmatrix} -\lambda & -3 \\ 0 & -1-\lambda \end{vmatrix} \mp 0 \begin{vmatrix} 3 & -3 \\ 1 & -1-\lambda \end{vmatrix} + 1 \begin{vmatrix} 3 & -\lambda \\ 1 & 0 \end{vmatrix}$$

$$-\lambda^3 - 2\lambda = 0 \mapsto \lambda^2 = 0 \;,\; \lambda_3 = -2$$

$$\lambda_1 = 0, \lambda_2 = 0 \;,\; \lambda_3 = -2$$

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For $\lambda=0$:

$$\left[\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 3 & 0 & -3 & 0 \\ 1 & 0 & -1 & 0 \end{array}\right]$$

$$\sim \left[\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \\ R2+3R1 \\ R3+R1 \end{array}$$

$-x_1 + x_3 = 0$ $\quad$ $x_3 = x_3$ $\quad$ $x_2 = x_2$

$x_3 = x_1$ $\quad$ ~~$x_3=3$~~

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = x_2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$$

Eigenvectors: $\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$

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For $\lambda = -2$

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$$\begin{pmatrix} -1 & 0 & 1 \\ 3 & 0 & -3 \\ 1 & 0 & -1 \end{pmatrix} - \begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 1 \\ 3 & 2 & -3 \\ 1 & 0 & 1 \end{pmatrix}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 2 & -6 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} \\ R2-3R1 \\ R3-R1 \end{array}$$

$x_1 + x_3 = 0$ $\quad$ $2x_2 - 6x_3 = 0$ $\quad$ $x_3 = x_3$

~~$x_1 + x_2$~~ $\quad$ $x_2 = 3x_3$

$x_3 = -x_1$

$$\vec{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -x_3 \\ 3x_3 \\ x_3 \end{pmatrix} = x_3 \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}$$

Eigenvector $= \begin{pmatrix} -1 \\ 3 \\ 1 \end{pmatrix}$

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	A.M	G.M
$\lambda = 0$	2	2
$\lambda = -2$	1	1
Σ	3	3

As $\Sigma A.M = \Sigma G.M$, this matrix is diagonalizable.