Homework#2

I. (12 Points) Questions on Limits and Continuity

a. Show that $\lim_{(x,y)\to(0,0)} f(x,y)$ with $f(x,y) = \frac{(2x-y)^2}{x-y}$ does not exist.

b. Evaluate if possible

i. $\lim_{(x,y)\to(0,0)} \frac{(sinx)(e^y-1)}{xy}$

ii. $\lim_{(x,y)\to(0,0)} |y|^x$

iii. $\lim_{(x,y)\to(2,1)} \frac{(x-2)(y-1)}{(x-2)^2+(y-1)^2}$

iv. $\lim_{(x,y)\to(0,1)} \tan^{-1}(\frac{x^2+1}{x^2+(y-1)^2})$

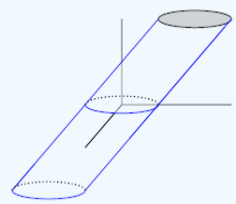
c. Discuss the continuity of the function

$$f(x,y) = \begin{cases} \dfrac{\sin(x^2-y^2)}{x^2-y^2}, & x^2 \neq y^2 \\ 1, & x^2 = y^2 \end{cases}$$

II. (16) Questions on Partial derivatives

a.

Consider the cylinder whose base is the radius-1 circle in the xy-plane centred at $(0,0)$, and which slopes parallel to the line in the yz-plane given by $z = y$.



When you stand at the point $(0,-1,0)$, what is the slope of the surface if you look in the positive y direction? The positive x direction?

b. For the function $f(x,y) = \begin{cases} \frac{cosx-cosy}{x-y} & if\ x \neq y \\ 0 & if\ \ x = y. \end{cases}$

i. Find $f_x(0,0)$.

ii. Find $f_y(x,y)$.

c. Let $f(x,y) = x^2 + y^3$, find the slope of the line tangent to the surface at $(-1,1,2)$, lying in

i. $x = -1$

ii. $y = 1$

d. Does a function $f(x,y)$ with continuous first partials derivatives throughout an open region R have to be continuous on R? Support your answer with the help of an example.

e. For the function $f(x,y) = \begin{cases} xy\frac{x^2-y^2}{x^2+y^2}, & if\ (x,y) \neq (0,0) \\ 0, & if\ \ (x,y) = (0,0) \end{cases}$

i. Show that $f_x(x,0) = x$ and $f_y(0,y) = -y$.

ii. Show that $f_{xy}(0,0) \neq f_{yx}(0,0)$.