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23i-2623
DS-B

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HOMEWORK #9

Q1: $A = \begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}$

① Eigenvalues: $\lambda^2 - tr(A)\lambda + \det A = 0$

$\lambda^2 - 1\lambda - 6 = 0$

$\lambda_1 = 3 \quad , \quad \lambda_2 = -2$

② $\lambda_1 = 3$ Case:

$A - 3I = \begin{pmatrix} -1 & 1 \\ 4 & -4 \end{pmatrix}$

$$\begin{pmatrix} -1 & 1 \\ 4 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$A\left[\begin{array}{cc|c} -1 & 1 & 0 \\ 4 & -4 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 4 & -4 & 0 \end{array}\right] \begin{array}{l} -R1 \\ R2-4R1 \end{array} \sim \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$x - y = 0$

$x = y$

$\vec{V_1} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ x \end{pmatrix} = x\begin{pmatrix} 1 \\ 1 \end{pmatrix}$

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$\lambda_2 = -2$ Case:

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$$A + 2I = \begin{pmatrix} 4 & 1 \\ 4 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 1 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\left[\begin{array}{cc|c} 4 & 1 & 0 \\ 4 & 1 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 4 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right] R2-R1$$

$$4x + y = 0$$
$$y = -4x$$

$$\vec{v_2} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -4x \end{pmatrix} = x\begin{pmatrix} 1 \\ -4 \end{pmatrix}$$

(a) $\begin{pmatrix} 2 & 1 \\ 4 & -1 \end{pmatrix}^{10}\begin{pmatrix} 5 \\ 1 \end{pmatrix}$

$A = PDP^{-1}$

$P = \begin{pmatrix} 1 & 1 \\ 1 & -4 \end{pmatrix}$, $D = \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$, $P^{-1} = \begin{pmatrix} +4/5 & +1/5 \\ +1/5 & -1/5 \end{pmatrix}$

$$\cancel{(A)^{10}} = \begin{pmatrix} 1 & 1 \\ 1 & -4 \end{pmatrix}\begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}^{10}\begin{pmatrix} 4/5 & 1/5 \\ 1/5 & -1/5 \end{pmatrix}\begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

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$$=\begin{pmatrix}1 & 1\\ 1 & -4\end{pmatrix}\begin{pmatrix}59049 & 0\\ 0 & 1024\end{pmatrix}\begin{pmatrix}4/5 & 1/5\\ 1/5 & -1/5\end{pmatrix}\begin{pmatrix}5\\ 1\end{pmatrix}$$

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$$=\begin{pmatrix}59049 & 1024\\ 59049 & -4096\end{pmatrix}\begin{pmatrix}4/5 & 1/5\\ 1/5 & -1/5\end{pmatrix}\begin{pmatrix}5\\ 1\end{pmatrix}$$

$$=\begin{pmatrix}(59049)(4/5)+(1024)(1/5) & (59049)(1/5)+(1024)(-1/5)\\ (59049)(4/5)+(-4096)(1/5) & (59049)(1/5)+(-4096)(-1/5)\end{pmatrix}\begin{pmatrix}5\\ 1\end{pmatrix}$$

$$=\begin{pmatrix}47452 & 47042.4\\ 46428 & 48066.4\end{pmatrix}\begin{pmatrix}5\\ 1\end{pmatrix}=\begin{pmatrix}(47452)(5)+(47042.4)(1)\\ (46428)(5)+(48066.4)(1)\end{pmatrix}$$

$$=\begin{pmatrix}284302.4\\ 280206.4\end{pmatrix}$$

(b) Diagonalization: $PDP^{-1}=A$

$$\begin{pmatrix}1 & 1\\ 1 & -4\end{pmatrix}\begin{pmatrix}3 & 0\\ 0 & -2\end{pmatrix}\begin{pmatrix}4/5 & 1/5\\ 1/5 & -1/5\end{pmatrix}=\begin{pmatrix}(1)(3)+(1)(0) & (1)(0)+(1)(-2)\\ (1)(3)+(-4)(0) & (1)(0)+(-4)(-2)\end{pmatrix}$$

$$=\begin{pmatrix}3 & -2\\ 3 & 8\end{pmatrix}$$

(c) $A^{10}=PD^{10}P^{-1}=\begin{pmatrix}1 & 1\\ 1 & -4\end{pmatrix}\begin{pmatrix}3 & 0\\ 0 & -2\end{pmatrix}^{10}\begin{pmatrix}4/5 & 1/5\\ 1/5 & -1/5\end{pmatrix}=\begin{pmatrix}47452 & 47042.4\\ 46428 & 48066.4\end{pmatrix}$