Muneeb Lone
DS-B
23i-2623
Day / Date

# HOMEWORK 5

Q1① $f(x,y) = x^2+4y^2-2x+8y$ , $x+2y=7 \Rightarrow g(x,y) = x+2y-7$

$\nabla f(x,y) = (2x-2)\hat{i} + (8y+8)\hat{j}$ $\nabla g(x,y) = \hat{i} + 2\hat{j}$

$(2x-2)\hat{i} + (8y+8)\hat{j} = \lambda(\hat{i}+2\hat{j})$

① $2x-2 = \lambda$ ② $8y+8 = 2\lambda$ $\rightarrow$ $8y+8 = 16$

$2(x-1) = \lambda$ $8(y+1) = 2\lambda$ $y+1 = \frac{16}{8} = 1$

$x = \frac{\lambda}{2}+1$ $y = \frac{\lambda}{4}-1$

$g\left(\frac{\lambda}{2}+1, \frac{\lambda}{4}-1\right) = \frac{\lambda}{2}+1 + 2\left(\frac{\lambda}{4}-1\right) = 7$

$\frac{\lambda}{2}+1+\frac{\lambda}{2}-2 = 7$

$\lambda = 7+1 = 8$

① $x = 5$ ② $y = 1$

C.P$(5, 1)$

$f_x = 2x-2$ $f_y = 8y+8$ $f_{xy} = 0$

$f_{xx} = 2$ $f_{yy} = 8$

$f_{xx} \cdot f_{yy} - (f_{xy})^2 > 0$ and $f_{xx} > 0$ @ $(5,1)$ so Minimum.

Minimum value: $f(5,1) = 25+4-10+8 = 27$

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(ii) $y = x^2$ from the point $(0,3)$:

$D(x,y) = \sqrt{(x-0)^2 + (y-3)^2}$

$D^2(x,y) = x^2 + (y-3)^2$

$D^2(x,x^2) = x^2 + (x^2-3)^2 = x^2 + x^4 - 6x^2 + 9 = \cancel{5x^3}\, x^4 - 5x^2 + 9$

$D^{2\prime}(x,x^2) = 4x^3 - 10x + 0$

$0 = 4x^3 - 10x$

$2x(2x^2-5) = 0$

$x = 0 \quad , x = \pm\sqrt{\frac{5}{2}}$

$y = 0 \quad , y = \frac{5}{2}$

$\underbrace{(0,0)}_{\text{Minimum Point}} , \left(\sqrt{\frac{5}{2}}, \frac{5}{2}\right) , \left(\sqrt{\frac{5}{2}}, \frac{5}{2}\right)$

$D^2(x,y) = x^2 + (y-3)^2$

$D(0,0) = \sqrt{0 + (0-3)^2}$

$D(0,0) = 3$ Ans.

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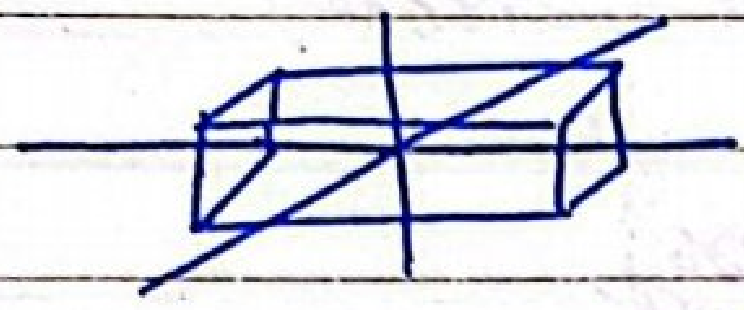
(iii) $(1,0,0), (0,1,0), (0,0,1); (0,0,0), x+y+z=1$



$V = xyz \qquad x+y+z=1$

$\nabla V = yz\,\hat{i} + xz\,\hat{j} + xy\,\hat{k} \qquad \nabla g = \hat{i} + \hat{j} + \hat{k}$

$\nabla V = \lambda \nabla g$

① $yz = \lambda$ ② $xz = \lambda$ ③ $xy = \lambda$

$yz = xz = xy \Rightarrow x = y = z$

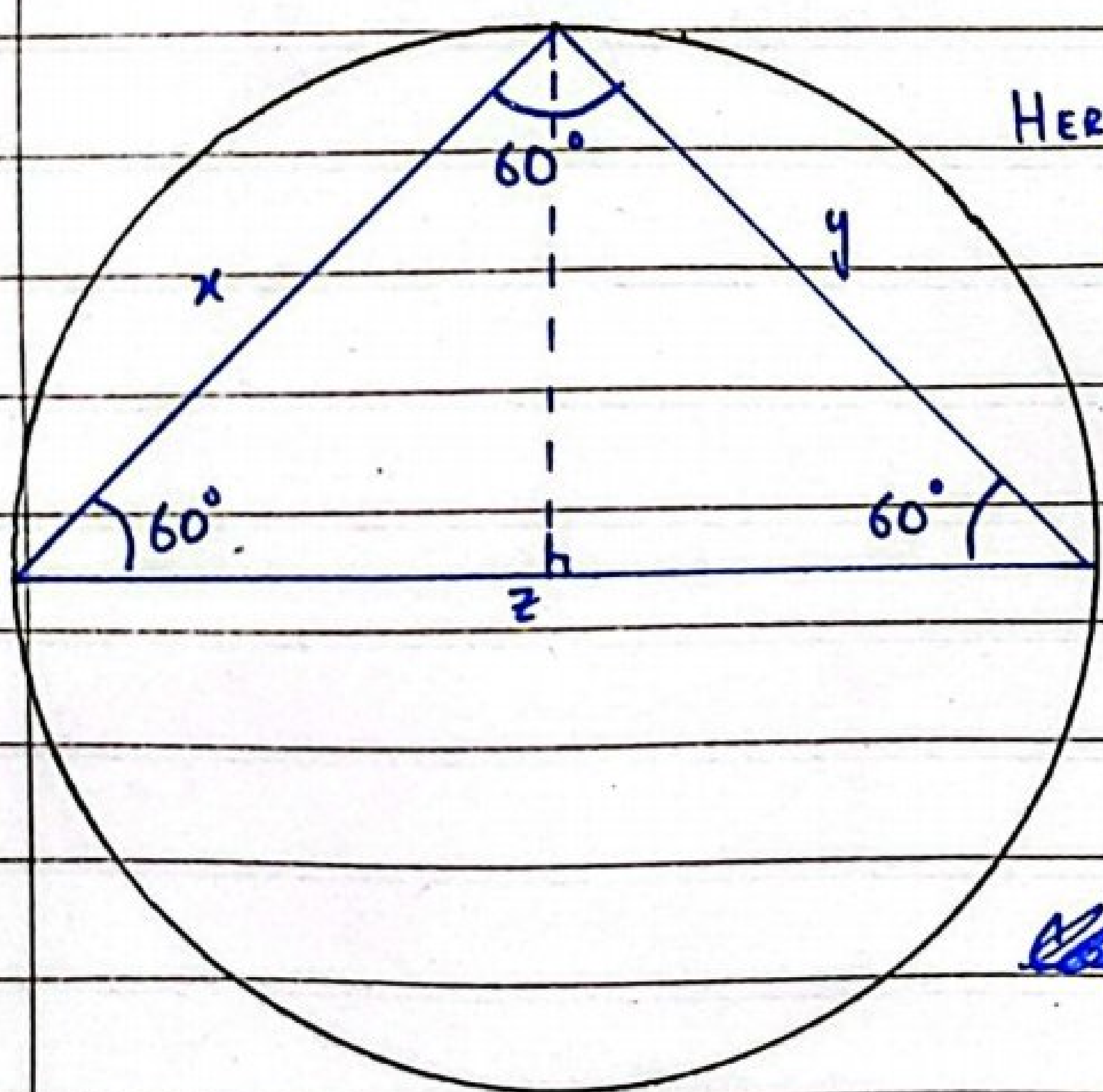
$g(x,x,x) = 3x - 1 = 0$

$x = \frac{1}{3}$

C.P $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

$V = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

(iv)




HERON'S FORMULA = $\sqrt{s(s-a)(s-b)(s-c)}$

Max

When $a = b = c$

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~~$P(x,y,z) = x+y+z$ and $x^2+y^2=r^2$~~
~~Perimeter Eq. Constraint (Circle)~~

~~$\nabla P = \hat{i}+\hat{j}+\hat{k}$ $\quad \nabla g = 2x\hat{i} + 2y\hat{j}$~~

$P = x+y+z \qquad g = a^2+b^2+c^2 = 4r^2$

$\nabla P = \hat{i}+\hat{j}+\hat{k} \qquad \nabla g = 2a\hat{i} + 2b\hat{j} + 2c\hat{k}$

$1 = 2a\lambda \qquad 1 = 2b\lambda \qquad 1 = 2c\lambda$

$a = \frac{1}{2\lambda} \qquad b = \frac{1}{2\lambda} \qquad c = \frac{1}{2\lambda}$

$g = \left(\frac{1}{2\lambda}\right)^2 + \left(\frac{1}{2\lambda}\right)^2 + \left(\frac{1}{2\lambda}\right)^2 - 4r^2$

$\frac{3}{4\lambda^2} - 4r^2 = 0$

$\frac{3}{4\lambda^2} = 4r^2$

$a = b = c = \frac{2r}{\sqrt{3}}$

$P = 3a = 2r\sqrt{3}$

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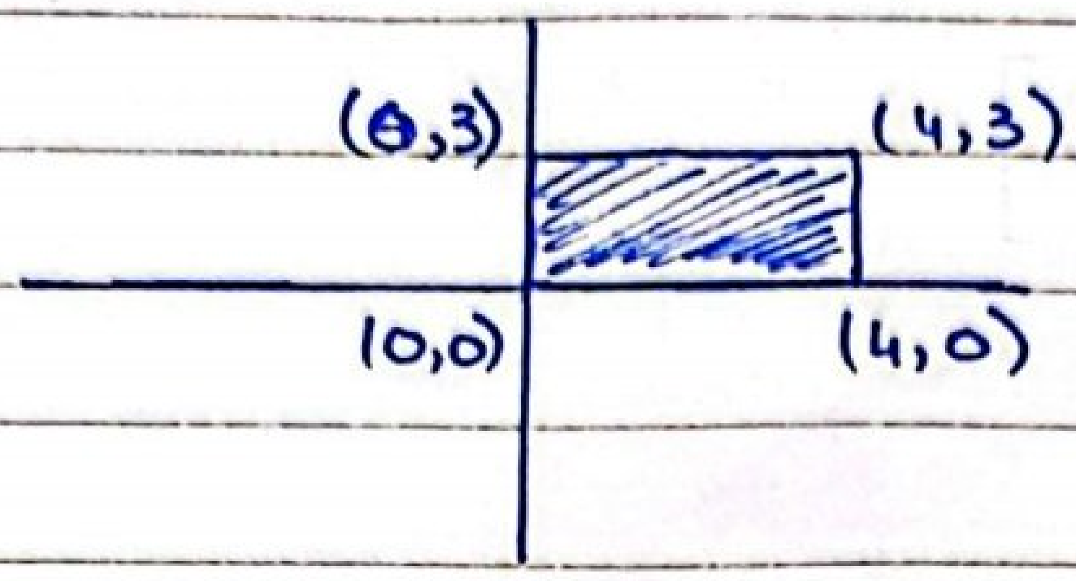
Q2 (i) $f(x,y) = \frac{1}{2}xy$ , Rectangle: $(0,0)\ (4,0)\ (4,3)\ (0,3)$




$$\text{Avg Val} = \frac{1}{\text{Area}} \iint_R f(x,y)\, dA$$

$$= \frac{1}{12} \int_0^4 \int_0^3 \frac{1}{2} xy \, dy\, dx$$

$$= \frac{1}{12} \cdot \frac{1}{2} \int_0^4 \left| \frac{xy^2}{2} \right|_0^3 dx$$

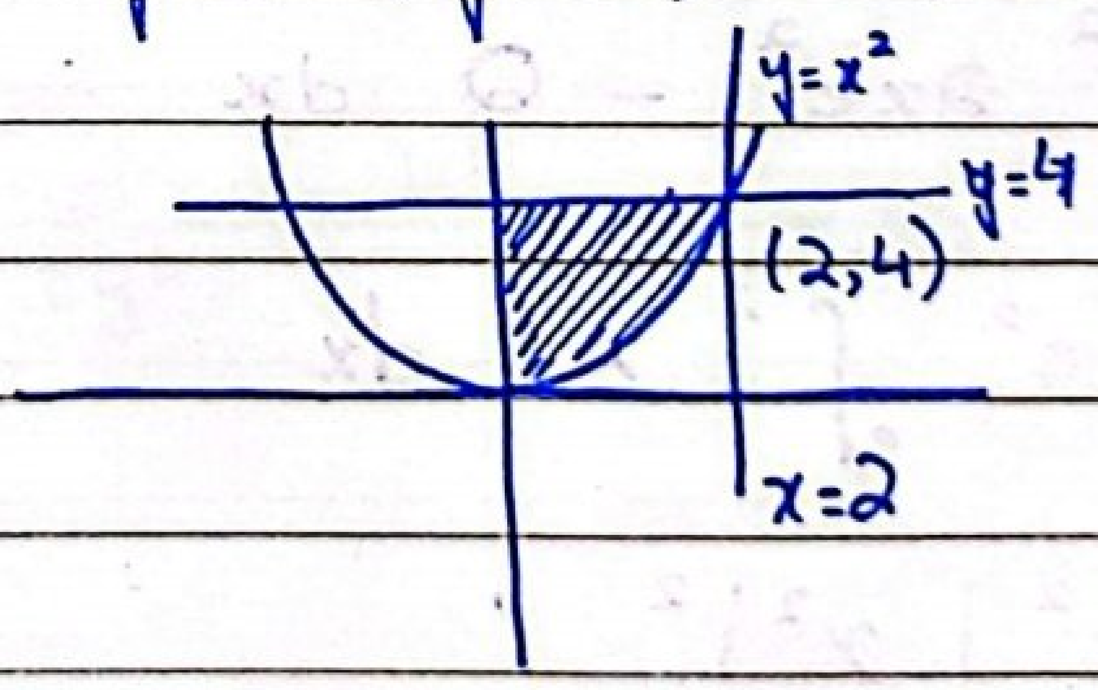
$$= \frac{1}{24} \int_0^4 \frac{9x}{2} \, dx$$

$$= \frac{1}{24} \left| \frac{9x^2}{4} \right|_0^4$$

$$= \frac{1}{24} \left[ \frac{9(\cancel{16})^4}{\cancel{4}1} \right] \cancel{= 1.5} = \frac{3}{2}$$

~~Question unclear. Easy~~

(ii) $\int_0^2 \int_{x^2}^4 x \cos y^2 \, dy\, dx$ $\qquad y = x^2$ , $y = 4$ , $x = 0$, $x = 2$




$$\int_0^4 \int_0^{\cancel{x}\sqrt{y}} x \cos y^2 \, dx\, dy$$

$$\int_0^4 \left[ \frac{x^2}{2} \cos y^2 \right]_0^{\sqrt{y}} dy$$

$$\int_0^4 \frac{(\sqrt{y})^2}{2} \cos y^2 \, dy$$

~~$u = y$~~ $\qquad$ ~~$dv = \cos y$~~

$u = y^2 \qquad \frac{du}{2y} = \cancel{2y}\, d\cancel{y}$

$$\frac{1}{2} \int_0^4 y \cos(y^2)\, dy$$

$$\frac{1}{2} \int_0^4 \frac{\cos(u)}{2}\, du = \frac{1}{4} \int_0^4 \cos(u)\, du$$

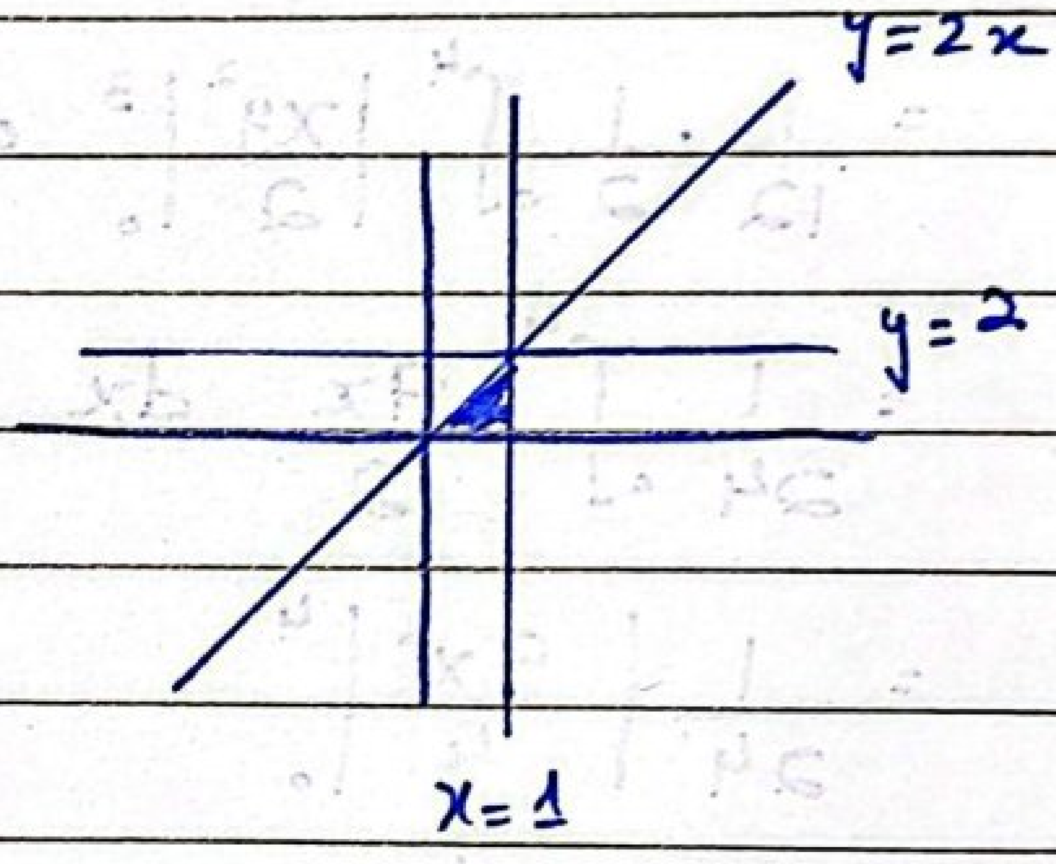
$\frac{1}{4}\left|\sin(u)\right|_0^4$

$\frac{1}{4}\left[\sin(4^2) - \sin(0)\right]$

$\frac{1}{4}\sin(16)$

The question was unclear so I've assumed the square was on the argument of the cos() function instead of the whole function.

(iii) $\int_0^2\int_{\frac{y}{2}}^{1} e^{y/x}\,dx\,dy$




$y=2x,\ y=0,\ y=2,\ x=1$

$\int_0^2\int_0^{2x} e^{y/x}\,dy\,dx$

$\int_0^2 \left|xe^{y/x}\right|_0^{2x} dx$

$\int_0^2 2xe^2 - 0\,dx$

$2e^2\int_0^2 x\,dx$

$\cancel{2}e^2\left|\frac{x^2}{\cancel{2}}\right|_0^2$

$e^2[4-0]$

$4e^2$ Ans.

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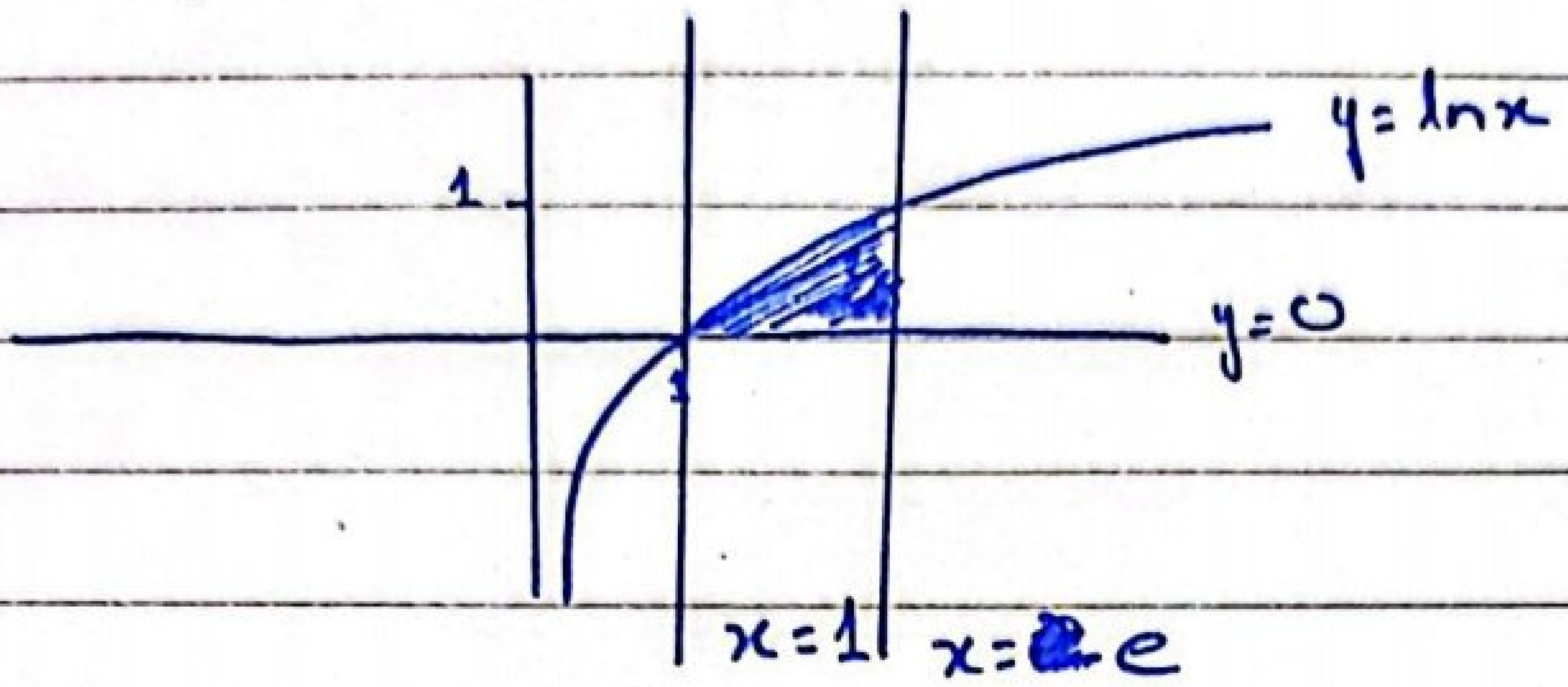
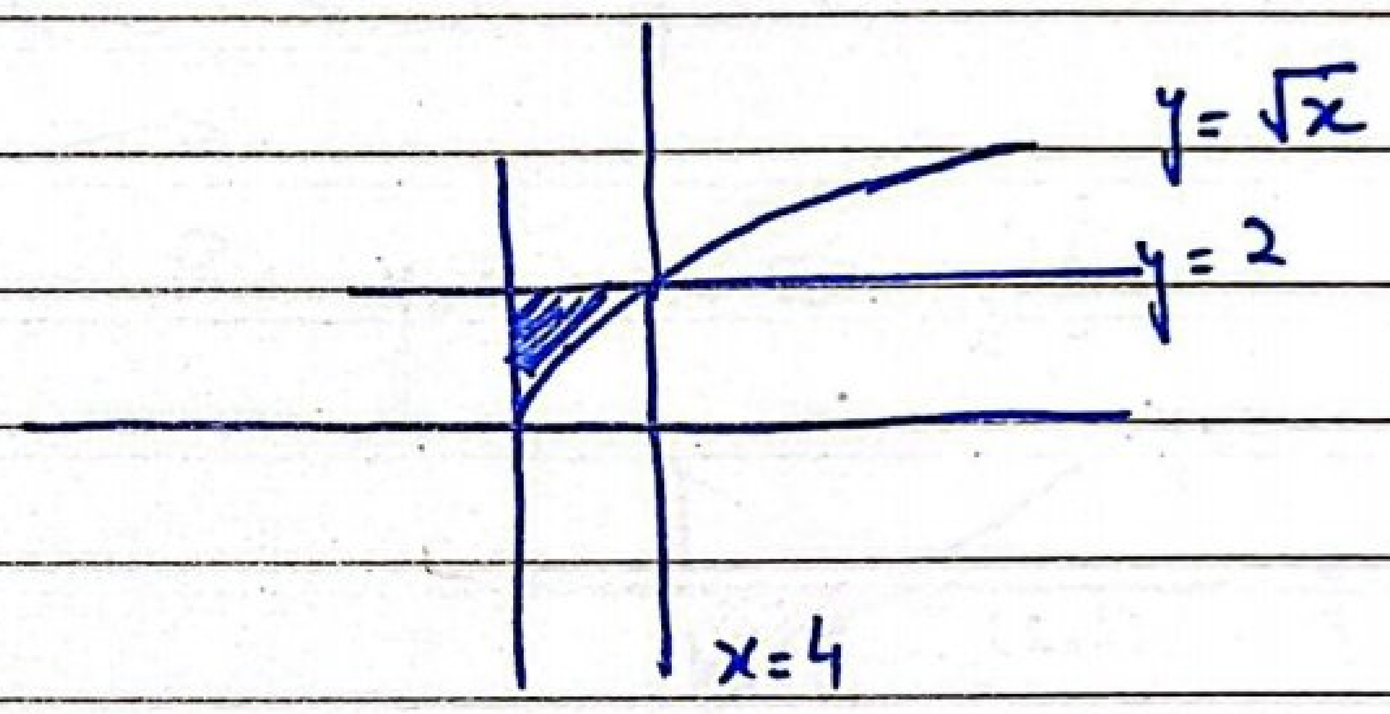
(iv) $\int_{1}^{e}\int_{0}^{\ln x} xy \; dy\, dx$

$y = \ln x \,,\; y = 0 \,,\; x = 1 \,,\; x = e$




$\int_{0}^{1}\int_{e^y}^{e} xy \; dx\, dy$

(v) $\int_{0}^{4}\int_{\sqrt{x}}^{2} \sin y^3 \; dy\, dx$




$\int_{0}^{2}\int_{0}^{y^2} \sin(y^3)\, dx\, dy$

$\int_{0}^{2} \left| x\sin(y^3) \right|_{0}^{y^2} dy$

$\int_{0}^{2} y^2 \sin(y^3)\; dy$

$u = y^3 \quad \frac{du}{dx} = 3y^2 \Rightarrow \frac{du}{3y^2} = dx$

$\int_{0}^{2} \frac{\cancel{y^2}\sin(\cancel{y^3})}{3\cancel{y^2}} \quad du$

$\frac{1}{3}\int_{0}^{2} \sin(u)\; du \Rightarrow \frac{1}{3}\left| -\cos(y^3) \right|_{0}^{2}$

$\frac{1}{3}\left[-\cos(8) - \cos(0)\right]$

$-\left[\frac{\cos 8}{3} + \frac{1}{3}\right]$ Ans.

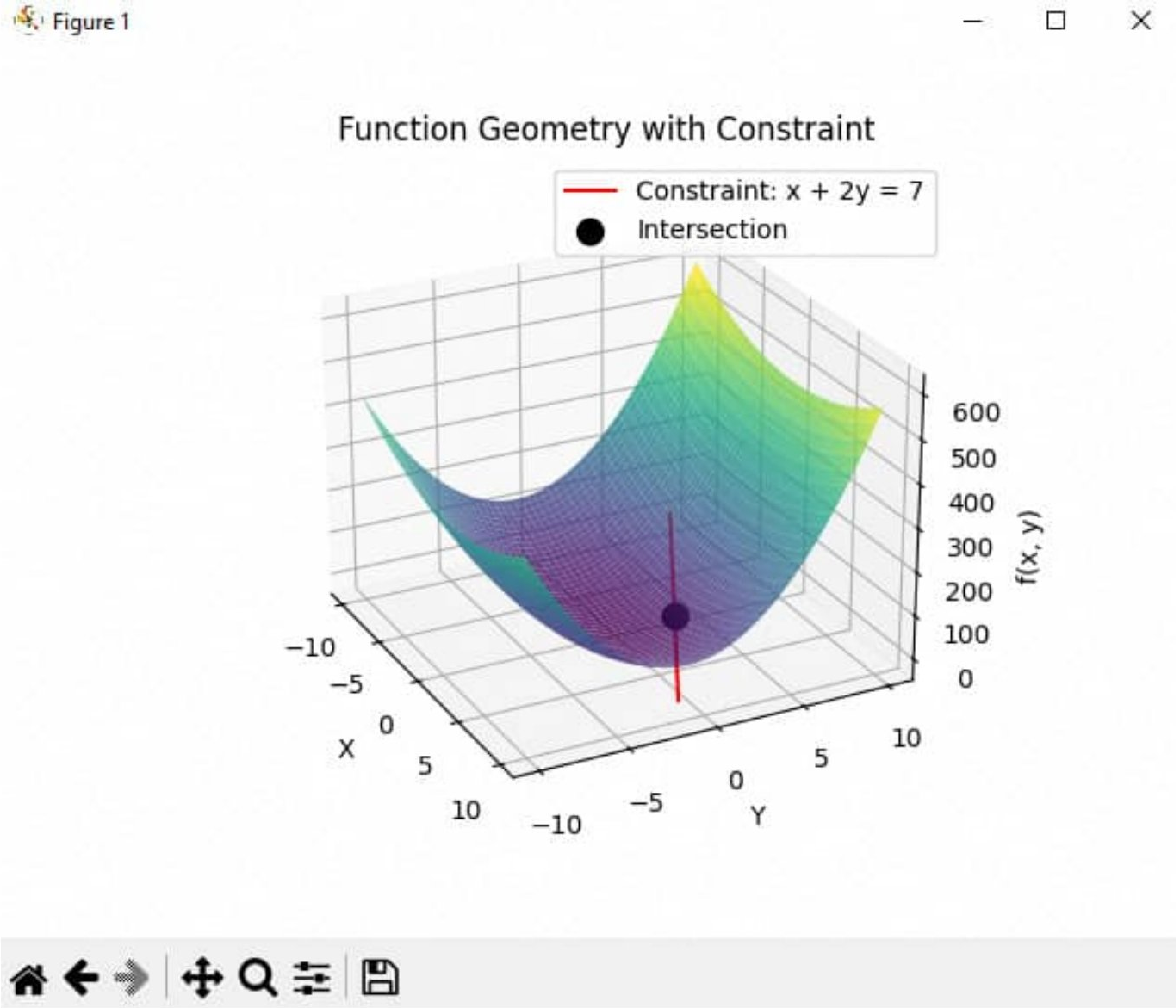
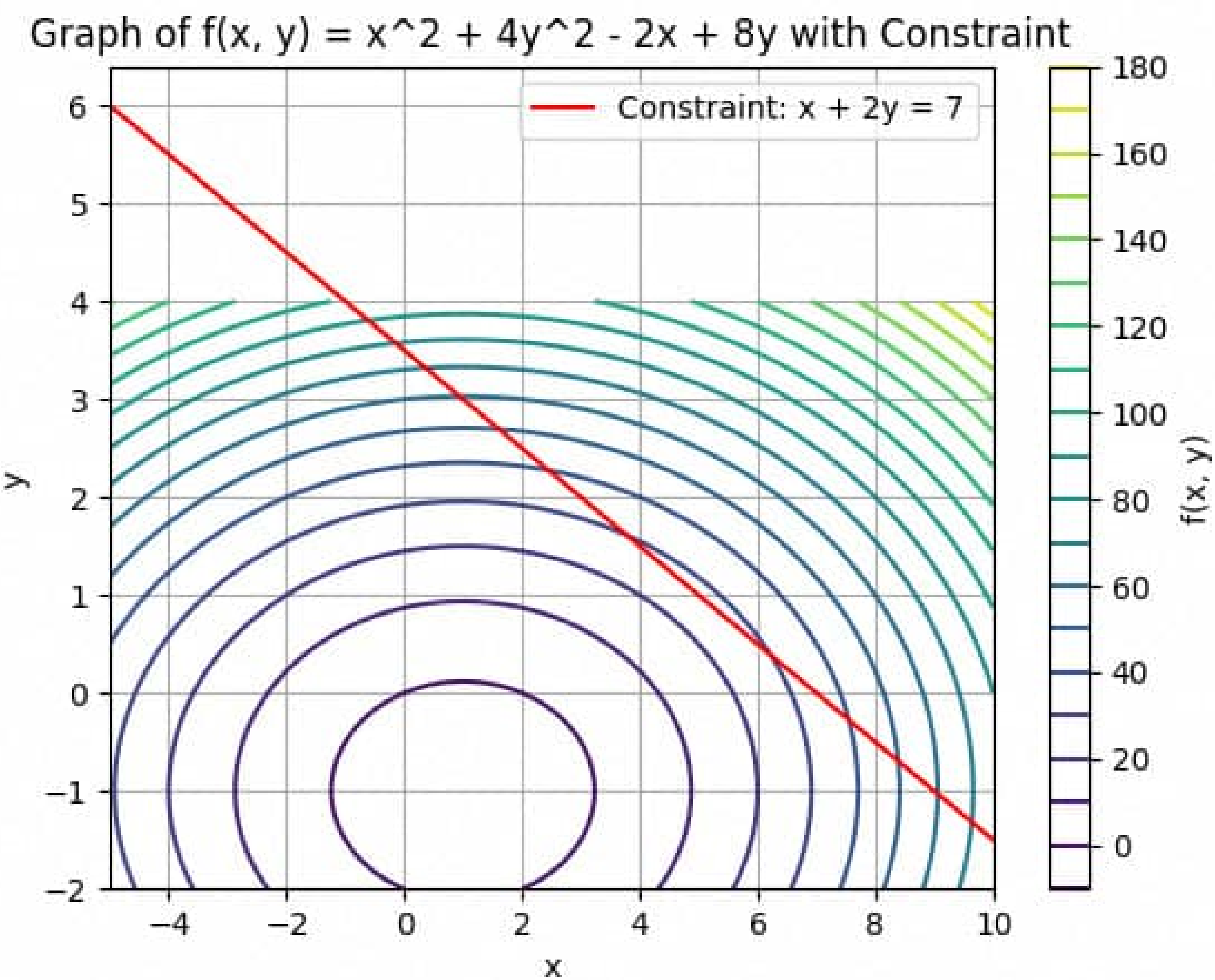


Figure 1
Function Geometry with Constraint
Constraint: x + 2y = 7
Intersection
600
500
400
300
200
100
0
f(x, y)
−10
−5
0
5
10
X
−10
−5
0
5
10
Y

Figure 1



Graph of f(x, y) = x^2 + 4y^2 - 2x + 8y with Constraint
Constraint: x + 2y = 7
y
x
f(x, y)
6
5
4
3
2
1
0
−1
−2
−4
−2
0
2
4
6
8
10
180
160
140
120
100
80
60
40
20
0