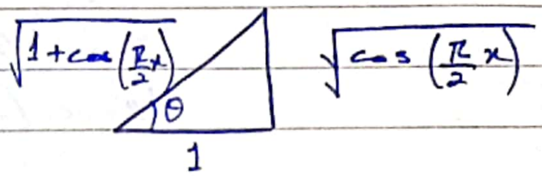
Date 4/12/23

ASSIGNMENT #08

Q: $\int \sqrt{1+\cos\left(\frac{\pi x}{2}\right)} \; dx$



$\tan(\theta) = \sqrt{\cos(\pi x/2)}$

$\tan^2(\theta) = \cos\left(\frac{\pi x}{2}\right)$ P

$dx = 2\tan(\theta)\sec^2(\theta) \; d\theta$

$\cos(\theta) = \dfrac{1}{\sqrt{1+\cos\left(\frac{\pi}{2}x\right)}}$

$\sec(\theta) = \sqrt{1+\cos\left(\frac{\pi}{2}x\right)}$

$\int \sqrt{1+\tan^2(\theta)} \; . \; 2\tan(\theta)\sec^2(\theta) \; d\theta$

$2\int \sqrt{\sec^2(\theta)} \; . \tan(\theta)\sec^2(\theta) \; d\theta$

$2\int \sec^2(\theta) \; . \tan(\theta) . \sec(\theta) \; d\theta$

$u = \sec^2(\theta)$

$du = 2\sec(\theta).(\sec(\theta).\tan(\theta)) \; d\theta$

$\cancel{2}\int \dfrac{\cancel{u} \; . \; \cancel{\tan(\theta)} . \sec(\theta)}{\cancel{2}\cancel{u} \; . \cancel{\tan(\theta)}} \; du$

$\int \sec(\theta) \; du$

$\int \sqrt{u} \; du$

$\int (u)^{1/2} \; du$

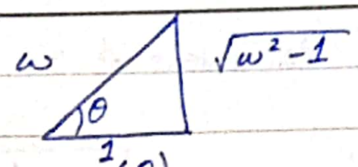
$\frac{2}{3} . u^{3/2} + C$

$\frac{2}{3} . \sec^{3/2}(\theta) + C$

$\frac{2}{3}\left[\sqrt{1+\cos\left(\frac{\pi}{2}x\right)}\right]^{\frac{3}{2}} + C \leftarrow$ Ans.

Date

Q: $\int \frac{\sqrt{x-2}}{\sqrt{x-1}}\,dx$



$w = \sqrt{x-1}$

$w^2 = x-1$

$2w\,dw = dx$

$= \int \frac{\sqrt{w^2-1}}{\cancel{w}}\,2\cancel{w}\,dw$

$= 2\int \sqrt{w^2-1}\,dw$

$= 2\int \sqrt{\sec^2(\theta)-1}\,dw$

$= 2\int \tan(\theta)\,\cancel{d\theta}\,\sec(\theta)\tan(\theta)\,d\theta$

$= 2\int \tan^2(\theta)\sec(\theta)\,d\theta$

$= 2\int [\sec^2(\theta)-1]\sec(\theta)\,d\theta$

$= 2\int \sec^3(\theta)-\sec(\theta)\,d\theta$

~~By parts~~

$\cancel{= 2\left[\int \sec^2(\theta)\sec(\theta)\,d\theta - \int \sec(\theta)\,d\theta\right]}$

$\cancel{= 2\{\sec\theta\tan\theta - \ln|\sec\theta + \tan\theta| - 2\ln}$

$= 2\left[\int \sec^3(\theta)\,d\theta - \int \sec(\theta)\,d\theta\right]$

$= 2\left\{\left[\frac{1}{2}\tan(\theta)\sec(\theta) + \frac{1}{2}\ln|\sec(\theta)+\tan\theta|\right] - \ln|\sec(\theta)+\tan(\theta)|\right\} + C$

$= \tan\theta\sec\theta + \ln|\sec(\theta)+\tan(\theta)| - 2\ln|\sec(\theta)+\tan(\theta)| + C$

$= \tan\theta\sec\theta - \ln|\sec(\theta)+\tan(\theta)| + C$

$= w\sqrt{w^2-1} - \ln|w + \sqrt{w^2-1}| + C$

$= \sqrt{x-1}\sqrt{x-2} - \ln\left|\sqrt{x-1} + \sqrt{x-2}\right| + C$

$w = \sec(\theta)$

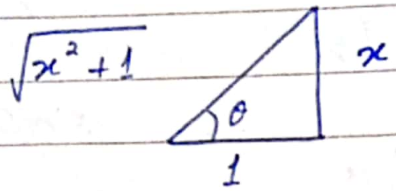
$dw = \sec(\theta)\tan(\theta)\,d\theta$

$\tan(\theta) = \sqrt{w^2-1}$

$\sec(\theta) = w$

Q3: $(x^2+1)^2 \dfrac{dy}{dx} = \sqrt{x^2+1}$; $y(0) = 1$

$$\frac{dy}{dx} = \frac{\sqrt{x^2+1}}{(x^2+1)^2}$$



$\Rightarrow \sin\theta = \dfrac{1}{\sqrt{x^2+1}}$

$$dy = \frac{1}{(\sqrt{x^2+1})^3}\, dx$$

$x = \tan(\theta)$

$dx = \sec^2(\theta)\, d\theta$

$$\int y = \int \frac{1}{(\sqrt{\tan^2\theta+1})^3} \sec^2(\theta)\, d\theta$$

$$y = \int \frac{1}{\sec^3\theta} \sec^2(\theta)\, d\theta$$

$$y = \int \frac{1}{\sec\theta}\, d\theta \Rightarrow y = \int \cos\theta\, d\theta$$

$$y = \sin(\theta) + C$$

$$y = \frac{1}{\sqrt{x^2+1}} + C$$

$y(0) = 1$

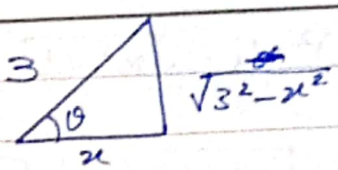
$$1 = \frac{1}{\sqrt{0+1}} + C$$

$\cancel{\sqrt{2} = 1 + C}$ $\quad 1 = C$

$\cancel{\sqrt{2} - 1 = C}$

$y = \sin(\theta) + 1$ ← Ans

Q4: $y = \dfrac{\sqrt{9-x^2}}{3}$



$\cancel{y}\ Y = \int \dfrac{\sqrt{9-x^2}}{3}\ dx$

$\sin(\theta) = \dfrac{x}{3}$

$\cancel{y=}\ Y = \dfrac{1}{3}\int \sqrt{3^2-x^2}\ dx$

$x = 3\sin(\theta)$

$dx = 3\cos(\theta)\ d\theta$

$Y = \dfrac{1}{3}\int \sqrt{9-(3\sin(\theta))^2}\ dx$

$\cancel{x=0}$

$= \dfrac{1}{3}\int \sqrt{9(1-\sin^2(\theta))}\ dx$

$\cancel{\sin(x) = L.P}$

$\cancel{L.P = 0}$

$= \dfrac{3}{3}\int \sqrt{\cos^2(\theta)}\ dx$

$\cancel{x=3}$

$x = 3\sin(\theta),\ 0 \le \theta \le \dfrac{\pi}{2}$

$= \int \cos^2(\theta)\ 3\cos(\theta)\ d\theta$

$= 3\int \cos^2(\theta)\ d\theta$

~~Bounds: x=0 to π/2 x=3~~ Bounds from 0 to $\dfrac{\pi}{2}$

$= 3\int \dfrac{1}{2}(1+\cos 2\theta)\ d\theta$

$= \dfrac{3}{2}(\theta) + \dfrac{3}{4}\sin(2\theta) + C$

$= \dfrac{3}{2}\left[\theta + \dfrac{1}{2}\sin(2\theta)\right]_0^{\frac{\pi}{2}}$

$= \dfrac{3}{2}\left[\left|\left(\dfrac{\pi}{2} + \dfrac{1}{2}\sin\left(\dfrac{2\pi}{2}\right)\right) - \left(0 + \dfrac{1}{2}\sin(0)\right)\right|\right]$

$= \dfrac{3}{2}\left[\left|\dfrac{\pi}{2} + 0 - (0+0)\right|\right]$

$= \dfrac{3\pi}{4}$ Av.