Homework # 6: Coordinate Vectors

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Key Concept

A **coordinate vector** represents the coordinates of a vector in terms of a specific basis. If you have a vector $\mathbf{v}$ and a basis $\mathbb{B} = \{\mathbf{b}_1, \mathbf{b}_2, \ldots, \mathbf{b}_n\}$, the vector $\mathbf{v}$ can be written as:

$$\mathbf{v} = c_1\mathbf{b}_1 + c_2\mathbf{b}_2 + \cdots + c_n\mathbf{b}_n$$

The coefficients $[\mathbf{v}]_{\mathbb{B}} = \begin{pmatrix} c_1 \\ c_2 \\ \cdot \\ \cdot \\ \cdot \\ c_n \end{pmatrix}$ form the *coordinate vector* of $\mathbf{v}$ with respect to the basis B.

Exercises

Exercise 1

Let $\mathbb{B} = \left\{ \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ -1 \end{pmatrix} \right\}$. Find

1. the coordinate vector of $\mathbf{v} = \begin{pmatrix} 4 \\ -3 \\ 7 \end{pmatrix}$ with respect to the basis $\mathbb{B}$ in $\mathbb{R}^3$.
2. the vector $\mathbf{u}$, such that $[\mathbf{u}]_{\mathbb{B}} = \begin{pmatrix} 5 \\ -4 \\ 1 \end{pmatrix}$

Exercise 2

Let $\mathbb{C} = \left\{1 + 2t + t^2, 3 - 3t, -t + 5t^2\right\}$.

1. Prove that $\mathbb{C}$ is basis of $\mathbb{P}_2(t)$.
2. Find the coordinate vector of $\mathbf{v} = 2 + 3t - t^2$ with respect to the basis $\mathbb{C}$.
3. Find the vector $\mathbf{u}$, such that $[\mathbf{u}]_{\mathbb{C}} = \begin{pmatrix} 5 \\ -4 \\ 1 \end{pmatrix}$

Exercise 3:

Consider the 2 dimensional subspace $W = \left\{ \begin{pmatrix} a & b \\ b & a \end{pmatrix} | a, b \in \mathbb{R} \right\}$ of $\mathbb{M}_{2\times 2}$.

1. Prove that $\mathbb{D} = \left\{ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \right\}$ is basis of W.
2. Check whether $\mathbb{E} = \left\{ \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}, \begin{pmatrix} -2 & 1 \\ 1 & -2 \end{pmatrix} \right\}$ is basis of W.
3. Find the coordinate vector of $\mathbf{v} = \begin{pmatrix} 5 & 3 \\ 3 & 5 \end{pmatrix}$ with respect to the basis $\mathbb{D}$ and $\mathbb{E}$
4. Find the vector $\mathbf{u}$, such that $[\mathbf{u}]_{\mathbb{E}} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$