MT1004 – Linear Algebra

Homework # 12

Question # 1

Let

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

(a) Without calculations, comment on the linear independency of columns and rows of A and invertibility of $A^T A$ and $A\,A^T$.
(b) Find the right inverse of A. (Hint: Use $A^T(A\,A^T)^{-1}$.)
(c) Find the standard matrix for the orthogonal projection on the row space of matrix A. (Hint: Use $A^T(A\,A^T)^{-1}A$.)

Question # 2

(a) Find a least-squares solution of $A\boldsymbol{x} = \boldsymbol{b}$ for

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \\ 1 & 2 \end{bmatrix}, \quad \boldsymbol{b} = \begin{bmatrix} 3 \\ -1 \\ 5 \end{bmatrix}$$

(Hint: Columns of A are orthogonal.)
(b) Find the least squares approximating line and the corresponding least-squares errors for the points (2, 3), (-4, 1), and (2, 5).

Question # 3

Construct a least-square model for the following data (Write the normal equations only. Do NOT solve the equations):
(a) Least squares approximating parabola for the points (1, 1), (2, -2), (3, 3), (4, 4).
(b) Least squares approximating plane for the points (1, 1, 1), (2, -2, 2), (3, 3, 3), (4, 4, 4).