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ASSIGNMENT #05

Q1: $\int \frac{x^2}{(x-1)(x^2+2x+1)}\,dx$

$\frac{x^2}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$

$\frac{x^2}{(x-1)(x+1)^2} = \frac{A(x+1)^2 + B(x-1)(x+1) + C(x-1)}{(x-1)(x+1)^2}$

$x^2 = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$

$x = 1$: $1 = 4A$, $A = 1/4$

$x = -1$: $1 = -2C$, $C = -1/2$

$1 = A + B$, $1 = 1/4 + B$, $B = 3/4$

$= \int \frac{3}{4(x+1)} - \frac{1}{2(x+1)^2} + \frac{1}{4(x-1)}\,dx$

$= \frac{3}{4}\int \frac{1}{x+1}\,dx - \frac{1}{2}\int \frac{1}{(x+1)^2}\,dx + \frac{1}{4}\int \frac{1}{x-1}\,dx$

$= \frac{1}{4}\left(\frac{2}{x+1} + \log(x-1) + 3\log(x+1)\right) + C$

Q2: $\int \frac{16x^3}{(4x^2-4x+1)}\,dx$

$4x+4$ / $4x^2-4x+1 \;|\; 16x^3$

$-16x^3 + 16x^2 - 4x$

$-16x^2 + 16x - 4$

$12x - 4$

$16\int \frac{x^3}{(2x-1)^2}\,dx$

$\frac{12x-4}{(2x-1)^2} = \frac{A}{2x-1} + \frac{B}{(2x-1)^2}$

$12x - 4 = A(2x-1) + B$

$x = 1/2$: $6 - 4 = A(0) + B$, $B = 2$

$12 = 2A$, $A = 6$

$\int 4x + 4 + \frac{6}{2x-1} + \frac{2}{(2x-1)^2}\,dx$

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$$= 16\int\left(\frac{x}{4} + \frac{3}{8(2x-1)} + \frac{1}{8(2x-1)^2} + \frac{1}{4}\right)dx$$

$$= 6\int\frac{1}{2x-1}\,dx + 2\int\frac{1}{(2x-1)^2}\,dx + 4\int x\,dx + 4\int 1\,dx$$

$$= \quad u = 2x-1\ ,\ du = 2\,dx$$

$$= 3\int\frac{1}{u}\,du + 2\int\frac{1}{(2x-1)^2}\,dx + 4\int x\,dx + 4\int 1\,dx$$

$$= 2x^2 + 4x + \frac{1}{1-2x} + 3\log(2x-1) + C$$

$$= \frac{8x^3 + 12x^2 - 18x + 6(2x-1)\log(1-2x) + 3}{4x-2} + C$$

Q3: Solve the initial value problem:

$$(3t^4 + 4t^2 + 1)\frac{dx}{dt} = 2\sqrt{3}\ ;\ x(1) = -\frac{\pi\sqrt{3}}{4}$$

$$\int 1\,\frac{dx}{dt} = \int 2\sqrt{3} \times \frac{1}{3t^4 + 4t^2 + 1}\,dt$$

$$x = 2\sqrt{3}\int\frac{1}{3t^4 + 4t^2 + 1}\,dt$$

$$= \sqrt{3}\int\frac{1}{t^2 + \frac{1}{3}}\,dt - \sqrt{3}\int\frac{1}{t^2+1}\,dt$$

$$= 3\tan^{-1}(\sqrt{3}t) - \sqrt{3}\tan^{-1}(t) + C$$

$t = 1$, $x = -\frac{\pi\sqrt{3}}{4}$

$$-\frac{\sqrt{3}\pi}{4} = \pi - \frac{\sqrt{3}}{4}\pi + C$$

$$C = -\pi$$

$$x = 3\tan^{-1}(\sqrt{3}t) - \sqrt{3}\tan^{-1}(t) - \pi \quad \text{Ans.}$$

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