Q1. Propositional Logic: Semantic Tableaux (10 Marks) [10]

Determine whether $A \to B$, $C \to D$, $B \vee D \to E$, $\neg E$ logically follows $\neg A \wedge \neg C$ or not

1. $A \to B$
2. $C \to D$
3. $B \vee D \to E$
4. $\neg E$
5. $\neg(\neg A \wedge \neg C)$

8 marks

1 Model

Implies, 3

6. $\neg(B \vee D)$

7. E

Closed
4, 7.

8. $\neg B$
9. $\neg D$

Implies, 2

10. $\neg C$

11. D

Closed
9, 11.

Implies, 1

12. $\neg A$

13. B

Closed
8, 13.

NOT-AND
5

14. $\neg\neg A$

Closed
12, 14.

15. $\neg\neg C$

Closed
10, 15.

2 marks

Yes, the given Premises logically follows the conclusion

Q2. Predicate Logic: Translations and Interpretations (10 Marks) [5+5]

$b(x)$ is true iff x is boat user
$P(y)$ is true iff y is Passenger.

In a seaside town, regulations state that every boat must have a life jacket for each passenger to ensure safety. One day, a boat is borrowed without the owner's permission by someone who is not aware that the boat is short on life jackets. When the coast guard stops the boat, they find that there are not enough life jackets for all passengers.

1 Express the following of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. [5]

(a) Boat users should ensure they have a life jacket for each passenger. [2.5]

1 $\forall x. \forall y. (b(x) \wedge P(y) \wedge U(x,y)) \rightarrow J(x,y))$

Negation.

$\equiv \neg(\forall x. \forall y. (b(x) \wedge P(y) \wedge U(x,y)) \rightarrow J(x,y)))$

$\equiv \exists x. \exists y. \neg ((b(x) \wedge P(y) \wedge U(x,y)) \rightarrow \neg J(x,y)))$

.5 1

(b) No boat user should be penalized for lacking life jackets if they were unaware of the shortage. [2.5]

1 $\neg(\exists x. \exists y.\ b(x) \wedge P(y) \wedge U(x,y) \wedge \neg\, \text{aware}(x) \wedge \text{Penalized}(x,y)))$

Negation

$\equiv \exists x. \exists y.\ b(x) \wedge P(y) \wedge U(x,y) \wedge \neg\, \text{aware}(x) \wedge \text{Penalized}(x,y).$

.5

2 Let domain Dx=Dy={-2, -1, 0, 1, 2}. Explain whether the following statements are true or not. If they are true then define the truth set. [5]

(a) $\forall x, \exists y \bullet x + y = 0$ [2.5]

$(\exists y.\ -2+y=0) \wedge (\exists y.\ -1+y=0) \wedge (\exists y.\ 0+y=0) \wedge$
$(\exists y.\ 1+y=0) \wedge (\exists y.\ 2+y=0)$

$(-2+2=0) \wedge (-1+1=0) \wedge (0+0=0) \wedge (1+(-1)=0)$
$\wedge (2+(-2)=0)$.

True.

(b) $\exists x, \forall y \bullet x + y = y$ [2.5]

$\cancel{\exists x.\ (x+-2=y) \wedge (x+-1=y)}$

$\exists x.\ (x+(-2)=-2) \wedge (x+(-1)=-1) \wedge (x+0=0)$
$\wedge (x+1=1) \wedge (x+2=2)$

True.

$(0+(-2)=-2) \wedge (0+(-1)=-1) \wedge (0+0=0)$
$\wedge (0+1=1) \wedge (0+2=2)$.

Q3. Predicate Logic: Transformational Proof (10 Marks) [10]

Show transformational proof of $\neg\exists x\exists y \bullet Q(x,y) \leftrightarrow Q(y,x) \iff \forall x\forall y \bullet (Q(x,y) \vee Q(y,x)) \wedge \forall y\forall x \bullet (\neg Q(y,x) \vee \neg Q(x,y))$

L.H.S.

$\equiv \forall x.\forall y.\neg(Q(x,y) \leftrightarrow Q(y,x))$ DeMorgans (2x)

$\equiv \forall x.\forall y.\neg((Q(x,y) \rightarrow Q(y,x)) \wedge (Q(y,x) \rightarrow Q(x,y)))$ Equivalence

$\equiv \forall x.\forall y.\neg((\neg Q(x,y) \vee Q(y,x)) \wedge (\neg Q(y,x) \vee Q(x,y)))$ Implies, 2(x)

$\equiv \forall x.\forall y.(\neg(\neg Q(x,y) \vee Q(y,x)) \vee \neg(\neg Q(y,x) \vee Q(x,y)))$ DeMorgan 1x

$\equiv \forall x.\forall y.((\neg\neg Q(x,y) \wedge \neg Q(y,x)) \vee (\neg\neg Q(y,x) \wedge \neg Q(x,y)))$ DeMorgan (2x)

$\equiv \forall x.\forall y.((Q(x,y) \wedge \neg Q(y,x)) \vee (Q(y,x) \wedge \neg Q(x,y)))$ Negation 2(x)

$\equiv \forall x.\forall y.(((Q(x,y) \wedge \neg Q(y,x)) \vee Q(y,x)) \wedge ((Q(x,y) \wedge \neg Q(y,x)) \vee \neg Q(x,y)))$ Distributive

$\equiv \forall x.\forall y.((Q(x,y) \vee Q(y,x)) \wedge (\neg Q(y,x) \vee Q(y,x)) \wedge (Q(x,y) \vee \neg Q(x,y)) \wedge (\neg Q(y,x) \vee \neg Q(x,y)))$ Distributive (2x)

$\equiv \forall x.\forall y(Q(x,y) \vee Q(y,x)) \wedge T \wedge T \wedge (\neg Q(y,x) \vee \neg Q(x,y)))$ Excluded Middle (2x)

$\equiv \forall x.\forall y.((Q(x,y) \vee Q(y,x)) \wedge (\neg Q(y,x) \vee \neg Q(x,y)))$ Simplification I

$\equiv \forall x.\forall y.(Q(x,y) \vee Q(y,x)) \wedge \forall x.\forall y.(\neg Q(y,x) \vee \neg Q x,y)$ Distributive

$\equiv \forall x.\forall y(Q(x,y) \vee Q(y,x)) \wedge \forall y.\forall x.(\neg Q(y,x) \vee \neg Q(x,y))$ Swap variables

Proved.

Q4. Predicate Logic: Natural Deduction Proof Marks) [10]

Proof the following by using natural deduction. you are only allowed to use inference rules, ... $P(x) \rightarrow Q(x) \wedge S(x)$, $\forall x \bullet P(x) \wedge R(x) \vdash_N \quad \forall x \bullet R(x) \wedge S(x)$

1. ∀x. P(x) → Q(x) ∧ S(x)] Premises
2. ∀x. P(x) ∧ R(x)

3.	xg	
4.	P(xg) → Q(xg) ∧ S(xg)	∀-E, 1
5.	P(xg) ∧ R(xg)	∀-E, 2
6.	P(xg)	∧-E, 5
7.	Q(xg) ∧ S(xg)	4, 6, →-E
8.	S(xg)	7, ∧-E
9.	R(xg)	5, ∧-E
10.	R(xg) ∧ S(xg)	8, 9, ∧-I
11.	∀x. R(x) ∧ S(x)	3-10, ∀-I.