

## HW#4

### Questions 1: (Directional Derivatives)

- i. The rate of change of a given functions  $f(x, y)$  at point  $P_0 = (1, 2)$  in the direction towards  $P_1 = (2, 3)$  is  $\sqrt{2}$  and in the direction towards  $P_2 = (1, 0)$  is  $-3$ . What is the rate of change  $f$  at  $P_0$  towards  $(0, 0)$ .
- ii. You are standing at a location where the surface of the earth is smooth. The slope in the southern direction is 4 and the slope in the south-eastern is  $\sqrt{2}$ . Find the slope in the eastern direction.
- iii. A hiker is walking on a mountain with height above the plane  $z = 0$  given by  $f(x, y) = 6 - xy^2$ . The positive  $x$ -axis points east and the positive  $y$ -axis points north, and the hiker starts from the point  $P(2, 1, 4)$ .
  - a. In what direction should the hiker proceed from  $P$  to ascend along the steepest path? What is the slope of the path?
  - b. Walking north from  $P$  will the hiker start to ascend or descend? What is the slope?
  - c. In what direction should the hiker walk from  $P$  to remain at the same height?

### Question 2:

1. Find three integers  $x, y, z$  whose sum is 32 and  $P = xyz$  is minimum.
2. A manufacturer wishes to make an open rectangular box of given volume  $V$  using the least possible material. Find the design and specifications.
3. Find the high and low points of the surface  $z = \sqrt{x^2 + y^2}$  with  $(x, y)$  varying over  $|x| \leq 1$  and  $|y| \leq 1$ .
4. Equal angle bends are made at equal distances from the two ends of a 100 m long fence so the resulting three segment fence can be placed along an existing wall to make an enclosure of trapezoidal shape. What is the largest possible area for such an enclosure?
5. In a certain community, there are two milk shops in competition, so that sales of each negatively affect the profits of the other. If milk shop A produces  $x$  liters of milk per month and milk shop B produces  $y$  liters per month, then the profits of the two shops are given by  $P = 2x - \frac{2x^2 + y^2}{10^6}$  and  $Q = 2y - \frac{4y^2 + x^2}{2 \times 10^6}$  respectively. Find the sum of the two profits if each shop independently sets its own production level to maximize its own profit and assumes that its competitor does likewise. Then, assuming cartel behavior, find the sum of the two profits if the two shops cooperate so as to maximize that sum.