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23i-2623
DS-B

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Homework #06

Q1: $\underbrace{x^2+y^2=a^2}_{\text{Circle in xy}}$ and $\underbrace{x^2+z^2=a^2}_{\text{Circle in xz}}$

$x^2+y^2 = x^2+z^2$

$y^2 = z^2$

$y^2 - z^2 = 0$

$z = \sqrt{a^2-x^2}$

$y = \sqrt{a^2-x^2}$

$$V = 8\int_0^a\int_0^{\sqrt{a^2-x^2}} z \; dy\,dx$$

$$= 8\int_0^a\int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2} \; dy\,dx$$

$$= 8\int_0^a \left[y\sqrt{a^2-x^2}\right]_0^{\sqrt{a^2-x^2}} dx$$

$$= 8\int_0^a a^2-x^2 \; dx$$

$$= 8\left[a^2x - \frac{x^3}{3}\right]_0^a$$

$$= 8\left[a^3 - \frac{a^3}{3}\right] = 8\left[\frac{2a^3}{3}\right]$$

Volume $= \frac{16a^3}{3}$ Ans.

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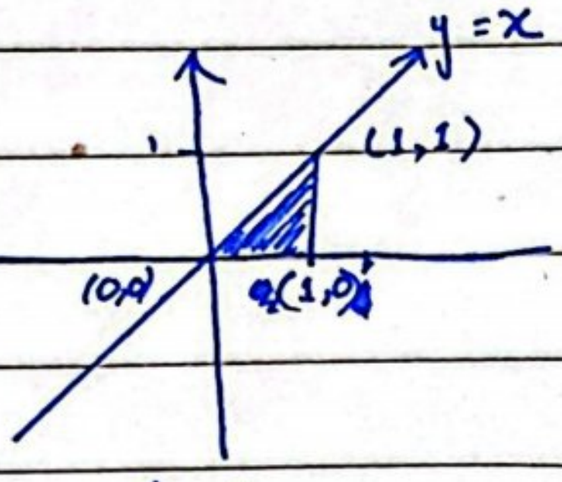
Q2. $\iint_D \frac{\sin x}{x}\, dA$, $D: y = x$, $x \geq 0$, $x = 1$



$0 \leq y \leq x$

$0 \leq x \leq 1$

$$\int_0^1 \int_0^x \frac{\sin x}{x}\, dy\, dx$$

$$\int_0^1 \left[y \frac{\sin x}{x} \right]_0^x dx$$

$$\int_0^1 \sin x\, dx$$

$$\left[-\cos x \right]_0^1$$

$$-\cos 1 + \cos(0)$$

$$V = 1 - \cos 1$$

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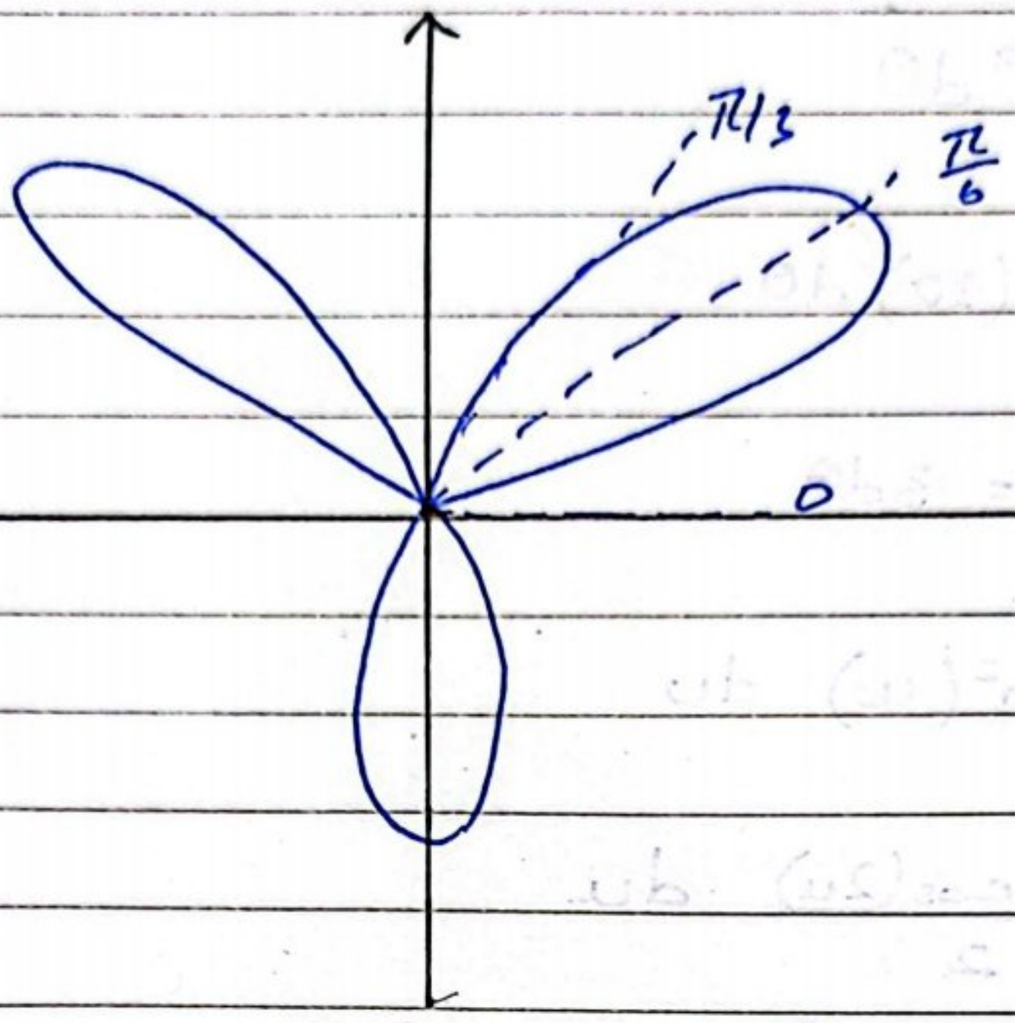
Q3: Petal Rose $r = \sin 3\theta$



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Q4: Area of one petal:

$$r = 0 \rightarrow \sin 3\theta = 0 \rightarrow \theta = 0, \pi/3$$

$$A = \int_0^{\frac{\pi}{3}} \frac{1}{2} r^2 \, d\theta$$

$$A = \frac{1}{2} \int_0^{\frac{\pi}{3}} \sin^2(3\theta) \, d\theta$$

$u = 3\theta \qquad du = 3d\theta$

$u = 0 \;, \; u = \pi$

$$A = \frac{1}{6} \int_0^{\pi} \sin^2(u) \, du$$

$$A = \frac{1}{6} \int_0^{\pi} \frac{1 - \cos(2u)}{2} \, du$$

$$A = \frac{1}{6} \left[\left(\frac{\pi}{2}\right) - \frac{\sin 2\pi - \sin(0)}{4} \right]$$

$$A = \frac{\pi}{12} \quad \text{Ans.}$$

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Q5: $\iint_D y\sqrt{x^2+y^2}\, dA$, $D: 0 \le x \le 1$, $0 \le y \le x$

$y = r\sin\theta$ ↓ $x = r\cos\theta$

$0 \le r\sin\theta \le r\cos\theta$ ↓ $0 \le r\cos\theta \le 1$

$r\sin\theta = r\cos\theta$ $\quad r = 0$, $r = \sec\theta$

$\theta = \frac{\pi}{4}$

$$\int_0^{\frac{\pi}{4}} \int_0^{\sec\theta} r^3 \sin\theta \; dr\, d\theta$$

$$\int_0^{\frac{\pi}{4}} \left| \frac{r^4}{4} \sin\theta \right| d\theta$$

$$\int_0^{\frac{\pi}{4}} \frac{\sec^4\theta \sin\theta}{4} \, d\theta$$

$$\frac{1}{4}\int_0^{\frac{\pi}{4}} \tan\theta \sec\theta \sec^2\theta \; d\theta$$

$u = \sec\theta \quad du = \tan\theta \sec\theta \, d\theta$

$$\frac{1}{4}\int_0^{\frac{\pi}{4}} u^2 \, du$$

$$\frac{1}{4}\left| \frac{u^3}{3} \right|_0^{\frac{\pi}{4}}$$

$$\frac{1}{12}\left| \sec^3\left(\frac{\pi}{4}\right) - \sec^3(0) \right|$$

$$\frac{1}{6}\left| \frac{1}{2}\sqrt{2} - 1 \right|$$

$\frac{\sqrt{2}}{6} - \frac{1}{12}$ Ans

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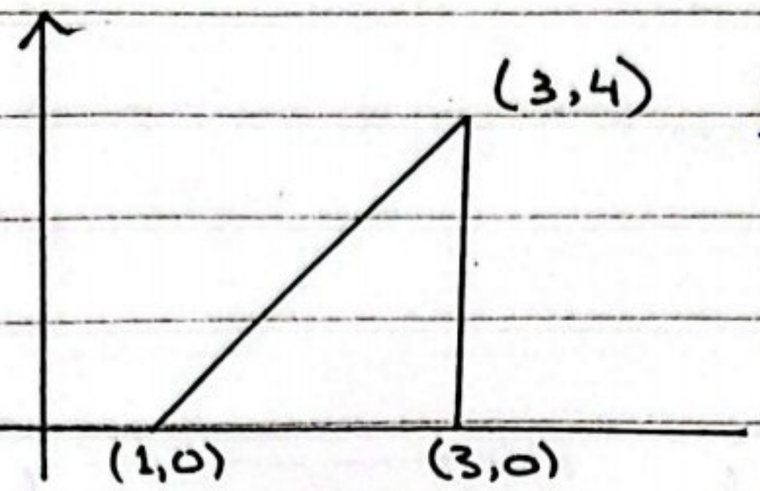
Q6: $P(x,y) = 2x^2 + 5y$



$y - y_1 = m(x - x_1)$

$y - 4 = \left(\frac{4-0}{3-1}\right)(x-3)$

$y - 4 = 2x - 6$

$y = 2x - 2$

a) $\int_1^3 \int_0^{2x-2} 2x^2 + 5y \; dy \; dx$

$\int_1^3 \left[2x^2 y + \frac{5}{2}y^2\right]_0^{2x-2} dx$

$\int_1^3 \left[2x^2(2x-2) + \frac{5}{2}(2x-2)^2\right] dx$

$\int_1^3 4x^3 - 4x^2 + \frac{5}{2}(4x^2 - 8x + 4)\, dx$

$\int_1^3 4x^3 - 4x^2 + 10x^2 - 20x + 10 \; dx$

$\int_1^3 4x^3 + 6x^2 - 20x + 10 \; dx$

$\left[\frac{4x^4}{4} + \frac{6x^3}{3} - \frac{20x^2}{2} + 10x\right]_1^3$

~~$[(3)^4 + 2(3)^3 - 10(3)^2 + 10(3)] - [1 + 2 - 10 + 10]$~~

$[x^4 + 2x^3 - 10x^2 + 10x]_1^3 = 72$

Population = 72000 people

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(b) Area of region: $\frac{1}{2}bh$

$A = \left(\frac{1}{2}\right)(2)(4)$

$A = 4$

Avg number of people per square mile = $\frac{\text{Population (thousands)}}{\text{Area (miles)}}$

Avg = $\frac{72}{4} = 18$

There are 18000 people per square mile on average.

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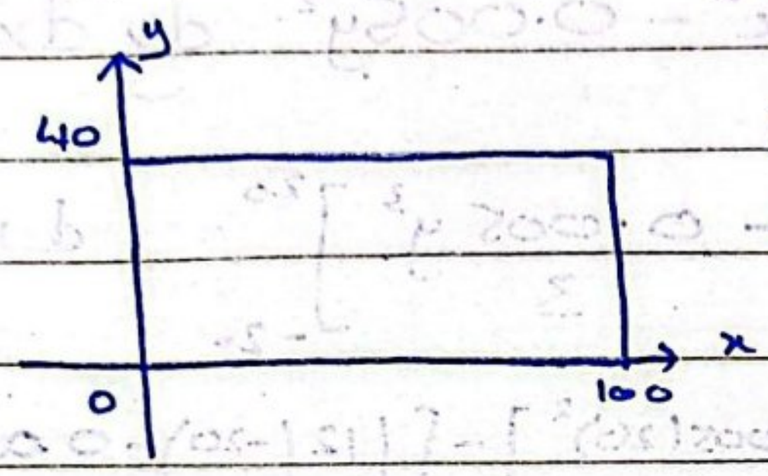
Q7: Surface Integral: $\int_A k(h-y)\,dA$

Width = 100 m, Height = 40m

$k = 10^4\ kg\,m^{-2}s^{-2}$



$x=0$, $x=100$

$y=0$, $y=40$

Total force $= \int_A k(h-y)\,dA$

$F = k\iint h-y\,dA$

$k\int_0^{100}\int_0^{40} h-y\,dy\,dx$

$k\int_0^{100}\left[hy-\frac{y^2}{2}\right]_0^{40}\,dx$

$k\int_0^{100} 40h-800\,dx$

$k\left[40hx-800x\right]$

$k\left[4000h-80000\right]$

$k\left[160000-80000\right]$

$80000k$

$F = 800,000,000$

$k\int_0^{100}\int_0^{h} h-y\,dy\,dx$

$k\int_0^{100}\left|hy-\frac{y^2}{2}\right|_0^{h}\,dx$

$k\int_0^{100} h^2-\frac{h^2}{2}\,dx$

$k\int_0^{100}\frac{h^3}{2}\,dx$

$k\left[\frac{h^2}{2}x\right]$

$F = 800,000,000$

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Q8: Base: $-30 \leq x \leq 30$

$-20 \leq y \leq 20$

Height $= h(x,y) = 12 - 0.003x^2 - 0.005y^2$

a) $$\int_{-30}^{30}\int_{-20}^{20} 12 - 0.003x^2 - 0.005y^2 \; dy\, dx$$

$$\int_{-30}^{30}\left[12y - 0.003x^2 y - \frac{0.005y^3}{3}\right]_{-20}^{20} dx$$

$$\int_{-30}^{30}\left[\left(12(20) - 0.003(20)x^2 - \frac{0.005(20)^2}{3}\right] - \left[\left(12(-20) - 0.003(-20)x^2 - \frac{0.005(20)^2}{3}\right)\right] dx$$

$$\int_{-30}^{30}\left[\frac{718}{3} - \frac{3x^2}{50}\right] - \left[-\frac{722}{3} + \frac{3x^2}{50}\right] dx$$

$$\int_{-30}^{30} \frac{718}{3} - \frac{3x^2}{50} + \frac{722}{3} - \frac{3x^2}{50} \; dx$$

$$\int_{-30}^{30} 480 - \frac{3x^2}{25} \; dx$$

$$\left[480x - \frac{3x^3}{75}\right]_{-30}^{30}$$

$$V = \left[\left(480(30) - \frac{3(30)^3}{75}\right) - \left(480(-30) - \frac{3(-30)^3}{75}\right)\right]$$

$V = 13320 + 13320$

$V = 26640$ Ans

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b) Area of region = b × h

$A = 60 \times 40 = 2400$

$$\text{Avg Height} = \frac{1}{A} \iint_D h(x,y)\, dA$$

$$= \left(\frac{1}{2400}\right)(26640)$$

$$= 11.1$$

The avg height is 11.1 m

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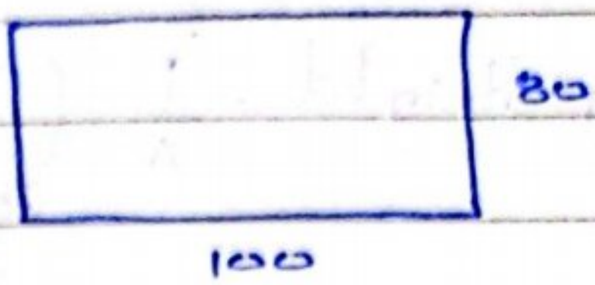
Q9: Surface Integral $= \int_A k(h-y)\,dA$

Width = 100m , Height = 80m

$k = 10^6\ m^{-2}s^{-2}$



$$F = \iint_D k(h-y)\,dA$$

$$k\int_0^{100}\int_0^{80} h-y\ dy\,dx$$

$$k\int_0^{100}\left[hy-\frac{y^2}{2}\right]_0^{80} dx$$

$$k\int_0^{100} 80h - 3200\ dx$$

$$k\left[80hx - 3200x\right]_0^{100}$$

$$k[8000h - 320{,}000]$$

$h = 80$

$$k[320000]$$

$$F = 320000 \times 10^6$$

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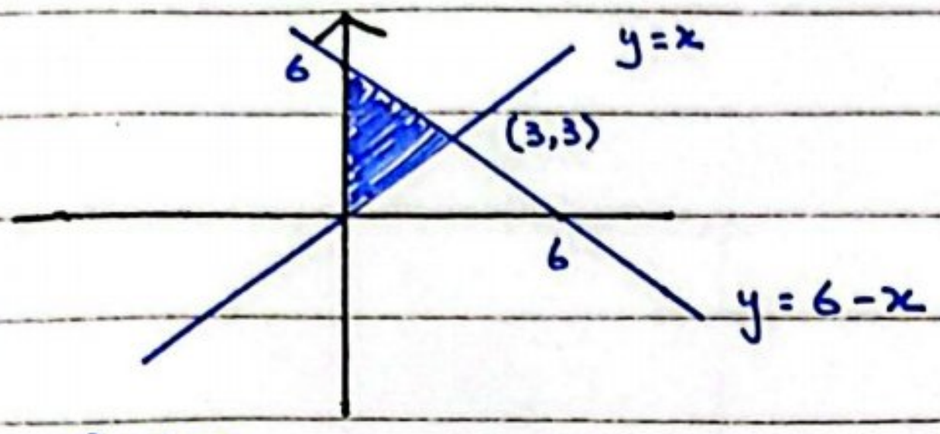
Q10: $h(x,y) = \frac{x}{y+2}$, $x=0$, $y=x$, $x+y=6$



$$V = \int_0^3 \int_x^{6-x} \frac{x}{y+2}\, dy\, dx$$

$$= \int_0^3 \left[x \ln(y+2)\right]_x^{6-x} dx$$

$$= \int_0^3 x\ln(6-x+2) - x\ln(x+2)\, dx$$

$$= \int_0^3 x\ln(8-x) - x\ln(x+2)\, dx$$

$$= \int_0^3 x\ln(8-x)\, dx - \int_0^3 x\ln(x+2)\, dx$$

$$\left[\frac{x^2}{2}\ln(8-x) + \frac{x^2}{4} + 32\ln(8-x) + 4x\right] - \left[\frac{x^2}{2}\ln(x+2) - \frac{x^2}{4} - 2\ln(x+2) + x\right]_0^3$$

$$\left|\frac{x^2}{2}\left(\ln(8-x) - \ln(x+2)\right) + \frac{2x^2}{4} + 32\ln(8-x) + 2\ln(x+2) + 3x\right|_0^3$$

$$\left[\frac{9}{2}\left(\ln(5) - \ln(5)\right) + \frac{18}{4} + 32\ln(5) + 2\ln(5) + 9\right] - \left[32\ln(8) + 2\ln(2)\right.$$

$$\frac{18}{4} + 34\ln(5) + 9 - 32\ln(8) - 2\ln(2)$$

$$\frac{27}{2} + 34\ln(5) - 32\ln(8) - 2\ln(2) \quad \text{Ans.}$$

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