Date

ASSIGNMENT #10

Q1: $\sum_{n=0}^{\infty}\left(\frac{5}{2^n}-\frac{1}{3^n}\right)$

$\sum_{n=0}^{\infty}\left(\frac{5}{2^n}\right)-\sum_{n=0}^{\infty}\left(\frac{1}{3^n}\right)$

$(5-1)+\left(\frac{5}{2}-\frac{1}{3}\right)+\left(\frac{5}{4}-\frac{1}{9}\right)+\left(\frac{5}{8}-\frac{1}{27}\right)+\ldots\ldots$

$\text{Sum} = \frac{5}{1-(\frac{1}{2})} \; \frac{-1}{1-(\frac{1}{3})} = 10-\frac{3}{2}=\frac{17}{2}$

Q2: $\sum_{n=1}^{\infty}\left(\frac{3}{n^2}-\frac{3}{(n+1)^2}\right)$

$S_k=\left(\frac{3}{1}-\frac{3}{4}\right)+\left(\frac{3}{4}-\frac{3}{9}\right)+\left(\frac{3}{9}-\frac{3}{16}\right)+\ldots+\left(\frac{3}{(k-1)^2}-\frac{3}{k^2}\right)+\left(\frac{3}{k^2}-\frac{3}{(k+1)^2}\right)$

$= 3-\left(\frac{3}{(k+1)^2}\right)$

$\lim_{k\to\infty} S_k = \lim_{k\to\infty}\left(3-\frac{3}{(k+1)^2}\right)=3$, series converges to 3.

Q3: $a_n=\left(\frac{3n+1}{3n-1}\right)^n$

$\lim_{n\to\infty}\left(\frac{3n+1}{3n-1}\right)^n = \lim_{n\to\infty}\left(e^{\ln\left(\frac{3n+1}{3n-1}\right)}\right)$

$= \lim_{n\to\infty} e^{\left(\frac{\ln(3n+1)-\ln(3n-1)}{\frac{1}{n}}\right)}$

$= \lim_{n\to\infty} e^{\left(\frac{\frac{3}{3n+1}-\frac{3}{3n-1}}{-\frac{1}{n^2}}\right)}$

$= e^{\frac{6}{9}} = e^{\frac{2}{3}} \Rightarrow$ Converges.