Question 1

You are designing a rectangular garden next to a wall. The garden has two vertical sides, one of which is the wall, and a flat top. The flat top is made of concrete that costs $8 per square foot, and the other two sides are made of wooden fencing, which costs $4 per linear foot. You have a budget of $800 for the garden's construction. Express the width 'w' as a function of the length 'l' of the garden. Also find the domain of the function.

Question 2

Taxicabs such as Uber and Lyft have a unique fare structure. Passengers are charged a flat fee of $3.00 for entering a cab and then an additional $0.75 for each quarter of a mile (or fraction thereof) traveled. Ignoring any additional charges, if 'x' represents the distance traveled in miles, write a function $C(x)$ that represents the cost of the taxi fare.

Using MATLAB, sketch the graph of C(x). Then, based on the graph of the fare function, find each of the following limits, if they exist:

i)   a) $\lim\limits_{x \to 0.30^-} C(x)$ b) $\lim\limits_{x \to 0.30^+} C(x)$ c) $\lim\limits_{x \to 0.30} C(x)$

ii)   a) $\lim\limits_{x \to 0.45^-} C(x)$ b) $\lim\limits_{x \to 0.45^+} C(x)$ c) $\lim\limits_{x \to 0.45} C(x)$

iii)   a) $\lim\limits_{x \to 0.75^-} C(x)$ b) $\lim\limits_{x \to 0.75^+} C(x)$ c) $\lim\limits_{x \to 0.75} C(x)$

Question 3

You are an automotive engineer developing a fuel-efficient hybrid car, the "EcoDrive 2024." The car's MPG (miles per gallon) varies with speed according to the following piecewise function:

$$\begin{cases} 100 - s \; if \; s \le 55mph \\ \dfrac{2200}{s} \; if \; s > 55mph \end{cases}$$

1. Find the speed for maximum MPG in electric mode and with the gasoline engine.
2. Calculate the MPG at specific speeds like 45 mph, 60 mph, and 70 mph.
3. Determine the speed at which the car transitions from electric to gasoline mode.
4. Discuss whether the MPG function is continuous at the transition speed (55 mph) and when approaching it from different directions. Provide reasons for your conclusions.

Question 4

Use the graph of the function f to decide whether the value of the given quantity exists. If it does, find it. If not, explain why.

a. $\displaystyle\lim_{x\to 10} f(x)$

b. $\displaystyle\lim_{x\to -2^+} f(x)$

c. $\displaystyle\lim_{x\to -8} f(x)$

d. $\displaystyle\lim_{x\to 6} f(x)$

e. $f(-8)$

f. the vertical asymptotes of the graph of f