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DS-B
23i-2623

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ASSIGNMENT #07

Q1: $\int \sin^5(x) \cos^{27}(x)\, dx$

$\int (\sin^2(x))^2 (\sin(x)) \cos^{27}(x)\, dx$

$\int (1-\cos^2 x)^2 (\sin x) \cos^{27}(x)\, dx$

$u = \cos x$, $du = -\sin x\, dx$

$dx = -\dfrac{du}{\sin x}$

$\int (1-u^2)^2 (\cancel{\sin x})(u^{27}) \cdot \left(\dfrac{-du}{\cancel{\sin x}}\right)$

$-\int (1-u^2)^2 (u^{27})\, du$

$-\int u^{27} - 2u^{29} + u^{31}\, du$

$-\left[\dfrac{u^{28}}{28} - \dfrac{u^{30}}{15} + \dfrac{u^{32}}{32}\right] + C$

$-\dfrac{\cos^{28}(x)}{28} + \dfrac{\cos^{30}(x)}{15} - \dfrac{\cos^{32}(x)}{32} + C$ ← Ans

Q2: $\int \sin^{20}(x) \cos^5(x)\, dx$

$\int \sin^{20}(x) (\cos^2(x))^2 \cos(x)\, dx$

$\int \sin^{20}(x) (1-\sin^2(x))^2 \cos(x)\, dx$ $\quad u = \sin(x)$

$du = \cos x\, dx$

$\int (u^{20})(1-u^2)^2\, du$

$\int (u^{20})(1^2 - 2u^2 - u^4)\, du$

$\int u^{20} - 2u^{22} - u^{24}\, du$

$\dfrac{u^{21}}{21} - \dfrac{2u^{23}}{23} - \dfrac{u^{25}}{25} + C$

$\dfrac{\sin^{21}(x)}{21} - \dfrac{2\sin^{23}(x)}{23} - \dfrac{\sin^{25}(x)}{25} + C$ Ans.

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Q3: $\int \sin^4(x)\cos^2(x)\, dx$

$$= \int (\sin^2(x))^2 (\cos^2(x))\, dx$$

$$= \int \left(\frac{1-\cos 2x}{2}\right)^2 \left(\frac{1+\cos 2x}{2}\right) dx$$

$$= \frac{1}{8}\int (1-\cos 2x)^2 (1+\cos 2x)\, dx$$

$$= \frac{1}{8}\int [1-2\cos 2x+\cos^2 2x][1+\cos 2x]\, dx$$

$$= \frac{1}{8}\int 1 \cancel{+\cos 2x} - \cancel{2}\cos 2x - \cancel{2}\cos^2 2x + \cancel{\cos^2 2x} + \cos^3 2x \}\, dx$$

$$= \frac{1}{8}\int 1 - \cos 2x - \cos^2(2x) + \cos^3(2x)\, dx$$

$$= \frac{1}{8}\left[\int 1\, dx - \int \cos 2x\, dx - \int \cos^2(2x)\, dx + \int \cos^3(2x)\, dx\right]$$

$$= \frac{1}{8}\left[x - \frac{\sin 2x}{2} - \frac{1}{2}\int (1+\cos 4x)\, dx + \int (1-\sin^2(2x))(\cos 2x)\, dx\right]$$

$u = \sin 2x,\ du = 2\cos 2x\, dx$

$$= \frac{1}{8}\left[x - \frac{u}{2} - \frac{1}{2}\int (1+\cos 4x)\, dx + \int \frac{(1-u^2)}{2}\, du\right]$$

$$= \frac{1}{8}\left[x - \frac{\sin 2x}{2} - \frac{1}{2}x - \frac{1}{4}\sin(4x) + \frac{1}{2}\left(\sin 2x - \frac{1}{3}\sin^3 2x\right)\right]$$

$$= \frac{1}{8}\left[\frac{x}{2} - \cancel{\frac{\sin 2x}{2}} - \frac{\sin 4x}{4} + \cancel{\frac{\sin 2x}{2}} - \frac{\sin^3 2x}{3}\right]$$

$$= \frac{1}{8}\left[\frac{x}{2} - \frac{\sin 4x}{4} - \frac{\sin^3 2x}{3}\right] \leftarrow \text{Ans}$$

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Q4. $\int \cot^4(x)\,dx$

$\int \frac{\cos^4(x)}{\sin^4(x)}\,dx$ $\cancel{u = \sin(x)}$

$\cancel{du = \cos(x)\,dx \Rightarrow dx = \frac{du}{\cos(x)}}$

$\int \frac{(\cos^2(x))^2}{(\sin^2(x))^2}$

$\int \cot^2(x)\cot^2(x)\,dx$

$\int \cot^2(x).(\csc^2(x)-1)\,dx$

$\int \cot^2(x)\csc^2(x) - \cot^2(x)\,dx$

$\int \cot^2(x)\csc^2(x)\,dx - \int \cot^2(x)\,dx$

$\int \cot^2(x)\csc^2(x)\,dx - \int \frac{1-\sin^2(x)}{\sin^2(x)}\,dx$

$u = \cot(x)$

$du = -\csc^2(x)\,dx$

$-\int u^2\,du - \int \frac{1}{\sin^2 x}\,dx - \int \frac{\sin^2(x)}{\sin^2(x)}\,dx$

$-\int \frac{u^3}{3} - \int \csc^2(x)\,dx - \int 1\,dx$

$\cancel{-\left(\frac{\cot^3(x)}{3} + (-\cot x) - (+x)\right)}$

$-\frac{\cot^3(x)}{3} + \cot(x) - x + C$ Ans.

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Q5: $\int \csc^3(x)\,dx$

$I = \int \csc^2(x) \cdot \csc(x)\,dx$

$u = \csc(x)$ $\quad\quad dv = \csc^2(x)$

$du = -\csc(x)\cot(x)\,dx$ $\quad\quad v = -\cot(x)$

$I = uv - \int v\,du$

$I = -\csc(x)\cot(x) - \int -\cot(x) \cdot -\csc(x)\cot(x)\,dx$

$I = -\csc(x)\cot(x) - \int +1\left(\cot^2 x\,\csc(x)\right)\,dx$

$\cancel{I = -\csc(x)\cot(x) + \int \csc(x)(\csc^2 - 1}$

$I = -\csc(x)\cot(x) - \int (\csc^2(x) - 1)(\csc(x))\,dx$

$I = \cancel{-\csc(x)}$ $\quad - \int \csc^3(x) - \csc(x)\,dx$

$- \int \csc^3(x)\,dx - \int \csc(x)\,dx$

$- [I + \ln|\csc x + \cot x|]$

$2I = -\csc(x)\cot(x) - \ln|\csc x + \cot x|$

$I = \frac{1}{2}\left[-\csc(x)\cot(x)\right] - \frac{1}{2}\left[\ln|\csc x + \cot x|\right] + C$

Q6. $\int \tan^{120} x \sec^4(x) \, dx$.

$\int \tan^{120}(x) \sec^2(x) \sec^2(x) \, dx$ $\qquad$ $\sec^2(x) = 1 + \tan^2(x)$

$\int \tan^{120}(x)\left(\tan^2(x) + 1\right)\left(\sec^2(x)\right) \, dx$

$\int \left(\tan^{122}(x) + \tan^{120}(x)\right) \sec^2(x) \, dx$

$u = \tan(x)$

$du = \sec^2(x) \, dx$

$dx = \dfrac{du}{\sec^2(x)}$

$\int \left(u^{122} + u^{120}\right) \cancel{\sec^2(x)} \cdot \dfrac{du}{\cancel{\sec^2(x)}}$

$\int u^{122} + u^{120} \, du$

$\dfrac{u^{123}}{123} + \dfrac{u^{121}}{121} + C$

$\dfrac{\tan^{123}(x)}{123} + \dfrac{\tan^{121}(x)}{121} + C$ A.