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# ASSIGNMENT #3

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Q1:

(i) This statement is false. For a set $\{v_1, v_2, v_3\}$ that is linearly dependent, $v_1$ and $v_2$ can have $V_3$ in their span however the same cannot be said for $v_1$ and $v_3$ having $v_2$ in their span.

Example: $T\{\vec{v_1}, \vec{v_2}, \vec{0}\}$

$$\vec{0} = 0.\vec{v_1} + 0.\vec{v_2}$$
$$\vec{v_1} \neq c.\vec{0} + c\vec{v_2}$$

(ii) While it is true that a set of ~~ve~~ $>n$ vectors from $\mathbb{R}^n$ is guaranteed to be linearly dependent, ~~the~~ ~~[illegible]~~ ~~[illegible]~~ ~~[illegible]~~ a set can still be linearly dependent even if contains $\leq n$ vectors.

Example: $v_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $v_2 = \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}$ $v_3 = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}$

This set contains $n=3$ vectors in $\mathbb{R}^{n=3}$. However it is linearly dependent as $v_1$ and $v_2$ lie on the same line (scalar multiples)

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(iii) This statement is true because if all columns of a matrix are pivot colums, those columns cannot be written as combinations of other colums and are hence, linearly independent

(iv) True because if u and v are linearly independent they cannot be written as scalar multiples of each other and the only solution to $c_1 u + c_2 v = 0$ is at $c_1 = c_2 = 0$. As the system itself is linearly dependent this can only mean that $\vec{w}$ can be written either as a scalar multiple of one ~~at~~ of the vectors or is in their span.

(v) This statement ~~has a~~ is true since having "at most one solution" requires the homogenous system $Ax = 0$ to have only the trivial solution, this implies that the columns of A must be linearly independent.

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Q#2 $u = \begin{bmatrix} h \\ -\frac{1}{2} \\ -\frac{1}{2} \end{bmatrix}$ $v = \begin{bmatrix} -\frac{1}{2} \\ h \\ -\frac{1}{2} \end{bmatrix}$ $w = \begin{bmatrix} -\frac{1}{2} \\ -\frac{1}{2} \\ h \end{bmatrix}$

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(i) $\left[\begin{array}{cc|c} h & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & h & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & h \end{array}\right]$

$\left[\begin{array}{cc|c} -\frac{1}{2} & -\frac{1}{2} & h \\ -\frac{1}{2} & h & -\frac{1}{2} \\ h & -\frac{1}{2} & -\frac{1}{2} \end{array}\right]$ Swap R1 , R3

$\left[\begin{array}{cc|c} 1 & 1 & -2h \\ 1 & -2h & 1 \\ -2h & 1 & 1 \end{array}\right]$ R1,R2,R3 * -2

$\left[\begin{array}{cc|c} 1 & 1 & -2 \\ 0 & -2h-1 & 1+2h \\ 0 & 1+2h & 1-4h^2 \end{array}\right]$ R2-R1, R3+2hR1

$\cancel{1}-4h^2 \quad \cancel{-1}-2h$

$-(4h^2+2h)=0$

$-4h^2=2k$

$\boxed{h=-\frac{1}{2}}$

$\left[\begin{array}{cc|c} 1 & 1 & -2 \\ 0 & -2h-1 & 1+2h \\ 0 & 0 & -4h^2-2h \end{array}\right]$ R3-R2

$u + v = -2w$ (1)

$(-2h-1)v = (1+2h)w$ (2)

$-(\cancel{1+2h})v = (\cancel{1+2h})w$

$w = -v$

For $h=-1/2$, $w$ is in the Span(u,v).

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(ii) $\left[\begin{array}{ccc|c} h & -1/2 & -1/2 & 0 \\ -\frac{1}{2} & h & -1/2 & 0 \\ -\frac{1}{2} & -1/2 & h & 0 \end{array}\right]$ Mixed

$\left[\begin{array}{ccc|c} 1 & 1 & -2h & 0 \\ 1 & -2h & 1 & 0 \\ -2h & 1 & 1 & 0 \end{array}\right]$ $*-2h$ <br> $*-2h$. <br> (Swap R1,R3)$^{*}-2h$

$\left[\begin{array}{ccc|c} 1 & 1 & -2h & 0 \\ 0 & -2h-1 & 1+2h & 0 \\ 0 & 1+2h & 1-2h & 0 \end{array}\right]$ R2-R1 <br> R3+2h(R1)

$~~1~~-2h-1-2h$

$-4h$

$$\begin{array}{c} \begin{array}{cccc} u & v & w & \end{array} \\ \left[\begin{array}{ccc|c} 1 & 1 & -2h & 0 \\ 0 & -2h-1 & 1+2h & 0 \\ 0 & 0 & -4h & 0 \end{array}\right] \end{array}$$ R3-R2

$u+v-2hw=0$

$(-2h-1)v+(1+2h)w=0$

~~$-4hw=0 \Rightarrow w=0$~~

$-4h=0$

$h=0$

The system is linearly dependent at all values of h ~~all~~ ~~the~~ as the system has w as a free variable for all values of h and hence ~~to~~ the homogenous equation has a non-trivial solution.

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Q3 $u = \begin{bmatrix} 0 \\ 3 \\ 1 \\ -1 \end{bmatrix}, v = \begin{bmatrix} 6 \\ 0 \\ 5 \\ 1 \end{bmatrix}, w = \begin{bmatrix} 4 \\ -7 \\ 1 \\ 3 \end{bmatrix}$

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$$\left[\begin{array}{ccc|c} 0 & 6 & 4 & 0 \\ 3 & 0 & -7 & 0 \\ 1 & 5 & 1 & 0 \\ -1 & 1 & 3 & 0 \end{array}\right]$$

(1)- $u + 5v + w = 0$

(2)- $3v + 2w = 0$

$$\left[\begin{array}{ccc|c} 1 & 5 & 1 & 0 \\ 3 & 0 & -7 & 0 \\ 0 & 6 & 4 & 0 \\ -1 & 1 & 3 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 5 & 1 & 0 \\ 0 & -15 & -10 & 0 \\ 0 & 6 & 4 & 0 \\ 0 & 6 & 4 & 0 \end{array}\right] \begin{array}{l} \\ R2 - 3R1 \\ \\ R4 + R1 \end{array}$$

$$\begin{array}{r} \\ \\ q3- \\ q4- \end{array} \begin{array}{c} \begin{array}{cccc} u & v & w & \end{array} \\ \left[\begin{array}{ccc|c} 1 & 5 & 1 & 0 \\ 0 & 3 & 2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \end{array} \begin{array}{l} \\ -(R2)/15 \\ R3 - 2R2 \\ R4 - 2R2 \end{array}$$

As $w$ is a free variable at all times this system is linearly dependent as it has a non-trivial solution.