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Calculus Assignment 6

Q1: ~~Form~~ $\int u\,dv = uv - \int v\,du$

$u = \{\ln(x)\}^n \quad , \quad dv = dx$

$du = n(\ln(x))^{n-1}\left(\frac{1}{x}\right)\,dx$

$v = x$

$$\int (\ln(x))^n\,dx = x(\ln(x))^n - n\int (\ln(x))^{n-1}\,dx$$

$\rightarrow \int (\ln(x))^5\,dx:$

$$\int (\ln(x))^5\,dx = x(\ln(x))^5 - 5\int (\ln(x))^4\,dx$$

$$\int (\ln(x))^4\,dx = x(\ln(x))^4 - 4\int (\ln(x))^3\,dx$$

$$\int (\ln(x))^3\,dx = x(\ln(x))^3 - 3\int \{(\ln(x))^2\}\,dx$$

$$\int (\ln(x))^2\,dx = x(\ln(x))^2 - 2\int \{(\ln(x))^1\}\,dx$$

$$\int (\ln(x))\,dx = \frac{1}{2}x(\ln(x))^2 - \int \frac{1}{2}x \cdot \frac{1}{x}\,dx$$

Simplify:

$$= \frac{1}{2}x(\ln(x))^2 - \frac{1}{2}\int dx$$

$$= \frac{1}{2}x(\ln(x))^2 - \frac{1}{2}x + C$$

$$\int (\ln(x))^2\,dx = x(\ln(x))^2 - 2\left(\frac{1}{2}x(\ln(x))^2 - \frac{1}{2}x + C\right)$$

$$= \cancel{x(\ln(x))^2} - \cancel{x(\ln(x))^2} + x - C$$

$$= x - C$$

Date

$\int(\ln(x))^3\,dx = x(\ln(x))^3 - 3(x - C)$

$= x(\ln(x))^3 - 3x + 3C$

$\int(\ln(x))^4\,dx = x(\ln(x))^4 - 4x + 4C$

$\int(\ln(x))^5\,dx = x(\ln(x))^5 - 5(x + C)$

$= x(\ln(x))^5 - 5x + 5C$

$\int(\ln(x))^5 = x(\ln(x))^5 - 5x + 5C + K$

where $5C$ is constant from every step and K is constant from final step

Q2: $\int\sqrt{a^2 - x^2}\,dx = \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a} + \frac{x}{2}\sqrt{a^2 - x^2}\right) + c$

$\int\sqrt{a^2 - x^2}\,dx$

$x = a\sin(\theta)$, $dx = a\cos(\theta)\,d\theta$

$\cancel{\int\sqrt{a^2 - a^2\sin^2(\theta)}(a\cos(\theta))\,d\theta}$ $\int\sqrt{a^2 - a^2\sin^2(\theta)}\;a\cos(\theta)\,d\theta$

$\int\sqrt{a^2(1 - \sin^2(\theta))}(a\cos(\theta))\,d\theta$

$\int\sqrt{a^2\cos^2\theta}\,(a\cos(\theta))\,d\theta$

$\int a^2\cos^2\theta\,d\theta$

$\cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$

$$= \int a^2\left(\frac{1}{2}\left(1+\cos(2\theta)\right)\right) d\theta$$

$$= \frac{a^2}{2}\int\left(1+\cos(2\theta)\right) d\theta$$

$$= \frac{a^2}{2}\left[\int 1 \, d\theta + \int 2\cos(2\theta)\, d\theta\right.$$

$$= \frac{a^2}{2}\left(\theta + \frac{1}{2}\sin(2\theta)\right) + C$$

$$\theta = \sin^{-1}\left(\frac{x}{a}\right)$$

$$= \frac{a^2}{2}\left(\sin^{-1}\left(\frac{x}{a}\right) + \frac{1}{2}\sin\left(2\sin^{-1}\left(\frac{x}{a}\right)\right)\right) + C$$

$$\sin(2\alpha) = 2\sin\alpha\cos\alpha$$

$$\cancel{= \frac{a^2}{2}\left(\sin^{-1}\left(\frac{x}{a}\right) + \sin\left(\sin^{-1}\left(\frac{x}{a}\right)\right)\left(\cos\left(\frac{x}{a}\right)\right)\right.}$$

$$= \frac{a^2}{2}\left(\sin^{-1}\left(\frac{x}{a}\right) + \sin\left(\sin^{-1}\left(\frac{x}{a}\right)\right)\cos\left(\sin^{-1}\left(\frac{x}{a}\right)\right)\right) + C$$

$$\sin^{-1}(\sin(x)) = x \quad , \quad \cos(\sin^{-1}(x)) = \sqrt{1-x^2}$$

$$= \frac{a^2}{2}\left(\sin^{-1}\left(\frac{x}{a}\right) + \cancel{\frac{x}{a}\left(1-x^2\right.} \frac{x}{a}\left(\sqrt{1-\left(\frac{x}{a}\right)^2}\right)\right) + C$$

$$\cancel{\frac{a^2}{2}\left(\sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{a}\sqrt{\frac{a^2-x^2}{a^2}}\right.}$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\theta = \sin^{-1}\left(\frac{x}{a}\right)$$

$$\cos\theta = \sqrt{1-\left(\frac{x}{a}\right)^2}$$

$$= \frac{a^2}{2}\left(\sin^{-1}\left(\frac{x}{a}\right) + \frac{x}{a}\sqrt{a^2-x^2} + C\right)$$

$$= \frac{a^2}{2}\left(\sin^{-1}\left(\frac{x}{a} + \frac{x}{2}\right)\right)\sqrt{a^2-x^2} + C$$

Date

Q3: $\int x \sin(\pi x)\cos(ex)\, dx$

$= \frac{1}{2}\int x\left(\sin((e+\pi)x) - \sin((e-\pi)x)\right) dx$

$= \frac{1}{2}\int \left(x \sin(\pi x + ex) - x \sin(ex - \pi x)\right) dx$

$= \frac{1}{2}\int (x\sin(\pi x + ex) - \frac{1}{2}\int x \sin(ex - \pi x)\, dx$

$= \frac{1}{2}\int \left(x \sin((e+\pi)x\right) dx - \frac{1}{2}\int x \sin((e-\pi)x)\, dx$

$u = x \quad , \quad dv = \sin((e+\pi)x)\, dx$

$du = dx \quad , \quad v = -\frac{\cos((e+\pi)x)}{e+\pi}$

$= \frac{-x\cos((e+\pi)x)}{2(e+\pi)} + \frac{1}{2(e+\pi)}\int \cos((e+\pi)x)\, dx - \frac{1}{2}\int x \sin(ex - \pi x)\, dx$

$u = (e+\pi)x \quad , \quad du = (e+\pi)\, dx$

$= \frac{-x\cos((e+\pi)x)}{2(e+\pi)} + \left(\frac{1}{2(e+\pi)^2}\right)\int \cos(u)\, du - \frac{1}{2}\int x \sin(ex - \pi x)\, dx$

$= \frac{-x\cos((e+\pi)x}{2(e+\pi)} + \frac{1}{2}\cdot\frac{1}{(e+\pi)^2} + \frac{\sin(u)}{2(e+\pi)^2} - \frac{1}{2}\int x \sin(ex - \pi x)\, dx$

$= \frac{-x\cos((e+\pi)x)}{2(e+\pi)} + \frac{\sin(u)}{2(e+\pi)^2} - \frac{1}{2}\int x \sin((e-\pi)x)\, dx$

$u = x \qquad dv = \sin((e-\pi))x\, dx$

$du = dx \qquad v = -\frac{\cos((e-\pi)x)}{e-\pi}$

$= \frac{x\cos((e-\pi)x)}{2(e-\pi)} - \frac{x\cos((e+\pi)x)}{2(e+\pi)} + \frac{\sin(u)}{2(e+\pi)^2} - \frac{1}{2(e-\pi}\int \cos(e-\pi)x\, d$

Date

* $s = (e-\pi)x$ and $ds = (e-\pi)\,dx$

$$= \frac{x\cos((e-\pi)x)}{2(e-\pi)} - \frac{x\cos((e+\pi)x)}{2(e+\pi)} + \frac{\sin(u)}{2(e+\pi)^2} - \frac{1}{2(e-\pi)^2}\int \cos(s)\,ds$$

$$= \frac{-\sin(s)}{2(e-\pi)^2} + \frac{\sin(u)}{2(e+\pi)^2} + \frac{x\cos((e-\pi)x)}{2(e-\pi)} - \frac{x\cos((e+\pi)x)}{2(e+\pi)} + C$$

$s = (e-\pi)x$, $u = (e+\pi)x$

$$= \frac{1}{2}\left(\frac{-\sin((e-\pi)x)}{(e-\pi)^2} + \frac{\sin((e+\pi)x)}{(e+\pi)^2} + \frac{x\cos((e-\pi)x)}{e-\pi} - \frac{x\cos((e+\pi)x)}{e+\pi}\right)$$

$+ C$ ⇐ Ans.

Q4: $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\sin x}{(1+\cos x)(2+\cos x)}\,dx$

$u = \cos x$, $u = \cos(0) = 1$, $u = \cos\left(\frac{\pi}{2}\right) = 0$

$du = -\sin(x)\,dx$

$$= -\int_1^0 \frac{1}{(1+u)(2+u)}\,du \Rightarrow \int_0^1 \frac{1}{(1+u)(2+u)}\,du$$

$$\frac{1}{(1+u)(2+u)} = \frac{A}{(1+u)} + \frac{B}{(2+u)}$$

$1 = A(2+u) + B(1+u)$

$u = -2$

$1 = B(-1)$

$B = -1$

$u = -1$

$1 = A$

$A = 1$

$$= \int_0^1 \frac{1}{1+u}\,du - \int_0^1 \frac{1}{2+u}\,du$$

Date

$s = u+2$ low $s = 2+0 = 2$, up $s = 2+1 = 3$

$ds = du$

$$= -\int_2^3 \frac{1}{s}\, ds + \int_0^1 \frac{1}{u+1}\, du$$

$$= -(\log(s))\Big|_2^3 + \int_0^1 \frac{1}{u+1}\, du$$

$$= -\log\left(\frac{3}{2}\right) + \int_0^1 \frac{1}{u+1}\, du$$

$p = u+1$, low $p = 1$, up $p = 2$

$dp = du$.

$$= -\log\left(\frac{3}{2}\right) + \int_1^2 \frac{1}{p}\, dp$$

$$= -\log\left(\frac{3}{2}\right) + \log(p)\Big|_1^2$$

$$= \log(2) - \log\left(\frac{3}{2}\right)$$

$$= \log\left(\frac{4}{3}\right)$$ Ans.