Assignment 2

Computational Fluid Dynamics- AE 706
Professor: **JC Mandal**

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Consider a physical domain between a ellipse (with semi-major axis a 1 m and semi-minor axis 0.5 m) and far field boundary located at 20m as shown in the Figure 1. You need to derive Transfinite interpolation mapping for generating grid in the physical domain. A (imaginary) cut is introduced so that physical domain becomes simply connected domain, which can then be mapped into unit square in the computational domain as shown in the Figure 1. Note that the co-ordinates of the points a and d are identical to the co-ordinates of b and c respectively.
(1) Derive the transfinite interpolation function/mapping for generating grid inside the physical domain showing every important steps.

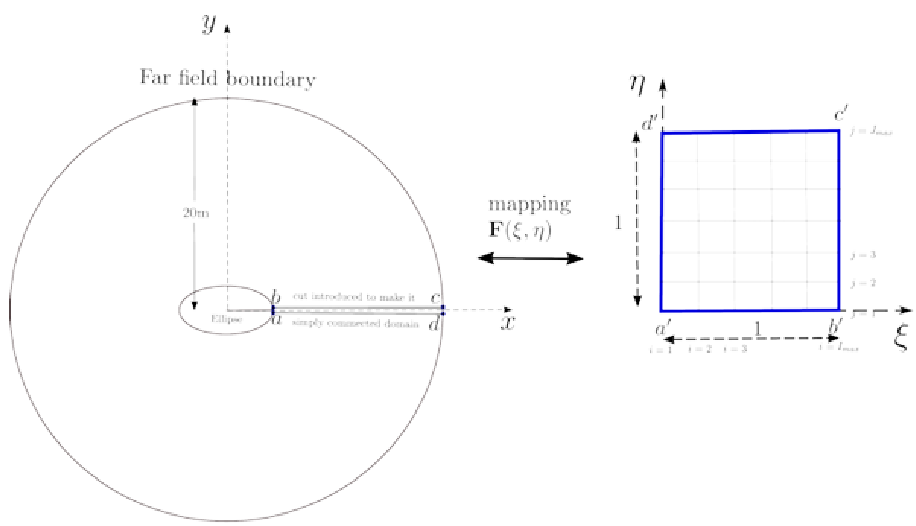

Figure 1: (i) Physical Domain & (ii) Computational Domain

Solution:

For $ab \rightarrow a'b'$:

$$F(\xi, 0) = \begin{bmatrix} \cos 2\pi\xi \\ \frac{1}{2} \sin 2\pi\xi \end{bmatrix} \tag{1}$$

For $cd \rightarrow c'd'$:

$$F(\xi, 1) = \begin{bmatrix} 20 \cdot \cos 2\pi\xi \\ 20 \cdot \sin 2\pi\xi \end{bmatrix} \tag{2}$$

For $ad \rightarrow a'd'$:

$$F(0, \eta) = \begin{bmatrix} 1 + 19\eta \\ 0 \end{bmatrix} \tag{3}$$

For $cb \rightarrow c'b'$:

$$F(1, \eta) = \begin{bmatrix} 1 + 19\eta \\ 0 \end{bmatrix} \tag{4}$$

And,

$$F(0,0) = \begin{bmatrix}1\\0\end{bmatrix}, \quad F(0,1) = \begin{bmatrix}20\\0\end{bmatrix}, \quad F(1,0) = \begin{bmatrix}1\\0\end{bmatrix}, \quad F(1,1) = \begin{bmatrix}20\\0\end{bmatrix} \tag{5}$$

By Transfinite Interpolation Function, we have:

$$P_\xi(F(\eta)) = (1-\xi)F(0,\eta) + \xi F(1,\eta) \tag{6}$$

$$P_\eta(F(\xi)) = (1-\eta)F(\xi,0) + \eta F(\xi,1) \tag{7}$$

$$P_{\xi\eta}(F) = (1-\xi)(1-\eta)F(0,0) + (1-\xi)\eta F(0,1) + \xi(1-\eta)F(1,0) + \xi\eta F(1,1) \tag{8}$$

Thus,

$$F(\xi,\eta) = P_\xi(F(\eta)) + P_\eta(F(\xi)) - P_{\xi\eta}(F) \tag{9}$$

Putting eq.(6), (7) and (8) in eq. (9), we get Now,

$$x(\xi,\eta) = [(1 + 19\eta - \xi - 19\eta\xi + \xi + 19\eta\xi + \cos 2\pi\xi \tag{10}$$

$$-\eta\cos 2\pi\xi + 20\eta\cos(2\pi\xi) - 1 + \xi + \eta - \xi\eta - 20\eta + 20\eta\xi - \xi\eta - 20\cdot\xi\eta] \tag{11}$$

$$y(\xi,\eta) = \left(\frac{1-\eta}{2} + 20\eta\right)\sin(2\pi\xi) \tag{12}$$

Thus,

$$x(\xi,\eta) = (1+19\eta)\cos 2\pi\xi \tag{13}$$

$$y(\xi,\eta) = \left(\frac{1+39\eta}{2}\right)\sin 2\pi\xi \tag{14}$$

So,

$$F(\xi,\eta) = \begin{bmatrix}x(\xi,\eta)\\y(\xi,\eta)\end{bmatrix} = \begin{bmatrix}(1+19\eta)\cos 2\pi\xi\\ \left(\frac{1+39\eta}{2}\right)\sin 2\pi\xi\end{bmatrix} \tag{15}$$

The above equation generates the grid when iterated 101 and 81 times, respectively. A Python code has been written to do this task and generate the required grids.

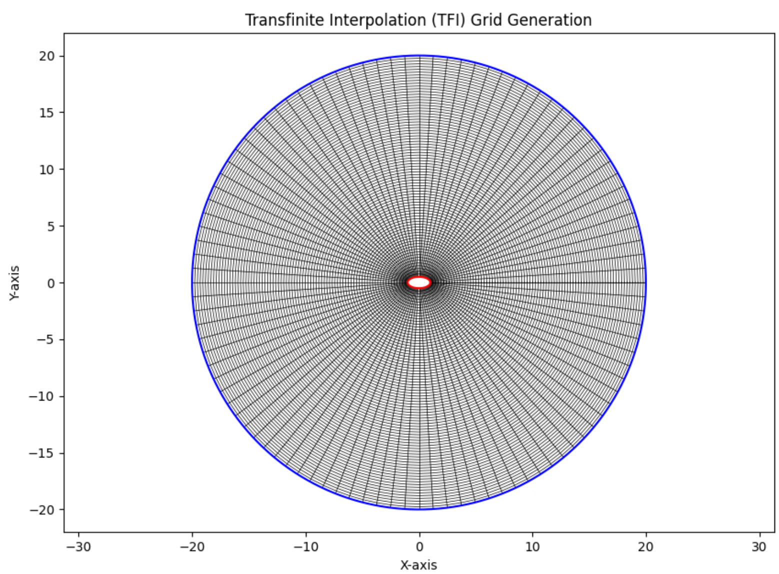

Figure 2: Grid Generated using Python Code