ASSIGNMENT-5

Computation of Lid-Driven Cavity Flow

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The goal of this assignment is to numerically solve the steady-state lid-driven cavity (as shown in Figure 1) flow using the Vorticity-Stream Function formulation. The flow is incompressible and two-dimensional. The velocity field is derived from the stream function, and the vorticity transport equation governs the evolution of vorticity in the domain.

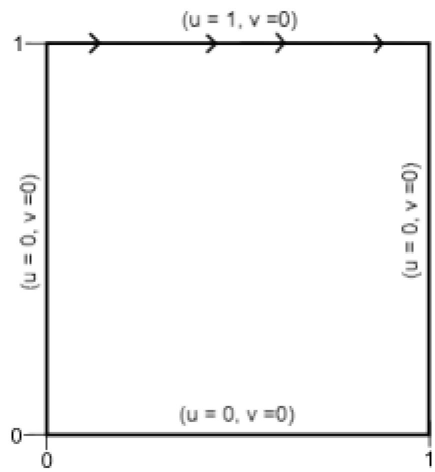


Figure1: Lid driven cavity

The following governing equations are to be solved

$$\frac{\partial w}{\partial t} = u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} = \boldsymbol{\nu}\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)$$

$$w = \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y}$$

$$u = \frac{\partial \psi}{\partial y},\ v =- \frac{\partial \psi}{\partial x}$$

and the stream function equation or Poisson equation

$$\left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = w\right)$$

Approximate all spatial derivatives using second-order central difference schemes, except for the spatial derivatives for the convective terms in the vorticity equation, which should be discretized using second order upwind differences.
Time marching using explicit Euler schemes. Only Point Jacobi or Gauss Seidel Methods are allowed to solve this problem.
Take Reynolds Number to be 100, σ_c=0.4 and σ_d=0.6. Take the grid size to be of 31x31 dimensions.

Boundary Conditions:

a) Top wall moving with u= u_{lid} = 1m/s. For all the other walls, no slip condition is valid.

b) At the top wall, $w = \frac{2\left(\psi_{i,j-1} - \psi_{i,j}\right)}{\Delta y^2} - \frac{2u_{lid}}{\Delta y}$

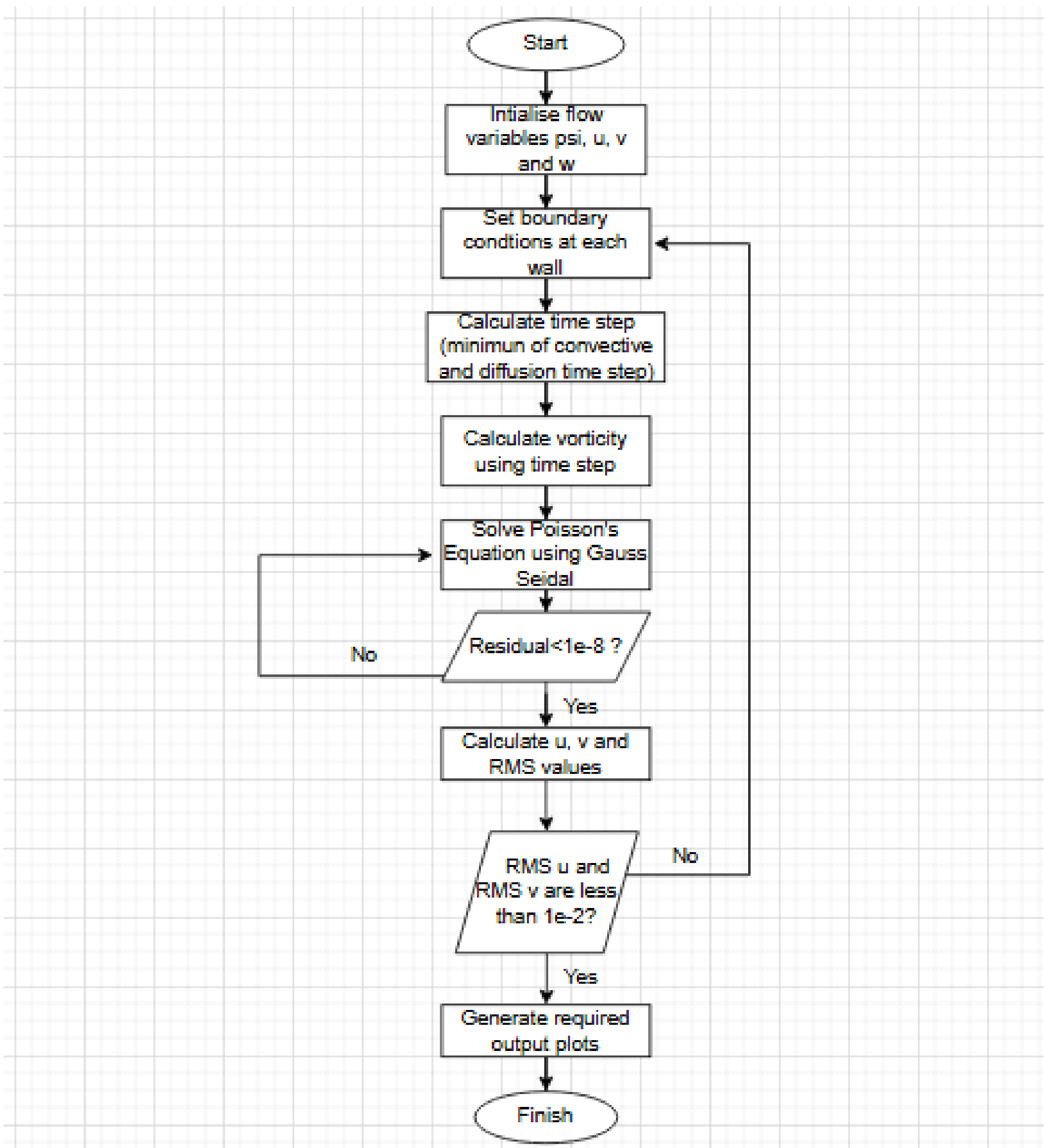
c) On the other walls it should be

$$\omega_{1,j} = -\frac{2(\psi_{2,j} - \psi_{1,j})}{\Delta x^2} \quad \text{(left wall)}$$

$$\omega_{1,j} = -\frac{2(\psi_{I-1,j} - \psi_{I,j})}{\Delta x^2} \quad \text{(right wall)}$$

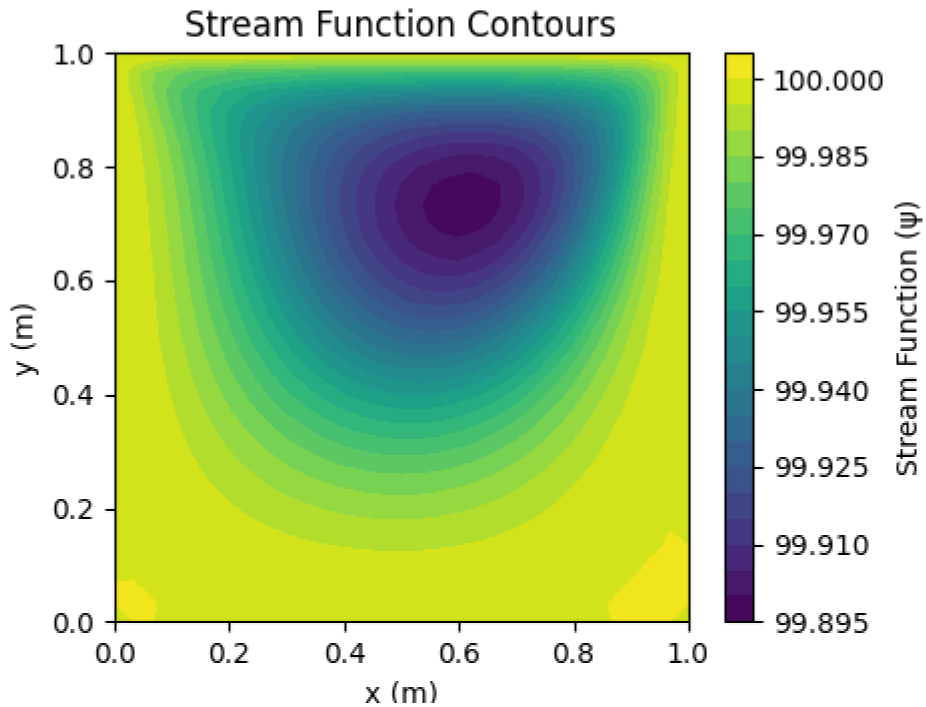
$$\omega_{1,j} = -\frac{2(\psi_{i,2} - \psi_{i,1})}{\Delta y^2} \quad \text{(bottom wall)}$$

Flowchart

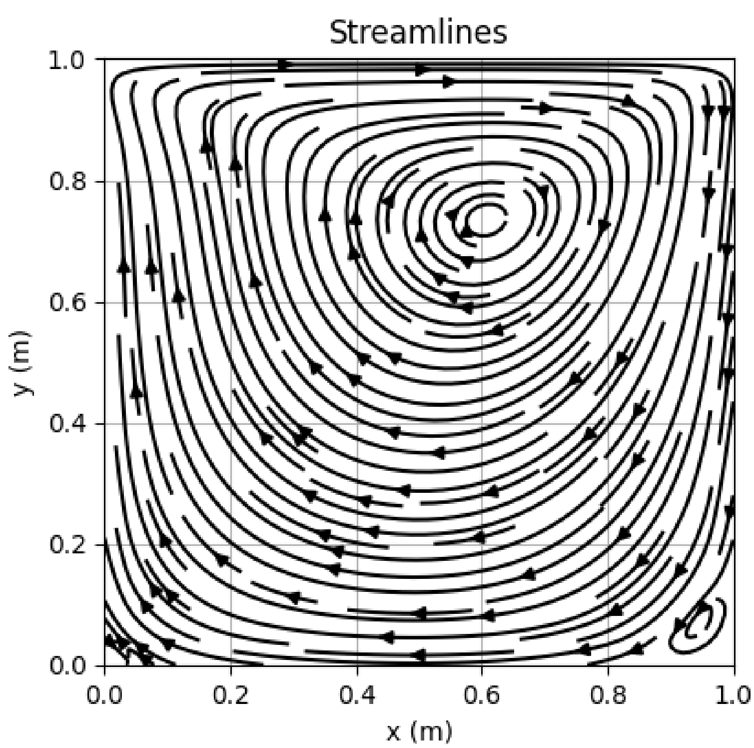


Solution:

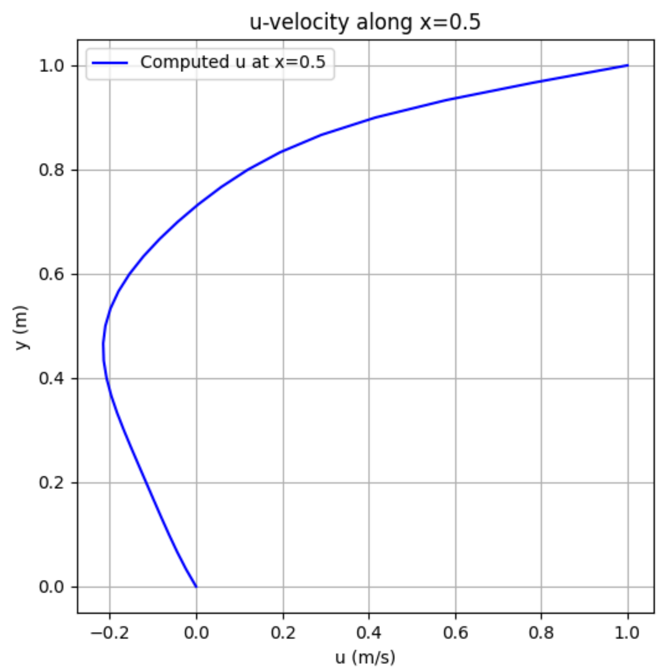
a) Stream Function Contour

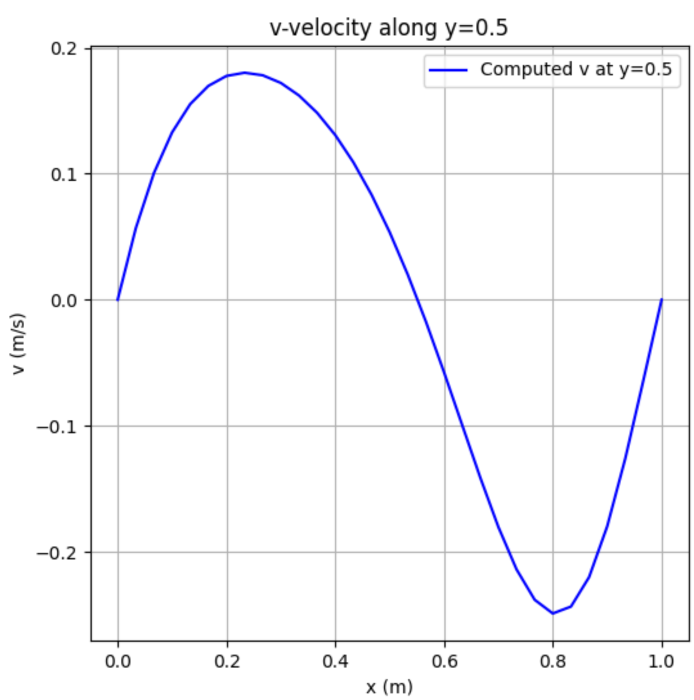


b) Streamline plot



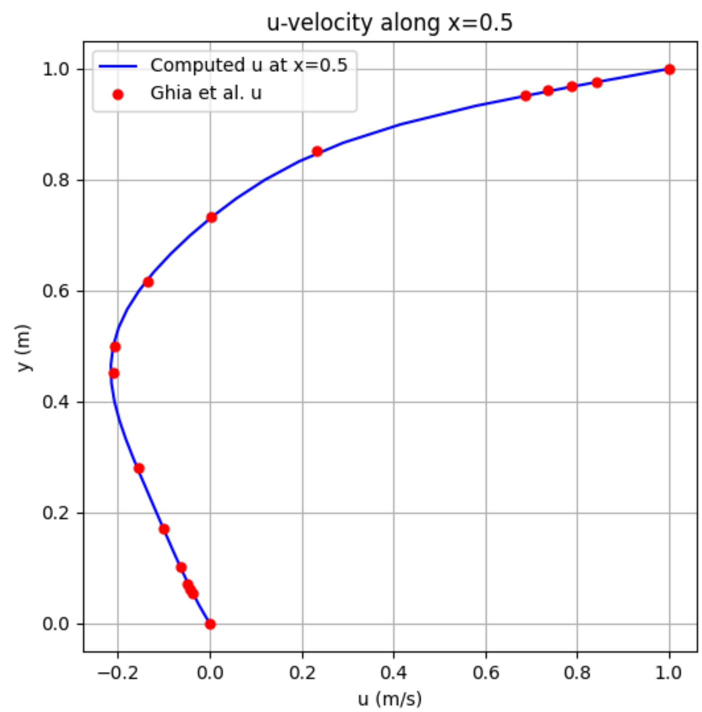
c) Velocities at x =0.5 and y = 0.5

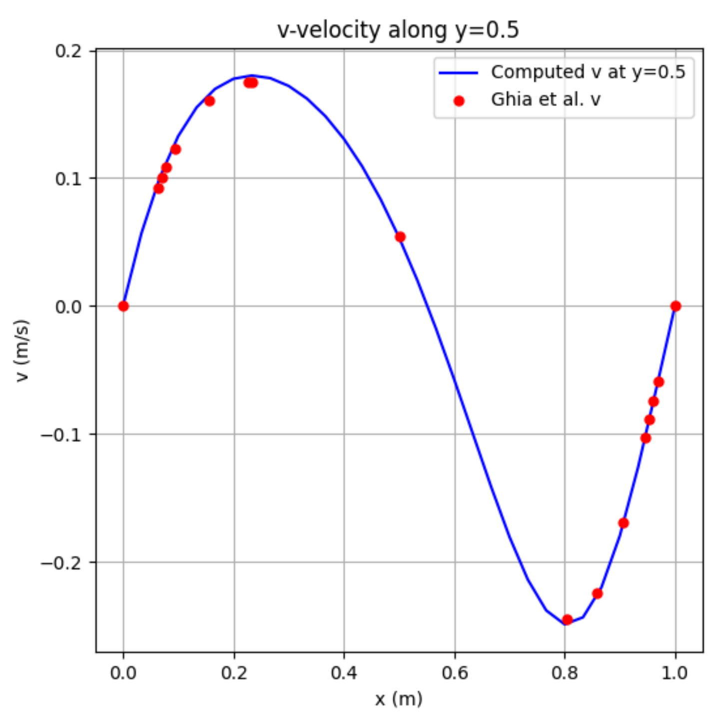




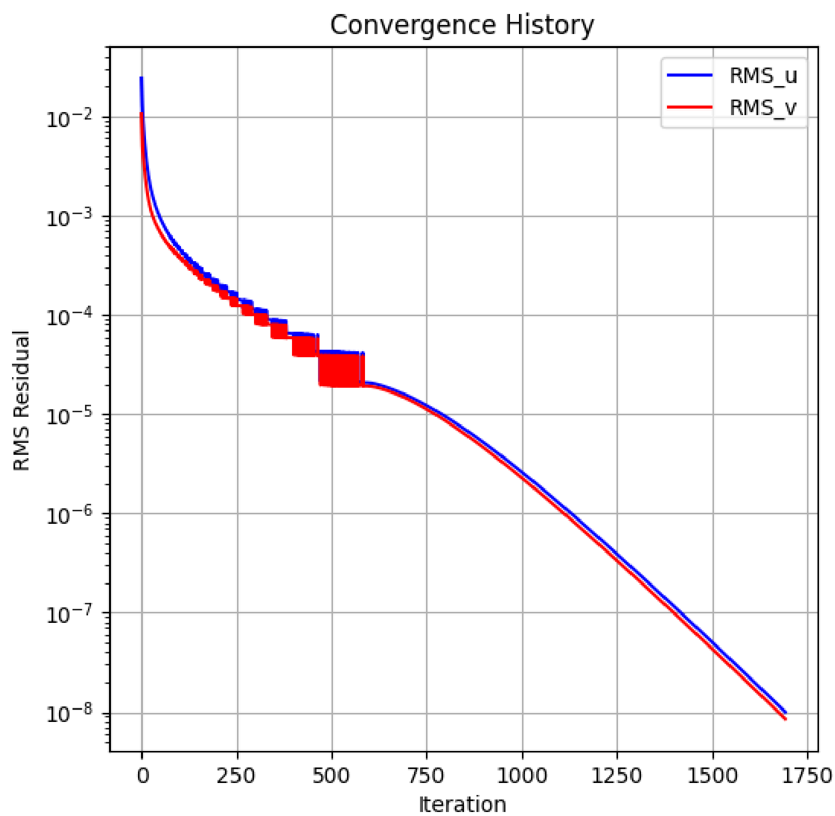
The above plots can be generated from the same code by commenting out the lines in u and v velocity plotting section where “Ghia. et al” plots are plotted.

d) Comparison of velocities with benchmark data





e) Convergence plots



Results:

a) The stream function and the streamline plots are in correspondence with each other.
b) Both plots also captured the central vortex perfectly.
c) The velocity plots generated are in excellent agreement with the reference date provided.
d) The convergence plot shows that residuals converged well below 2000 iterations.