ASM - MST 1

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3rd Question

Evaluate the normal and shearing stresses on the planes, whose normal has the following direction cosines:

(a) $n_x = n_y = \frac{1}{\sqrt{2}}, n_z = 0$

(b) $n_x = 0, n_y = n_z = \frac{1}{\sqrt{2}}$

(c) $n_x = n_y = n_z = \frac{1}{\sqrt{3}}$

for a point which has 200 MPa, -100 MPa, -100 MPa normal stresses in x,y and z direction and -200 MPa is the value of $\tau_{xy}, \tau_{yz}, \tau_{zx}$.

Solution

(a) $n_x = n_y = \frac{1}{\sqrt{2}}, n_z = 0$

Using Cauchy's equation,

$$T_x^n = n_x\sigma_x + n_y\tau_{yx} + n_z\tau_{zx}$$

$$T_y^n = n_x\tau_{xy} + n_y\sigma_y + n_z\tau_{zy}$$

$$T_x^n = n_x\tau_{xz} + n_y\tau_{yz} + n_z\sigma_z$$

$$T_x^n = (\frac{1}{\sqrt{2}} \cdot 200) - (\frac{1}{\sqrt{2}} \cdot 200) - (0 \cdot 200)$$

$$T_x^n = 0 \;\; MPa$$

$$T_y^n = -(\frac{1}{\sqrt{2}} \cdot 200) - (\frac{1}{\sqrt{2}} \cdot 100) - (0 \cdot 200)$$

$$T_y^n = -\frac{300}{\sqrt{2}} \;\; MPa$$

$$T_z^n = -(\frac{1}{\sqrt{2}} \cdot 200) - (\frac{1}{\sqrt{2}} \cdot 200) - (0 \cdot 100)$$

$$T_z^n = -\frac{400}{\sqrt{2}} \;\; MPa$$

$$\sigma_n = n_xT_x^n + n_yT_y^n + n_zT_z^n$$

$$\sigma_n = (\frac{1}{(\sqrt{2}} \cdot 0) - (\frac{1}{\sqrt{2}} \cdot \frac{300}{\sqrt{2}}) - (0 \cdot \frac{400}{\sqrt{2}})$$

$$\sigma_n = \frac{-300}{2}$$

$$\sigma_n = -150 \ \ MPa$$

Normal and shear components of resultant stress vector are:

$$|T^n|^2 = \sigma_n^2 + \tau_n^2$$

$$\tau_n^2 = |T^n|^2 - \sigma_n^2$$

The components of resultant stress vector are:

$$|T^n|^2 = T_x^{n2} + T_y^{n2} + T_z^{n2}$$

$$|T^n|^2 = 0 + \frac{300}{2}^2 + \frac{400}{2}^2$$

$$|T^n|^2 = \frac{250000}{2}$$

$$|T^n|^2 = 125000$$

$$\tau_n^2 = 125000 - (-150)^2$$

$$\tau_n^2 = 102500$$

$$\tau_n = 320.16 \ \ MPa$$

(b) $n_x = 0, n_y = n_z = \frac{1}{\sqrt{2}}$
Using Cauchy's equation,

$$T_x^n = n_x \sigma_x + n_y \tau_{yx} + n_z \tau_{zx}$$

$$T_y^n = n_x \tau_{xy} + n_y \sigma_y + n_z \tau_{zy}$$

$$T_x^n = n_x \tau_{xz} + n_y \tau_{yz} + n_z \sigma_z$$

$$T_x^n = (0 \cdot 200) - (\frac{1}{\sqrt{2}} \cdot 200) - (\frac{1}{\sqrt{2}} \cdot 200)$$

$$T_x^n = -\frac{400}{\sqrt{2}} \ \ MPa$$

$$T_y^n = -(0 \cdot 200) - (\frac{1}{\sqrt{2}} \cdot 100) - (\frac{1}{\sqrt{2}} \cdot 200)$$

$$T_y^n = -\frac{300}{\sqrt{2}} \ \ MPa$$

$$T_z^n = -(0 \cdot 200) - (\frac{1}{\sqrt{2}} \cdot 200) - (\frac{1}{\sqrt{2}} \cdot 100)$$

$$T_z^n = -\frac{300}{\sqrt{2}} \ \ MPa$$

$$\sigma_n = n_x T_x^n + n_y T_y^n + n_z T_z^n$$

$$\sigma_n = (0 \cdot \frac{400}{\sqrt{2}}) - (\frac{1}{\sqrt{2}} \cdot \frac{300}{\sqrt{2}}) - (\frac{1}{\sqrt{2}} \cdot \frac{300}{\sqrt{2}})$$

$$\sigma_n = \frac{-600}{2}$$

$$\sigma_n = -300 \ \ MPa$$

Normal and shear components of resultant stress vector are:

$$|T^n|^2 = \sigma_n^2 + \tau_n^2$$

$$\tau_n^2 = |T^n|^2 - \sigma_n^2$$

The components of resultant stress vector are:

$$|T^n|^2 = T_x^{n2} + T_y^{n2} + T_z^{n2}$$

$$|T^n|^2 = \frac{400}{2}^2 + \frac{300}{2}^2 + \frac{300}{2}^2$$

$$|T^n|^2 = \frac{340000}{2}$$

$$|T^n|^2 = 170000$$

$$\tau_n^2 = 170000 - (-350)^2$$

$$\tau_n^2 = 102500$$

$$\tau_n = 282.84 \ \ MPa$$

(c) $n_x = n_y = n_z = \frac{1}{\sqrt{3}}$

Using Cauchy's equation,

$$T_x^n = n_x \sigma_x + n_y \tau_{yx} + n_z \tau_{zx}$$

$$T_y^n = n_x \tau_{xy} + n_y \sigma_y + n_z \tau_{zy}$$

$$T_x^n = n_x \tau_{xz} + n_y \tau_{yz} + n_z \sigma_z$$

$$T_x^n = (\frac{1}{\sqrt{3}} \cdot 200) - (\frac{1}{\sqrt{3}} \cdot 200) - (\frac{1}{\sqrt{3}} \cdot 200)$$

$$T_x^n = -\frac{200}{\sqrt{3}} \ \ MPa$$

$$T_y^n = -(\frac{1}{\sqrt{3}} \cdot 200) - (\frac{1}{\sqrt{3}} \cdot 100) - (\frac{1}{\sqrt{3}} \cdot 200)$$

$$T_y^n = -\frac{500}{\sqrt{3}} \ \ MPa$$

$$T_z^n = -(\frac{1}{\sqrt{3}} \cdot 200) - (\frac{1}{\sqrt{3}} \cdot 200) - (\frac{1}{\sqrt{3}} \cdot 100)$$

$$T_z^n = -\frac{500}{\sqrt{3}} \ \ MPa$$

$$\sigma_n = n_x T_x^n + n_y T_y^n + n_z T_z^n$$

$$\sigma_n = -(\frac{1}{\sqrt{3}} \cdot \frac{200}{\sqrt{3}}) - (\frac{1}{\sqrt{3}} \cdot \frac{500}{\sqrt{3}}) - (\frac{1}{\sqrt{3}} \cdot \frac{500}{\sqrt{2}})$$

$$\sigma_n = \frac{-1200}{3}$$

$$\sigma_n = -400 \ \ MPa$$

Normal and shear components of resultant stress vector are:

$$|T^n|^2 = \sigma_n^2 + \tau_n^2$$

$$\tau_n^2 = |T^n|^2 - \sigma_n^2$$

The components of resultant stress vector are:

$$|T^n|^2 = T_x^{n2} + T_y^{n2} + T_z^{n2}$$

$$|T^n|^2 = \frac{200}{2}^2 + \frac{500}{2}^2 + \frac{500}{2}^2$$

$$|T^n|^2 = \frac{540000}{3}$$

$$|T^n|^2 = 180000$$

$$\tau_n^2 = 180000 - (-400)^2$$

$$\tau_n^2 = 20000$$

$$\tau_n = 141.42 \ \ MPa$$