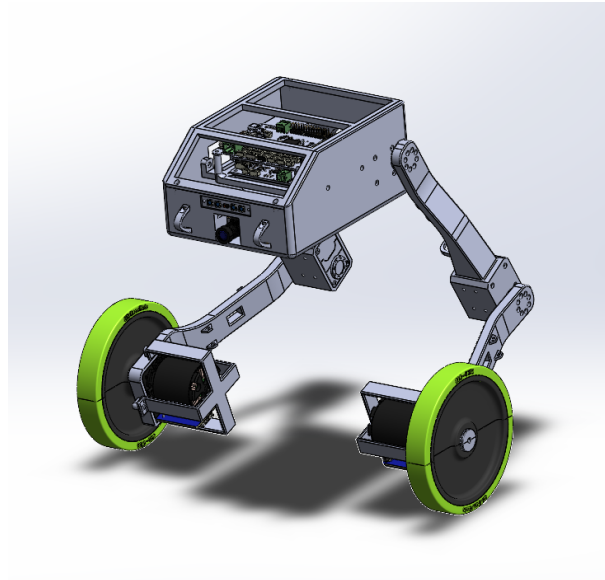


## Design and Control of a Multi-Joint Two-Wheeled Inverted Pendulum Robot for Agile Motion Learning



Master's thesis M-MM/JJJJ-XXXXX

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Hannover, 14. January 2024

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## Declaration of Authorship

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I, *Mohamed Barakat*, hereby affirm that the Master's thesis entitled *Design and Control of a Multi-Joint Two-Wheeled Inverted Pendulum Robot for Agile Motion Learning* was written independently, that no references and aids other than those indicated were used, that all passages of the thesis which were taken over literally or analogously from other sources are marked as such and that the thesis has not yet been presented to any examination board in the same or similar form.

I hereby agree to the transmission of my work also to external services for plagiarism checking by plagiarism software.

Hannover, 14. January 2024

(Mohamed Barakat)



# Eigenständigkeitserklärung

Hiermit erkläre ich, dass ich die vorliegende Arbeit selbstständig und eigenhändig sowie ohne unerlaubte fremde Hilfe und ausschließlich unter Verwendung der aufgeführten Quellen und Hilfsmittel angefertigt habe.

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Ort, Datum

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Mohamed Barakat

## Abstract

This thesis addresses the challenge of input trajectory transfer in heterogeneous multi-agent systems, where each agent has distinct system dynamics.

The concept of transfer learning has been applied to MAS to accelerate the learning process of a target agent by leveraging previously learned trajectories from a different source agent. However, directly transferring input trajectories between agents with dissimilar dynamics can lead to unwanted behaviour. This can be overcome by applying a mapping that transforms the input trajectory of the source system into an appropriate input trajectory for a target system so that their outputs match. Obtaining such an input transfer map from simple experiments is difficult, so previous research often focuses on output transfer, adaptive control or strong similarity assumptions. Those approaches either require extensive model knowledge or have limited usability.

This thesis provides a comprehensive understanding of trajectory transfer, distinguishing the input and output transfer cases and highlighting their similarities. Those insights are then used to propose a simple, data-driven method to estimate a dynamic input transfer map for SISO systems that addresses the aforementioned problem. Structural similarity in the systems is leveraged to simplify the estimation process further. The transfer map is estimated as a lifted system matrix and a transfer function. The performance of the estimated dynamic input transfer map is compared to a static map using a simple gain and to the direct transfer of the input trajectories.

It is shown that input and output transfer are equivalent under certain conditions. An input transfer map based on this assumption was able to significantly reduce the differences between a source and a target system in three simulated scenarios. This even includes scenarios where the systems have nonlinear, non-minimum phase dynamics.

Compared to a static map, the performance of a dynamic input transfer is superior, especially in cases where the estimated transfer map is applied to input trajectories which are not part of the training data. Furthermore, the dynamic input transfer map can have a model order lower than the theoretically expected order of the transfer system, given that the source and target systems are related. Therefore, the expected order only provides an upper limit for the order of an estimated transfer map, and not a recommended choice.

Nonetheless, this thesis alone does not fully explore the potential and limitations of the proposed method. Future research is necessary to address these topics further.

**Keywords**— input transfer, trajectory transfer, transfer learning, multi-agent systems

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# 1 Introduction

## 2 Table of content Draft

Questions to answer in each section

1. what is going to be in here?
  2. how long or how elaborate?
  3. what is the purpose (the take home message)?
1. Introduction
    - a) Background and Motivation
      - i. Discuss the evolution and significance of robotics in various industries.
      - ii. Emphasize the need for advancements in robotic stability and mobility.
    - b) Problem Statement
      - i. Define the specific challenges in designing a legged self-balancing robot.
    - c) Objectives(Outline the primary goals of the thesis)
  2. Literature Review
    - a) Overview of Robotics
    - b) Previous Work in Self-Balancing Robots
    - c) Control Strategies
      - i. what is going to be in here? a comparison of the control strategies used.
      - ii. how long or how elaborate? not too detailed but with more explanation of the control theory used in the project
      - iii. what is the purpose (the take home message)? pros and cons of the different and why would we prefer one of them over the other depending on the applications
  3. Design and Development of the Robot
    - a) Mechanical Design
      - i. Initial calculations
        - A. what is going to be in here? -> torque initial calculations
        - B. how long or how elaborate? -> two or three scenarios
        - C. what is the purpose (the take home message)? for choosing the correct motors
      - ii. design
        - A. what is going to be in here?-> CAD design and the explanations of the challenges
        - B. how long or how elaborate? detailed explanations of the reason behind the design decision

- C. what is the purpose (the take home message)? assembling the robot with fitting parts to match the new model requirements
  - iii. Modeling
    - A. what is going to be in here?-> the figures for the new model and the new COG and MOI calculations and the equations of motion.
    - B. how long or how elaborate? 4 to 5 pages explaining the equations in details
    - C. what is the purpose (the take home message)? showing the calculations for the new model and it would influence the equations of motion.
- b) Electrical Design
  - i. what is going to be in here? Component diagram showing the choice of all the components and there intended use and why we chose each of these components
  - ii. how long or how elaborate? detailed explanation of the requirement boards for operating the robot, the choice of components based calculations for the motors.
  - iii. what is the purpose (the take home message)? show how the Electrical design is configured in the optimal way to operate the robot
- c) Software and Control
  - i. Control Algorithm
    - A. what is going to be in here?->flowchart of the Control Algorithm
    - B. how long or how elaborate?-> detailed explanation of the used control theory
    - C. what is the purpose (the take home message)? -> how the control is implemented
  - ii. Firmware
- d) Safety
  - i. what is going to be in here? different design changes for safety measures(motors covers, wire routing, body bumper, distance sensor , algorithm safety, electrical safety )
  - ii. how long or how elaborate? 1 or two pages max that include the
  - iii. what is the purpose (the take home message)? the safety measures taken to minimize crashes, failure
- 4. Experimental Setup and Methodology
  - a) Simulation Environment
  - b) Physical Prototype Testing
  - c) Data Collection and Analysis
- 5. Results and Discussion
  - a) Simulation Results
  - b) Real-world Performance
  - c) Comparison and Analysis
- 6. Conclusion and Future Work
  - a) Summary of Findings
  - b) Contributions
  - c) Recommendations for Future Research

## **3 Mechanical Design**

## **4 Electronic Design**

## 5 Modelling and Simulation

In this pivotal chapter, we meticulously derive the center of gravity and the moment of inertia for the two-wheeled self-balancing robot. These parameters are the linchpins of our dynamic analysis, serving as the critical variables within the equations of motion that govern the robot's behavior. By calculating these values with precision, we can substitute them into our dynamic equations, thereby tailoring the model to reflect the true dynamics of the robot. This process not only enhances the accuracy of our simulations but also ensures that the control strategies developed are based on a robust and representative model of the robot's physical capabilities. The careful derivation of these parameters is a testament to the thoroughness of our approach, ensuring that the resulting model is both reliable and predictive of the robot's real-world performance.

## 5.1 Mathematical Modelling

- **TWIPR Model:** Explanation of the Two-Wheeled Inverted Pendulum Robot (TWIPR) model.
- **Focus on 2D Dynamics:** Discussion on the scope limited to 2D dynamics and plans for future expansion to 3D dynamics and controller synthesis.
- **Assumptions and Parameters:** Detailing assumptions such as considering motor angles as parameters.
- **Model Derivation for  $l_{cg}$  and  $I_y$ :** Derivation of the models for center of gravity length ( $l_{cg}$ ) and inertia around the y-axis ( $I_y$ ).
- **Integration into TWIPR Model:** Integration of derived models into a new TWIPR framework.
- **Linear Model Derivation:** Derivation of the linear model from the integrated TWIPR model.

## 5.2 Numerical Simulation

- **Discrete Numerical Simulation:** Elaboration on the process of discrete numerical simulation, including the discrete double integration method to arrive at the state vector.

## 5.3 Simulation Environment

- **Overview of the Simulation Environment:** A brief overview of the simulation environment, referencing David's Bachelor Thesis for details.
- **Integration of the Model:** Description of integrating the new TWIPR model into the simulation environment and a summary of the individual components of the model.

## 5.4 Controller Synthesis

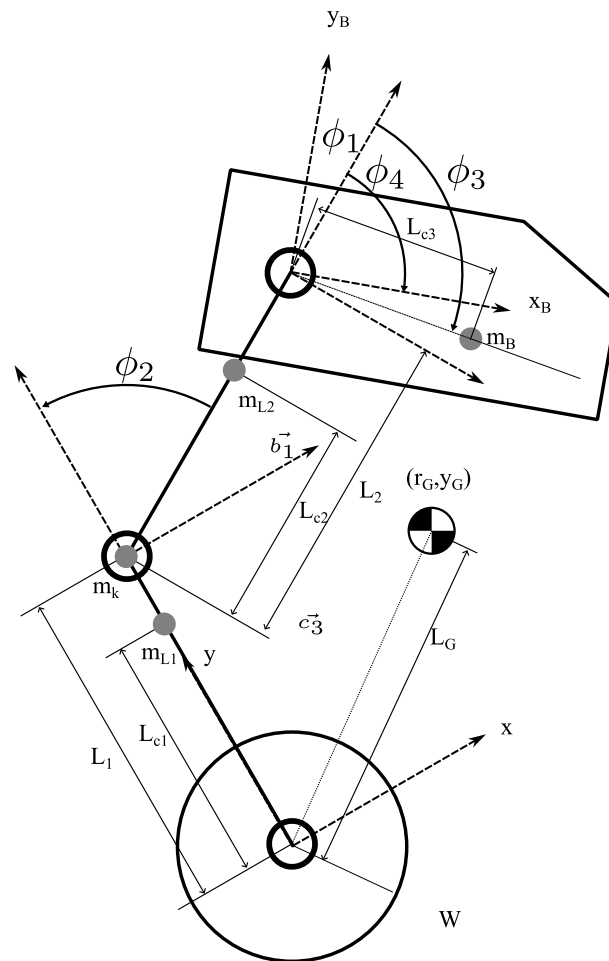
- **State-Space Controllers:** Introduction to two state-space controllers: Linear Quadratic Regulator (LQR) and Pole-Placement.
- **Configuration Specificity:** Explanation of how these controllers are specific to one configuration (knee angle and hip angle) of the robot.
- **Controller Retuning Algorithm:** Presentation of an algorithm for retuning the controllers as the configuration changes.

## 5.5 Simulation Analysis

- **Controller Responses:** Analysis of step response and responses to other inputs using one of the controllers in different configurations.
- **Impact of Non-Retuning:** Discussion on the effects of not retuning the controllers.
- **Influence of Leg Configuration:** Analysis of how leg configuration influences the robot's behavior.

- **Controller Setting Comparisons:** Comparative study of different controller settings and a comparison between LQR and Pole-Placement controllers.
- **Optimal Controller Configuration:** Conclusion on which controller configuration might be best suited for such a robot.
- Modeling allows for predictive analysis and understanding of the robot's behavior.

In order to predict the behavior of the robot under different settings, the modeling procedure entails constructing mathematical representations of the robot's dynamics and control systems.



**Figure 5.1:** Figure illustrating the mechanical model with the local coordinate system to determine the center of gravity.

### Center of Gravity calculations

## 5.6 Center of Gravity

- in that section the calculations of the overall center of gravity
- **Significance in Dynamics:**The stability of an object is directly impacted by the COG position, which also affects how it reacts to forces and moments from the outside world.
- maintain equilibrium,
- accurately determining the COG is crucial for predicting and controlling dynamic behavior



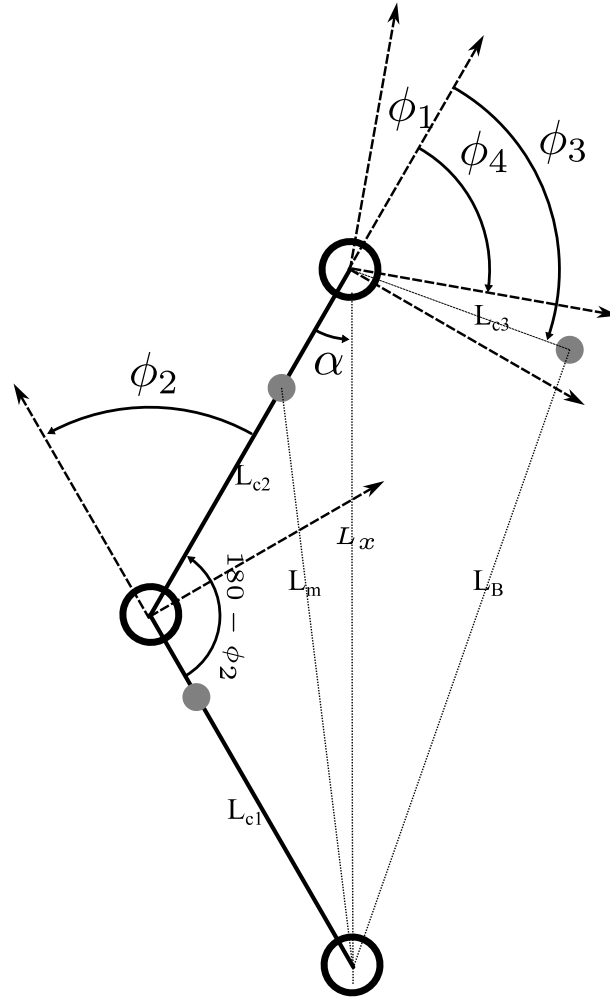
in the above figure in order to simplify the derivation of the center of

$$x_{CG} = \frac{m_{L1} \cdot 0 + m_K \cdot 0 + m_{L2} \cdot L_{C2} \cdot \sin(\phi_2) + m_B \cdot (L_2 \cdot \sin(\phi_2) + L_{C3} \cdot \sin(\phi_2 + \phi_3))}{m_{L1} + m_{L2} + m_K + m_B} \quad (5.1)$$

$$y_{CG} = \frac{m_{L1} \cdot L_{C1} + m_K \cdot L_1 + m_{L2} \cdot (L_1 + L_{C2} \cdot \cos(\phi_2)) + m_B \cdot (L_1 + L_2 \cdot \cos(\phi_2) + L_{C3} \cdot \cos(\phi_2 + \phi_3))}{m_{L1} + m_{L2} + m_K + m_B} \quad (5.2)$$

$$L_G = \sqrt{x_{CG}^2 + y_{CG}^2} \quad (5.3)$$

$$\theta = \arctan\left(\frac{x_{CG}}{y_{CG}}\right) \quad (5.4)$$



**Figure 5.2:** Schematic representation detailing the requisite angles and lengths for calculating the moment of inertia.

## 5.7 Moment of inertia

Moment of inertia calculations

$$L_m = \sqrt{L_1^2 + L_{C2}^2 - L_1 L_{C2} \cos(180 - \phi_2)} \quad (5.5)$$

$$L_x = \sqrt{L_1^2 + L_2^2 - L_1 L_2 \cos(180 - \phi_2)} \quad (5.6)$$

$$\alpha = \cos^{-1} \left( \frac{l_2^2 + l_x^2 - l_1^2}{2l_2 l_x} \right) \quad (5.7)$$

$$L_b = \sqrt{L_x^2 + L_{C3}^2 - L_x L_{C3} \cos(180 - \alpha - \phi_3)} \quad (5.8)$$

$$I_{L1} = \frac{1}{12} m_{L1} (a_1^2 + b_1^2) \quad (5.9)$$

$$I_{L2} = \frac{1}{12} m_{L1} (a_2^2 + b_2^2) \quad (5.10)$$

$$I_K = \frac{1}{2} m_K R_m^2 \quad (5.11)$$

$$I_B = \frac{1}{12} m_B (a_B^2 + b_B^2) \quad (5.12)$$

$$I = I_{L1} + m_{L1} L_{C1}^2 + I_K + m_K L_1^2 + I_{L2} + m_{L2} L_m^2 + I_B + m_B L_b^2 \quad (5.13)$$

## 5.8 Equation of motion

Equation of motion

## 5.9 Dynamics of the Two-Wheeled Inverted Pendulum Robot

Given the functions  $B_i : \mathbb{R} \rightarrow \mathbb{R}$ ,  $C_{ij} : \mathbb{R} \rightarrow \mathbb{R}$ ,  $D_{ij} : \mathbb{R} \rightarrow \mathbb{R}$ , and  $V_i : \mathbb{R} \rightarrow \mathbb{R}$ ,  $i, j \in \{1, 2, 3\}$ , the equations of motion are given by:

$$\ddot{s} = \frac{\sin(\Theta)}{V_1(\Theta)} (-C_{11}(\Theta) + C_{12}\dot{\Theta}^2 + C_{13}(\Theta)\dot{\psi}^2) - \frac{D_{11}(\Theta)}{V_1(\Theta)}\dot{s} + \frac{D_{12}(\Theta)}{V_1(\Theta)}\dot{\Theta} + \frac{B_1(\Theta)}{V_1(\Theta)}(\tau_L + \tau_R) \quad (5.14)$$

$$\ddot{\Theta} = \frac{\sin(\Theta)}{V_1(\Theta)} (C_{21} - C_{22}(\Theta)\dot{\Theta}^2 - C_{23}(\Theta)\dot{\psi}^2) + \frac{D_{21}(\Theta)}{V_1(\Theta)}\dot{s} - \frac{D_{22}(\Theta)}{V_1(\Theta)}\dot{\Theta} - \frac{B_2(\Theta)}{V_1(\Theta)}(\tau_L + \tau_R) \quad (5.15)$$

$$\ddot{\psi} = \frac{\sin(\Theta)}{V_2(\Theta)} (C_{31}(\Theta)\dot{\Theta}\dot{\psi} - C_{32}(\Theta)\dot{\psi}\dot{s}) - \frac{D_{33}(\Theta)}{V_2(\Theta)}\dot{\psi} - \frac{B_3}{V_2(\Theta)}(\tau_L - \tau_R) \quad (5.16)$$

The equations of motion are derived in [44]. The functions  $B_i : \mathbb{R} \rightarrow \mathbb{R}$ ,  $C_{ij} : \mathbb{R} \rightarrow \mathbb{R}$ ,  $D_{ij} : \mathbb{R} \rightarrow \mathbb{R}$ , and  $V_i : \mathbb{R} \rightarrow \mathbb{R}$ ,  $i, j \in \{1, 2, 3\}$ , are given by:

$$C_{11}(\Theta) = m_B^2 l^2 \cos(\Theta), \quad (5.17)$$

$$C_{12} = (I_2 + m_B l^2) m_B l, \quad (5.18)$$

$$C_{13}(\Theta) = (I_2 + m_B l^2) m_B l + m_B l (I_3 - I_1 - m_B l^2) \cos^2(\Theta), \quad (5.19)$$

$$C_{21} = (m_B + 2m_W + \frac{2J}{r^2}) m_B l, \quad (5.20)$$

$$C_{22}(\Theta) = m_B^2 l^2 \cos(\Theta), \quad (5.21)$$

$$C_{23}(\Theta) = m_B^2 l^2 + (m_B + 2m_W + \frac{2J}{r^2})(I_3 - I_1 - m_B l^2) \cos(\Theta). \quad (5.22)$$

$$C_{31}(\Theta) = 2(I_3 - I_1 - m_B^2) \cos(\Theta), \quad (5.23)$$

$$C_{31} = m_B l, \quad (5.24)$$

$$D_{11}(\Theta) = \frac{(I_2 + m_B^2) 2c_\alpha}{r^2} - \frac{m_B \cos(\Theta) 2c_\alpha}{r}, \quad (5.25)$$

$$D_{12}(\Theta) = \frac{(I_2 + m_B^2) 2c_\alpha}{r} - 2m_B \cos(\Theta) c_\alpha, \quad (5.26)$$

$$D_{21}(\Theta) = \frac{(m_B + 2m_W + \frac{2J}{r^2}) 2c_\alpha}{r} + \frac{m_B \cos(\Theta) 2c_\alpha}{r^2}, \quad (5.27)$$

$$D_{22}(\Theta) = \frac{(m_B + 2m_W + \frac{2J}{r}) 2c_\alpha + m_B \cos(\Theta) 2c_\alpha}{r}, \quad (5.28)$$

$$D_{33}(\Theta) = \frac{d^2}{2r^2 c_\alpha}, \quad (5.29)$$

$$B_1 = \frac{(I_2 + m_B^2) \frac{1}{r} + m_B \cos(\Theta)}{r}, \quad (5.30)$$

$$B_2 = \frac{m_B l}{r} - \cos(\Theta) + m_B + 2m_W + \frac{2J}{r^2}, \quad (5.31)$$

$$B_3 = \frac{d}{2r}, \quad (5.32)$$

$$V_1 = (m_B + 2m_W + \frac{2J}{r^2})(I_2 + m_B^2) - m_B^2 \cos^2(\Theta), \quad (5.33)$$

$$V_2 = I_3 + 2K + (m_W + \frac{J}{r^2}) \frac{d^2}{2} - (I_3 - I_1 - m_B^2) \sin^2(\Theta). \quad (5.34)$$

**Table 5.1:** Parameters of the mechanical system

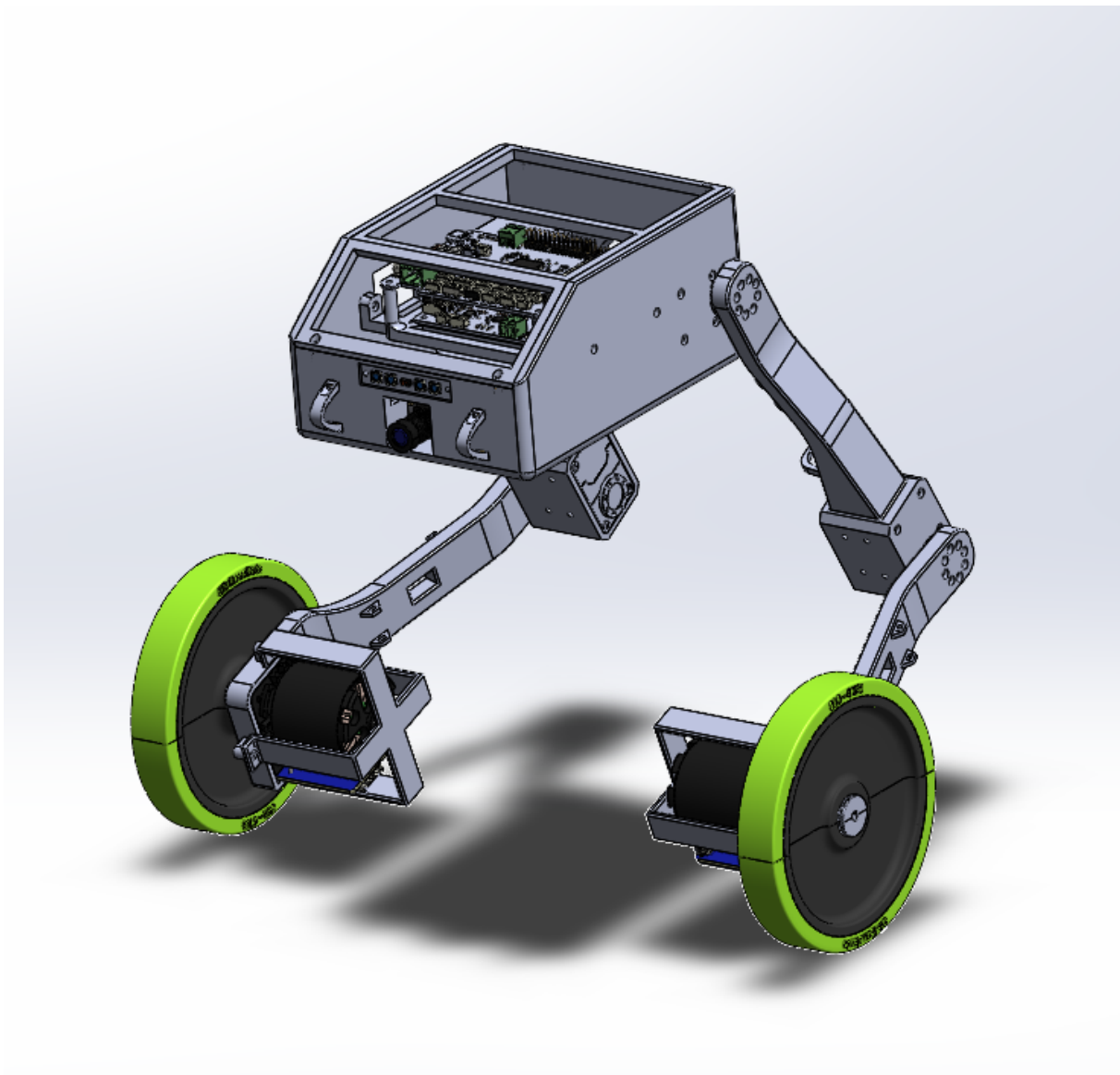
Parameter	Value	Description
$m_B$	2.5 kg	mass of the pendulum body
$m_W$	0.636 kg	mass of a wheel
$l$	0.026 m	distance between the wheel axis and the pendulum's center of gravity
$d$	-	distance between the two wheels
$J$	$5.175e^{-4}$ kgm <sup>2</sup>	moment of inertia of a wheel w.r.t. reference frame {C} in direction of $c_2$
$K$	-	moment of inertia of a wheel w.r.t. reference frame {C} in direction of $c_3$
$I_1$	-	moment of inertia of pendulum's body w.r.t. reference frame {B} in direction of $b_1$
$I_2$	0.0165 kgm <sup>2</sup>	moment of inertia of pendulum's body w.r.t. reference frame {B} in direction of $b_2$
$I_3$	-	moment of inertia of pendulum's body w.r.t. reference frame {B} in direction of $b_3$
$c_\alpha$	$4.630e^{-4}$ Nms	viscous friction coefficient

## **6 Mechanical Assembly**

## **7 Firmware and Testing**

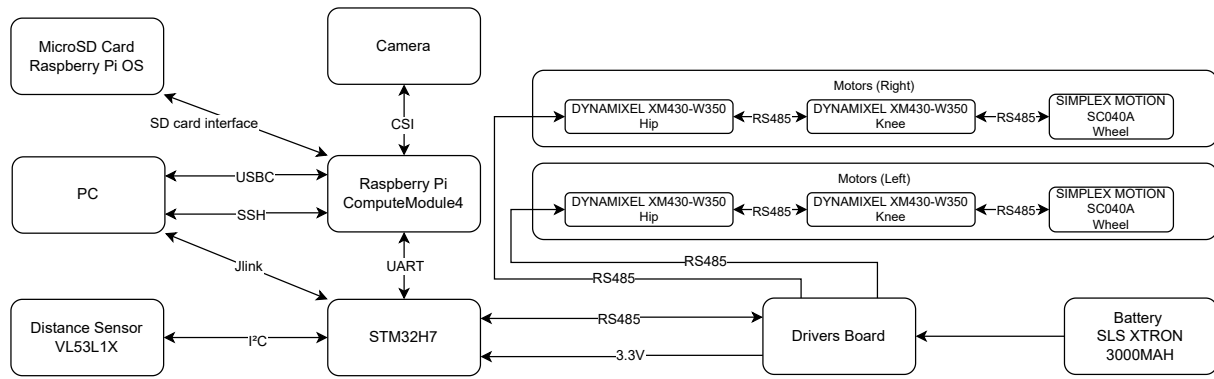
## 8 Design

This comprehensive chapter unfolds the intricate details of the new design of our two-wheeled self-balancing robot, an advanced piece of engineering that incorporates additional degrees of freedom to enhance its movement capabilities. Through an exploration of the various motions the robot can perform, we delve into the intricate design considerations of each component, ensuring that they align with the overall functional and aesthetic vision.



**Figure 8.1:** Schematic representation detailing the requisite angles and lengths for calculating the moment of inertia.

- The Robot new design.
- The added degrees of freedom.
- Different motions that can be performed.
- Discussing each component of the robot and things taken into account while designing it.
  - Body
    - \* overall design inspiration
    - \* For the body: the consideration for including all the necessary components in a compact form is to optimize the use of space and at the same time distribute the weight equally.
    - \* The body includes the hip motors, the battery, camera, sensor, a rack that includes the motors drivers board, the micro-controller board attached to the Raspberry Pi.
    - \* fastening features that were specifically designed in order to easily mount the battery, organize the cable between the boards and the rest of the robot parts.
    - \* Features for modular design and easy printing.
  - Thigh
    - \* curvature of that joint to give room for the motors
    - \* cable management
    - \* motor cover to insure its fixation.
  - Calf
    - \* curvature of that joint and the thigh joint combined make enough room for the wheel motor so the it have clearance from the body.
    - \* cable management
    - \* motor mount and additional frame.



**Figure 8.2:** Schematic representation detailing the requisite angles and lengths for calculating the moment of inertia.

- electronic components
  - Motors taking into count the needed torque and speed
  - comparison between the BLDC and Geared Robotic
  - RS-485 communications protocol compared to others
    - \* for the knee and hip motors high torque and low speed is needed.(calculations the show the weights and the needed torques)
    - \* for the wheel motors high speed and low torque is needed.
  - Boards
    - \* STM high performance h7 constum board connected with rasperrypi
    - \* driver board that provide the power for 6 motors.
  - modules
    - \* camera module.
    - \* distance sensor.



## 9 Safety

- hardware.
  - motors
    - \* Hip motor is enclosed in the body.
    - \* the knee motors are enclosed in cover.
    - \* the wheels motors have a
  - wiring
    - \* where the wires are neatly fastened in a predetermined route so that it wouldn't be caught in the robot movement which would cause damage to the robot and also to protect them from damage.
    - \* the correct type were used to avoid overheating upon the draw of current from the divers.
    - \* Rs-485 connection between motors where used to avoid additional cables.
  - proximity sensor
    - \* placed in the front and back side of the robot would help to detect crashing in trivial positions such as running into a wall and that could be easily prevented by proximity sensor monitoring the distance between the robot the and the obsticals in it direction.
  - body safety additional parts in case of impact to protect the internal components.
- software.

## **Bibliography**