Geodetic Exercise 02

Input data:

- A table of 9 points with Geographic coordinates in a source datum (ED50) and in a target datum (WGS84). Coordinates are in the format of degrees, minutes, and seconds (DMS).
- Datums' parameters: "source: https://epsg.io/1311"
    - ED50: a = 6378388, f = 1.0 / 297
    - WGS84: a= 6378137, f=1.0/298.257223563

Requirements:

- Read the input data
- Calculate the 7-transformation parameter between the 2 systems.

References for calculations:

- EPSG Guidance Note 7-2 Coordinate conversions and transformations including formulas

Steps:

- Create 2 comma-separated text files for both source and target coordinates.
- Change from (DMS) format into the degrees format and save it into text files.
- Transform the coordinates from the geographic format into the cartesian format and save them into text files.
- Formulate the formulas between the 2 coordinate systems and use the least squares to solve for the 7 transformation parameters. (please find the equation in handwriting at the end of the document Fig.2 and Fig. 3).
- Matrices are populated using the 9 common points in the 2 systems.
- Note: when using the 9 points, the residuals were very big and the final transformation parameters were not accurate. However, when omitting point (8), the results were satisfying as shown in table (1). Perhaps, further check for this point is needed.

Table 1: the differences in calculating the seven parameters when using 8 and 9 points

	Dx	Dy	Dz	Rx	Ry	Rz	scale
With 9 points	1566.917682	73.052551	2131.797471	0.000095	-0.000028	0.000167	-0.000438
With 8 points	-89.687860	-93.799149	-123.355869	0.000000	0. 000000	0. 000001	0.000001

- Export the derived transformation parameters into a text file.
- Display the matrices A and b and the residuals and the computed target coordinates using the derived parameters on the screen to go through the steps visually.
- For further steps, the standard deviation for each derived parameter could be calculated. Additionally, the indirect transformation from XYX to Lat, Long, h could be done so that when we have geographic coordinates in ED50, we could use the transformation parameters to transform to the geographic coordinates in WGS 84.
- Figure 1 shows a screenshot of the program running.

Program architecture:

- In the Geodetic.Exercise.Shared, there are 2 classes (Geographic and Cartesian). The Geographic class contains some operations on the geographic data such as loading from files(which is being used in Ex01 and Ex02), converting DMS into degrees and Cartesians, etc.
- In Geodetic.Exercise.2, there is the program.cs and the Parameters class which contains all the operations of calculating the 7 parameters.

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This program is to take 2 text files of 8 common geographic coordinates in a source (ED50) and a target (WGS84) and it u
ses least squares to solve for the 7 transformation parameters..
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please type the path to the source coordinates(dms) text file:
source_positions_dms.txt
please type the path to the target coordinates(dms) text file:
target_positions_dms.txt
please type the path to save the transformation Parameters file:
transformationParameters.txt
__Source__
Pt(1), X: 3602160.609551804, Y: -181010.885535306, Z: 5242987.765409547
Pt(2), X: 3885884.5607094145, Y: -117701.08830873616, Z: 5039577.2514158385
Pt(3), X: 4088966.7792036324, Y: 134191.04911337554, Z: 4876936.464971203
Pt(4), X: 3918931.377523862, Y: 2034.6430950471997, Z: 5015530.873813304
Pt(5), X: 3820878.1128398976, Y: 259120.39669689845, Z: 5083543.594459484
Pt(6), X: 3989344.7898404547, Y: 69729.4817514321, Z: 4959548.995763461
Pt(7), X: 3756445.501300127, Y: -48038.507536248944, Z: 5137266.214556692
Pt(9), X: 3988680.5859797914, Y: -101503.6390129049, Z: 4959576.21593617
__Target__
Pt(1), X: 3602075.2505890583, Y: -181107.62833175051, Z: 5242870.987855185
Pt(2), X: 3885799.661559758, Y: -117797.9642934771, Z: 5039460.178480487
Pt(3), X: 4088882.287907643, Y: 134094.2595868675, Z: 4876819.219089943
Pt(4), X: 3918846.594452799, Y: 1937.8860110580129, Z: 5015413.782814331
Pt(5), X: 3820793.3739467626, Y: 259024.0097680369, Z: 5083426.609858567
Pt(6), X: 3989260.155562283, Y: 69632.79503089104, Z: 4959431.825851601
Pt(7), X: 3756360.4876690116, Y: -48135.25318776363, Z: 5137149.268482287
Pt(9), X: 3988595.7896593283, Y: -101600.60624246676, Z: 4959459.071966563
Transformation Parameters:
tX: -89.687860, tY: -93.799149, tZ: -123.355869
rX: 0.000000, rY: 0.000000, rZ: 0.000001, dS: 0.000001
```

```
Transformed Pt(1): X: 3602075.567562, Y: -181107.642395, Z: 5242870.966940
Target Pt(1): X: 3602075.250589, Y: -181107.628332, Z: 5242870.987855
Transformed Pt(2): X: 3885799.824507, Y: -117797.985472, Z: 5039460.198520
Target Pt(2): X: 3885799.661560, Y: -117797.964293, Z: 5039460.178480
Transformed Pt(3): X: 4088882.103126, Y: 134094.309597, Z: 4876819.207223
Target Pt(3): X: 4088882.287908, Y: 134094.259587, Z: 4876819.219090
Transformed Pt(4): X: 3918846.590598, Y: 1937.870023, Z: 5015413.790059
Target Pt(4): X: 3918846.594453, Y: 1937.886011, Z: 5015413.782814
Transformed Pt(5): X: 3820793.005873, Y: 259024.020764, Z: 5083426.593837
Target Pt(5): X: 3820793.373947, Y: 259024.009768, Z: 5083426.609859
Transformed Pt(6): X: 3989260.038855, Y: 69632.738792, Z: 4959431.841631
Target Pt(6): X: 3989260.155562, Y: 69632.795031, Z: 4959431.825852
Transformed Pt(7): X: 3756360.549874, Y: -48135.217489, Z: 5137149.283160
Target Pt(7): X: 3756360.487669, Y: -48135.253188, Z: 5137149.268482
Transformed Pt(8): X: 3988595.965742, Y: -101600.595478, Z: 4959459.063028
Target Pt(8): X: 3988595.789659, Y: -101600.606242, Z: 4959459.071967
```

Fig.1: A screen shot of the running program

$$\begin{pmatrix} X_T \\ Y_T \\ Z_T \end{pmatrix} = M \begin{pmatrix} 1 & +r_Z & -r_Y \\ -r_Z & 1 & +r_X \\ +r_Y & -r_X & 1 \end{pmatrix} \begin{pmatrix} X_S \\ Y_S \\ Z_S \end{pmatrix} + \begin{pmatrix} t_X \\ t_Y \\ t_Z \end{pmatrix}$$

$$M = 1 + ds$$

$$X_T = (1+ds)(X_S + r_Z \cdot Y_S - r_Y \cdot Z_S) + t_X$$

$$X_T = X_S + r_Z \cdot Y_S - r_Y \cdot Z_S + ds \cdot X_S + \cancel{ds \cdot r_Z \cdot Y_S} - \cancel{ds \cdot r_Y \cdot Z_S} + t_X$$

$$Y_T = (1+ds)(-X_S \cdot r_Z + Y_S + Z_S \cdot r_X) + t_Y$$

$$Y_T = -X_S \cdot r_Z + \cancel{Y_S} + Z_S \cdot r_X - \cancel{ds \cdot X_S \cdot r_Z} + ds \cdot Y_S + \cancel{ds \cdot Z_S \cdot r_X} + t_Y$$

$$Z_T = (1+ds)(r_Y \cdot X_S - r_X \cdot Y_S + Z_S) + t_Z$$

$$Z_T = r_Y \cdot X_S - r_X \cdot Y_S + \cancel{Z_S} + \cancel{ds \cdot r_Y \cdot X_S} - \cancel{ds \cdot r_X \cdot Y_S} + ds \cdot Z_S + t_Z$$

$$\Delta X = r_Z \cdot Y_S - r_Y \cdot Z_S + ds \cdot X_S + t_X$$

$$\Delta Y = Z_S \cdot r_X - X_S \cdot r_Z + ds \cdot Y_S + t_Y$$

$$\Delta Z = -Y_S \cdot r_X + X_S \cdot r_Y + ds \cdot Z_S + t_Z$$

Fig.2: Least squares formula

Design matrix
"Coefficients of the 7 unknowns"

$$\text{Residuals} \rightarrow b = A \cdot P \leftarrow \text{The 7 unknowns}$$

$$\overbrace{T_x \quad T_y \quad T_z}^{\text{Translation}} \quad \overbrace{\omega_x \quad \omega_y \quad \omega_z}^{\text{Rotation}} \quad \overbrace{s}^{\text{Scale}}$$

$$\underbrace{\begin{array}{ccccccc} T_x & T_y & T_z & \omega_x & \omega_y & \omega_z & s \\ \end{array}}$$

$$\underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 & -Z_s & Y_s & X_s \\ 0 & 1 & 0 & Z_s & 0 & -X_s & Y_s \\ 0 & 0 & 1 & -Y_s & X_s & 0 & Z_s \\ & & & & & & \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} T_x \\ T_y \\ T_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}}_{P}$$

$$P = (A^T A)^{-1} A^T b$$

Fig.3: Matrix population for one point