The LUA-PHYSICAL library

$Version\ 0.1$

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Abstract

lua-physical is a pure Lua library which provides functions and object for doing computation with physical quantities. This package provides a standard set of units of the SI and the imperial system. It is possible to give a number a mesurement uncertainty.

is also integrated and is calculated by gaussian error propagation. The package includes some $\,$

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1 Introduction

The author of this package is a teacher at the Kantonsschule Zug, Switzerland, a high-school. The main use of this package is to write physics problem sets and integrate the calculation directly into the luatex-file. The package is now in use for more than two years and a lot of bugs have been found and crushed. Nevertheless it could be possible that some bugs are still there, living uncovered. Therefore I recommend not to use this library productively in industry or science. If one does so, it's the responsability of the user to check results for plausability. If the user finds some bugs, please report them on github.com or directly to the author.

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2 Basic usage

Since this package is pure lua library one has to require it explicitly by calling require("physical"). For printing results the siunitx package is used. It's recommended to define a shortcut like \q or \Qty to convert the lua quantity object to a siunitx expression. An example preamble is shown in the following.

```
\usepackage{siunitx}
2
3
      % configure siunitx
4
      \sisetup{
5
        output-decimal-marker = {.},
6
        per-mode = symbol,
        separate-uncertainty = false,
7
8
        add-decimal-zero = true,
9
        exponent-product = \cdot,
10
        round-mode = off
11
12
13
      % load lua-physical
14
      \begin{luacode*}
15
        physical = require("physical")
16
      \end{luacode*}
17
18
      % shortcut for printing physical quantities
19
      \mbox{\ensuremath{\mbox{newcommand}}} [1] {\%}
20
        \directlua{tex.print(physical.Quantity.tosiunitx(#1, "scientific-
               notation=fixed,exponent-to-prefix=false"))}%
21
```

Listing 1: basic preamble

Given the preamble one can use now units in lua code and insert results in the latex code.

```
1 \begin{luacode}
2    s = 10 * _m
3    t = 2 * _s
4    v = s/t
5 \end{luacode}
6
7 A car travels $\q{s}$ in $\q{t}$. calculate its velocity.
8    $$
9    v=\frac{s}{t} = \frac{\q{s}}{\q{t}} = \q{v} = \q{v:to(_km/_h)}
10    $$
```

Listing 2: basic usage

A car travels 10 m in 2 s. Calculate its velocity.

$$v = \frac{s}{t} = \frac{10 \,\mathrm{m}}{2 \,\mathrm{s}} = 5 \,\mathrm{m/s} = 18 \,\mathrm{km/h}$$

3 Supported Units

3.1 Base Units

Quantity	Unit Symbol Dim.		Dim.	Definition		
number	_	_1	1	The dimensionless number one.		
length	meter	_m	L	The distance light travels in vacuum during $1/299792458$ second.		
mass	kilogram	_kg	М	The mass of the international protoype of the kilogram.		
time	second	_S	Т	Is 9 192 631 770 times the period of the radiation from the transition between the two hyperfine levels of the ground state of caesium-133.		
electric current	ampere	_A	I	The constant current which, if maintained in two straight parallel conductors of infinte length, of negligible circular crosss-section, and placed 1 m apart in vacuum, would produce between these conductors a force equal to $2 \cdot 10^{-7}$ N/m.		
thermody- namic temperature	kelvin	_K	Θ	Is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water.		
amount of substance	mole	_mol	N	Amount of substance that contains as many particles as there are atoms in 0.012 kg of carbon-12.		
luminous intensity	candela	_cd	J	the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequenc $540 \cdot 10^{12} \text{Hz}$ and has a radiant intensity in that direction of $(1/683) \text{W/sr}$		
informa- tion	Bit	_bit	В	The smallest amount of information.		
currency	Euro	_EUR	L	The value of one Euro.		

Table 1: Base units of the International System of Units (SI) and the base unit of information and currency.

3.2 Derived Units

Quantity	Unit	Symbol	Dimension	Definition
plane angle	radian	_rad	1	_1
solid angle	steradian	_sr	1	_rad^2
frequency	hertz	_Hz	T^{-1}	1/_s
force	newton	_N	$ m MLT^{-2}$	_kg*_m/_s^2
pressure	pascal	_Pa	${ m M}{ m L}^{-1}{ m T}^{-2}$	_N/_m^2
energy	joule	_J	${ m M}{ m L}^2{ m T}^{-2}$	_N*_m
power	watt	_W	${ m M}{ m L}^2{ m T}^{-3}$	_J/_s
electric charge	coulomb	_C	ΤΙ	_A*_s
electric potential difference	volt	_V	${ m M}{ m L}^2{ m T}^{-3}{ m I}^{-1}$	_J/_C
capacitance	farad	_F	$ m L^{-2}M^{-1}T^4I^2$	_C/_V
electric resistance	ohm	_Ohm	$L^2 M T^{-3} I^{-2}$	_V/_A
electric conductance	siemens	_\$	$L^{-2} M^{-1} T^3 I^2$	_A/_V
magnetic flux	weber	_Wb	$L^2 \mathrm{M} \mathrm{T}^{-2} \mathrm{I}^{-1}$	_V*_s
magnetic flux density	tesla	_T	$ m M T^{-2} I^{-1}$	_V*_s
inductance	henry	_H	$L^2 M T^{-2} I^{-2}$	_Wb/_A
Celsius temperature	degree Celsius	_degC	Θ	_K
luminous flux	lumen	_lm	J	_cd*_sr
illuminance	lux	_lux	$\mathrm{L}^{-2}\mathrm{J}$	_lm/_m^2
activity	becquerel	_Bq	T^{-1}	1/_s
absorbed dose	gray	_Gy	$\mathrm{L}^2\mathrm{T}^{-2}$	_J/_kg
dose equivalent	sievert	_Sv	$L^2 T^{-2}$	_J/_kg
catalytic activity	katal	_kat	$\mathrm{T}^{-1}\mathrm{N}$	_mol/_s

Table 2: Derived units of the International System of Units (SI) 7

There are a few units with dimension 1. The unit Bel is only available with prefix decibel, because $_B$ is the unit byte.

Quantity	Unit	Symbol	Dim.	Definition
	percent $\%$	_percent	1	1e-2*_1
	permille $\%$	_permille	1	1e-3*_1
	parts-per-million	_ppm	1	1e-6*_1
	parts-per-billion	_ppb	1	1e-9*_1
	parts-per-trillion	_ppt	1	1e-12*_1
	parts-per-quadrillion	_ppq	1	1e-15*_1
	decibel	_dB	1	_1
plane angle	degree	_deg	1	(Pi/180)*_rad
	arc minute	_arcmin	1	_deg/60
	arc second	_arcsec	1	_arcmin/60
	gradian	_gon	1	(Pi/200)*_rad
	turn	_tr	1	2*Pi*_rad
solid angle	spat	_sp	1	4*Pi*_sr
length	astronomical unit	_au	L	149597870700*_m
	lightyear	_ly	L	_c*_a
	parsec	_pc	L	(648000/Pi)*_au
	angstrom	_angstrom	L	1e-10*_m
	fermi	_fermi	L	1e-15*_m
area	are	_ar	L^2	1e2*_m^2
	hectare	_hectare	L^2	1e4*_m^2
	barn	_barn	L^2	1e-28*_m^2
volume	liter	_L	L^3	0.001*_m^3
	metric teaspoon	_tsp	L^3	0.005*_L
	metric tablespoon	_Tbsp	L^3	3*_tsp
time	minute	_min	Т	_60*_s
	hour	_h	Т	_60*_min
	day	_d	Т	_24*_h
	week	_wk	Т	_7*_d
	year	_a	Т	365.25*_d
	svedberg	_svedberg	Т	1e-13*_s
mass	tonne	_₽	M	1000*_kg

Table 3: Units outside of the International System of Units (SI)

Quantity	Unit	Symbol	Dim.	Definition
length	inch	_in	L	0.0254*_m
	thou	_th	L	0.001*_in
	pica	_pica	L	_in/6
	point	_pt	L	_in/72
	hand	_hh	L	4*_in
	foot	_ft	L	12*_in
	yard	_yd	L	3*_ft
	rod	_rd	L	5.5*_yd
	chain	_ch	L	4*_rd
	furlong	_fur	L	10*_ch
	mile	_mi	L	8*_fur
	league	_lea	L	3*_mi
	nautical mile	_nmi	L	1852 * _m
	nautical league	_nlea	L	3*_nmi
	cable	_cbl	\mathbf{L}	_nmi/10
	fathom	_ftm	L	6*_ft
velocity	knot	_kn	$ m L^1T^{-1}$	_nmi/_h
area	acre	_ac	L^2	43560*_ft^2
volume	gallon	_gal	L^3	4.54609*_L
	quart	_qt	L^3	_gal/4
	pint	_pint	L^3	_qt/2
	cup	_cup	L^3	_pint/2
	gill	_gi	L^3	_pint/4
	fluid ounce	_fl_oz	L^3	_gi/5
	fluid dram	_fl_dr	L^3	_fl_oz/8

Table 4: Imperial units

Quantity	Unit	Symbol	Dim.	Definition
mass	grain	_gr	M	64.79891*_mg
	pound	_1b	M	7000*_gr
	ounce	_oz	M	_lb/16
	dram	_dr	M	_1b/256
	stone	_st	M	14*_lb
	quarter	_qtr	M	2*_st
	${\bf hundred weight}$	_cwt	M	4*_qtr
	long ton	_ton	M	20*_cwt

Table 5: Imperial units

Unit	Symbol	Dim.	Definition
U.S. survey inch	_in_US	L	_m/39.37
U.S. survey hand	_hh_US	L	4*_in_US
U.S. survey foot	_ft_US	L	3*_hh_US
U.S. survey link	_li_US	L	0.66*_ft_US
U.S. survey yard	_yd_US	L	3*_ft_US
U.S. survey rod	_rd_US	L	5.5*_yd_US
U.S. survey chain	_ch_US	L	4*_rd_US
U.S. survey furlong	_fur_US	L	10*_ch_US
J.S. survey mile	_mi_US	L	8*_fur_US
J.S. survey league	_lea_US	L	3*_mi_US
U.S. survey fathom	_ftm_US	L	72*_in_US
U.S. survey cable	_cbl_US	L	120*_ftm_US
	J.S. survey inch J.S. survey hand J.S. survey foot J.S. survey link J.S. survey yard J.S. survey rod J.S. survey chain J.S. survey furlong J.S. survey mile J.S. survey league J.S. survey fathom	J.S. survey inchin_US J.S. survey handhh_US J.S. survey footft_US J.S. survey linkli_US J.S. survey yardyd_US J.S. survey rodrd_US J.S. survey chainch_US J.S. survey furlongfur_US J.S. survey milemi_US J.S. survey leaguelea_US J.S. survey fathomftm_US	J.S. survey inchin_US L J.S. survey handhh_US L J.S. survey footft_US L J.S. survey linkli_US L J.S. survey yardyd_US L J.S. survey rodrd_US L J.S. survey chainch_US L J.S. survey furlongfur_US L J.S. survey milemi_US L J.S. survey leaguelea_US L J.S. survey fathomftm_US L

Table 6: U. S. customary units

Quantity	Unit	Symbol	Dim.	Definition
currency	Afghan afghani	_AFN	\mathbf{C}	0.012*_EUR
	Albanian lek	_ALL	С	0.008*_EUR
	Armenian Dram	_AMD	С	0.0018*_EUR
	Angolan Kwanza	_AOA	С	0.0028*_EUR
	Argentine Peso	_ARS	С	0.021*_EUR
	U.S. dollar	_USD	C	0.89*_EUR
	Japanese yen	_JPY	С	0.008*_EUR
	British pound	_GBP	С	1.17*_EUR
	Australian dollar	_AUD	С	0.63*_EUR
	Canadian dollar	_CAD	С	0.66*_EUR
	Swiss franc	_CHF	С	0.88*_EUR
	Chinese yuan	_CNY	С	0.13*_EUR
	Swedish krona	_SEK	С	0.094*_EUR
	New Zealand dollar	_NZD	С	0.60*_EUR

Table 7: Currency units based on exchange rates from 7.3.2019, 21:00 UTC.

4 Lua Documentation

In this chapter, the following shortcuts will be used.

```
1 local D = physical.Dimension
2 local U = physical.Unit
3 local N = physical.Number
4 local Q = physical.Quantity
```

The term number refers to a lua integer or a lua float number. By string a lua string is meant and by bool a lua boolean.

4.1 physical.Quantity

The quantity class is the main part of the library. Each physical Quantity and all units are represented by an instance of this class.

Q.new(q=nil)

Copy Constuctor

Parameters

```
q: Q or number, optional
    Optional argument is either Q, a number or nil.
return: Q
    The created Q instance
```

Note

As an argument it takes Q, number or nil. If Q is given, a copy of it is made and returned. If a number is given, the function creates a dimensionless quantity with that value. In the case nil is given, the quantity _1 is returned.

Example

```
1 myOne = Q()
2 myNumber = Q(42)
3 myLength = Q(73*_m)
```

Q.defineBase(symbol,name,dimension)

This function is used to declare the base units. Units are represented as Q instances.

Parameters

symbol : string

symbol of the base quantity

name: string

name of the base quantity

dimension: D

Instance of the $\ensuremath{\mathtt{D}}$ class, which represents the dimension of the quantity.

return: Q

The created ${\tt Q}$ instance.

Note

The function creates a global variable, an underscore concatenated with the symbol argument, e. g. m becomes the global variable _m.

The name is used for example in the siunitx conversion function, e.g meter will be converted to \meter.

Each quantity has a dimension associated with it. The argument dimension allows any dimension to be associated to base quantities. By default, the SI convention is used.

Example

```
1 Q.defineBase("m", "meter", L)
2 Q.defineBase("kg", "kilogram", M)
```

Quantity.define(symbol, name, q, tobase=nil, frombase=nil)

Creates a new derived unit from an expression of other units. For affine quantities like the temperature in celcius, one can give convertion functions to and from base units.

Parameters

symbol: string

Symbol of the base quantity

name: string

Name of the base quantity

q: physical.Quantity
Definition of the unit

tobase: function, optional

to convert a quantity to base units

frombase: function, optional

to convert a quantity from the base units

return: Quantity
The defined quantity

Examples

```
1 \quad \texttt{Q.define("L", "liter", \_dm^3)}
2 Q.define("Pa", "pascal", _N/_m^2)
3 Q.define("C", "coulomb", _A*_s)
5 Q.define(
6
      "degC",
      "celsius",
7
      _K,
8
9
      function(q)
10
       q.value = q.value + 273.15
11
       return q
12
      end,
      function(q)
13
        q.value = q.value - 273.15
14
15
        return q
16
      end
17 )
```

Quantity.definePrefix(symbol,name,factor)

Defines a new prefix.

```
symbol : string, Symbol of the base quantity
name : string, Name of the base quantity
factor : number, the factor which corresponds to the prefix
```

```
1 Q.definePrefix("c", "centi", 1e-2)
2 Q.definePrefix("a", "atto", 1e-18)
```

Quantity.addPrefix(prefixes, units)

Create several units with prefixes from a given unit.

```
prefixes : string, list of unit symbols
units : Quantity, list of quantities
```

```
1 Q.addPrefix({"n","u","m","k","M","G"},{_m,_s,_A})
```

Quantity.to(self,q,usefunction=false)

Converts the quantity self to the unit of the quantity q. If the boolean usefunction is true, the convertion function is used for conversion.

```
self : Quantity
q : Quantity
usefunction : Bool
```

```
1  s = 1.9 * _km
2  print( s:to(_m) )
3  1900 * _m
4
5  T = 10 * _degC
6  print( T:to(_K) )
7  10 * _K
8  print( T:to(_K,true) )
9  283.15 * _K
```

Quantity.tosiunitx(self,param,mode)

Converts the quantity into a siunity string.

```
\label{eq:param:quantity} $$param: string $$mode: Number, 0:\SI, 1:\num, 2:\si $$
```

```
1  s = 1.9 * _km
2
3  print( s:tosiunitx() )
4  \SI{1.9}{\kilo\meter}
5
6  print( s:tosiunitx(nil,1) )
7  \num{1.9}
8
9  print( s:tosiunitx(nil,2) )
10  \si{\kilo\meter}
```

Quantity.isclose(self,q,r)

Checks if this quantity is close to another one. The argument ${\tt r}$ is the maximal relative deviation.

```
self : Quantity
q : Quantity, Number
```

r: Number

```
1  s_1 = 1.9 * _m
2  s_2 = 2.0 * _m
3  print( s_1:isclose(s_2,0.1) )
4  true
5  print( s_1:isclose(s_2,0.01) )
6  false
```

Quantity.min(q1, q2, ...)

Returns the smallest quantity of several given ones. The function returns q1 if the Quantities are equal.

```
q1: Quantity, Number, first argument
```

q2: Quantity, Number, second argument

```
1 s_1 = 15 * _m
2 s_2 = 5 * _m
3 print(s_1:min(s_2))
4 5 * _m
```

Quantity.max(q1, q2, ...)

Returns the biggest quantity of several given ones. The function returns q1 if the Quantities are equal.

```
q1: Quantity, Number, first argument
```

q2: Quantity, Number, second argument

```
1 s_1 = 15 * _m
2 s_2 = 5 * _m
3 print(s_1:max(s_2))
4 15 * _m
```

Quantity.abs(q)

Returns the absolute value of the given quantity q.

```
{\bf q} : Quantity, Number, {\bf argument}
```

```
1 U = -5 * _V

2 print(U)

3 -5 * _V

4 print(U:abs())

5 5 * _V
```

Quantity.sqrt(q)

Returns the square root of the given quantity.

q: Quantity, Number argument

```
1 A = 25 * _m^2
2 s = sqrt(A)
3 print(s)
4
```

Quantity.log(q, base)

Returns the logarithm of the given quantitiy. If no base is given, the natural logarithm is calculated.

 ${\tt q}: {\tt Quantity}, {\tt Number} \ {\tt dimensionless} \ {\tt argument}$

base: Quantity, Number dimensionless argument

```
1 I = 1 * _W/_m^2

2 I_0 = 1e-12 * _W/_m^2

3 print(10 * (I/I_0):log(10) * _dB)

4 120 * _dB
```

Quantity.exp(q)

Returns the value of the exponential function of the given quantitiy.

q: Quantity, Number dimensionless argument

```
1 x = 2 * _1
2 print(x:exp())
3 7.3890560989307
```

Quantity.sin(q)

Returns the value of the sinus function of the given quantitiy.

q: Quantity, Number dimensionless argument

```
1 alpha = 30 * _deg
2 print( alpha:sin() )
3 0.5
```

Quantity.cos(q)

Returns the value of the cosinus function of the given quantity. The quantity has to be dimensionless.

q: Quantity, Number dimensionless argument

```
1 alpha = 60 * _deg
2 print( alpha:cos() )
3 0.5
```

Quantity.tan(q)

Returns the value of the tangent function of the given quantity. The quantity has to be dimensionless.

q: Quantity, Number dimensionless argument

```
1 alpha = 45 * _deg
2 print( alpha:tan() )
3 1
```

Quantity.asin(q)

Returns the value of the arcus sinus function of the given quantity. The quantity has to be dimensionless.

q: Quantity, Number dimensionless argument

```
1 x = 0.5 * _1
2 print( x:asin():to(_deg) )
3 30 * _deg
```

Quantity.acos(q)

Returns the value of the arcus cosinus function of the given quantity. The quantity has to be dimensionless.

q: Quantity, Number dimensionless argument

```
1 x = 0.5 * _1
2 print( x:acos():to(_deg) )
3 60 * _deg
```

Quantity.atan(q)

Returns the value of the arcus tangent function of the given quantity. The quantity has to be dimensionless.

q: Quantity, Number dimensionless argument

```
1 x = 1 * _1
2 print( x:atan():to(_deg) )
3 45 * _deg
```

Quantity.sinh(q)

Returns the value of the hyperbolic sine function of the given quantity. The quantity has to be dimensionless. Since lua doesn't implement the hyperbolic functions the following formula is used

$$\sinh(x) = 0.5 \cdot e^x - 0.5/e^x \quad .$$

q: Quantity, Number dimensionless argument

```
1 x = 1 * _1
2 print(x:sinh())
3 1.1752011936438
```

Quantity.cosh(q)

Returns the value of the hyperbolic cosine function of the given quantity. The quantity has to be dimensionless. Since lua doesn't implement the hyperbolic functions the following formula is used

$$\cosh(x) = 0.5 \cdot e^x + 0.5/e^x \quad .$$

q: Quantity, Number dimensionless argument

```
1 x = 1 * _1
2 print(x:cosh())
3 1.5430806348152
```

Quantity.tanh(q)

Returns the value of the hyperbolic tangent function of the given quantity. The quantity has to be dimensionless. Since lua doesn't implement the hyperbolic functions the following formula is used

$$\tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad .$$

q: Quantity, Number dimensionless argument

```
1 x = 1 * _1
2 print(x:tanh())
3 0.76159415595576
```

Quantity.asinh(q)

Returns the value of the inverse hyperbolic sine function of the given quantity. The quantity has to be dimensionless. Since lua doesn't implement the hyperbolic functions the following formula is used

$$asinh(x) = \ln\left(x + \sqrt{x^2 + 1}\right) \quad .$$

q: Quantity, Number dimensionless argument

```
1 x = 1 * _1
2 print(x:asinh())
3 0.88137358701954
```

Quantity.acosh(q)

Returns the value of the inverse hyperbolic cosine function of the given quantity. The quantity has to be dimensionless. Since lua doesn't implement the hyperbolic functions the following formula is used

$$a\cosh(x) = \ln\left(x + \sqrt{x^2 - 1}\right) \quad , x > 1 \quad .$$

q: Quantity, Number dimensionless argument bigger than or equal to one.

```
1 x = 2 * _1
2 print(x:acosh())
3 1.3169578969248
```

Quantity.atanh(q)

Returns the value of the inverse hyperbolic cosine function of the given quantity. The quantity has to be dimensionless. Since lua doesn't implement the hyperbolic functions the following formula is used

$$\operatorname{atanh}(x) = \ln\left(\frac{1+x}{1-x}\right) \quad , -1 < x < 1 \quad .$$

 ${\tt q}: {\tt Quantity}, {\tt Number} \ {\rm dimensionless} \ {\rm argument} \ {\rm with} \ {\rm magnitude} \ {\rm smaller} \ {\rm than} \ {\rm one}.$

```
1 x = 0.5 * _1
2 print(x:atanh())
3 0.54930614433405
```

4.2 physical.Dimension

All physical quantities do have a physical dimension. For example the quantity Area has the dimension L^2 (length to the power of two). In the SI-System there are seven base dimensions, from which all other dimensions are derived. Each dimension is represented by an n-tuple, where n is the number of base dimensions. Each physical quantity has an associated dimension object. It is used two check if two quantities can be added or subtraced and if they are equal.

Dimension.new(q=nil)

Constructor of the Dimension class.

Parameters

 ${\tt q}: {\tt Dimension} \ {\rm or} \ {\tt string}, \ {\rm optional}$

The name or symbol of the dimension. If q is a dimension, a copy of it is made. If no argument ist given, a dimension *zero* is created.

return: Dimension

The created Quantity object

Notes

_

Examples

```
1  V_1 = D("Velocity")
2  L = D("L")
3  V_2 = D(L/T)
```

4.3 physical.Unit

The task of this class is keeping track of the unit term. The unit term is a fraction of units. The units in the enumerator and denominator can have an exponent.

Unit.new(u=nil)

Copy Constructor. It copies a given unit object. If nothing is given, an empty unit is created.

Parameters

```
u: Unit
```

The unit object which will be copied.

return : Unit

The created Unit object

Unit.new(symbol, name, prefixsymbol=nil, prefixname=nil)

Constructor. A new Unit object with symbol is created. The prefixsymbol and prefixname are optional.

Parameters

```
symbol: String
```

The symbol of the unit.

 ${\tt name}: {\tt String}$

The name of the unit.

prefixsymbol : String

The optional symbol of the prefix.

prefixname : String

The optional name of the prefix.

return : Unit

The created Unit object

Unit.tosiunitx(self)

The unit term will be compiled into a string, which the Latex package siunitx can understand.

Parameters

return: String

The siunitx representation of the unit term.