NAME: ________________________________ DATE: ____________

Question 1: (2 marks)
Please describe in less than 100 words which of the Newton-Cotes methods you would use to integrate function *f(x)=cos(x)+sin(2x)* over [0,2π] and why.

Question 2: (12 marks)
Given the following integral:

$$\int_0^2 \int_{-1}^0 x^4 e^{y^2}\, dx\, dy$$

1) Please show the plot of the function $f(x, y) = x^4 e^{y^2}$ in Matlab. [0.5 Marks]

2) Fill in the following table:

Number of Gauss Points in each direction	Gaussian Integration
1	
3	
5	

[1.5 Marks]

3) Using Integrate_analytic_function_2D.m (Tutorial 10) as a template, code adaptive refinement for the Rectangle, Trapezium and Simpson's Rules, where the number of intervals in x and y directions are progressively increased from 8 to 80 in steps of 2. The following types of refinement should be implemented for each rule:
   a. Only increase the number of intervals in the x direction in steps of 2, leaving the number of intervals in the y direction constant at 8.
   b. Only increase the number of intervals in the y direction in steps of 2, leaving the number of intervals in the x direction constant at 8.
   c. Simultaneously increase the number of intervals in the x and y direction from 8 to 80 in steps of 2.

   Show your Matlab code.

[4 Marks]

4) Make one figure for each rule where you show how the solution changes with progressive refinement by plotting the number of intervals vs the values of the integral. The three types of refinement (a, b and c) must be shown clearly in each plot.

[2 Marks]

5) Please discuss the different behaviours of the refinement curves in the three plots from 4). Specifically, explain why each refinement strategy shows different behaviours in the same rule, and which refinement strategy and rule are more accurate and why.

[4 Marks]

Question 3: (2 marks)

In lecture 10 we have seen how to extend the trapezium rule from 1D to 2D. Please describe in simple steps how you would proceed to derive the 3D trapezium rule from the 2D formulation, without deriving the whole formula, in order to solve the following integral:

$$\int_e^f \int_c^d \int_a^b f(x,y,z)dxdydz$$