Solving Equations (continued)

Linear Equations

Recap of Yesterday's lectures:

- Significant figure and decimal places
- Factorising algebraic expressions
- Percentage and Ratios
- Introduction to algebra
- Indices and laws of indices

$$a^m \times a^n = a^{m+n}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$(a^m)^n = a^{mn}$$

$$a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$$

$m=1$, $$a^{1/n} = \sqrt[n]{a}$$

$$a^{-n} = \frac{1}{a^n}$$

Solving Linear Equations

Example : Solve $3x-13=2x+9$.

Solution

$$3x-13=2x+9$$

$$3x-2x = 9+13$$

$$x = 22$$

Q. Solve $\dfrac{x-3}{4}=1$

Soln : $\dfrac{x-3}{4}=1$

$$x-3 = 1 \times 4$$

$$x-3 = 4$$

$$x = 4+3 = 7$$

$$\underline{\underline{x = 7}}$$

Q. $\dfrac{2x+4}{5} = \dfrac{x-3}{2}$

Soln: $\dfrac{2x+4}{5} = \dfrac{x-3}{2}$

$$2(2x+4) = 5(x-3)$$

$$4x+8 = 5x-15$$

$$8+15=5x-4x$$

$$23 = x$$

$$\Rightarrow x = 23$$

Solving Simultaneous Equations

Example: Solve using elimination method:

$$x+3y=14 \quad — (1)$$
$$2x-3y=-8 \quad — (2)$$

Solution

$$x+3y=14 \quad — (1)$$
$$+\ 2x-3y=-8 \quad — (2)$$

Add (1) and (2)

$$3x+0 = 6$$

$$\Rightarrow 3x=6$$

$$x = \frac{6}{3} = 2$$

$\overline{3} =$

Substitute $x=2$ in eq 1 to find y

$$2+3y=14$$

$$3y = 14-2$$

$$3y = 12$$

$$y = \frac{12}{3} = 4$$

So, $x=2$ and $y=4$

Q Solve $a+2b=7$ using substitution
$5a+3b=0$

Soln:

$$a+2b=7 \quad -1$$

$$5a+3b=0 \quad -2$$

make a the subject of formula in 1

$$a + 2b = 7$$

$$\rightarrow \boxed{a = 7 - 2b} \quad —— \quad (*)$$

Substitute $a = 7 - 2b$ in 2

$$5(7 - 2b) + 3b = 0$$

$$35 - 10b + 3b = 0$$

$$35 - 7b = 0$$

$$35 = 7b$$

$$\frac{35}{7} = b$$

$$5 = b$$

$$\Rightarrow b = 5$$

Put $b = 5$ in $(*)$ to find a.

$\rightarrow \quad a = 7 - 2b = 7 - 2(5)$
$= 7 - 10 = -3$
$\Rightarrow a = -3$

$a = -3, \; b = 5$

Q: Solve $2x + 5y = 18$ using elimination
$3x + 3y = 18$

Solution:

$2x + 5y = 18$ — (1)
$3x + 3y = 18$ — (2)

multiply eq. (1) by 3 and eq (2) by 2

$6x + 15y = 54$ — (3)

eq (3) − (4) $\quad - \; 6x + 6y = 36$ — (4)

$0 + 9y = 18$

$$\Rightarrow 9y = 18$$

$$y = 18/9 = 2$$

Put $y = 2$ in (1) to find x

$$2x + 5y = 18$$

$$2x + 5(2) = 18$$

$$2x + 10 = 18$$

$$2x = 18 - 10$$

$$2x = 8$$

$$x = 8/2 = 4$$

$$\underline{\underline{x = 4,\ y = 2}}$$

Solving Quadratic Equations

Quadratic equations are of the form

$$ax^2+bx+c=0$$

where a, b and c are real numbers

i.e. $a, b, c \in \mathbb{R}$ ← set of real numbers.

Solving Quadratic Equations by factorisation

Q. $x^2+8x+12=0$

→ $(x+6)(x+2)=0$

$\Rightarrow x+6=0$ or $x+2=0$

$\Rightarrow \underline{\underline{x=-6}}$ or $\underline{\underline{x=-2}}$

Q. $6x^2+5x-4=0$

$$\begin{array}{ll} 3x & +4 \\ 2x & -1 \end{array}$$

$-4 = -4 \times 1$
$= 4 \times -1$
$= -2 \times 2$

$8x - 3x = 5x$

$(3x+4)(2x-1) = 0$

$3x+4=0$ or $2x-1=0$

$\Rightarrow 3x = -4$ or $2x = 1$

$x = \frac{-4}{3}$ $\qquad$ $x = \frac{1}{2}$

$x = -4/3$ or $x = \frac{1}{2}$

Q $x^2 = 81$

Soln:

$x^2 = 81$

1st method: take sq root of both sides

$$x = \pm\sqrt{81}$$

$$x = +\sqrt{81} \text{ or } -\sqrt{81}$$

$$x = 9 \text{ or } x = -9$$

2nd (Preferred) method

$$x^2 = 81$$

$$\underbrace{x^2 - 81}_{} = 0$$

$$x^2 - 9^2 = 0$$

using difference of two squares

$$x^2 - b^2 = (x-b)(x+b)$$

$$\Rightarrow x^2 - 9^2 = (x-9)(x+9) = 0$$

$$\Rightarrow (x-9) = 0 \text{ or } x+9 = 0$$

$\underline{\underline{x = 4}}$ or $\underline{\underline{x = -4}}$

General formula for quadratic equations

$$ax^2 + bx + c = 0 \quad \text{——} (**)$$

obtained by applying completing the square to $(**)$.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$\Rightarrow x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

or

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}.$$

<u>Q</u> Solve using the quadratic formula

$$4x^2 + 4x + 1 = 0 \leftarrow \checkmark$$

Solution

$$ax^2 + bx + c = 0$$

$$a = 4 \ , \ b = 4 \ , \ c = 1$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-4 \pm \sqrt{4^2 - 4(4)(1)}}{2(4)}$$

$$x = \frac{-4 \pm \sqrt{16 - 16}}{8} = \frac{-4 \pm \sqrt{0}}{8}$$

$$x = \frac{-4}{8} = -\tfrac{1}{2} \quad (\text{twice})$$

$$ax^2 + bx + c = 0 \ , \ a, b, c \in \mathbb{R}$$

- If the values of a, b and c are such that

$b^2 - 4ac$ is positive (that is, $b^2 - 4ac > 0$)
then the formula will give two distinct real roots.

- If $b^2 - 4ac = 0$, then there will be a single root known as a repeated root or an equal root.

$$4x^2 + 4x + 1 = 0$$

$$\begin{array}{ll} 2x & +1 \\ 2x & +1 \end{array}$$

$$2x + 2x = 4x$$

$$(2x+1)(2x+1) = 0$$

$$\Rightarrow 2x+1 = 0 \quad \text{or} \quad 2x + = 0$$

$x = -\frac{1}{2}$ (twice)

- If $b^2 - 4ac$ is negative, that is

$$b^2 - 4ac < 0,$$

then the formula will require us to find the square root of a negative number. This has no solution in the real line.

So, if $b^2 - 4ac < 0$, then the quadratic equation has no real roots

The quantity $\boxed{b^2 - 4ac}$ is known as the discriminant. This is because it allows us distinguish between the three possible cases:

$b^2 - 4ac > 0$ — two distinct real roots

$b^2 - 4ac = 0$ — equal or repeated roots

$b^2 - 4ac < 0$ — no real roots

Solving a quadratic and linear simultaneous equation

Q. A quadratic curve and straight line are defined by

$$y = x^2 + 6x - 8$$

$$y = 7x + 12$$

Solution:

$$y = x^2 + 6x - 8 \quad \text{— (1)}$$
$$y = 7x + 12 \quad \text{— (2)}$$

Substitute for y in (1) using (2)

$$7x+12 = x^2+6x-8$$

$$x^2+6x-8 = 7x+12$$

$$x^2+6x-8-7x-12=0$$

$$x^2+6x-7x-8-12=0$$

$$x^2-x-20=0$$

$$(x-5)(x+4)=0$$

$$\Rightarrow x-5=0 \quad \text{or} \quad x+4=0$$

$$\Rightarrow x=5 \quad \text{or} \quad x=-4$$

Substitute for x in eq 2 to find y

$$y = 7x+12$$

$y = 7x + 12$

when $x = 5$,

$$y = 7(5) + 12 = 35 + 12 = 47$$

when $x = -4$, $y = 7(-4) + 12 = -28 + 12$

$$= -16$$

$$y = -16$$

Solutions are $x = 5,\ y = 47$

$x = -4,\ y = -16$