Sets

A set is an unordered collection of clearly defined objects, things or states.

- We denote sets by capital letters
- Objects of set are contained in braces

e.g.

$$A = \{2, 4, 6, 8, ---\}$$ ← set of positive even numbers

↑ infinite set

$$B = \{0, 1, 2\}.$$

↑ finite set.

The objects of a set are called elements or members.

The null or empty set : $\{\ \}$ or $\emptyset$

(A set may have no elements.)

Notation: Given a set A we write

$x \in A$ to mean x is an element of set A or x is a member of set A

$x \notin A$ means x is not an element of A.

$\notin$ not an element of

Eg. Let B be the set
$$B = \{0, 1, 2, 3\}.$$

Then $0 \in B$, $2 \in B$

$$4 \notin B$$

The Cardinality of a set

The cardinality of a set is the number of distinct elements in that set.

If A is a set, then we denote cardinality of A by

$$|A| \quad \text{or} \quad \mathrm{Card}(A)$$

e.g. $A = \{1, 2, 3, 4\}$.

$|A| = 4$ because it has 4 distinct members

$B = \{1, \underline{1, 2, 3, 2, 4}\}$.

$|B| = 4$

$$A = B$$

$$\{1,2,3,4\} = \{4,3,2,1\} = \{1,1,2,3,3,4\}$$

We can also talk about the cardinality of the empty set.

$$|\{\ \}| = 0$$

$\longrightarrow |\{1,2,3,4,3\}|$

$\longrightarrow |\{1,2,3,4\}|$

Equal sets

Two sets are equal if they contain the same distinct elements.

$$A = \{1, 2, 3, 4 3\}$$

$$B = \{1, 2, 3, 4\}$$

Then A and B contain the same distinct elements.

$$A = B$$

Example of sets already seen

$\mathbb{N}$ – set of natural nos

$\mathbb{Z}$ – set of integers

$\mathbb{Q}$ – set of Rational numbers

$\mathbb{R}$ — set of real numbers

$\mathbb{C}$ — set of complex number

When all elements of one set A are contained in another set B, we say A is a subset of B and we write

$$A \subseteq B.$$

$$A = \{0, 1, 2\}, \quad B = \{-1, 0, 1, 2\}.$$

$$\subseteq \longrightarrow \boxed{A \subseteq B}$$

$$|A| = 3 < 4 = |B|$$

$$\Longrightarrow A \subseteq B$$

proper subset

<u>Remark</u> : Every set is a subset of itself

$$B \subseteq B$$

<u>Universal set</u>

The set containing all the objects of interest in a particular context is called a <u>universal set</u>. It's denoted $\mathcal{U}$ or $\mathcal{E}$

$\mathcal{U}$ = the set of whole number

<u>Complement of a set</u>

Given a set $X \subseteq \mathcal{U}$, we can define a new set, called the <u>complement</u> of X denoted $\bar{X}$ (or X')

$$\overline{X} = \mathcal{U} \setminus X = \mathcal{U} - X$$

set-difference

that is, it contains all elements in $\mathcal{U}$ that are not in set X.

e.g

$$\mathcal{U} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$$

$$X = \{2, 3, 4, 5\}.$$

$$\overline{X} = \{0, 1, 6, 7, 8, 9\} = \mathcal{U} - X$$

Intersection of a set

The intersection of the sets X and Y

denoted $X \cap Y$ contains all elements common to both X and Y.

$\cap$ ↑ Intersect

Mathematically, we write this as

$$X \cap Y = \{x : x \in X \text{ and } x \in Y\}$$

↑ such that

$$= \{x \mid x \in X \text{ and } x \in Y\}.$$

Eg Let → $X = \{0, 1, 2, 3, 4, 5, \underline{6}, 7, 8, 9\}$

$Y = \{0, 3, 6, 9, 12, 15\}$

$W = \{11, 12, 13\}$

→ $Z = \{2, 4, 6, 8, 10, 12\}$

State a) $Y \cap Z$ b) $X \cap Z$ c) $X \cap (Y \cap Z)$

d) $X \cap W$ ✓

Solns:

a) $Y \cap Z = \{6, 12\}$

a) $Y \cap Z = \{6, 12\}$ ✓

b) $X \cap Z = \{2, 4, 6, 8\}$

c) $X \cap (Y \cap Z) = X \cap \{6, 12\}$.

$= \{6\}$.

d) $X \cap W = \{\ \}$ or $\emptyset$

Union of a set

The union of sets X and Y denoted $X \cup Y$, contains all elements in both X and Y.

Mathematically as

$$X \cup Y = \{x : x \in X \text{ or } x \in Y \text{ or both}\}$$

e.g. $X = \{0, 1, 2, 3\}$

Eg $X = \{0, 1, 2, 3\}$

$Y = \{0, 3, 6, 9, 12\}$.

$$X \cup Y = \{0, 1, 2, 3, 6, 9, 12\}$$

$$= \{0, 0, 1, 2, 3, 3, 6, 9, 12\}.$$

$$|X \cup Y| = 7$$

Q. List the elements of the following finite sets

a) $A = \{x : x$ is a positive integer, greater than 5 and less than 20$\}$.

b) $B = \{x : x$ is odd and x is greater than 5 and less than 20$\}$.

Soln:

$$A = \{6, 7, \ldots, 19\}.$$

$$B = \{7, 9, 11, 13, 15, 17, 19\}$$

Q. Given $A = \{5, 6, 7, 9\}$, $B = \{0, 2, 4, 6, 8\}$ and $\mathcal{U} = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

List the elements of the following sets

a) $\overline{A}$ b) $\overline{\overline{A}}$ c) $\overline{A} \cup \overline{B}$ d) $\overline{A} \cap \overline{B}$

e) $A \cup B$ f) $\overline{A \cup B}$ g) $\overline{A \cap B}$

Soln

a) $\overline{A} = \mathcal{U} - A = \{0, 1, 2, 3, 4, 8\} \leftarrow$

b) $\overline{\overline{A}} = \mathcal{U} - \overline{A} = \{5, 6, 7, 9\} = A$

$$\Rightarrow \overline{\overline{A}} = A$$

c) $\overline{A} = \{0, 1, 2, 3, 4, 8\} \Leftarrow$

$\overline{B} = U - B = \{1, 3, 5, 7, 9\} \Leftarrow$

$\overline{A} \cup \overline{B} = \{0, 1, 2, 3, 4, 5, 7, 8, 9\}.$

d) $\overline{A} \cap \overline{B} = \{1, 3\}$

$U = \{0, 1, \ldots, 9\}.$

e) $A \cup B = \{0, 2, 4, 5, 6, 7, 8, 9\}.$

f) $\overline{A \cup B} = U - (A \cup B)$

$= \{1, 3\}.$

g) $\overline{A \cap B}$

$A \cap B = \{6\}$

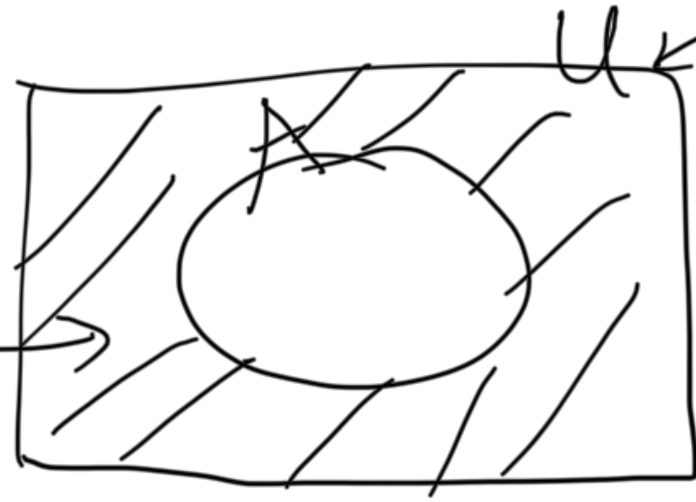
$\overline{A \cap B} = U - \{6\}$

$= \{0, 1, 2, 3, 4, 5, 7, 8, 9\}.$

Venn diagrams

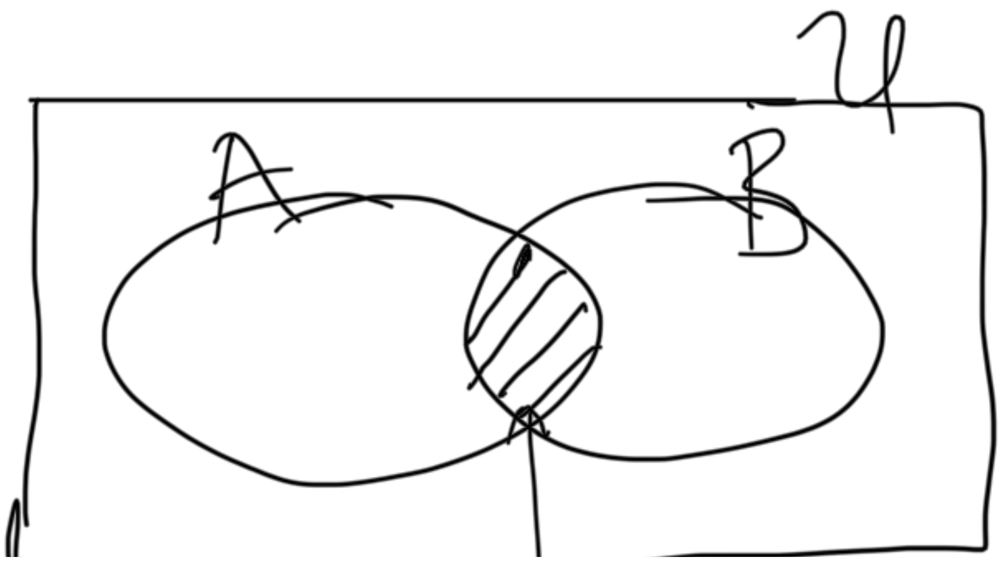
Sets can be represented diagramatically using Venn diagrams.

e.g. set $A \subseteq \mathcal{U}$

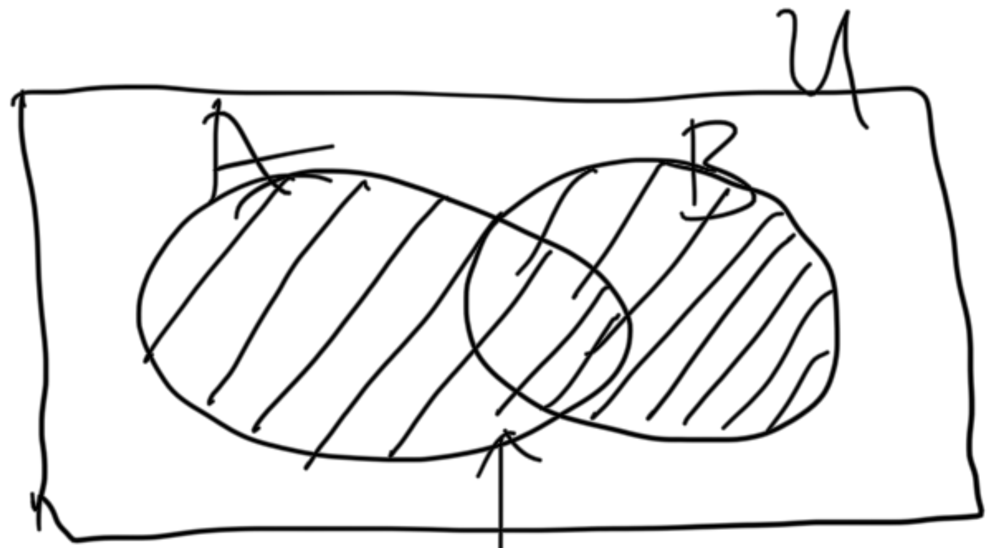


rectangle represents Universal set.

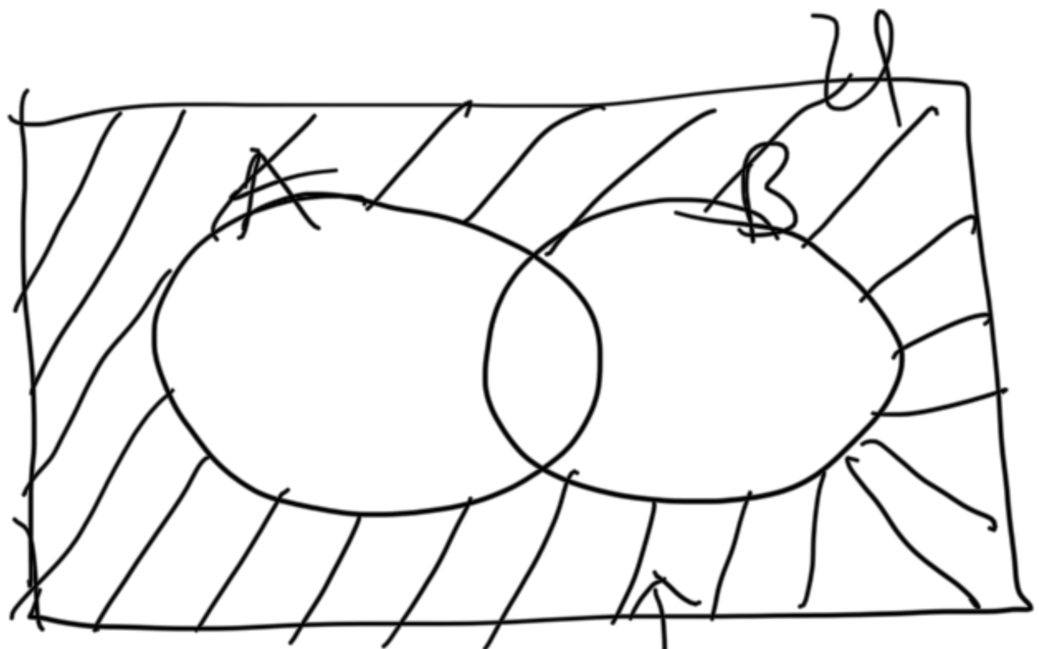
shaded region is $\bar{A}$



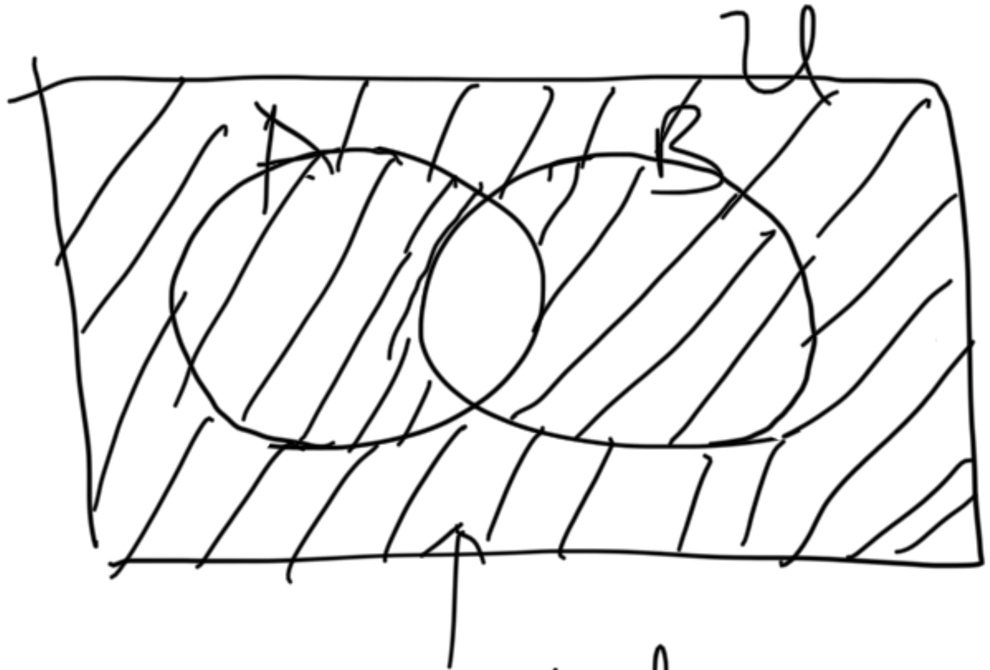
$A \cap B$



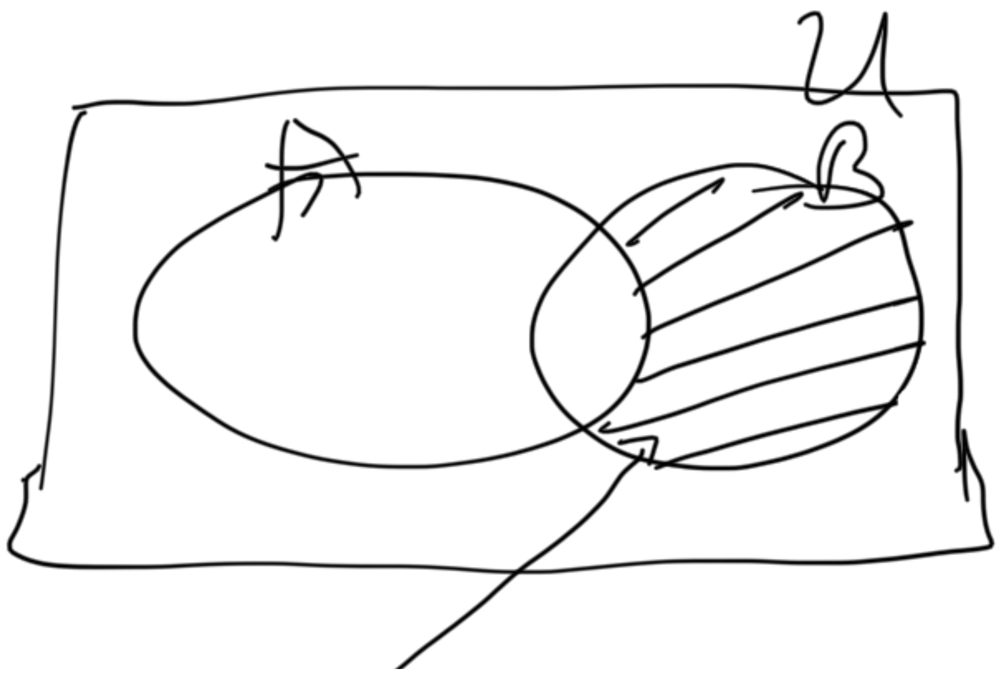
shaded region is $A \cup B$



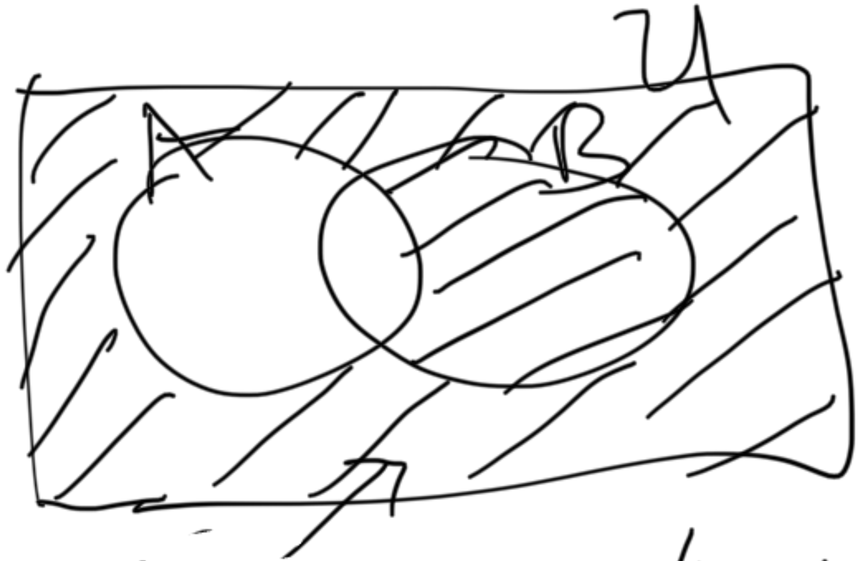
$\overline{A \cup B}$ - shaded region



shaded - region is $\overline{A \cap B}$



$\bar{A} \cap B$ is shaded region



Region shaded is $\bar{A}$

A' or $\bar{A}$

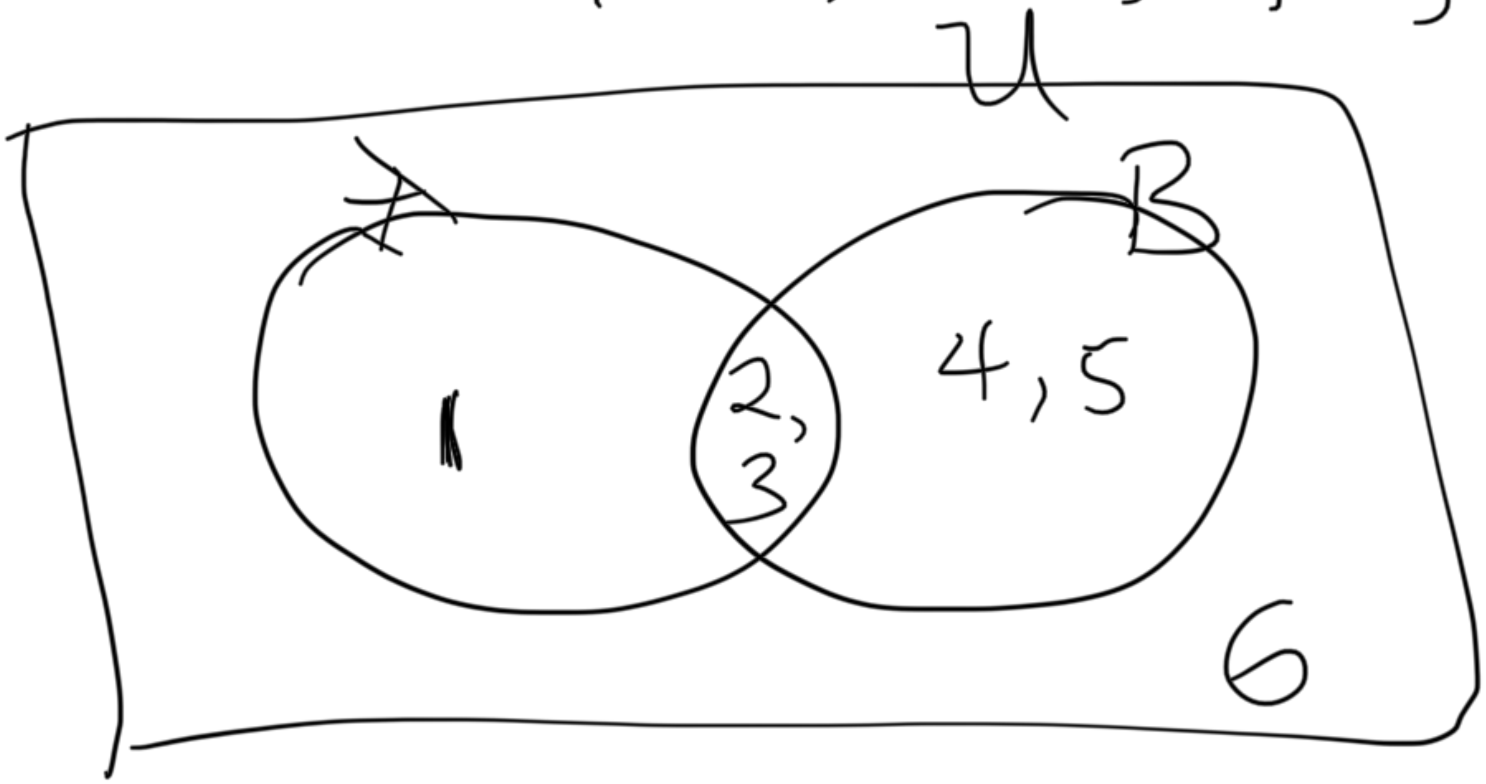
$$\bar{A} = U - A$$

Represent these sets in a venn diagram

$A = \{1, 2, 3\}$

$B = \{2, 3, 4, 5\}$.

$U = \{1, 2, 3, 4, 5, 6\}$.



Number Bases

The decimal system

$$253 = 200 + 50 + 3$$
$$= 2(100) + 5(10) + 3(1)$$
$$= 2(10^2) + 5(10^1) + 3(10^0)$$

When we use 10 as a base we say we're writing in base 10 or <u>decimal</u> <u>system</u>.

In the decimal system, there are 10 digits they are

0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

$$\overset{3}{5}\,\overset{2}{1}\,\overset{1}{9}\,\overset{0}{2}_{10} = 5000 + 100 + 90 + 2$$

$$= \underline{5 \times 10^{3}} + \underline{1 \times 10^{2}} + \underline{9 \times 10^{1}} + \underline{2 \times 10^{0}}$$

<u>Binary System</u>

A binary uses base 2.

it has only 2 digits 0 and 1

e.g. 101_2, 11_2 are binary numbers.

Binary numbers have important applications in computer science.

Converting from Binary to decimal

Ex. Convert 1101_2 to decimal.

$$\overset{3}{1}\overset{2}{1}\overset{1}{0}\overset{0}{1}_2 = 1\times2^3+1\times2^2+0\times2^1+1\times2^0$$

$$= 8+4+0+1$$

$$1101_2 = 13_{10}$$

Example: Convert 1001_2 to base 10 (decimal)

Soln

$$1001_2 = 1\times2^3 + 0\times2^2 + 0\times2^1 + 1\times2^0$$
$$= 8 + 0 + 0 + 1$$
$$= 9_{10}$$

$$1001_2 = 9_{10}$$

Convert decimal to binary

Example: Convert 37_{10} to base 2 (binary)

Soln

2	37
2	18 r 1
2	9 r 0
2	4 r 1

$$
\begin{array}{r|l}
2 & 2 \; r\, 0 \\
2 & 1 \; r\, 0 \\
 & 0 \; r\, 1
\end{array}
$$

$$37_{10} = 100101_2$$

Same

$37 \div 2 = \underline{18} \; r \; 1$

$18 \div 2 = 9 \; r \; 0$

$9 \div 2 = 4 \; r \; 1$

$4 \div 2 = 2 \; r \; 0$

$2 \div 2 = 1 \; r \; 0$

$1 \div 2 = 0 \; r \; 1$

$37 = 100101_2$

<u>2nd method</u>

$$37 = 32 + \overset{2^2}{4} + \overset{2^0}{1}$$

$= 1(2^5) + 0(2^4) + 0(2^3) + 1(2^2) + 0(2^1)$
$+ 1(2^0)$

$\Rightarrow 37_{10} = 100101_2$

Q a) Convert 39_{10} to base 2

b) convert 111001110_2 to base 10

c) Convert 241_{10} to base 2.

Solutions:

a) $39 = 32 + 4 + 2 + 1$

$= 1(2^5) + 0(2^4) + 0(2^3) + 1(2^2)$
$+ 1(2^1) + 1(2^0)$

$39 = \underline{100111_2}$ ✓

OR

$39 \div 2 = 19 \text{ r } 1$

$19 \div 2 = 9 \text{ r } 1$

$9 \div 2 = 4 \text{ r } 1$

$4 \div 2 = 2 \text{ r } 0$

$2 \div 2 = 1 \text{ r } 0$

$1 \div 2 = 0 \text{ r } 1$

$\Rightarrow 39_{10} = 100111_2$ ✓

b) $\overset{8}{1}\overset{7}{1}\overset{6}{1}\overset{5}{0}\overset{4}{0}\overset{3}{1}\overset{2}{1}\overset{1}{1}\overset{0}{0}_2 = 1\times2^8 + 1\times2^7 + 1\times2^6 + 0 + 0 +$

$1\times2^3 + 1\times2^2 + 1\times2^1 + 0\times2^0$

$= 2^8 + 2^7 + 2^6 + 2^3 + 4 + 2$

$= 462$

C) 241_{10} to base 2

$$\underline{241} = 128 + 64 + 32 + 16 + 1$$

$$= 1(2^7) + 1(2^6) + 1(2^5) + 1(2^4) + 0(2^3) +$$

$$0(2^2) + 0(2^1) + 1(2^0)$$

$$= \underline{11110001}_2$$