Numbers

BIDMAS / BODMAS, SUBSTITUTION

INDICES

Numbers:

Number systems:-

Natural numbers : $\mathbb{N}$ or N

$1, 2, 3, 4, \ldots$ or $0, 1, 2, 3, 4, \ldots$

Integers: $\mathbb{Z}$ - set of integers

$-3, -2, -1, 0, 1, 2, 3, \ldots$

Rational numbers : $\mathbb{Q}$ - set of rational numbers.

contains fractional numbers.

p/q, $p, q \in \mathbb{Z}$ and $q \neq 0$

belongs to
or is member of

$2/3$, $\frac{4}{3}$, $10/10 = 1$

$9 = 9/1$ ✓ fraction

$\mathbb{Z} \subset \mathbb{Q}$

subset

$9/1 = 9$ ✓ , $-9 = -9/1$

Real numbers : $\underline{\mathbb{R}}$ — set of real numbers

$\mathbb{Q} \cup \mathbb{I}\mathbb{Q}$

union

irrational numbers

$\mathbb{I}\mathbb{Q}$ — irrational numbers

e.g

$\sqrt{2}, \sqrt{3}, \sqrt{5}, \sqrt{7}, \pi$

surd

$-\sqrt{2}, -\sqrt{3}$ etc.

$\sqrt{4} = 2$

Numbers: Addition, Subtraction and division

$3 + 7 = 10$

$-3+7=4$ $\quad$ $-7+3=-4$

$- + + = -$ if the larger number is $-$ve

$- + + = +$ if the larger number is $+$ve.

$-8+5=-3$ $\quad$ $-5+8=\underline{\underline{3}}$ ✓

$$\left\{\begin{array}{l} - \times - = + \\ - \times + = - \\ + \times - = - \\ + \times + = + \end{array}\right.$$

$-2 \times -3 = 6$

$$-2 \times 3 = -6$$

$$2 \times -3 = -6$$

$$2 \times 3 = 6$$

$$- \div - = +$$

$$- \div + = -$$

$$+ \div - = -$$

$$+ \div + = +$$

$$-4 \div -2 = \frac{+4}{+2} = \underline{\underline{2}}$$

$$-4 \div 2 = \frac{-4}{2} = -2$$

$$4 \div -2 = \frac{4}{-2} = -2$$

$$4 \div 2 = \frac{4}{2} = \underline{\underline{2}}$$

Prime numbers and Factorisation

factors or divisors : divides a number without remainders

e.g 2 is a factor of 4 because $4 \div 2 = 2$.

→ Prime numbers are numbers with exactly two factors, 1 and itself.

e.g: 2, 3, 11 are examples of prime factors

1 is **not** a prime number because it has only one factor - itself.

2 is the smallest prime number we can have.

$$-2 = -2 \times 1 \qquad \underbrace{-2, -1, 1, 2}_{\underline{4 \text{ factors}}}$$
$$= 2 \times -1$$

Can there be any other even prime number?

NO, 2 is the only even prime number
All even numbers are divisible by 2.

All the other prime numbers are odd numbers except 2.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29,

31, ...

types of Primes : $\begin{cases} \text{Mersene Primes} \\ \text{Fermat's Primes.} \end{cases}$

Prime factorization of numbers

Example

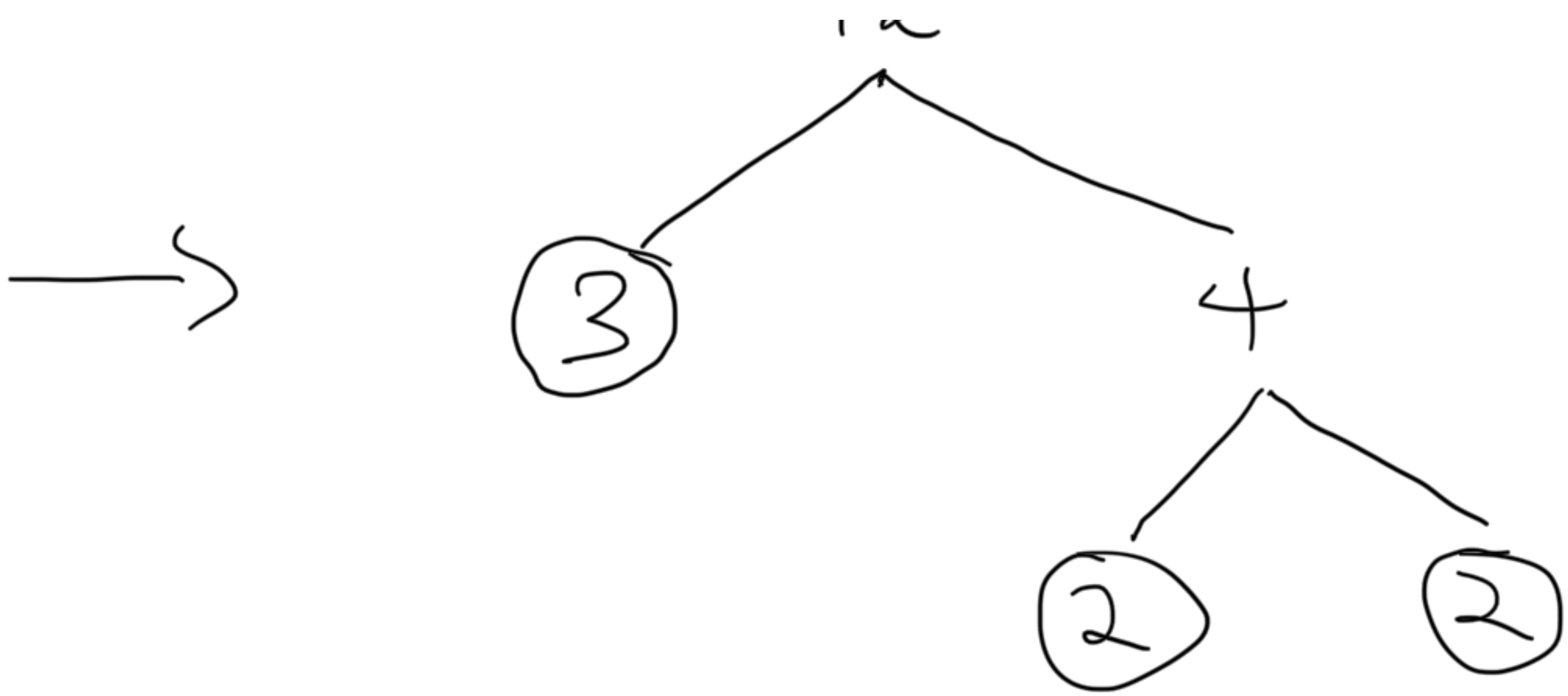
$$\left.\begin{aligned} 12 &= 1 \times 12 \\ \rightarrow \quad &= 2 \times 6 \\ &= 3 \times 4 \end{aligned}\right\} \text{factorization of 12}$$

factors of 12 are 1, 2, 3, 4, 6 and 12.

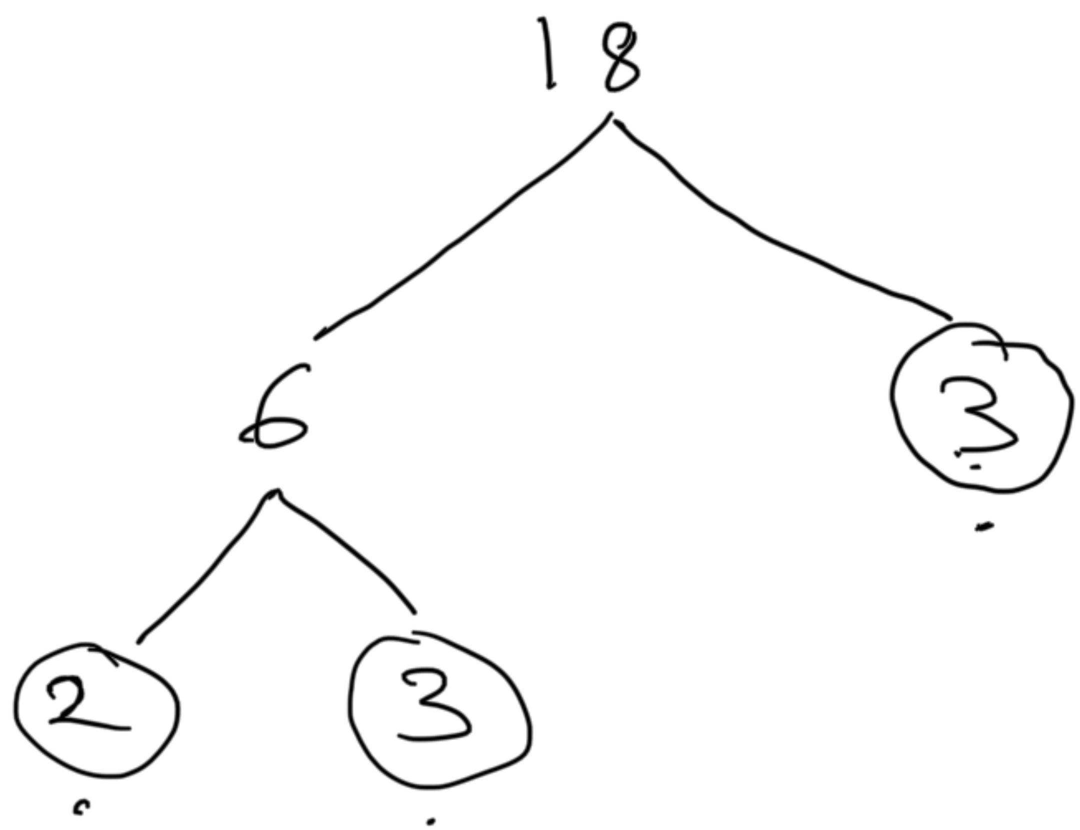
$$12 = 2 \times 6 = 2 \times 2 \times 3$$

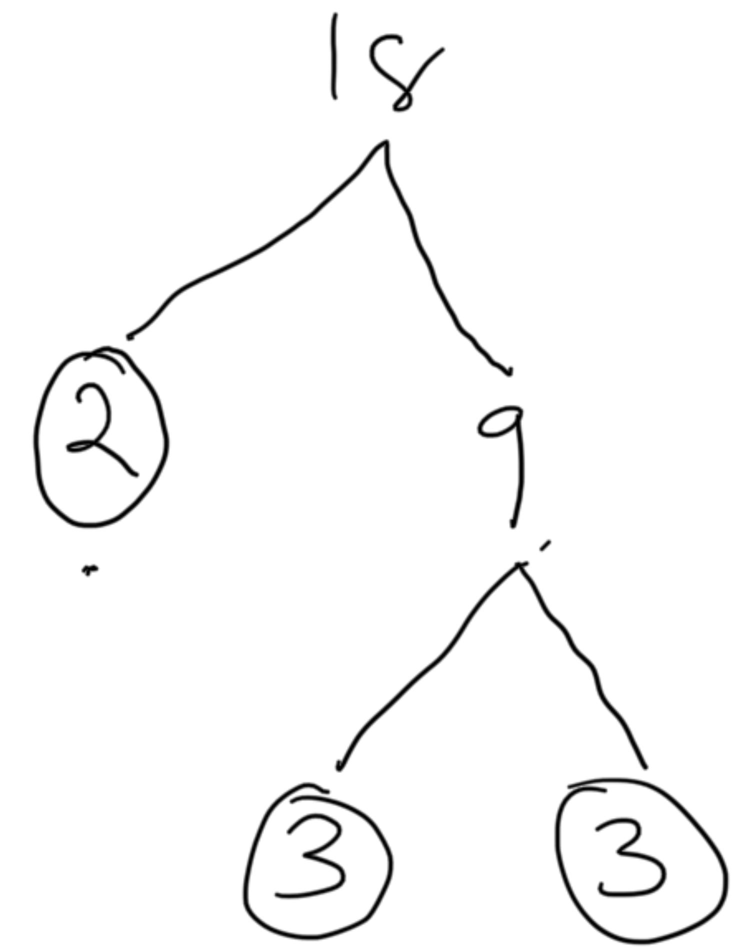
$$12 = 2 \times 2 \times 3$$

12



$12 = 2\times2\times3$



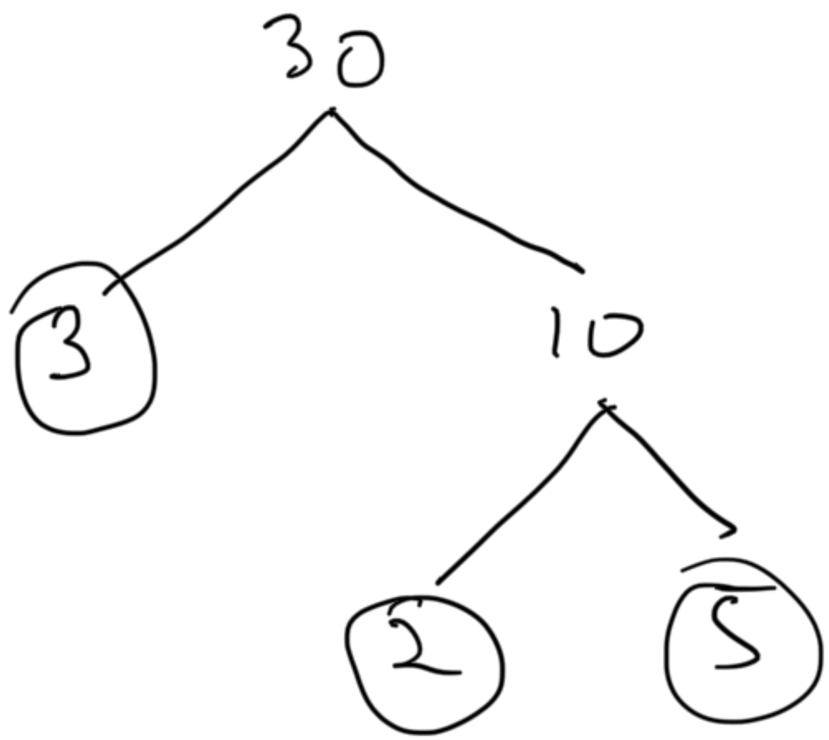


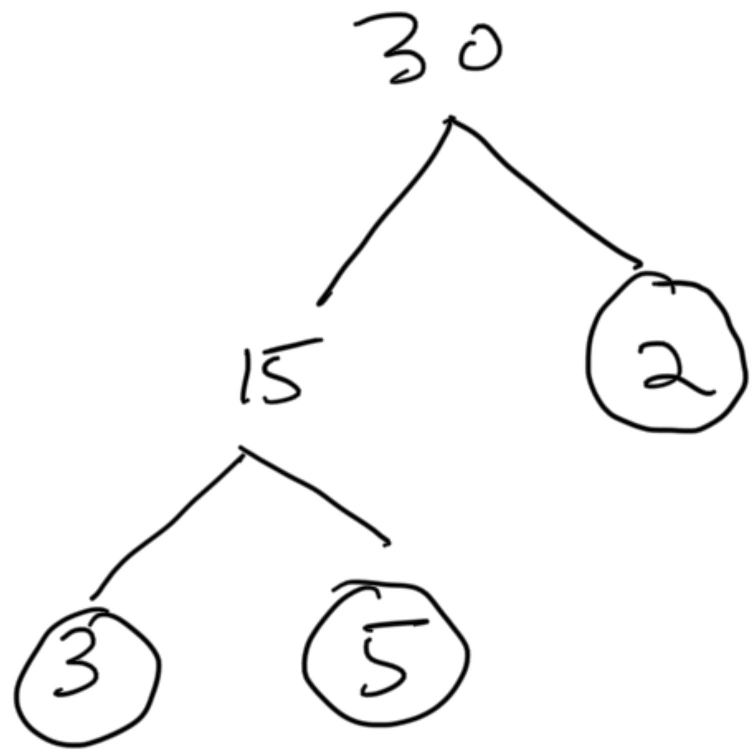
$18 = 2 \times 3 \times 3$

HCF and LCM

Highest Common factors (HCF)

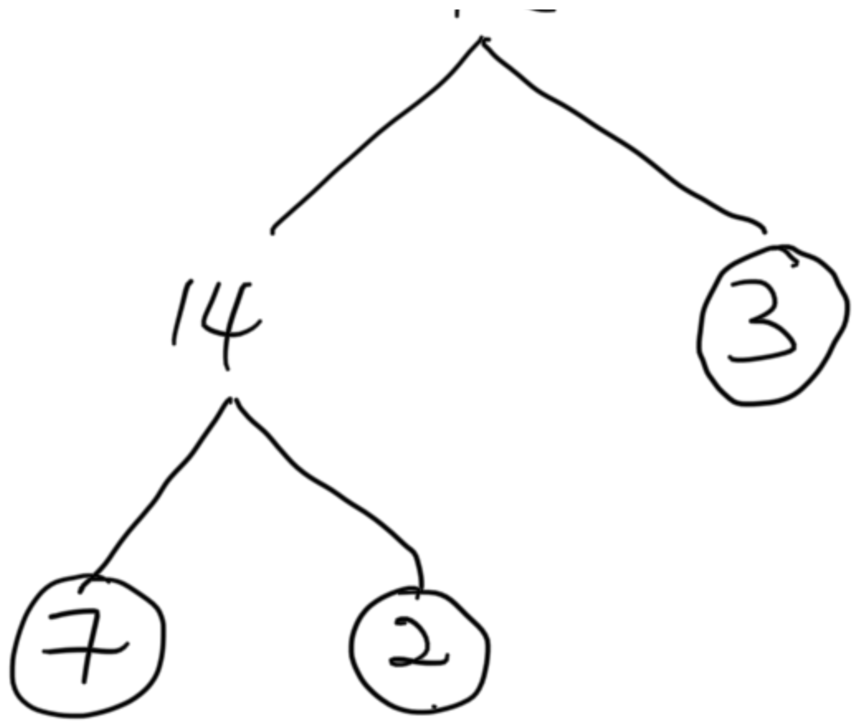
E.g Factorise the two numbers 30 and 42.





$30 = 2 \times 3 \times 5$

42



$$30 = 2 \times 3 \times 5$$
$$42 = 2 \times 3 \times 7$$

$$42 = 2 \times 3 \times 7$$

$$HCF(30, 42) = 2 \times 3 = 6$$

→

$$\begin{aligned} 30 &= 1 \times 30 \\ &= 2 \times 15 \\ &= 5 \times 6 \\ &= 3 \times 10 \end{aligned} \qquad \begin{aligned} 42 &= 1 \times 42 \\ &= 2 \times 21 \\ &= 3 \times 14 \\ &= 6 \times 7 \end{aligned}$$

the factors they have in common are

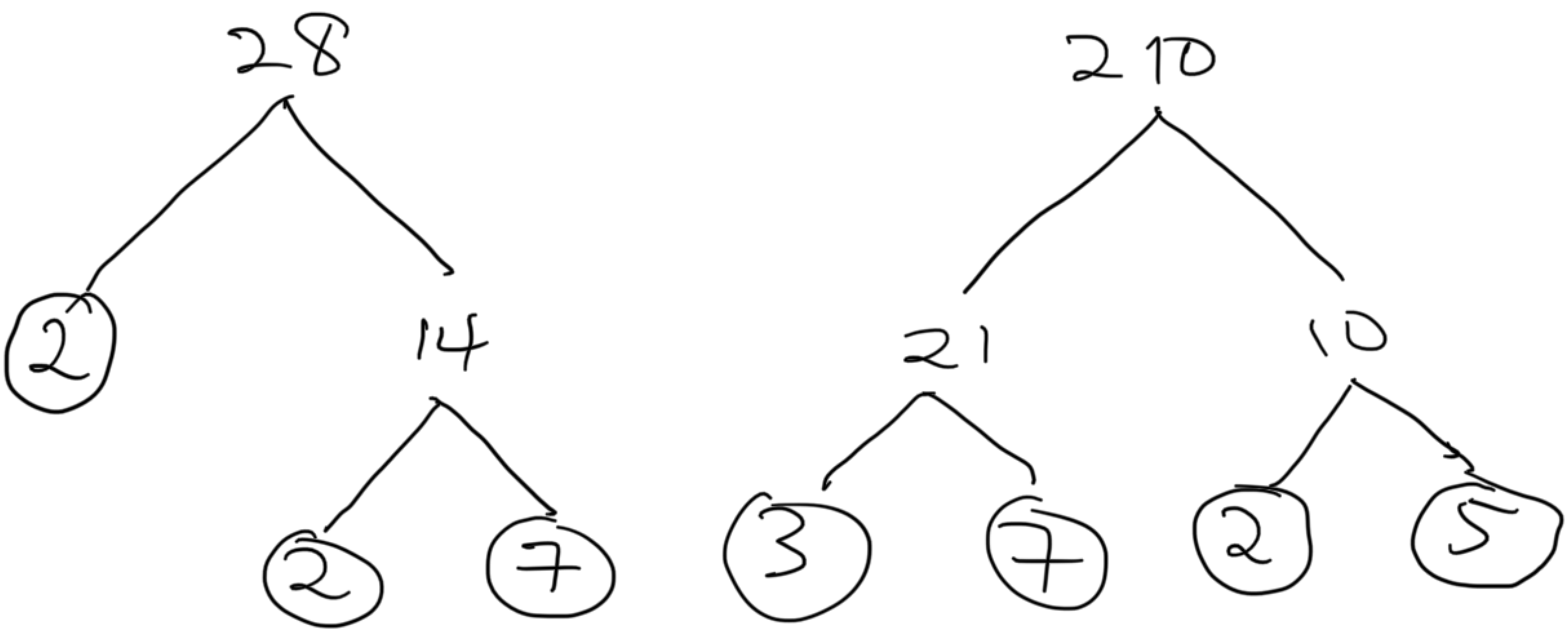
1, 2, 3, 6

$$12 = 2 \times 2 \times 3$$
$$18 = 2 \times 3 \times 3$$

$$\text{HCF}(12, 18) = 2 \times 3 = \underline{6}$$

Find HCF of 28 and 210

<u>Solution</u>:



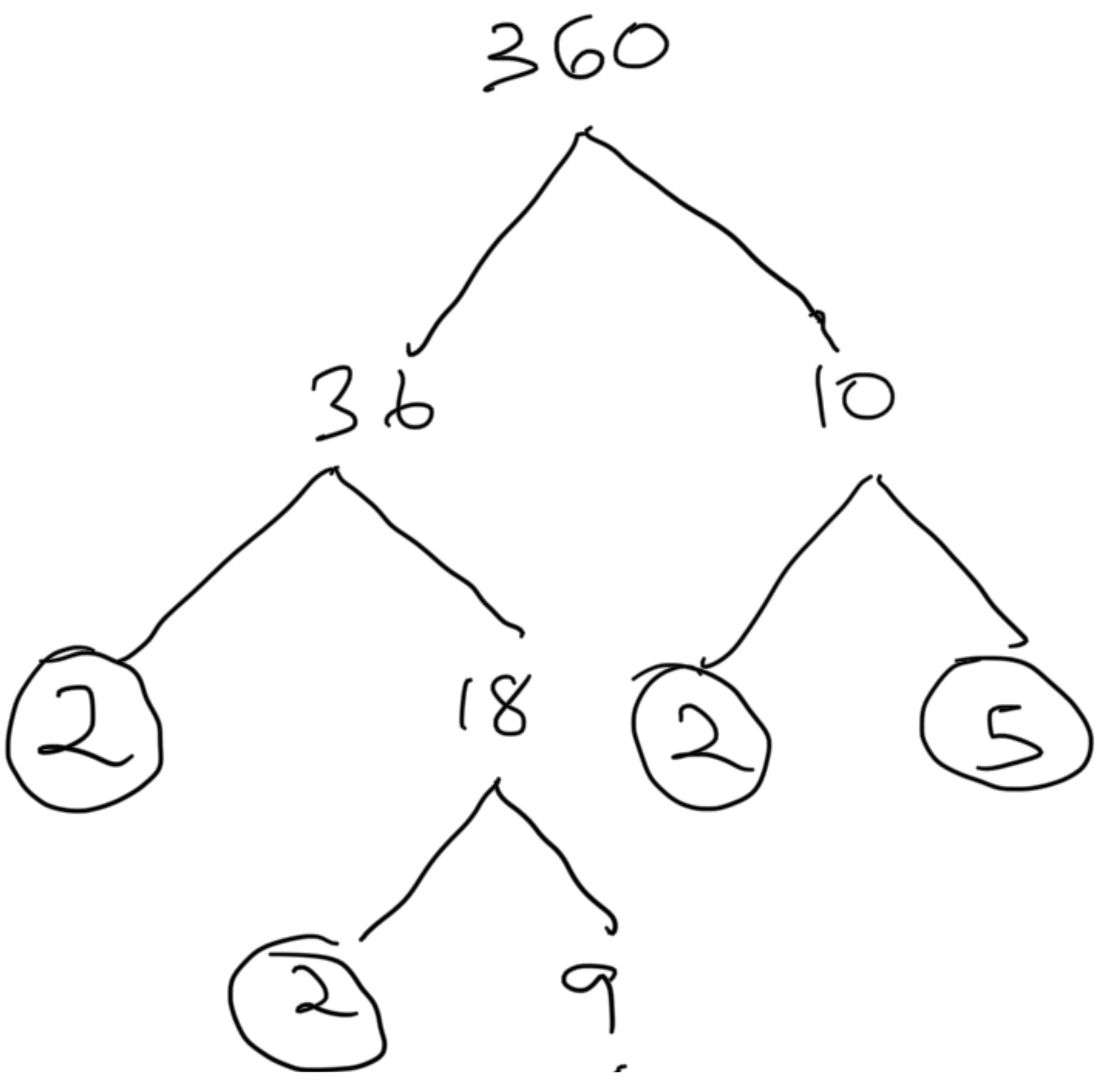
$\Rightarrow 28 = 2 \times 2 \times 7$

$\Rightarrow 210 = 2 \times 3 \times 5 \times 7$

$$28 = 2 \times 2 \times 7$$
$$210 = 2 \times 3 \times 5 \times 7$$

$$\Rightarrow HCF(28, 210) = 2 \times 7 = \underline{\underline{14}}$$

Find HCF of 28, 210 and 360



3 3

$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$

$210 = 2 \times 3 \times 5 \times 7$

$28 = 2 \times 2 \times 7$

$HCF(28, 210, 360) = 2$

Lowest Common Multiple (LCM)

Eg

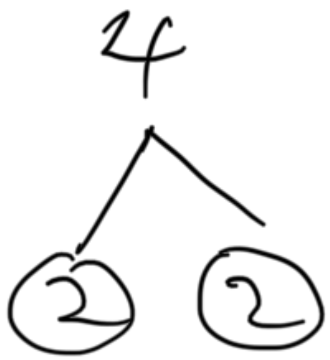
Find the LCM of 4 and 6.

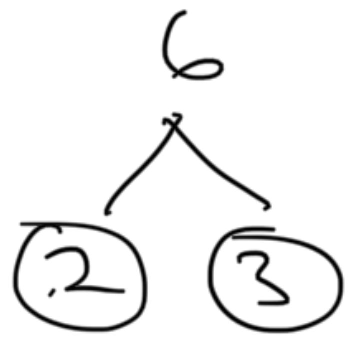
multiples of 4 – 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, ...

multiples of 6 – 6, 12, 18, 24, 30, 36, 42, 48, ...

$LCM(4, 6) = 12$

Prime factorization method:





$\Rightarrow 4 = 2 \times 2$

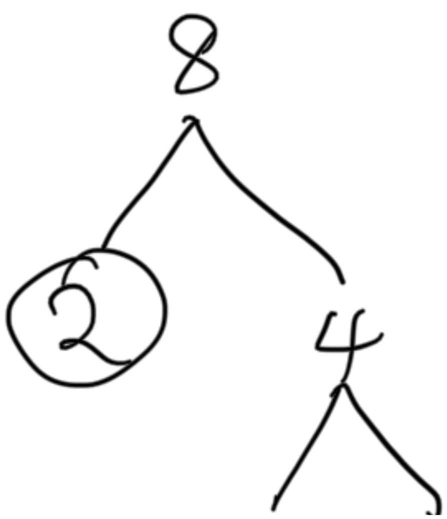
$6 = 2 \times 3$

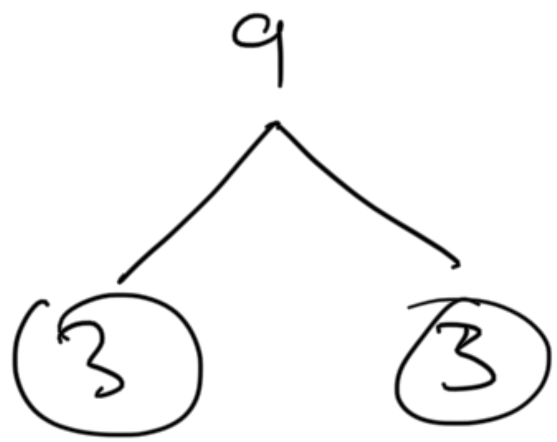
$4 = 2 \times 2$

$6 = 2 \times 3$

$LCM(4,6) = \underline{2 \times 2} \times 3 = \underline{12}$

Find $LCM(8,9)$





2 2

$8 = 2 \times 2 \times 2$

$9 = 3 \times 3$

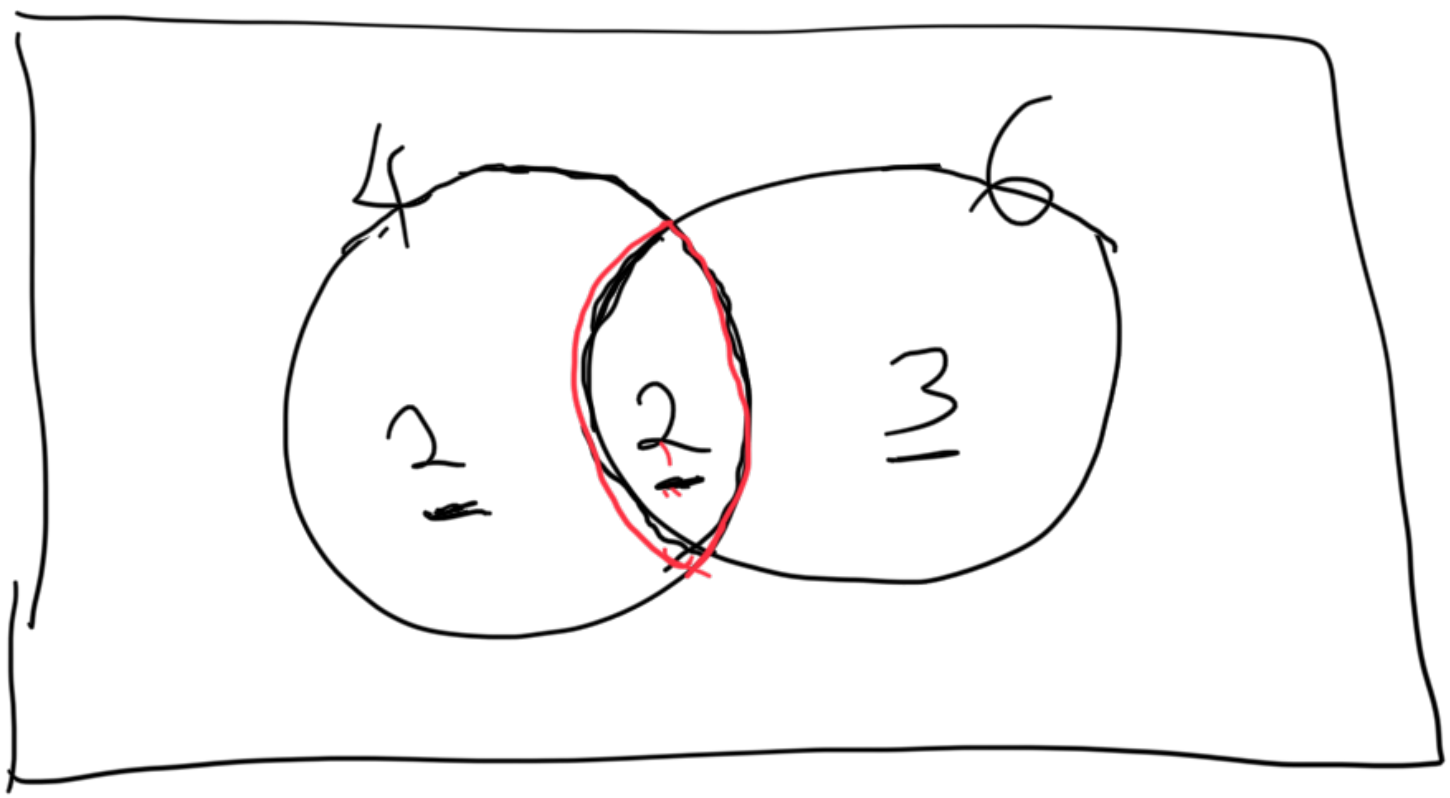
$8 = 2 \times 2 \times 2$

$9 = 3 \times 3$

$LCM(8,9) = 2 \times 2 \times 2 \times 3 \times 3$

$= \underline{\underline{72}}$

$LCM(4,6)$

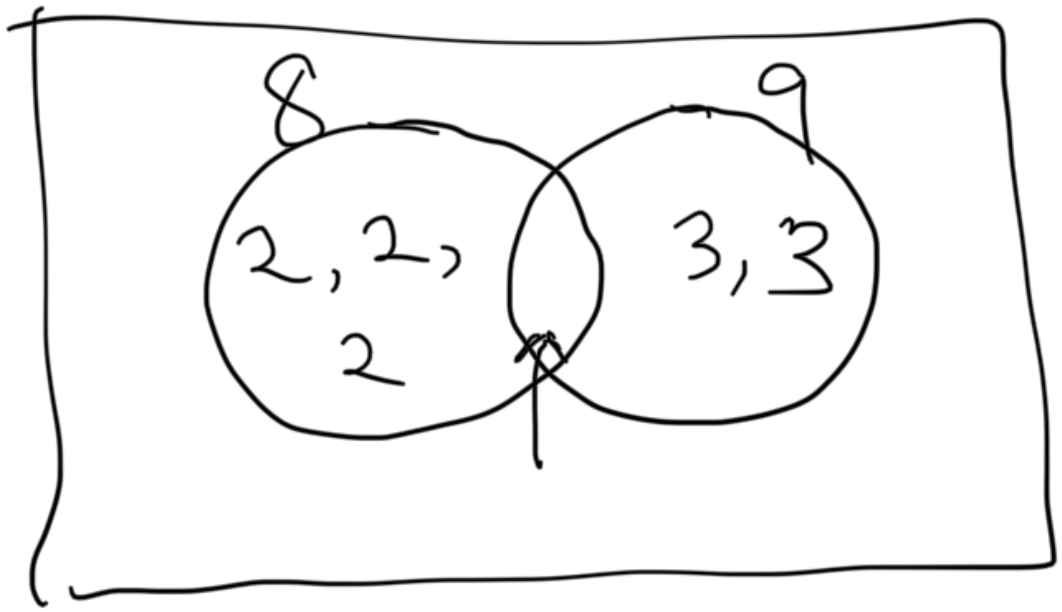


$HCF(4,6) = 2$

$$LCM(4,6) = 2 \times 2 \times 3$$
$$= \underline{\underline{12}}$$

$HCF(8,9)$ & $LCM(8,9)$

$$8 = 2 \times 2 \times 2$$
$$9 = 3 \times 3$$



$$HCF(8,9) = \underline{1}$$

$$LCM(8,9) = 2 \times 2 \times 2 \times 3 \times 3$$
$$= \underline{\underline{72}}.$$

Remark: If two numbers are coprime (that is, their HCF is 1) then their LCM is the product of the two numbers.

product of the two numbers.

$$LCM(3,5) = 3 \times 5$$

$$LCM(9,10) = 9 \times 10 = \underline{\underline{90}}$$