Significant figures and Decimal Places

Significant figures

Suppose to write 953 to two significant figures; we can use no more than two non-zero digits

950 953 960

953 = 950 to 2 s.f.

$9\,5\,|\,3 = 950$

E.g. Write 98432 to 3 sig figures

Solution

$98\,4\,|\,3\,2 = 98400$ to 3 s.f.

Write 63415.23 to 4 s.f.

Soln:

$$63415.23 = 63420 \text{ to 4 s.f.}$$

Write 0.00004562 to 2 sig figures

Soln:

$$0.00004562 = 0.000046 \text{ 2 sig. fig.}$$

Write 795.3 to 2 sig fig.

Solution:

$$795.3 = \underline{800} \text{ (2 sig fig)}$$

Question: A number is given to 2 sig. figures as 67.

a) What is the max. value the number could have?

b) What is the minimum value the number could have?

Solution:

a) $67.5 = 68$ to 2 sig figs

$67.49 = 67$ to 2 s.f.

$67.4999\ldots$ = max value.

b) min $66.5 = 67$ to 2 sig fig.

If x was the number:

$$66.5 \leq x < 67.5$$

Decimal Places

Q. Write 63.5261 to 2 dec. places

Solution:

$63.52|61 = 63.53$ (to 2 d.p.)

Q. Write 1.98 to 1 d.p.

Soln

$$1.9\overline{8} = 2.0$$

Q. Write 75.999 to
a) 4 s.f. b) 2 d.p.

Soln:

a) $75.99\overline{9} = 76.00$

b) $75.99\overline{9} = 76.00$

Percentage and Ratio's

Percentage

Percent means out of 100.

Percentage is just a fraction with 100 as the denominator.

e.g $\frac{19}{100} = 19\%$

↑ percent.

Q: Convert $5/8$ into a percentage

Soln $\frac{5}{8} \times \frac{100}{1}^{25} = \frac{125}{2} = 62.5\%$

2

You can change any fraction to a percentage by multiplying by 100.

Q: Express 17.5% as a decimal.

$17.5\% = \frac{17.5}{100} = 0.175$

Q Calculate 27% of 90

Solution:

$$27\% \text{ of } 90 = \frac{27}{10\cancel{0}} \times 9\cancel{0}$$

$$= 2.7 \times 9$$

$$= \underline{24.3}$$

Q. A television set is advertised as £315. The retailer offers a 10% discount. How much do you pay for the television?

Solution:

1st method: 10% of 315

$$= \frac{10}{10\cancel{0}} \times 315 = 31.5$$

So, new price = 315 − 31.5

= £283.50

2nd method: discount of 10% of 100%

$100 - 10 = 90\%$ — new price percentage.

90% of $315 = \frac{90}{100} \times 315$

$= 9 \times 31.5$

$= £283.50$

Percentage change

When a quantity changes, we can also calculate the percentage change of the quantity.

$$\text{Percentage change} = \frac{\text{change}}{\text{original value}} \times 100$$

$$= \frac{\text{new value} - \text{original value}}{\text{original value}} \times 100$$

If the change is positive then there has been an increase. If the change is negative, then there has been a decrease in the quantity.

Q. A microwave oven is reduced in price from £149.95 to £135. Calculate the percentage change in price.

Solution:

Original value = 149.95
new value = 135

$$\text{Percentage change} = \frac{135 - 149.95}{149.95} \times 100$$

$$= \frac{-14.95}{149.95} \times 100$$

$$= -9.97\% \quad (2\text{dp})$$

$\simeq -10\%$

The negative result indicates a decrease in price.

Q. A worker earns £500 per week. She received a 6% increase. Calculate her new weekly wage.

Solution:

add 6% to 100%

106% of 500

$= \frac{106}{100} \times 500 = £530$

Ratios

Ratio is just an alternative way of expressing fractions.

E.g. Suppose we wish to divide £100 between two people Lee and Ling in the ratio 6:4.

This means that for every £4 Ling receives Lee gets £6.

So, every £10 is divided as £6 for Lee and £4 for Ling.

So $\frac{6}{10}$ for Lee and $\frac{4}{10}$ for Ling.

$\frac{6}{10}$ of 100 $= \frac{6}{10} \times 100 = 60$ for Lee

$\frac{4}{10} \times 100 = 40$ for Ling.

Q. Divide 180 in the ratio 3:2

Soln:

$3 + 2 = 5$

$\frac{3}{5}$ of $180 = \frac{3}{\cancel{5}_{1}} \times \frac{36}{\cancel{180}} = 108$

$\frac{2}{5}$ of $180 = \frac{2}{\cancel{5}_{1}} \times \frac{36}{\cancel{180}} = 72$

The number is divided into 108 and 72.

Q. Divide 250 cm in the ratio 1:3:4

Solution:

$1+3+4 = 8$

$\frac{1}{8}$, $\frac{3}{8}$ and $\frac{4}{8}$

$\frac{1}{8}$ of $250 = \frac{1}{\cancel{8}_{4}} \times \frac{125}{\cancel{250}} = 31.25$ cm

$\frac{3}{8}$ of 250 = 3 × 31.25 = 93.75cm

$\frac{4}{8}$ of 250 = 4 × 31.25 = 125cm

So, 250 is divided into 31.25cm, 93.75cm and 125cm.

2 : 3 is the same as 4 : 6

2 : 3 is the same 6 : 9 and so on.

This stems from equivalent fractions.

2 : 3 and 4 : 6 are equivalent ratios

e.g. $\frac{3}{5}$: 2 is equivalent 3 : 10 by multiplying by 5.

Q: Divide 380 kg in the ratio 3/4 : 1/5

Solution:

$$20\times \frac{3}{4} : \frac{1}{5}^{\times 20} = 15 : 4$$

$$LCM(4,5) = 20$$

$$15 + 4 = 19$$

$\frac{15}{19}$ and $\frac{4}{19}$.

$$\frac{15}{19} \text{ of } 380 = \frac{15}{\cancel{19}_{1}} \times \cancel{380}^{20} = 300 \text{ kg}.$$

$$\frac{4}{19} \text{ of } 380 = 80 \text{ kg}$$

Algebra

We use letters to represent numbers.

$$x + y = y + x$$

$$x - y \neq y - x \quad \text{e.g} \quad 2 - 3 \neq 3 - 2.$$

$$x \times y = xy = yx = y \times x$$

$$x \div y = \frac{x}{y}$$

$$y \div x = \frac{y}{x}.$$

Indices:

$$3^2 = 3 \times 3$$

$$3^3 = 3 \times 3 \times 3$$

$a^2 = a \times a$

$a^m = \underbrace{a \times a \times \cdots a}_{m \text{ of them}}$

$a^2 b^4 = a \times a \times b \times b \times b \times b$

↑ in expanded form.

Q: Simplify

a) -5^2 b) $(-5)^2$

Soln

a) $-(5^2) = -(5 \times 5) = -25$

b) $(-5)^2 = -5 \times -5 = 25$

Q Evaluate $(-3)^3$

Solution:

$(-3) - 7 \times 7 \times -3$

$= -27$

Q. Express the following compactly using indices

a) $xxxyyx$ b) $abccba$

Soln

a) $xxxyyx = x \times x \times x \times x \times y \times y \times x$

$= x^4 y^2$

b) $abccba = a^2 b^2 c^2$ ✓

$= (abc)^2$ ✓

Substitution and Formula:

Q. Find the value of $a + 7b + 3c$ when

$\Rightarrow$ $a=1, b=2$ and $c=3$.

Solution:

$a=1, b=2, c=3$

$a+7b+3c = 1+7(2)+3(3)$

$= 1+14+9$

$= 24$

Q. If $x=4$, find the value of

a) $8x^4$ b) $(8x)^3$

Soln:

a) $8x^4 = 8(4^4) = 2048$

b) $(8x)^3 = 8x \times 8x \times 8x$

$= 8(4) \times 8(4) \times 8(4) = 32^3$

$= 32768.$

OR

$$(8x)^3 = (8(4))^3 = 32^3 = 32768$$

Q. Find $-x^2$ when $x = -4$.

Soln:

$$-x^2 = -(-4)^2$$
$$= -16$$

Q. Find the value of x^2+3x when
a) $x=2$ b) $x=-2$.

Solution:

a) $x=2$

$$x^2+3x = 2^2+3(2)$$
$$= 4+6 = 10$$

b) $x=-2$

$$(-2)^2 + 3(-2) = 4 - 6 = \underline{\underline{-2}}$$

Q Find the value of $\frac{3x^2}{4} + 5x$ when $x = -3$

Soln:

$$x = 3$$

$$\frac{3(-3)^2}{4} + 5(-3)$$

$$= \frac{27}{4} - 15 = \frac{-33}{4} = -8.25$$

Using Algebraic formula

Q: Use the formula $A = lb$ to find A when $l = 10$ and $b = 2.5$

Soln

$$A = l \times b = 10 \times 2.5 = \underline{\underline{25}}$$

Q Use the formula $y = x^2 + 3x + 4$ to find

y when $x = -2$.

Solution; $x = -2$

$$y = x^2 + 3x + 4$$

$$y = (-2)^2 + 3(-2) + 4$$

$$= 4 - 6 + 4$$

$$= 2$$

Area $= l \times b$