AM Functions 24/09/2021

A function is a rule that receives an input and produces a single output.

e.g. a rule that adds 2 to the input

- let x be the input

input rule output

$$\xrightarrow{x} \boxed{\text{add } 2} \xrightarrow{x+2}$$

Note: For a rule to be a function, it is necessary that only one output is produced for any given input.

We usually denote functions, input and

output using letters.

e.g.

$$f : x \longrightarrow y \qquad \text{or} \qquad f(x) = y$$

function input output

e.g.

$$f : x \longrightarrow x+2$$

or

$$f(x) = x+2$$

if $x = 3$,

$$f(3) = 3+2 = 5$$

e.g. $h : t \longrightarrow t^2$

or

$$h(t) = t^2$$

if $t = 4$

$$h(4) = 4^2 = 16$$

Q. A function multiplies the input by 5. Write down the function in mathematical notation

Soln: let f be the function and x be input

$$f(x) = 5x$$

Q: A function divides the input by 6 and then adds 3 to the result. Write the function in mathematical notation.

Soln:

let h be the function and let t be the input.

$$\boxed{h(t) = \frac{t}{6} + 3}$$

b) Find $h(2)$ and $h(-12)$

Solution

$$h(t) = \frac{t}{6} + 3$$

$t = 2$, $h(2) = \frac{2}{6} + 3$

$$= \frac{1}{3} + 3 = \underline{3\tfrac{1}{3}}$$

$t = -12$, $h(-12) = \frac{-\cancel{12}^{\,2}}{\cancel{6}_{\,1}} + 3$

$$h(-12) = -2 + 3 = \underline{1}$$

Composite functions

Sometimes we may wish to apply two or more

functions one after the other.
The output of one function becomes the input of the next function.

<u>Ex</u> Let $f(x) = 2x$ and $g(x) = x+3$

We want to find

$$f(g(x)) \quad , \quad g(f(x))$$

composite function

$$x \longrightarrow \boxed{\;g\;} \xrightarrow{x+3} \boxed{\;f\;} \longrightarrow 2(x+3)$$

$$f(g(x)) = f(x+3) = 2(x+3) = 2x+6$$

Q. Given $f(t) = t^2+1$, $g(t) = \frac{2}{t}$ and $h(t) = 2t$

Determine each of the following composite fcns:

a) $f(g(t))$ b) $g(h(t))$ c) $f(h(t))$ d) $f(g(h(t)))$

Soln:

a) $f(g(t)) = f\left(\frac{2}{t}\right) = \left(\frac{2}{t}\right)^2 + 1 = \frac{4}{t^2} + 1$

b) $g(h(t)) = g(2t) = \frac{\cancel{2}^{1}}{\cancel{2}_{1}t} = \frac{1}{t}$

c) $f(h(t)) = f(2t) = (2t)^2 + 1$ ✓

$= 4t^2 + 1$

d) $f(g(h(t))) = f(g(2t)) = f\left(\frac{2}{2t}\right)$

$$= f\left(\frac{1}{t}\right)$$

$$= \left(\frac{1}{t}\right)^2 + 1$$

$$= \frac{1}{t^2} + 1$$

Inverse of a function

Let f be the function defined by

$$f(x) = y$$

The function g that receives y as an input and generates an output of x, if it exists, is called the inverse function of f.

$$g(f(x)) = g(y) = x. \Rightarrow g \text{ is the inverse of } f.$$

It is denoted by $f^{-1}(x)$ i.e. $f^{-1}(x)$

... is denoted by $f^{-1}(x)$, i.e $g(x) = f^{-1}(x)$

Example: The function f and g are defined by $f(x) = 2x$ and $g(x) = \frac{x}{2}$

a) Verify that f is an inverse of g

b) Verify that g is an inverse of f.

Solution:

a) We need to show $f(g(x)) = x$

$$f(g(x)) = f\left(\frac{x}{2}\right) = \cancel{2}\left(\frac{x}{\cancel{2}}\right)$$

$$= x$$

$\Rightarrow$ f is the inverse of g

b) We need to show $g(f(x)) = x$

$$g(f(x)) = g(\underline{2x}) = \frac{\cancel{2}x}{\cancel{2}} = x$$

$\Rightarrow$ g is the inverse of f.

<u>Q</u>. If f is an inverse of h, does this mean h is also an inverse of f?

<u>Yes</u> ✓

<u>Q</u> Find the inverse of $h(t) = 3t - 4$

<u>Solution:</u>

let $y = h(t) = 3t - 4$

$y = 3t - 4$

make t the subject of formula.

$y = 3t - 4$

$3t - 4 = y$

$3t = y + 4$

$t = \frac{y+4}{3}$

$\Rightarrow y = \frac{t+4}{3}$

$\Rightarrow h^{-1}(t) = \frac{t+4}{3}$

Q. Find the inverse of $h(t) = \frac{2}{t} - 3$

Solution:

$$\text{let } y = \frac{2}{t} - 3$$

$$y = \frac{2}{t} - 3$$

$$\frac{2}{t} - 3 = y$$

$$\frac{2}{t} = \frac{y+3}{1}$$

$$2(1) = t(y+3)$$

$$\frac{2}{} = t$$

$$\boxed{y+3}$$

$$\Rightarrow h^{-1}(t) = \frac{2}{t+3}$$

Graphs of functions

$$\boxed{y = mx + c}$$

Q1. Plot a graph of $y = 2x - 1$ for $-3 \le x \le 3$

Soln:

x - input, $-3 \le x \le 3$

$[-3, 3]$

$\hookrightarrow \{x \in \mathbb{R} : -3 \le x \le 3\}$

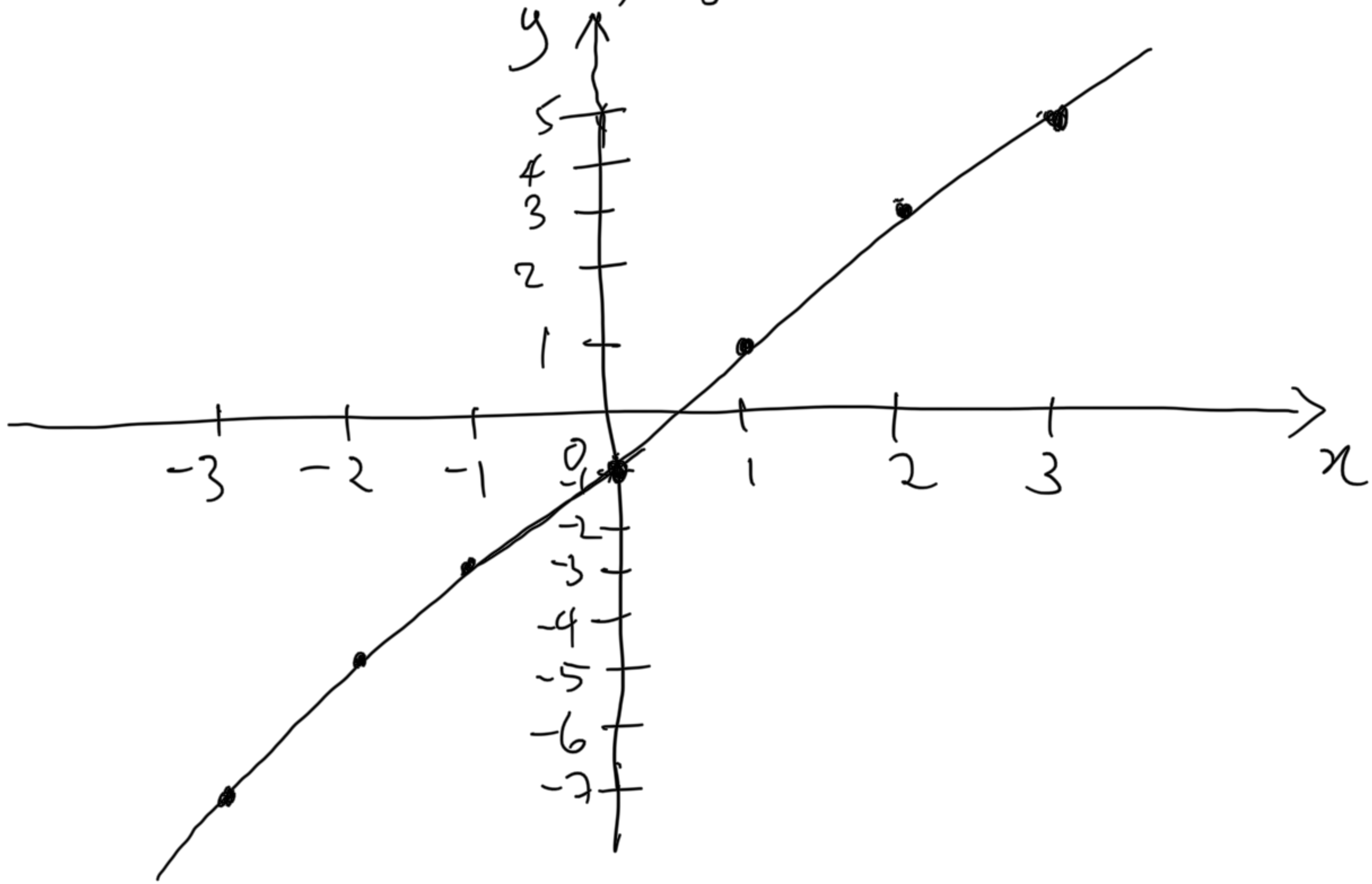
x	-3	-2	-1	0	1	2	3

output → $y = 2x-1$ | -7 | -5 | -3 | -1 | 1 | 3 | 5

when $x = -3$, $y = 2(-3) - 1 = -7$

$x = -2$, $y = 2(-2) - 1 = -5$

$x = -1$, $y = 2(-1) - 1 = -3$



Q. Plot the graph of $y = x^2 - 3x + 2$ for

$$\boxed{-2 \le x \le 3}$$

ax^2+bx+c

$a<0$ $a>0$

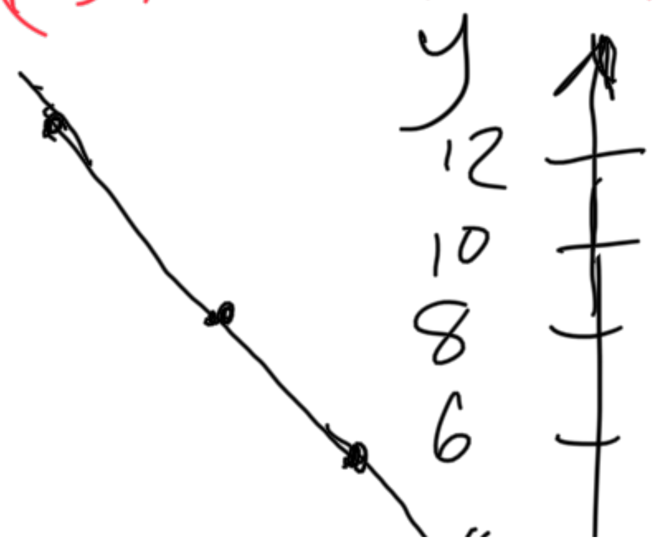
x	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2	2.5	3
$y = x^2 - 3x + 2$	12	8.75	6	3.75	2		0		0		2

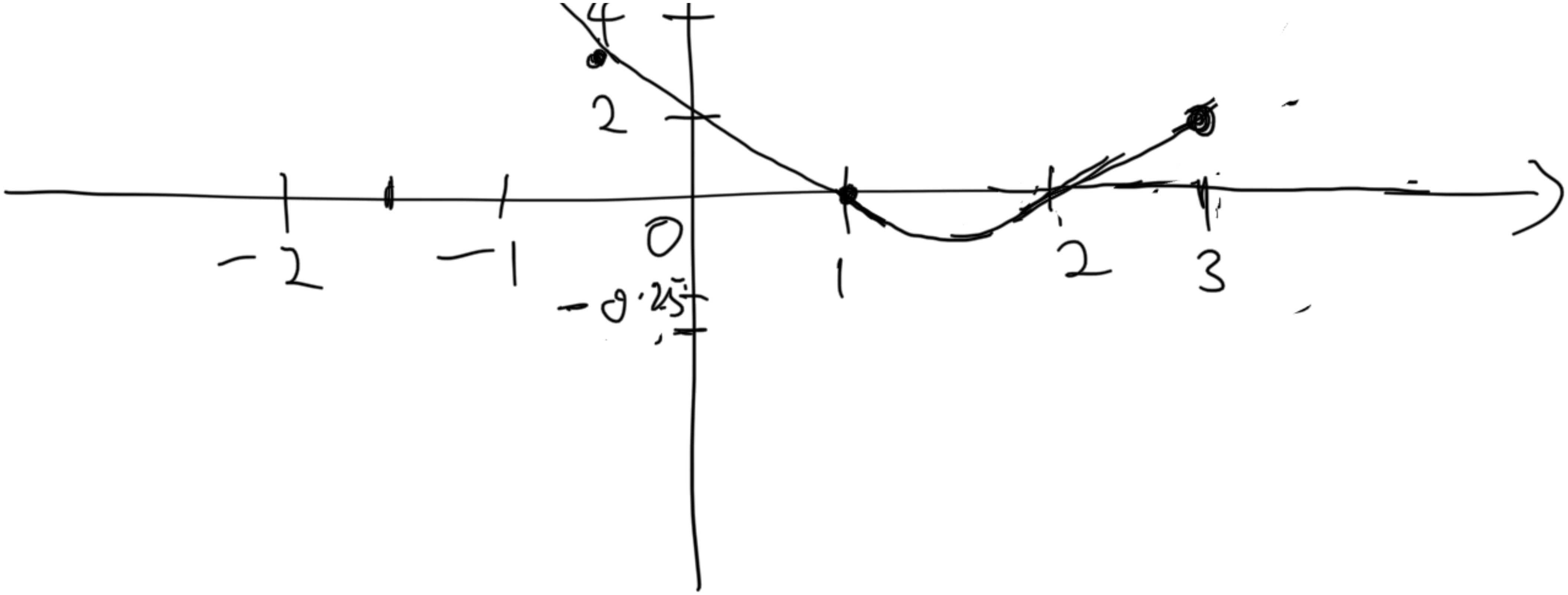
$x = 2$

$4 - 6 + 2 = 0$

$9 - 9 + 2 = 2$

$x = 1.5 \Rightarrow (1.5)^2 - 3(1.5) + 2 = -\frac{1}{4} = -0.25$





Domain of a function

Range of a function

<u>Example</u>

let $f(x) = 2x - 1$, for $-3 \le x \le 3$

then, domain of f is the values x can take:

$[-3, 3]$

includes -3 includes $+3$

$$[-3, 3] = \{x \in \mathbb{R} : -3 \leq x \leq 3\}.$$

$$-2 < x \leq 5 \implies (-2, 5]$$

-2 not included

5 included

$$(-2, 5] = \{x \in \mathbb{R} : -2 < x \leq 5\}.$$

$$-4 < x < 5 = (-4, 5) = \{x \in \mathbb{R} : -4 < x < 5\}$$

-4 not included

5 not included

Range of a function

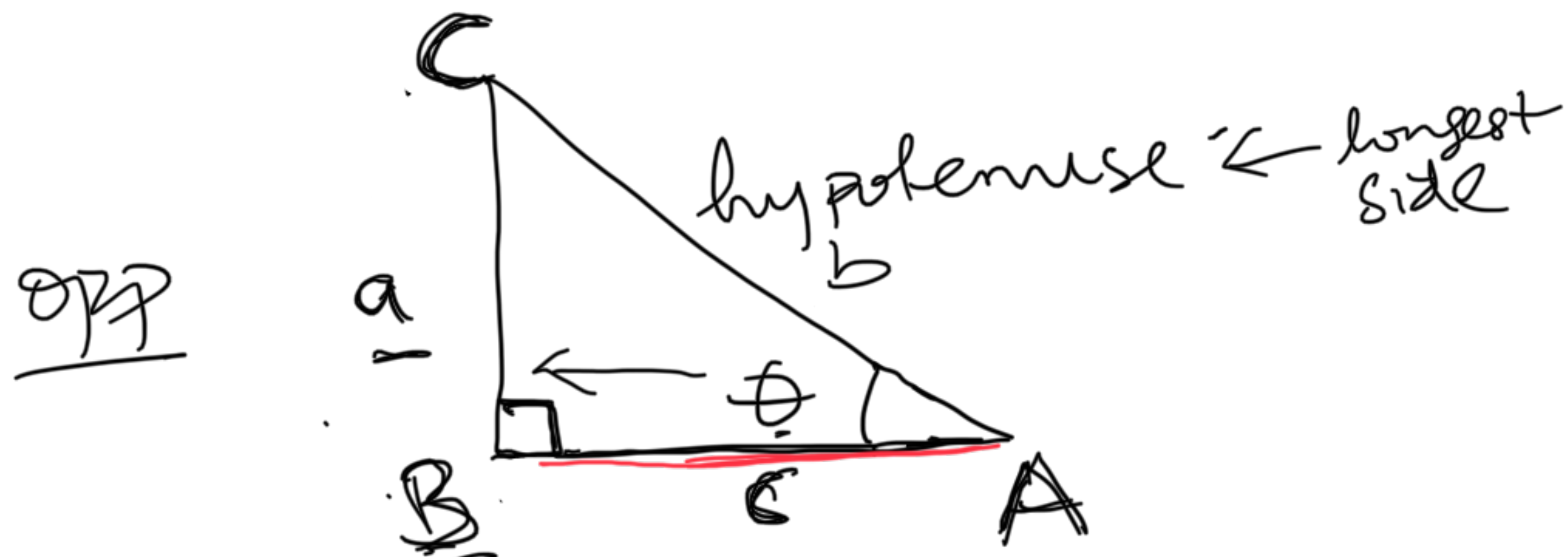
e.g.) $f(x) = 2x - 1, \quad -3 \leq x \leq 3.$

Range of f : Set of values of our output.

$$\text{Range of } f = \{ y : y = f(x) \}$$
$$= \{ -7, -5, -3, -1, 1, 3, 5 \}.$$

Trigonometry

Consider the right angled triangle ABC with right angle at B.



→ $\boxed{\text{Pythagoras' theorem: } b^2 = a^2 + c^2}$

Defns

Sine θ, $\sin\theta = \dfrac{\text{opp}}{\text{hyp}} = \dfrac{a}{b}$

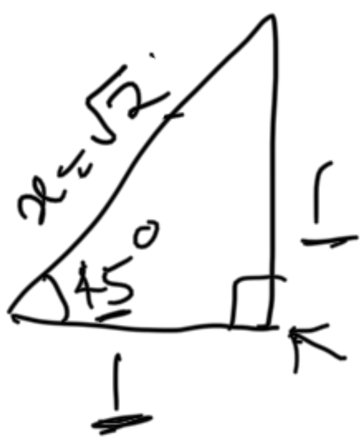
Cosine θ, $\cos\theta = \dfrac{\text{adj}}{\text{hyp}} = \dfrac{c}{b}$

tangent θ, $\tan\theta = \dfrac{\text{opp}}{\text{adj}} = \dfrac{a}{c}$

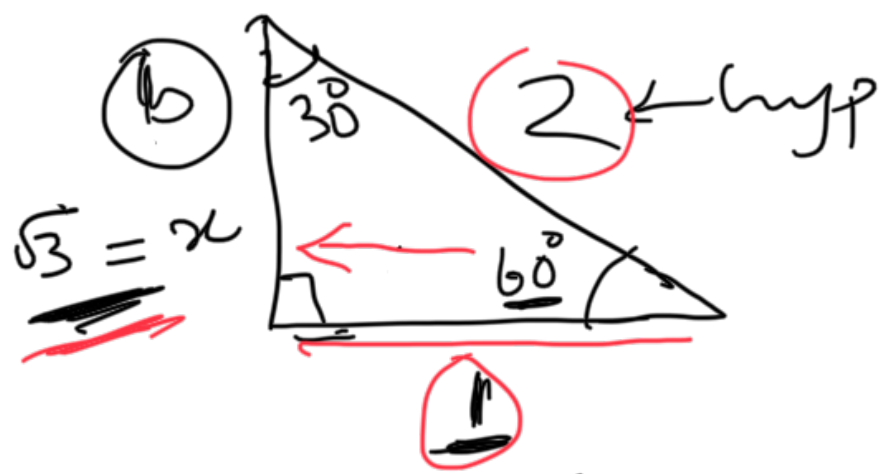
SOH CAH TOA

Q:

(a)



(b)



Using the triangles above, write down

[illegible]

expressions for

a) $\sin 45^\circ$, $\cos 45^\circ$ and $\tan 45^\circ$

b) $\sin 30^\circ$, $\cos 30^\circ$ and $\tan 30^\circ$

c) $\sin 60^\circ$, $\cos 60^\circ$ and $\tan 60^\circ$

Soln : For a)

$$x^2 = 1^2 + 1^2$$
$$x^2 = 2$$
$$x = \sqrt{2}$$

For b)

$$2^2 = x^2 + 1^2$$
$$x^2 = 2^2 - 1^2$$
$$x^2 = 4 - 1$$
$$x^2 = 3 \Rightarrow x = \sqrt{3}$$

a) $\rightarrow \sin 45^\circ = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

rationalised

$\rightarrow \cos 45^\circ = \frac{adj}{hyp} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\Rightarrow \sin 45^\circ = \cos 45^\circ = \frac{\sqrt{2}}{2}$

$\tan 45^\circ = \frac{opp}{adj} = \frac{1}{1} = 1$

$\sin 30^\circ = \frac{opp}{hyp} = \frac{1}{2}$

$\cos 30^\circ = \frac{adj}{hyp} = \frac{\sqrt{3}}{2}$

$\tan 30^\circ = \frac{opp}{adj} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\sin 60^\circ = \dfrac{opp}{hyp} = \dfrac{\sqrt{3}}{2}$

$\cos 60^\circ = \dfrac{adj}{hyp} = \dfrac{1}{2}$

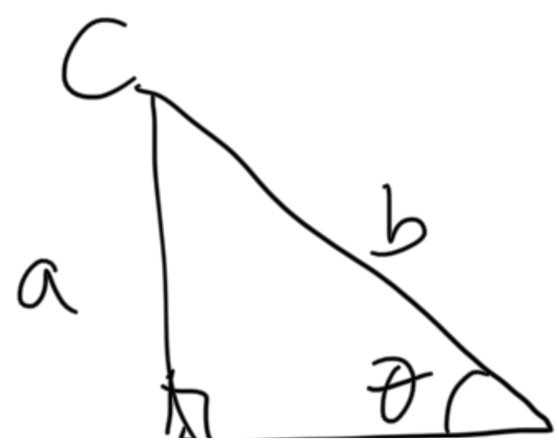
$\tan 60^\circ = \dfrac{opp}{adj} = \dfrac{\sqrt{3}}{1} = \underline{\underline{\sqrt{3}}}$

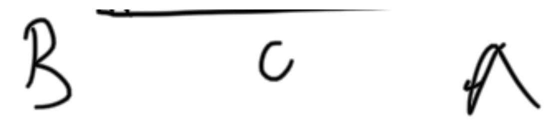
$\sin 30^\circ = \cos 60^\circ = 1/2$

$\sin 60^\circ = \cos 30^\circ = \sqrt{3}/2$

Q Prove $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$.

<u>Soln</u>





$$\sin\theta = \frac{a}{b}, \quad \cos\theta = \frac{c}{b}$$

$$\tan\theta = \frac{a}{c}$$

$$\frac{\sin\theta}{\cos\theta} = a/b \div c/b$$

$$= \frac{a}{\cancel{b}_1} \times \frac{\cancel{b}^1}{c} = \frac{a}{c} = \tan\theta$$

$$\Rightarrow \tan\theta = \frac{\sin\theta}{\cos\theta}.$$

Q. Find angle θ in the following right angle triangle.

a)

b)

<u>Soln</u>

a) $\tan\theta = \frac{1}{2}$

$\theta = \tan^{-1}(1/2)$

$= 26.57 \ (2dp)$

b) $\cos\theta = \frac{3}{7}$

$\theta = \cos^{-1}(3/7)$

$= 64.62 \ (2dp).$