Indices

Laws of Indices

1st law: $a^m \times a^n = a^{m+n}$

$\underset{\text{base}}{a}$, $m \leftarrow$ index or power.

e.g. $3^2 \times 3^3 = 3^{2+3} = 3^5$

Q. Simplify $a^4 b^5 b^2 a^3$

Solution:

$$a^4 b^5 b^2 a^3 = \underbrace{a^4 \times a^3} \times \underbrace{b^5 \times b^2}$$

$$= a^{4+3} \times b^{5+2}$$

$$= \underline{\underline{a^7 b^7}}$$

2nd law of indices

$$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$$

$a^{\cdot\cdot}$

e.g. $3^4 \div 3^2 = 3^{4-2} = \underline{3^2}$

Q Simplify $\dfrac{a^7}{a^5}$.

Soln: $\dfrac{a^7}{a^5} = a^{7-5} = a^2$

$$3^4 \div 3^2 = \frac{3^4}{3^2} = \frac{3 \times 3 \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3}} = \underline{3^2} = 9$$

Q simplify $\dfrac{x^4}{x^4}$

Soln: $1 = \dfrac{x^4}{x^4} = x^{4-4} = x^0$

$$\Rightarrow x = 1$$

In general, $a^0 = 1$ for all $a \in \mathbb{R}$.

$$\sqrt{2}^0 = 1, \quad 0.5^0 = 1, \quad 16^0 = 1$$

3rd law : $(a^m)^n = a^{mn}$

e.g $(3^2)^3 = 3^{2\times3} = 3^6$ ✓

$$\parallel$$

Check: $3^2 \times 3^2 \times 3^2$ expanded form.

$= 3^6$ ✓

Q Simplify $(x^4)^5$

Solution:

$$(x^4)^5 = x^{4\times5} = x^{20}$$

Q. Simplify $(2a^2)^4$

Soln:

1st method: $(2a^2)^4 = 2a^2 \times 2a^2 \times 2a^2 \times 2a^2$

$$= 2^4 \times (a^2)^4$$

$$= 16a^8$$

2nd method:

$$(2a^2)^4 = 2^4 \times (a^2)^4$$

$$= \underline{\underline{16a^8}}$$

generalise this to

$$(a^m b^n)^k = (a^m)^k (b^n)^k = a^{mk} b^{nk}$$

Q. Simplify $\dfrac{(x^3)^3}{x^2}$

Solution:

$(x^3)^3 \quad x^{3\times3} \quad x^9 \quad x^{9-2} \quad x^7$

$$\frac{(x\ )}{x^2} = \frac{x}{x^2} = \frac{x}{x^2} = x = \underline{\underline{x}}$$

Q Simplify $(t^4)^2(t^3)^2$

Solution:

$$(t^4)^2(t^3)^2 = t^{4\times 2} \times t^{3\times 2}$$

$$= t^8 \times t^6$$

$$= t^{8+6} = t^{14}$$

Negative powers

$$a^{-m} = \frac{1}{a^m} \qquad \text{if } m = 1$$

Note: $a^1 = a$

$x^1 = x$ and so on

$$a^{-1} = \frac{1}{a}$$

e.g. $\frac{1}{2} = 2^{-1}$, $\frac{1}{4} = 4^{-1}$

Q Evaluate a) 2^{-5} b) $1/3^{-4}$

Solutions

a) $2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

b) $\frac{1}{3^{-4}} = 3^{-(-4)} = 3^4 = 81$

Q. Rewrite each of the following using only positive powers

a) 7^{-3} b) x^{-5} c) $\frac{1}{x^{-9}}$

Solutions:

a) $7^{-3} = \frac{1}{7^3}$ b) $x^{-5} = \frac{1}{x^5}$

c) $\frac{1}{x^{-a}} = x^{-(-a)} = x^{a}$

Q Show that $\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$.

Proof;

$$\left(\frac{a}{b}\right)^{m} = \frac{a^{m}}{b^{m}}$$

$$\left(\frac{a}{b}\right)^{-1} = \frac{a^{-1}}{b^{-1}} = \frac{\frac{1}{a}}{\frac{1}{b}} = \frac{1}{a} \times \frac{b}{1} = \frac{b}{a}$$

$$\text{OR} = a^{-1} \times \frac{1}{b^{-1}}$$

$$= \frac{1}{a} \times b^{-(-1)}$$

$$= \frac{1}{a} \times b = \frac{b}{a}$$

$$\left(\frac{a}{b}\right)^{-1} = \frac{b}{a}$$

Fractional Powers

4th law: $\boxed{x^{m/n} = \sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m}$

if $m=1$, then

$$x^{1/n} = \sqrt[n]{x}$$

e.g. $m=1$, $n=2$

$$x^{1/2} = \sqrt{x}$$

$$x^{1/3} = \sqrt[3]{x}$$

Q. Evaluate a) $81^{1/2}$ b) $27^{1/3}$ c) $81^{-1/2}$

Solutions

a) $81^{1/2} = \sqrt{81} = 9$

b) $27^{1/3} = \sqrt[3]{27} = 3$

c) $81^{-1/2} = \frac{1}{81^{1/2}} = \frac{1}{9}$ ✓

Q. Write each of the following using a single index -

a) $\sqrt{x^3}$ b) $(\sqrt{x})^3$

Solution:

a) $\sqrt{x^3} = (x^3)^{1/2} = x^{3/2}$

b) $(\sqrt{x})^3 = (x^{1/2})^3 = x^{3/2}$

Q. Show that

a) $\left(\frac{a}{b}\right)^{-m/n} = \left(\frac{b}{a}\right)^{m/n}$

b) $\left(\frac{a}{b}\right)^{m/n} = \frac{a^{m/n}}{}$

$b^{m/n}$

Solutions:

a) $\left(\frac{a}{b}\right)^{-m/n} = \left(\left(\frac{a}{b}\right)^{-1}\right)^{m/n}$

$= \left(\frac{b}{a}\right)^{m/n}$ ✓

b) $\left(\frac{a}{b}\right)^{m/n} = (ab^{-1})^{m/n} = a^{m/n} \times (b^{-1})^{m/n}$

$= a^{m/n} \times b^{-m/n}$

$\frac{a}{b} = a \times \frac{1}{b} = ab^{-1}$

$= a^{m/n} \times \frac{1}{b^{m/n}}$

$= \frac{a^{m/n}}{b^{m/n}}$

Simplifying Algebraic Expressions

Q: Simplify: $3x + 7x - 2x$

Q Simplify $3x + 7x - 2x$

Soln:

$$3x + 7x - 2x = 8x$$

Q Simplify $3x + 2y$.

$3x$ and $2y$ are not like terms so we can't simplify further.

Q. Simplify $x + 7x - x^2$

Soln:

$$x + 7x - x^2 = 8x - x^2$$

so, we can't simplify further as $8x$ and x^2 are not like terms.

Q. Simplify $ab + a^2 - 7b^2 + 9ab + 8b^2$

Soln:

$$ab + 9ab + 8b^2 - 7b^2 + a^2$$

$= \underline{\underline{10ab + b^2 + a^2}}$

Q. Simplify a) $xy + yx$

b) $7x^2 - 11x^3 + 14x^2 + y^3$

Soln:

a) $xy = yx$ so, $xy + yx = \underline{\underline{2xy}}$

Note: $xy = yx$

b) $\underline{7x^2} - 11x^3 + \underline{14x^2} + y^3$

$= 7x^2 + 14x^2 - 11x^3 + y^3$

$= \underline{\underline{21x^2 - 11x^3 + y^3}}$

Q Expand a) $a(b+c)$ b) $(a+b)(c-d)$

c) $6(2x+4)$

Solutions:

a) $a(b+c) = a\times b + a\times c$
$= ab + ac$

b) $(a+b)(c-d) = a(c-d) + b(c-d)$
$= ac - ad + bc - bd$

c) $6(2x+4) = 12x + 24$

Factorisation

Q. Factorise a) $8x - x^2$
b) $3x + 12$

Solutions

a) $8x - x^2 = x(8-x)$

b) $3x + 12 = 3(x+4)$

4 Factorise a) $8x^2 - 12x$
b) $5x^2 - 15x^3$
c) $6x + 3x^2 + 9xy$

Solutions:

a) $8x^2 - 12x = 4x(2x - 3)$

$$\frac{{}^{2}\cancel{8}x^{\cancel{2}}}{\cancel{4}\cancel{x}_{1}}, \quad \frac{{}^{3}\cancel{12}\cancel{x}}{\cancel{4}\cancel{x}_{1}}$$

b) $5x^2 - 15x^3 = 5x^2(1 - 3x)$

$$\frac{{}^{3}\cancel{15}x^{\cancel{3}\,1}}{\cancel{5}\cancel{x^2}_{1}}$$

c) $6x + 3x^2 + 9xy = 3x(2 + x + 3y)$

$$\frac{{}^{2}\cancel{6}\cancel{x}}{\cancel{3}\cancel{x}} \quad \frac{\cancel{3}x^{\cancel{2}}}{\cancel{3}\cancel{x}} \quad \frac{{}^{3}\cancel{9}\cancel{x}y}{\cancel{3}\cancel{x}}$$

Factorising Quadratic expressions

A equation of the form

$ax^2+bx+c=0$ is called a quadratic equation.

the highest power of x is 2.

e.g $2x^2+1$,

$2x^2+x+1$, are quadratic expressions

Factorise a) $x^2+8x+12$
b) $x^2+10x+25$
c) x^2-121
d) x^2-5x+6
e) $2x^2-200$

Solutions

a) $x^2+8x+12$ factors of 12

a) $x^2+8x+12$

factors of 12

$= (x+6)(x+2)$

1	12	$=13$ ✗
$+6$	$+2$	$=8$ ✓
3	4	$=7$ ✗

b) $x^2+10x+25$

$= (x+5)(x+5)$

factors of 25

1	25	✗
5	5	$=10$ ✓

d) x^2-5x+6

factors of 6

$-3, -2 = -5$ ✓

$= (x-3)(x-2)$

c) x^2-121 ← a difference of two squares.

$= x^2-11^2$

$= (x+11)(x-11)$

$\boxed{a^2-b^2 = (a+b)(a-b)}$

e) $2x^2-200 = 2(x^2-100)$

$= 2(x^2-10^2)$

$$= 2(x+10)(x-10)$$

Factorise a) $2x^2+11x+12$

b) $4x^2+6x+2$

c) $6x^2+7x-3$

<u>Solution</u>:

a) $2x^2+11x+12$

Scissors method:

$$\underline{2x^2}+\underline{11x}+\underline{12}$$

factors of 12

3	4
6	2

$$\begin{array}{ccc} \underline{2}x & & +3 \\ x & & +4 \end{array}$$

$$8x+3x=11x$$

$$2x^2+11x+12 = \underline{(2x+3)(x+4)}$$

$2x^2 + 11x + 12 =$

$2x^2 + 11x + 12$

$2 \times 12 = 24$

find factors of 24 that add together to give 11

factors of 24

$12, 2$ ✗

$8, 3$ ✓

$6, 4$ ✗

$$2x^2 + 11x + 12 = \underbrace{2x^2 + 8x} + \underbrace{3x + 12}$$

$$= 2x(x+4) + 3(x+4)$$

$$= (x+4)(2x+3)$$ ✓

b) $4x^2 + 6x + 2$

$$\begin{array}{ccc} 2x & & +2 \\ & \times & \\ 2x & & +1 \end{array}$$

$$2x + 4x = 6x$$

$$= (2x+2)(2x+1)$$

$$\begin{array}{ccc} 4x & & +2 \\ & \times & \\ x & & +1 \end{array}$$

$$4x + 2x = 6x$$

$$(4x+2)(x+1)$$

$$2(2x+1)(x+1)$$

c) $6x^2 + 7x - 3$

$$\begin{array}{ccc} 3x & & -1 \\ & \times & \\ 2x & & +3 \end{array}$$

$$9x - 2x = 7x$$

$$(3x-1)(2x+3)$$

Simplifying Algebraic fractions

Q. Simplify

a) $18x^2$ b) 5 c) $5x$

$\frac{\ldots}{6x}$ $\frac{\ldots}{25 + 15x}$ ... $\frac{\ldots}{25x + 10y}$

d) $\frac{2(x-1)}{(x+3)(x-1)}$ e) $\frac{x+2}{x^2+3x+2}$

f) $\frac{3x+xy}{x^2+5x}$

Solutions:

a) $\frac{\cancel{18}^{3} \cancel{x^2}^{x}}{\cancel{6x}_{1}} = \frac{3x}{1} = \underline{\underline{3x}}$

b) $\frac{5}{\underline{25+15x}} = \frac{\cancel{5}^{1}}{\cancel{5}_{1}(5+3x)} = \frac{1}{\underline{\underline{5+3x}}}$

c) $\frac{5x}{25x+10y} = \frac{{}^{1}\cancel{5}x}{\cancel{5}_{1}(5x+2y)} = \frac{x}{\underline{\underline{5x+2y}}}$

d) $\dfrac{2\cancel{(x-1)}^{1}}{(x+3)\cancel{(x-1)}} = \dfrac{2}{x+3}$

e) $\dfrac{x+2}{x^2+3x+2} = \dfrac{\cancel{x+2}^{1}}{(x+1)\cancel{(x+2)}_{1}} = \dfrac{1}{x+1}$

$$x^2+3x+2$$
$$=(x+1)(x+2)$$

f). $\dfrac{3x+xy}{x^2+5x} = \dfrac{\cancel{x}^{1}(3+y)}{\cancel{x}_{1}(x+5)} = \dfrac{3+y}{x+5}$

Solving Equations

linear equation: $ax+b=c$

is a linear equation

a, b and c are numbers

x is what we want

Q Solve $4x+8=12$ to find :

Soln :

$$4x+8=12$$

subtract 8 from both sides

$$4x+\cancel{8}-\cancel{8}=12-8$$

$$4x=4$$

divide both sides by 4

$$\frac{\cancel{4}x}{\cancel{4}}=\frac{4}{4}$$

$$x=1$$

$$4x+8=12$$

$$4x=12-8$$

$$4x=4$$

$$x = \frac{4}{4} = 1$$

Q

$$5x + 17 = 4x - 3$$

$$5x - 4x + 17 = -3$$

$$x + 17 = -3$$

$$x = -3 - 17 = -20$$

$$x = -20$$