Question 1 Percentages, Fractions, Order of Operations, Rounding, Standard forms.

a) Find

i. $42\frac{1}{2}\%$ as a decimal

ii. $42\frac{1}{2}\%$ as a fraction in simplest form

iii. $42\frac{1}{2}\%$ of 600

[3]

Solutions

i. $42\frac{1}{2}\% = 0.452$
ii. $42\frac{1}{2}\% = \frac{425}{1000} = \frac{17}{40}$
iii. $42\frac{1}{2}\% \; of \; 60 = \frac{42.5}{100} \times 60 = \frac{51}{2} = 25.5$

b) Round the following numbers

i. 0.0041993 to 3 significant figures

ii. 23.4598 to 3 decimal places

[2]

Solutions

i. $0.0041993 \approx 0.00420 \; (3 \; s.f.)$
ii. $23.4598 \approx 23.450 \; (3 \; d.p.)$

c) Subtract 4.2×10^{-1} from 2.4×10^{2} and give your answer in standard form. You must show your working

[2]

Solution

$2.4 \times 10^2 = 240, \quad 4.2 \times 10^{-1} = 0.42$

Hence,

$$240 - 0.42 = 239.58 = 2.3958 \times 10^2$$

d) Insert brackets to make the following equation true

$$6 - 4^2 + 2 \times 5 = 30$$

[1]

Solution

$$((6 - 4)^2 + 2) \times 5 = 30$$

e) A number is 1550 correct to 3 significant figures. Find

i. The smallest possible number

ii. The largest possible number

[2]

Solutions

i. The smallest possible number is 1545
ii. The largest possible number is 1555

Question 2 Algebraic expressions, Substitution and Ratio

a) Expand and simplify the following

i. $(x - y) - 8(x + y)$

ii. $(x - y)^2 - 8(x + y)$

iii. Evaluate the expression $(x - y)^3(x - 2y)$ when $x = 2$ and $y = -1$

[3]

Solutions

i. $(x - y) - 8(x + y) = x - y - 8x - 8y = -7x - 9y$
ii. $(x - y)^2 - 8(x + y) = x^2 - 2xy + y^2 - 8x - 8y$
iii. $(x - y)^3(x - 2y) = \big(2 - (-1)\big)^3\big(2 - 2(-1)\big) = 27 \times 4 = 108$

b) Simplify the following expressions, giving answers in their simplest form

i. $\frac{(4x+5)(x+3)}{(x+3)(x+5)}$

ii. $\frac{x^2+5x+6}{x^2-4x-12}$

[3]

Solutions

i. $\frac{(4x+5)(x+3)}{(x+3)(x+5)} = \frac{4x+5}{x+5}$

ii. $\frac{x^2+5x+6}{x^2-4x-12} = \frac{(x-3)(x+2)}{(x-6)(x+2)} = \frac{x-3}{x-6}$

c) Simplify the following expressions

i. $\frac{4x}{3} \div \frac{2}{9}$

ii. $(a^2b^5) \times (a^{-3}b^{-2}c)$

[2]

Solutions

i. $\frac{4x}{3} \div \frac{2}{9} = \frac{4x}{3} \times \frac{9}{2} = 2x \times 3 = 6x$

ii. $(a^2b^5) \times (a^{-3}b^{-2}c) = a^{-1}b^3c$

d) Two brothers, Mike and Vince, share a sum of money in the ratio 3: 8. Vince gets £40 more than Mike. Calculate how much the brothers share. You must show your working.

[2]

Solution

i. First Method:
Let x be Mike's share. Then Vince's share is $x + 40$.
Thus,

$$\frac{x}{x+40} = \frac{3}{8}$$
$$8x = 3x + 120$$
$$5x = 120$$
$$x = 24$$

So, Vince's share is £64 and Mike's share is £24. The amount the brothers share is £88.

ii. Second Method:

The difference between the numbers in the ratio is

$$8 - 3 = 5$$

And since Vince earns £40 more than Mike, we have

$$40 \div 5 = 8$$

So, Mike's share is $3 \times 8 = £24$ and Vince's share is $8 \times 8 = £64$. The amount the brothers share is £88.

Question 3 Factors and Multiples

a) Express the numbers 120, 150 and 360 as a product of primes and find their lowest common multiple.

[2]

Solutions:

$$120 = 2^3 \times 3 \times 5$$
$$150 = 2 \times 3 \times 5^2$$
$$360 = 2^3 \times 3^2 \times 5$$

$$LCM = 2^3 \times 3^2 \times 5^2 = 1800$$

Question 4 Linear, Simultaneous and Quadratic Equations

a) Solve the following equations. You must show full working.

i. $4x - 6 = -5x + 2$

ii. $3(2x - 8) = -5(2x + 8)$

[2]

Solutions

i. Now,

$$4x - 6 = -5x + 2 \Longrightarrow 9x = 8 \Longrightarrow x = \frac{8}{9}$$

ii. $3(2x - 8) = -5(2x + 8) \Longrightarrow 6x - 24 = -10x - 40 \Longrightarrow 16x = -16 \Longrightarrow x = -1.$

b) Solve the following simultaneous equations

$$4x - 3y = 11$$
$$10x + 2y = -1$$

[2]

Solutions

$$4x - 3y = 11. \quad (1)$$
$$10x + 2y = -1. \quad (2)$$

Multiply equation (1) *by* 2 and equation (2) *by* 3

$$8x - 6y = 22. \quad (3)$$
$$30x + 6y = -3. \quad (4)$$

Adding (3) and (4), we get

$$38x = 19 \implies x = \frac{1}{2}$$

Substituting this in (1), we have

$$2 - 3y = 11 \implies 3y = -9 \implies y = -3$$

So, $x = \frac{1}{2}$ and $y = -3$.

c) Factorise the following quadratic expressions.

i. $x^2 - 6x + 9$

ii. $4x^2 + 3x$

iii. $4x^2 - 5x - 6$

[3]

Solutions

i. $x^2 - 6x + 9 = (x - 3)(x - 3)$

ii. $4x^2 + 3x = x(4x + 3)$

iii. $4x^2 - 5x - 6 = (4x + 3)(x - 2)$

d) Hence or otherwise, solve the following quadratic equations

i. $x^2 - 6x + 9 = 0$

ii. $4x^2 + 3x = 0$

iii. $4x^2 - 5x - 6 = 0$

[3]

Solutions

i. $x^2 - 6x + 9 = (x - 3)(x - 3) = 0 \implies x = 3 \; twice$

ii. $4x^2 + 3x = x(4x + 3) = 0 \implies x = 0 \; or \; x = -\frac{3}{4}$

iii. $4x^2 - 5x - 6 = (4x + 3)(x - 2) = 0 \implies x = -\frac{3}{4}$ *or* $x = 2$.

Question 5 Indices

a) Given that $2^3 = 8$, express 8^{2x+4} in the form 2^y where y is an expression in terms of x.

[1]

Solution

$$8^{2x+4} = 2^{3(2x+4)} = 2^{6x+12}$$

b)

i. Evaluate $9^2 \div 9^5$

ii. Evaluate $125^{-\frac{2}{3}}$

iii. Given that $27\sqrt{3} = 3^a$, find the value of a.

iv. Simplify $(16x^{12})^{\frac{3}{4}}$

[4]

Solutions

i. $9^2 \div 9^5 = 9^{-3}$

ii. We have,

$$125^{-\frac{2}{3}} = \frac{1}{5^2} = \frac{1}{25}$$

iii. Now,

$$3^{\frac{7}{2}} = 3^a \implies a = \frac{7}{2}$$

iv. $(16x^{12})^{\frac{3}{4}} = 2^3 \times x^9 = 8x^9$

Question 6 Sets and Number bases

a) Suppose A, B and C are subsets of a universal set $\mathcal{E}$ as follows:

$\mathcal{E} = \{x: x \text{ is an integer and } -3 \leq x \leq 15\}$

$A = \{x:$ x is a multiple of 3$\}$

$B = \{x: x > 5\}$

$C = \{x: x \text{ is an even number}\}$

List the following sets

i. $A \cap B$

ii. $A \cup \bar{C}$

iii. $(A \cap B) \cup \bar{C}$

[4]

Solutions

$$\mathcal{E} = \{-3, -2, -1, 0, 1, 2, \dots, 15\}$$

$$A = \{-3, 0, 3, 6, 9, 12, 15\}$$

$$B = \{6, 7, 8, 9, 10, 11, 12, 13, 14, 15\}$$

$$C = \{-2, 0, 2, 4, 6, 8, 10, 12, 14\}$$

$$\bar{C} = \{-3, -1, 1, 3, 5, 7, 9, 11, 13, 15\}$$

i. $A \cap B = \{6, 9, 12, 15\}$

ii. $A \cup \bar{C} = \{-3, -1, 0, 1, 3, 5, 6, 7, 9, 11, 12, 13, 15\}$

iii. $(A \cap B) \cup \bar{C} = \{-3, -1, 1, 3, 5, 6, 7, 9, 11, 12, 13, 15\}$

The shaded region is $\bar{X} \cap (\bar{Y} \cup Z)$. Award 2 marks for correct shading.

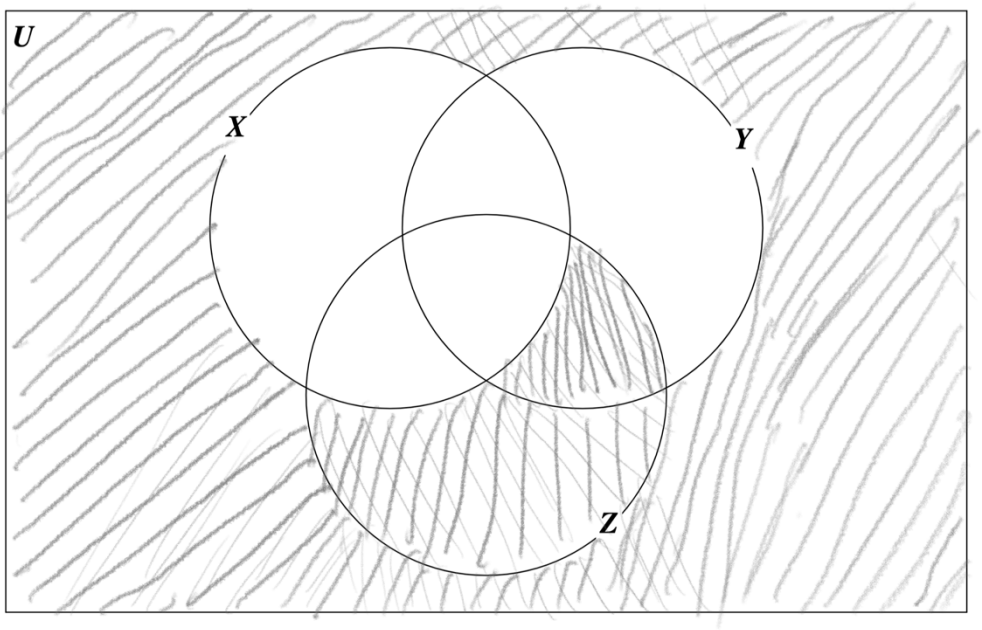
Award 1 mark for a good attempt at shading this region, e.g. $\bar{Y} \cup Z$ or $\bar{X} \cap \bar{Y}$ or $\bar{X} \cap \bar{Y} \cap \bar{Z}$

b) Draw and shade a Venn diagram to represent the following general set

$$\bar{X} \cap (\bar{Y} \cup Z)$$

[2]

The shaded region is $\bar{X} \cap (\bar{Y} \cup Z)$. Award 2 marks for correct shading.



c) Convert

i. $456B_{16}$ to decimal
ii. 653_{10} to base 7

[2]

Solutions

i. $456B_{16} = 4 \times 16^3 + 5 \times 16^2 + 6 \times 16 + B = 17771_{10}$

ii. 653_{10} to base 7

$$653_{10} \div 7 = 93\ r\ 2$$
$$93 \div 7 = 13\ r\ 2$$
$$13 \div 7 = 1\ r\ 6$$
$$1 \div 7 = 0\ r\ 1$$

So,

$$653_{10} = 1622_7$$

Question 7 Linear, Simultaneous and Quadratic Equations

a) Solve the following equations. You must show full working.

iii. $4x - 6 = -5x + 2$

iv. $3(2x - 8) = -5(2x + 8)$

[2]

b) Solve the following simultaneous equations

$$4x - 3y = 11$$
$$10x + 2y = -1$$

[2]

c) Factorise the following quadratic expressions.

iv. $x^2 - 6x + 9$

v. $4x^2 + 3x$

vi. $4x^2 - 5x - 6$

[3]

d) Hence or otherwise, solve the following quadratic equations

iv. $x^2 - 6x + 9 = 0$

v. $4x^2 + 3x = 0$

vi. $4x^2 - 5x - 6 = 0$

[3]

Question 8 Sequences and Series

a) Given the sequence $11, 7, 3, -1, \ldots$

i. Find an expression for the nth term of the sequence

ii. Find an expression for the sum of the first n terms of the sequence

iii. Find S_{10}, the sum of the first 10 terms

[3]

b) Given the following sum $\sum_{i=0}^{3}(-2)^i(3i - 1)$

i. Write out the sum in full

ii. Find the value of the sum

[2]

c) Write the following sum using sigma notation

$$(2 \times 3)^2 + (3 \times 4)^2 + (4 \times 5)^2 + \cdots + (20 \times 21)^2$$

[2]

d) The second term of a geometric progression is 24. The sum to infinity of this progression is 150.

i. Write down two equations in a and r where a is the first term and r is the common ratio.

ii. Solve your equations to find the possible values of a and r.

[3]

Question 9 Graphs and functions

a)

i. Plot the graph of $y = 4x + 1$ for $-5 \leq x \leq 5$ and show where it cuts the axes

ii. On the same grid, plot the graph of $x^2 - 4$ for $-5 \leq x \leq 5$

iii. From your graph or otherwise, state the coordinates of the points of intersection of the two functions.

[4]

b) Given the following functions $f(x) = 4x + 1$ and $g(x) = \frac{2}{x+1}, x \neq 1$

i. Evaluate $f(3)$

ii. Evaluate $g(f(4))$

iii. Find an expression for $f^{-1}(x)$

iv. Find an expression for $f^{-1}(\frac{2}{x+1})$

[4]

Question 10 Trigonometry

a) Triangle ABC has angle $A = 90°$ and sides $a = 13\ cm$ and $b = 5\ cm$

i. Find the length of side c.

ii. Find angles B and C, give your answers to the nearest degree.

[3]

Solutions

i. By Pythagoras' theorem,

$$c = \sqrt{13^2 - 5^2} = \sqrt{144} = 12$$

ii. For angle B

$$\sin B = \frac{5}{13} \Rightarrow B = \sin^{-1}\left(\frac{5}{13}\right) = 23^{\circ}\ (nearest\ degree)$$

So, angle $C = 90 - 23 = 67^{\circ}\ (to\ the\ nearest\ degree)$

c) Triangle PQR has angle $Q = 35^{\circ}$ and $R = 105^{\circ}$ and side $p = 120\ cm$. Find the length of side r. Give your answer to three significant figures.

[2]

Solutions

We have

$$\frac{r}{\sin R} = \frac{p}{\sin P}$$

$$\frac{r}{\sin 105} = \frac{120}{\sin 40}$$

$$r = 120 \times \frac{\sin 105}{\sin 40} = 180\ cm\ (3\ sig.fig.)$$