Lecture 1: Propositional Logic

Section 1.1

Propositions

- A *proposition* is a declarative sentence that is either true or false.
- Examples of propositions:
  a) The Moon is made of green cheese.
  b) Trenton is the capital of New Jersey.
  c) Toronto is the capital of Canada.
  d) $1 + 0 = 1$
  e) $0 + 0 = 2$
- Examples that are not propositions.
  a) Sit down!
  b) What time is it?
  c) $x + 1 = 2$
  d) $x + y = z$

Propositional Logic

- Propositions that cannot be expressed in terms of simpler propositions are called **atomic propositions** .

- Constructing Propositions
  - Propositional Variables: *p*, *q*, *r*, *s*, …
  - The proposition that is always true is denoted by **T** and the proposition that is always false is denoted by **F**.

Propositional Logic

- Constructing Propositions
  - Propositional Variables: $p, q, r, s, \ldots$
  - The proposition that is always true is denoted by **T** and the proposition that is always false is denoted by **F**.
  - Compound Propositions: constructed from logical connectives and other propositions
    - Negation $\neg$
    - Conjunction $\wedge$
    - Disjunction $\vee$
    - Implication $\rightarrow$
    - Biconditional $\leftrightarrow$

Compound Propositions: Negation

Definition 1 Let p be a proposition. The *negation of* p, denoted by $\neg p$ (also denoted by $\overline{p}$), is the statement

"It is not the case that p."

The proposition $\neg p$ is read "not p." The truth value of the negation of p, $\neg p$, is the opposite of the truth value of p.

Remark: The notation for the negation operator is not standardized. Although $\neg p$ and $\overline{p}$ are the most common notations used in mathematics to express the negation of p, other notations you might see are $\sim p$, $-p$, p', Np, and $!p$.

Compound Propositions: Negation

- The *negation* of a proposition p is denoted by $\neg p$ and has this truth table:

p	$\neg p$
T	F
F	T

- **Example** : If p denotes "The earth is round.", then $\neg p$ denotes "It is not the case that the earth is round," or more simply "The earth is not round."

Conjunction & Disjunction

Definition 2 Let p and q be propositions. The *conjunction* of p and q, denoted by $p \wedge q$, is the proposition "p and q." The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

Definition 3 Let p and q be propositions. The *disjunction* of p and q, denoted by $p \vee q$, is the proposition "p or q." The disjunction $p \vee q$ is false when both p and q are false and is true otherwise.

Conjunction

- The *conjunction* of propositions p and q is denoted by $p \wedge q$ and has this truth table:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

- **Example** : If p denotes "I am at home." and q denotes "It is raining." then $p \wedge q$ denotes "I am at home and it is raining."

Disjunction

- The *disjunction* of propositions p and q is denoted by $p \vee q$ and has this truth table:

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

- **Example** : If p denotes "I am at home." and q denotes "It is raining." then $p \vee q$ denotes "I am at home or it is raining."

The Connective Or in English

- In English "or" has two distinct meanings.
  - **"Inclusive Or"** - In the sentence "Students who have taken CSE230 or MAT120 may take this class," we assume that students need to have taken one of the prerequisites, but may have taken both. This is the meaning of disjunction. For $p \lor q$ to be true, either one or both of p and q must be true.
  - **"Exclusive Or"** - When reading the sentence "Soup or salad comes with this entrée," we do not expect to be able to get **both** soup and salad. This is the meaning of Exclusive Or (Xor). In $p \oplus q$, one of p and q must be true, but not both. The truth table for $\oplus$ is:

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F