COLLEGE
Tanta Engineering

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الجبر

المحاضرة الثانية - مبدأ الاستنتاج الرياضي

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Mathematical Induction principle (M I P)

* Sum. Notation: $\sum_{i=1}^{n} a_i = a_1 + a_2 + \dots + a_n$

$\Rightarrow \sum_{i=1}^{n} K = K + K + K + \dots = nK$

$\Rightarrow \sum_{i=m_0}^{n_0} a_i = 0 \quad ; m_0 > n_0$

A method to prove some thierums contains an integer say (n) as "n" is positev

Consider $C(n)$ $n \in N$ is $\ni$

① $C(1)$ is true (base)

(step) ② If $C(K)$, $K \in n$ is true implise $C(K+1)$ is also true

then $C(n)$ is true $\forall n \in N$

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Ex: use MIP to prove the following

1- $\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$

At $n=1$; L.H.S $= 1^2 = 1$

R.H.S $= \frac{1 \times 2 \times 3}{6} = 1$

$\therefore C(1)$ is true

Asume $C(K)$: $1^2 + 2^2 + \dots + K^2 = \frac{K \times K+1 \times 2K+1}{6}$

R.T.P: $C(K+1) = 1^2 + 2^2 + \dots + (K+1)^2 = \frac{(K+1)(K+2)(2K+3)}{6}$

L.H.S $= 1^2 + 2^2 + \dots + K^2 + (K+1)^2 = \frac{(K)(K+1)(2K+1)}{6} + (K+1)^2$

$= \frac{(K+1)(2K^2 + K + 6K + 6)}{6}$

$=$ R.H.S

$$C(n): \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}^n = \begin{bmatrix} \sin nx & -\sin nx \\ \sin nx & \cos nx \end{bmatrix}$$

$$C(1): L.H.S = \begin{pmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{pmatrix}, \quad R.H.S: \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}$$

Assume $C(K): \begin{bmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{bmatrix}^K = \begin{pmatrix} \cos Kx & -\sin Kx \\ \sin Kx & \cos Kx \end{pmatrix}$

$$C(K+1): \begin{bmatrix} \sin(K+1)x & -\sin(K+1)x \\ \sin(K+1)x & \cos(K+1)x \end{bmatrix}$$

$$L.H.S = \begin{pmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{pmatrix}^K \cdot \begin{pmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{pmatrix}^1$$

$$= \begin{pmatrix} \cos Kx & -\sin Kx \\ \sin Kx & \cos Kx \end{pmatrix} \cdot \begin{pmatrix} \cos x & -\sin x \\ \sin x & \cos x \end{pmatrix}$$

$$= \begin{pmatrix} \cos Kx \cdot \cos x - \sin Kx \cdot \sin x & -\sin x \cdot \cos Kx - \cos x \cdot \sin Kx \\ \sin Kx \cdot \cos x + \cos Kx \cdot \sin x & -\sin x \cdot \sin Kx + \cos x \cdot \cos Kx \end{pmatrix}$$

$$= \begin{pmatrix} \cos(K+1)x & -\sin(K+1)x \\ \sin(K+1)x & \cos(K+1)x \end{pmatrix} = R.H.S$$

Prove that $2^n > n^2 \quad \forall n \geq 5$

At $n = 5$: ~~C(5)~~

$L.H.S = 2^5 = 32$

$R.H.S = 5^2 = 25$

$\therefore C(5)$ is true

Assume that $2^K > K^2 \quad \forall K \in n$

R.T.P $2^{(K+1)} > (K+1)^2$

$L.H.S = 2^K \cdot 2 > 2K^2$

$2^{K+1} - (K+1)^2 > 2K^2 - (K+1)^2 = K^2 - 2K - 1$

$K = \frac{2 \pm \sqrt{4+4}}{2} = 1 \pm \sqrt{2}$

$2^{K+1} - (K+1)^2 > [K - 1 - \sqrt{2}][K - 1 + \sqrt{2}] > 0$

$\therefore 2^{K+1} > (K+1)^2$

Prove that $\frac{1}{n+1} + \frac{1}{n+2} \cdots\cdots + \frac{1}{2n} > \frac{13}{24}$ $: n > 1$

At $n = 2$

$L.H.S = \frac{1}{3} + \frac{1}{4} = \frac{7}{12} \times \frac{2}{2} = \frac{14}{24} > \frac{13}{24}$

At $n = K$

Asume that $S_K = \frac{1}{K+1} + \frac{1}{K+2} + \cdots + \frac{1}{2K} > \frac{13}{24}$, $K > 1$

At $n = K+1$

R.T.P: $\frac{1}{K+2} + \frac{1}{K+3} + \cdots + \frac{1}{2K+1} + \frac{1}{2K+2} > \frac{13}{24}$

$S_{K+1} - S_K = \frac{1}{2K+2} + \frac{1}{2K+1} - \frac{1}{K+1} = \frac{2K+1 + 2K+2 - 2(2K+2)}{(2K+2)(2K+1)} = \frac{1}{(2K+2)(2K+1)}$

$\therefore S_{K+1} > S_K$

$\therefore S_{K+1} > \frac{13}{24}$