

سنتر فيوتشر

Subject:..... رياضه ايرادي

Chapter:..... تابع نظريه ذات الحدين

Mob: 0112 3333 122

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Expand  $\frac{3x}{(x-2)(2x+1)}$  and find coefficient

$x^n$ , find Condition for Convergence

$$\frac{3x}{(x-2)(2x+1)} = \frac{A}{x-2} + \frac{B}{2x+1}$$

$$x=2 \quad A = \frac{6}{5}$$

$$x = -\frac{1}{2} \quad B = \frac{-3/2}{-5/2} = 3/5$$

$$f(x) = \frac{6/5}{x-2} + \frac{3/5}{2x+1}$$

$$= \frac{-3}{5} \left[ \frac{1}{1-x/2} \right] + \frac{3}{5} \left( \frac{1}{1+2x} \right)$$

$$= \frac{3}{5} \left[ \frac{1}{1+2x} - \frac{1}{1-x/2} \right]$$

شرط التقارب  $\rightarrow \frac{|x|}{2} < 1$  ,  $|2x| < 1$   
 $|x| < \frac{1}{2}$

$$f(x) = \frac{3}{5} \left[ (1-2x+(2x)^2 - \dots + (-2x)^n) - (1+\frac{x}{2} - \dots + (\frac{x}{2})^n) \right]$$

مع  $x^n$   $\odot$   $\frac{3}{5}(-2)^n - (\frac{1}{2})^n$

Expand

$$\frac{8x+3}{(x-4)^2(x+3)}$$

Find Coefficient  $x^0$ 

$$\frac{8x+3}{(x-4)^2(x+3)} = \frac{A}{x+3} + \frac{B}{x-4} + \frac{C}{(x-4)^2}$$

$$x=4 \quad C = \frac{35}{7} = 5$$

$$x=-3 \quad A = \frac{-21}{49} = -3/5$$

$$\therefore 8x+3 = A(x-4)^2 + B(x-4)(x+3) + C(x+3)$$

$$\begin{array}{l} x=0 \\ 3 = 16A - 12B + 3C \end{array}$$

$$3 = 16\left(\frac{-3}{5}\right) - 12B + 15$$

$$3 - \frac{27}{5} = -12B$$

$$B = 1/5$$

$$\therefore \frac{8x+3}{(x-4)^2(x+3)} = \frac{-3/5}{x+3} + \frac{1/5}{x-4} + \frac{5}{(x-4)^2}$$

(12)

C

Condition for Convergence

$$= -\frac{1}{5} \left[ \frac{1}{1+x/3} \right] - \frac{1}{20} \left( \frac{1}{1-x/4} \right) + \frac{5}{16} \left( \frac{1}{1-x/4} \right)^2$$

$$\left| \frac{x}{3} \right| < 1 \Rightarrow |x| < 3$$

$$\left| \frac{x}{4} \right| < 1 \Rightarrow x < 4$$

$\therefore$  Condition  $|x| < 3$

$$\begin{aligned} & -\frac{1}{5} \left[ 1 - \frac{x}{3} + \dots + (-1)^n \left( \frac{x}{3} \right)^n \right] \\ & -\frac{1}{20} \left[ 1 + \frac{x}{4} + \dots + \left( \frac{x}{4} \right)^n \right] \\ & + \frac{5}{16} \left[ 1 + 2 \frac{x}{4} + 3 \frac{x^2}{16} + \dots + (n+1) \left( \frac{x}{4} \right)^n \right] \end{aligned}$$

Coefficient  $x^n$

من  $x^n$  جملو

$$-\frac{1}{5} \left[ -\frac{1}{3} \right]^n - \frac{1}{20} \left( \frac{1}{4} \right)^n + \frac{5}{16} (n+1) \cdot \left( \frac{1}{4} \right)^n$$

(10)

(3)

Find coefficient  $x^2$  in the expansion

$$f(x) = \frac{5x + 1 + \frac{2}{x}}{(x+2)(x^2+1)}$$

$$\frac{5x^2 + x + 2}{x(x+2)(x^2+1)} = \frac{A}{x} + \frac{B}{x+2} + \frac{C(x+1)}{x^2+1}$$

$$\underline{x=0} \quad A = 1 \quad \textcircled{1}$$

$$x = -2 \quad B = \frac{20}{(-2)(5)} = -2$$

$$x(x+2)(x^2+1) \text{ ' } \textcircled{1} \text{ '}$$

$$5x^2 + x + 2 = A(x+2)(x^2+1) + Bx(x^2+1) + (Cx+D)(x)(x+2)$$

$$5x^2 + x + 2 = A[x^3 + 2x^2 + x + 1] + Bx^3 + Bx + Cx^3 + Dx^2 + 2Cx^2 + 2Dx$$

مقارنة معاملات  $x$

$x^3$

$$\textcircled{3} = A + B + C$$

$$C = 1$$

$\textcircled{4}$

$x^2$

$$5 = 2A + D + 2C$$

$$D = 1$$



$$f(x) = \frac{1}{x} + \frac{-2}{2+x} + (1+x) \frac{1}{1+x^2}$$

$$= \frac{1}{x} - \left[ \frac{1}{1+\frac{x}{2}} \right] + (1+x) \left( \frac{1}{1+x^2} \right)$$

$$= \frac{1}{x} - \left[ 1 - \frac{x}{2} + \left(\frac{x}{2}\right)^2 + \dots + \left(-\frac{x}{2}\right)^n \right]$$

$$+ (1+x) \left[ 1 - x^2 + x^4 - \dots + (-1)^n x^{2n} \right]$$

$$- \left(-\frac{1}{2}\right)^n \quad \text{معامل } x^n$$

$$(-1)^n \quad \text{معامل } x^{2n+1}$$

$$(-1)^n \quad \text{معامل } x^{2n}$$

$$\begin{array}{c} \text{زوج } n \\ \text{زوج } n \end{array} \rightarrow x^n$$

$$\therefore \text{Coefficient } x^{2n} = (-1)^n - \left(-\frac{1}{2}\right)^{2n}$$

$$x^{2n+1} = (-1)^n - \left(-\frac{1}{2}\right)^{2n+1}$$

$$(-1)^n - \left(\frac{1}{2}\right)^{2n} \quad \text{معامل } x^{2n}$$

$$(-1)^n + \left(\frac{1}{2}\right)^{2n+1} \quad \text{معامل } x^{2n+1}$$

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Find coefficient  $x^n$  in the expansion

$$\frac{1-x}{1+x+x^2}$$

$$\therefore 1+x+x^2 = \frac{1-x^3}{1-x}$$

$$\frac{1-x}{1+x+x^2} = \frac{(1-x)(1-x)}{(1-x^3)} = (1-x)^2 \cdot \frac{1}{1-x^3}$$

$$= (1-2x+x^2)(1+x^3+x^6+\dots+x^{3n})$$

$$1 \text{ } \rightarrow x^{3n} \text{ } \text{term}$$

$$-2 \text{ } \rightarrow x^{3n+1} \text{ } \text{term}$$

$$1 \text{ } \rightarrow x^{3n+2} \text{ } \text{term}$$

Find coefficient  $x^n$  in the expansion

$$\frac{(1+x)^3}{(1-x+x^2)^2}$$

$$1-x+x^2 = \frac{1-(-x)^3}{1-(-x)} = \frac{1+x^3}{1+x}$$

(7)

$$f(x) = \frac{(1+x)^3}{(1+x^3)^2} = (1+x)^3 \cdot \frac{1}{(1+x^3)^2}$$

$$= [1 + 3x + 3x^2 + x^3] [1 - 2x^3 + \dots + (n+1)(-x^3)^n]$$

$$= (1 + 3x + 3x^2 + x^3) [1 - 2x^3 + \dots + (n+1)(-1)^n x^{3n}]$$

$$(-1)^n [1] = n(-1)^{n-1} + (n+1)(-1)^n \quad \left. \begin{array}{l} \text{so } x^{3n} \text{ term} \\ 3(n+1)(-1)^n \text{ so } x^{3n+1} \text{ term} \\ 3(n+1)(-1)^n \text{ so } x^{3n+2} \text{ term} \end{array} \right\}$$

Expand  $\sqrt{1+x}$  Find Coefficient  $x^n$

$$(1+x)^{-1/2} = 1 + \frac{1}{2}x + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!} x^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!} x^3$$

$$+ \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})(-\frac{7}{2})}{4!} x^4 + \dots + \frac{(-\frac{1}{2})(-\frac{3}{2}) \dots (-\frac{2n+1}{2})}{n!} x^n$$

(2)



$$\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\left(-\frac{7}{2}\right) \cdots \left(-\frac{(2n+1)}{2}\right)}{n!} x^n$$

$$= \frac{(-1)^n \quad 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdots (2n+1) \cdot 2n}{2^n \quad n! \quad \begin{array}{ccc} \cancel{2} & \cancel{4} & \cancel{6} \\ \cancel{2 \cdot 1} & \cancel{2 \cdot 2} & \cancel{2 \cdot 3} \end{array} \quad \begin{array}{c} \cancel{2n} \\ \cancel{2 \cdot n} \end{array}} x^n$$

$$= \frac{(-1)^n (2n)!}{2^n \cdot n! \cdot 2^n \cdot n!} x^n$$

$$= \frac{(-1)^n (2n)!}{2^{2n} (n!)^2} x^n$$

so  $x^n$  is

$$\frac{(-1)^n (2n)!}{2^{2n} \cdot (n!)^2} \neq$$

(1)

Find coeff  $x^n$  in the expans  $\frac{1}{\sqrt{1-x}}$

$$(1-x)^{-1/2} = 1 + \frac{1}{2}(x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2 + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3!}x^3$$

$$+ \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{4!}x^4 - \dots - \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2}) \dots (-\frac{(2n-3)}{2})}{n!}x^n$$

$$\frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2}) \dots (-\frac{(2n-3)}{2})}{n!} x^n$$

$$= \frac{(-1)^{n-1} (1)(3)(5) \dots (2n-3)}{2^n n!}$$

$$= \frac{(-1)^{n-1}}{n! 2^n} \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \dots (2n-3)(2n-2)}{2 \cdot 4 \cdot 6 \dots (2n-2)(2n-1)(2n)}$$

$$= \frac{(-1)^{n-1}}{n! 2^n} \frac{(2n-2)!}{2^{n-1} (n-1)!}$$

$$= \frac{(-1)^{n-1} (2n-2)!}{2^{n-1} n! (n-1)!}$$

#

Q

٢ الفرص بين الأعداد التي ليس لها أعداد  
تتزايد  $n$  سالبة

$$n = \pm \frac{\text{العدد لعدد بعد الواحد}}{y}$$

$$Z = \pm \frac{y}{(m) \text{ العال المشترك الموهود في المقام}}$$

$$Z = \pm y/m$$

لواحد كل موهيت او سالبة  $Z$  سالبة

لواحد كل ~~كل~~ تتزايد  $Z$  موهيت

$$\frac{(1+Z)^n}{\text{لحد بلات رت}}$$

الطريقا تتزايد  $n$  سالبة

لواحد كل موهيت  $Z$  سالبة

لواحد متزايد  $Z$  موهيت

اذا تكرر الواحد في البسط  $n = \frac{1}{2}$

(١٠) (٤)

Find S

$$S = 1 + \frac{1}{4} + \frac{1 \cdot 3}{4 \cdot 8} + \frac{1 \cdot 3 \cdot 5}{4 \cdot 8 \cdot 12} + \dots$$

(11)

$$S = 1 + \frac{1}{4} + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2! \cdot 4^2} (-2)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3! \cdot 4^3} (-2)^3$$

$$1 + \frac{1}{4} + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!} \left(\frac{-2}{4}\right)^2 + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!} \left(\frac{-2}{4}\right)^3$$

$$= 1 + \left(\frac{-1}{2}\right)\left(\frac{-2}{4}\right) + \dots$$

$$= \left[1 + \left(\frac{-2}{4}\right)\right]^{-\frac{1}{2}} = \left(\frac{1}{2}\right)^{-\frac{1}{2}} = \sqrt{2} \quad \#$$

Find

$$S = \frac{1}{5} - \frac{1 \cdot 4}{5 \cdot 10} + \frac{1 \cdot 4 \cdot 7}{5 \cdot 10 \cdot 15} - \frac{1 \cdot 4 \cdot 7 \cdot 10}{5 \cdot 10 \cdot 15 \cdot 20}$$

$$= \frac{1}{5} - \frac{(-\frac{1}{3})(-\frac{4}{3})}{2! \cdot 5^2} (-3)^2 + \frac{(-\frac{1}{3})(-\frac{4}{3})(-\frac{7}{3})}{3! \cdot 5^3} (-3)^3$$

(11)

$$S = \frac{1}{5} - \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{3}{5}\right)^2}{2!} + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)\left(-\frac{3}{5}\right)^3}{3!}$$

$$= \frac{1}{5} - \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(\frac{3}{5}\right)^2}{2!} - \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)\left(\frac{3}{5}\right)^3}{3!} - \dots$$

$$= -\frac{\left(-\frac{1}{3}\right)\left(\frac{3}{5}\right)}{1!} - \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(\frac{3}{5}\right)^2}{2!} - \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)\left(\frac{3}{5}\right)^3}{3!} - \dots$$

نظر ١ ، نتج ١

$$1 - 1 - \left(-\frac{1}{3}\right)\left(\frac{3}{5}\right) - \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(\frac{3}{5}\right)^2}{2!} - \dots$$

$$= 1 - \left[ 1 + \left(-\frac{1}{3}\right)\left(\frac{3}{5}\right) + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(\frac{3}{5}\right)^2}{2!} - \dots \right]$$

$$= 1 - \left[ \left(1 + \frac{3}{5}\right)^{-\frac{1}{3}} \right] = 1 - \left(\frac{8}{5}\right)^{-\frac{1}{3}}$$

$$= 1 - \sqrt[3]{5/8}$$

١٥



Find

$$S = 1 + \frac{4}{14} + \frac{4 \cdot 7}{14 \cdot 21} + \frac{4 \cdot 7 \cdot 10}{14 \cdot 21 \cdot 28} + \dots$$

$$= 1 + \frac{4}{2! \cdot 7} + \frac{4 \cdot 7}{3! \cdot 7^2} + \frac{4 \cdot 7 \cdot 10}{4! \cdot 7^3} + \dots$$

$$= 7 \left[ 7 + \frac{4}{2! \cdot 7^2} + \frac{4 \cdot 7}{3! \cdot 7^3} + \frac{4 \cdot 7 \cdot 10}{4! \cdot 7^4} + \dots \right]$$

$$= 7 \left[ 7 + \frac{1 \cdot 4}{2! \cdot 7^2} + \frac{1 \cdot 4 \cdot 7}{3! \cdot 7^3} + \frac{1 \cdot 4 \cdot 7 \cdot 10}{4! \cdot 7^4} + \dots \right]$$

$$= 7 \left[ 7 + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)(-3)^2}{2! \cdot 7^2} + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)(-3)^3}{3!} + \dots \right]$$

$$7 \left[ -\frac{1}{3} \left(-\frac{3}{7}\right) + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{3}{7}\right)^2}{2!} + \dots + 1 - 1 \right]$$

$$= 7 \left[ 1 + \left(-\frac{1}{3}\right)\left(-\frac{3}{7}\right) + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{3}{7}\right)^2}{2!} + \dots - 1 \right]$$

$$= 7 \left[ \left(1 - \frac{3}{7}\right)^{-\frac{1}{3}} - 1 \right]$$

$$= 7 \left[ \left(\frac{4}{7}\right)^{-\frac{1}{3}} - 1 \right] = 7 \left[ \sqrt[3]{7/4} - 1 \right]$$

Find

$$S = \frac{1}{3} + \frac{1 \cdot 3}{3 \cdot 6} + \frac{1 \cdot 3 \cdot 5}{3 \cdot 6 \cdot 9} + \dots$$

ب

$$= \frac{1}{3} + \frac{(-1/2)(-3/2)}{2! \cdot 3^2} (-2)^2 + \frac{(-1/2)(-3/2)(-5/2)}{3! \cdot 3^3} (-2)^3$$

$$= \left(-\frac{1}{2}\right)\left(-\frac{2}{3}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!} \left(\frac{-2}{3}\right)^2 + \dots$$

نخرج اخرج 1

$$1 + \left(-\frac{1}{2}\right)\left(-\frac{2}{3}\right) + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!} \left(\frac{-2}{3}\right)^2 - \dots - 1$$

$$= \left[\left(-\frac{2}{3}\right)^{-1/2} - 1\right] = \left(\frac{1}{3}\right)^{-1/2} - 1 = \sqrt{3} - 1$$

Find S باب

$$S = 1 - \frac{25}{64} + \frac{(25)(45)}{64 \cdot 96} - \frac{(25)(45)(65)}{64 \cdot 96 \cdot 128}$$

$$S = 2 - \frac{1}{3} - \frac{1}{6} - \frac{5}{6 \cdot 9} - \frac{5 \cdot 7}{6 \cdot 9 \cdot 12} \dots$$

$$S = \frac{11}{6} + \frac{5 \cdot 7}{6 \cdot 12} + \frac{5 \cdot 7 \cdot 9}{6 \cdot 12 \cdot 18} + \dots$$

(5)

• multi nomial

$$[1 + ax + by + cz]^n$$

$$x^{r_1} y^{r_2} z^{r_3}$$

لوع درمصل

2-4

$$r_1 + r_2 + r_3 \leq n$$

$$L = n - (r_1 + r_2 + r_3)$$

$$\frac{n!}{r_1! r_2! r_3! L!} \cdot a^{r_1} b^{r_2} c^{r_3}$$

Find Coefficient  $x^2 y^3 z^4$  in The expansion

$$\left[1 + \frac{1}{6}x + 3y - 2z\right]^{15}$$

$$L = 15 - (2 + 3 + 4) = 6$$

$$\therefore \text{Coff} = \frac{15!}{2! 3! 4! 6!} \left(\frac{1}{6}\right)^2 (3)^3 (-2)^4$$

Find coff  $x^2 y^3 z^3$  in  $\left[2 + 3x - y + \frac{1}{2}z\right]^{10}$

$$2^{10} \left[1 + \frac{3x}{2} - \frac{y}{2} + \frac{1}{4}z\right]^{10}$$

$$2^{10} \left[ \frac{10!}{2! 3! 3! 2!} \left(\frac{3}{2}\right)^2 \left(-\frac{1}{2}\right)^3 \left(\frac{1}{4}\right)^3 \right] \neq$$

$$n < r_1 + r_2 + r_3$$

لا حجة له

نوع المعاد = ٩

Find Coeff  $x^3 y^4 z^7$

if  $(1 + 3x + 5y + z)^9$

مع  $x^3 y^4 z^7$  هو المصغ

Find Coeff  $x^6 y^5 z^8$  in the expan

$(1 + 2x^2 + y^5 + 3z^4)^7$

let  $x^2 = t$        $y^5 = v$        $z^4 = u$

عز معال

$t^3 v \cdot u^2$

$(1 + 2t + v + 3u)^7$

هو

$$\frac{7!}{3! \cdot 1! \cdot 2! \cdot 1!} (2)^3 (1)^1 (3)^2$$

$$= \frac{7!}{12} (8)(9) \neq$$

(١٦)