سنتر فيوتشر

Subject: اعدادى رياضة

Chapter: الدوال الزائدية

hyperbolic functions

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Hyperbolic function الدوال الزائدية

$$\sinh x = \frac{e^{x} - e^{-x}}{2}$$

$$\cosh x = \frac{e^{x} + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{1}{\tanh x}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}, \quad \operatorname{csch} x = \frac{1}{\sinh x}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\coth^2 x - 1 = \operatorname{csch}^2 x$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$= 2\cosh^2 x - 1$$

$$= 1 + 2\sinh^2 x$$

تذكر خواص اللوغاريتمات

$$\ln x + \ln y = \ln xy$$

$$\ln (x/y) = \ln x - \ln y$$

$$\ln x^y = y \ln x$$

$$e^{\ln \square} = \square \quad \text{و} \quad \ln e^{\square} = \square$$

$$\ln (x-y) = \quad \text{تقل}$$

$$\ln (x+y) = \quad \text{تقل}$$

تقل

$$\frac{\ln x}{\ln y} = \log_y x$$

$$\log_b x = y \qquad x = b^y$$

$$\log_x x = 1$$

$$\log x = y \qquad 10^y = x$$

$$\log_e x = \ln x \qquad \log_y x = \frac{\ln x}{\ln y}$$

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Prove that

$$\cosh^2 x - \sinh^2 x = 1$$

$$\because L.H.S = \cosh^2 x - \sinh^2 x$$

$$= \left(\frac{e^x + e^{-x}}{2}\right)^2 - \left(\frac{e^x - e^{-x}}{2}\right)^2$$

$$= \frac{\cancel{e^{2x}} \cancel{+ e^{-2x}} + 2 - (\cancel{e^{2x}} \cancel{+ e^{-2x}} - 2)}{4} = \frac{4}{4}$$

$$= 1$$

$$e^x \cdot e^{-x} = e^0 = 1$$

Prove that $\cosh x + \sinh x = e^x$

$$\because \cosh x + \sinh x = \frac{e^x + e^{-x}}{2} + \frac{e^x - e^{-x}}{2}$$

$$= \frac{e^x \cancel{+ e^{-x}} + e^x \cancel{- e^{-x}}}{2} = \frac{2 e^x}{2}$$

$$= e^x$$

Prove $\cosh x - \sinh x = e^{-x}$

$$\cosh x - \sinh x = \frac{e^{x} + e^{-x}}{2} - \left(\frac{e^{x} - e^{-x}}{2}\right)$$

$$= \frac{\cancel{e^{x}} + e^{-x} - \cancel{e^{x}} + e^{-x}}{2} = \cancel{\frac{2 e^{-x}}{2}}$$

$$= e^{-x}$$

$$* \ \cosh^2 x - \sinh^2 x = 1$$

$$\because \cosh x + \sinh x = e^{x}$$ نثبت مرحلة

$$\cosh x - \sinh x = e^{-x}$$

بالضرب

$$\cosh^2 x - \sinh^2 x = e^{x} e^{-x} = 1$$

Prove that

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\because 1 - \tanh^2 x = 1 - \left[\frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}\right]^2$$

$$= \frac{(e^{x} + e^{-x})^2 - (e^{x} - e^{-x})^2}{(e^{x} + e^{-x})^2}$$ وحد المقامات

$$1-\tanh^2 x = \frac{e^{2x}+e^{-2x}+2-[e^{2x}+e^{-2x}-2]}{(e^x+e^{-x})^2}$$

$$= \frac{4}{(e^x+e^{-x})^2} = \frac{2^2}{(e^x+e^{-x})^2}$$

$$= \left(\frac{1}{\cosh x}\right)^2 = \operatorname{sech}^2 x$$

Prove that

$$\sinh 2x = 2\sinh x\cosh x$$

$$2\sinh x\cosh x = \frac{\cancel{2}[e^x - e^{-x}](e^x+e^{-x})}{\cancel{2}\cdot 2}$$

$$= \frac{e^{2x}-e^{-2x}}{2} = \sinh 2x$$

$$\sinh \square = \frac{e^{\square} - e^{-\square}}{2}$$

موضة حرة:

Prove

$$\underbrace{\sinh^{-1} x = \ln\left[x+\sqrt{1+x^2}\right]}_{\text{ط}}$$

نفرض $\sinh^{-1} x = y$

$\therefore \quad x = \sinh y$

$$x = \frac{e^{y}-e^{-y}}{2}$$

بالضرب × $2e^{y}$ للطرفين:

$$2xe^{y} = e^{2y} - \cancel{e^{y}}\,\cancel{e^{-y}}_{1}$$

$$e^{2y} - 2xe^{y} - 1 = 0$$

$$(e^{y})^2 - 2x(e^{y}) - 1 = 0$$

بالقانون العام:

$$az^2+bz+c$$

$$z = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$a=1 \qquad b=-2x \qquad c=-1$

$$e^{y} = \frac{2x \pm \sqrt{4x^2-4(-1)}}{2}$$

$$e^{y} = \frac{\cancel{2}x \pm \cancel{2}\sqrt{x^2+1}}{\cancel{2}}$$

خذ ln للطرفين:

$$y = \ln\left[x \pm \sqrt{x^2+1}\right]$$

مرفوض

$\because \sinh^{-1} x = \ln\left[x + \sqrt{x^2+1}\right]$

Prove that

$\cosh^{-1} x = \ln\left[x \pm \sqrt{x^2-1}\right]$

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let $y = \cosh^{-1} x$

$\cosh y = x$

$\frac{e^{y} + e^{-y}}{2} = x$ بالضرب $2e^{y}$

$e^{2y} + 1 = 2x e^{y}$

$(e^{y})^2 \cancel{e^{2y}} - 2x e^{y} + 1 = 0$ معادلة من الدرجة الثانية

$e^{y} = \frac{2x \pm \sqrt{4x^2 - 4}}{2} = \cancel{\frac{2x \pm 2\sqrt{x^2-1}}{2}}$

$e^{y} = x \pm \sqrt{x^2-1}$

نأخذ ln للطرفين

$y = \ln\left[x \pm \sqrt{x^2-1}\right]$

Show that

$$\tanh^{-1} x = \frac{1}{2} \ln \left[\frac{1+x}{1-x} \right]$$

الحل

$$\tanh y = X$$

$$y = \tanh^{-1} x$$

$$\frac{e^{y} - e^{-y}}{e^{y} + e^{-y}} = \frac{x}{1}$$

بالضرب في $\frac{e^{y}}{e^{y}}$

$$\frac{e^{2y} - 1}{e^{2y} + 1} = \frac{x}{1}$$

$$\therefore X e^{2y} + X = e^{2y} - 1$$

$$1 + x = e^{2y} - X e^{2y}$$

$$1 + X = e^{2y} \left[1 - X \right]$$

$$e^{2y} = \frac{1+x}{1-X}$$

∴ خد ln للطرفين

$$2y = \ln \left[\frac{1+x}{1-X} \right]$$

$$y = \frac{1}{2} \ln \left[\frac{1+x}{1-X} \right]$$

$$\therefore \tanh^{-1} x = \frac{1}{2} \ln \left(\frac{1+X}{1-X} \right)$$

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Find logarithm form for $\coth^{-1}(x)$

اوجد الصورة اللوغاريتمية

$$y = \coth^{-1} x$$

$$\coth y = x$$

$$\frac{e^{y} + e^{-y}}{e^{y} - e^{-y}} = \frac{x}{1}$$

بالضرب $\frac{e^{y}}{e^{y}}$

$$\frac{e^{2y} + 1}{e^{2y} - 1} = \frac{x}{1}$$

$$\therefore X e^{2y} - X = e^{2y} + 1$$

$$X e^{2y} - e^{2y} = x + 1$$

$$e^{2y} [X - 1] = X + 1$$

$$e^{2y} = \frac{x+1}{X-1}$$

$$2y = \ln \left[\frac{x+1}{X-1} \right]$$

$$\therefore y = \frac{1}{2} \ln \left(\frac{x+1}{X-1} \right) = \cancel{\frac{1}{2} \ln \left[\frac{1-x}{1+x} \right]}$$

Prove that

$$\operatorname{sech}^{-1} x = \ln\left[\frac{1 \pm \sqrt{1-x^2}}{x}\right]$$

$$y = \operatorname{sech}^{-1} x$$

$$\operatorname{sech} y = x \qquad \therefore \cosh y = \frac{1}{x}$$

$$\frac{e^{y} + e^{-y}}{2} = \frac{1}{x}$$

بالضرب $2e^{y}$

$$e^{2y} + 1 = \frac{2}{x} e^{y}$$

$$e^{2y} - \frac{2}{x} e^{y} + 1 = 0$$

$$x e^{2y} - 2e^{y} + x = 0$$

$$e^{y} = \frac{\cancel{2} \pm \cancel{2}\sqrt{\cancel{4} - \cancel{4}x^2}}{\cancel{2}x} =$$

$$\therefore e^{y} = \frac{1 \pm \sqrt{1-x^2}}{x}$$

ضع ln للطرفين

$$y = \ln\left[\frac{1 \pm \sqrt{1-x^2}}{x}\right]$$

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Prove that

$$\operatorname{Csech}^{-1} x = \ln\left[\frac{1+\sqrt{1+x^2}}{x}\right]$$

let $y = \operatorname{Csech}^{-1} x$

$$\operatorname{Csech} y = x$$

$$\frac{1}{\sinh y} = x \qquad \sinh y = \frac{1}{x}$$

$$\therefore x\left[\frac{e^{y} - e^{-y}}{2}\right] = 1$$

بالضرب $2e^{y}$

$$x\left[e^{2y} - 1\right] = 2e^{y}$$

$$x e^{2y} - 2e^{y} - x = 0$$

$$e^{y} = \frac{2 \pm \sqrt{4 + 4x^2}}{2}$$

$$e^{y} = \frac{\cancel{2} \pm \cancel{2}\sqrt{1+x^2}}{\cancel{2}}$$

$$e^{y} = 1 \pm \sqrt{1+x^2}$$

مرفوض

$$y = \ln\left[1 + \sqrt{1+x^2}\right]$$

Prove that $\sinh^{-1} x = \operatorname{csch}^{-1}\left(\frac{1}{x}\right)$

$y = \sinh^{-1} x$

$\sinh y = x$

$\operatorname{csch} y = \frac{1}{x}$

$y = \operatorname{csch}^{-1}\left(\frac{1}{x}\right)$

$\therefore \sinh^{-1} x = \operatorname{csch}^{-1}\left(\frac{1}{x}\right)$

Prove that $\tanh^{-1}(x) = \coth^{-1}\left(\frac{1}{x}\right)$

let $\tanh^{-1} x = y$

$\tanh y = x$

$\coth y = \frac{1}{x}$

$y = \coth^{-1}\left(\frac{1}{x}\right)$

$\therefore \tanh^{-1} x = \coth^{-1}\left(\frac{1}{x}\right)$

Prove that $\sin^{-1}x + \cos^{-1}x = \pi/2$

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$y = \sin^{-1}x$ $\quad \therefore \sin y = x$

$\cos\left(\frac{\pi}{2} - y\right) = x$

$\frac{\pi}{2} - y = \cos^{-1}x$

$\frac{\pi}{2} = y + \cos^{-1}x$

$\therefore \frac{\pi}{2} = \sin^{-1}x + \cos^{-1}x$

$\sin(90 - x) = \cos x$

$\cos(90 - x) = \sin x$

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show that

$\tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}$

الحل

$y = \tan^{-1}x$ $\quad \tan y = x$

$\cot\left(\frac{\pi}{2} - y\right) = x$

$\frac{\pi}{2} - y = \cot^{-1}x$

$\frac{\pi}{2} = y + \cot^{-1}x$

$\frac{\pi}{2} = \tan^{-1}x + \cot^{-1}x$

$\tan(90 - x) = \cot x$

$\cot(90 - x) = \tan x$

Prove that

$$\tanh^{-1}\left(\frac{x^2-1}{x^2+1}\right) = \ln x$$

let $\tanh^{-1}\left(\frac{x^2-1}{x^2+1}\right) = y$

$$\frac{x^2-1}{x^2+1} = \tanh y = \frac{e^y - e^{-y}}{e^y + e^{-y}}$$

بالضرب $\frac{e^y}{e^y}$

$$\frac{x^2-1}{x^2+1} = \frac{e^{2y}-1}{e^{2y}+1}$$

$$(x^2+1)(e^{2y}-1) = (x^2-1)(e^{2y}+1)$$

$$\cancel{x^2 e^{2y}} + e^{2y} - x^2 \cancel{-1} = \cancel{x^2 e^{2y}} - e^{2y} + x^2 \cancel{+1}$$

$$\cancel{2}x^2 = \cancel{2}e^{2y}$$

$$x^2 = e^{2y}$$

ناخد ln للطرفين

$$2y = \ln x^2$$

$$\cancel{2}y = \cancel{2}\ln x$$

$$y = \ln x$$