COLLEGE
Tanta Engineering

إعدادي 2020

تفاضل

قاعدة لوبيتال

سنتر فيوتشر

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Subject: إعدادى رياضة

Chapter: النهايات باستخدام قاعدة لوبيتال

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النهايات باستخدام قاعدة لوبيتال

$e^{-\infty} = 0$ $\qquad$ $e^{\infty} = \infty$ $\qquad$ $\ln 0 = -\infty$

$\ln \infty = \infty$ $\qquad$ $\ln 1 = 0$ $\qquad$ $\sec 90 = \infty$

$\csc 0 = \infty$ $\qquad$ $\tan 90 = \infty$

اذا كان الناتج $\frac{0}{0}$ او $\frac{\infty}{\infty}$ نفاضل البسط لوحده
ونفاضل المقام لوحده

$$+ \lim_{x \to 0} \frac{\sin^{-1} x}{x} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{\frac{1}{\sqrt{1-x^2}}}{1} = 1$$

$$+ \lim_{x \to 0} \frac{e^{x} - e^{-x}}{x} = \frac{1-1}{0} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{e^{x} + e^{-x}}{1} = 1+1 = 2 \quad (1)$$

* $\lim_{x \to 0} \frac{1-\cos x}{x^2} = \lim_{x \to 0} \frac{\sin x}{2x} = \frac{0}{0}$

$\lim_{x \to 0} \frac{\cos x}{2} = 1/2$

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* $\lim_{x \to 0} \frac{x - \tan x}{\sin x} = \frac{0}{0}$

$\lim_{x \to 0} \frac{1 - \sec^2 x}{\cos x} = \frac{1-1}{1} = \frac{0}{1} = 0$

$\lim_{x \to 0} \frac{e^{x^2} - 1}{\cos x - 1} = \frac{1-1}{1-1} = \frac{0}{0}$

$\lim_{x \to 0} \frac{2x\, e^{x^2}}{-\sin x} = \frac{0}{0}$

$\lim_{x \to 0} \frac{2e^{x^2} + 2x(2x)e^{x^2}}{-\cos x} = \frac{2+0}{-1}$

$= -2$

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اذا كان الناتج $\infty - \infty$ او $0 \cdot \infty$
نحول المسالة إلى صورة كسرية ونطبق لوبيتال

$$\lim_{x \to \pi/2} \sec x - \tan x = \infty - \infty$$

$$\lim_{x \to \pi/2} \left[\frac{1 - \sin x}{\cos x} \right] = \frac{0}{0}$$

$$\lim_{x \to \pi/2} \frac{-\cos x}{-\sin x} = \frac{0}{1} = 0$$

$$\lim_{x \to 1} \left[\frac{1}{x-1} - \frac{1}{\ln x} \right]$$

$$= \frac{1}{0} - \frac{1}{0} = \infty - \infty$$

$$\lim_{x \to 1} \left[\frac{\ln x - x + 1}{(x-1)\ln x} \right] = \frac{0}{0}$$

$$\lim_{x \to 1} \frac{\frac{1}{x} - 1}{(x-1)\frac{1}{x} + \ln x}$$

نختصر قبل التعويض ②

$$\lim_{x \to 1} \frac{1 - x}{(x-1) + x\ln x} = \frac{0}{0}$$

$$\lim_{x \to 1} \frac{-1}{1 + x \cdot \frac{1}{x} + \ln x} = \frac{-1}{1+1+0}$$

$$= -\frac{1}{2}$$

$$\lim_{x \to 0} \left[\frac{1}{x} - \frac{1}{\sin x} \right] = \infty - \infty$$

$$\lim_{x \to 0} \frac{\sin x - x}{x \sin x} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{\cos x - 1}{\sin x + x \cos x} = \frac{0}{0}$$

$$\lim_{x \to 0} \frac{-\sin x}{\cos x + \cos x - x \sin x}$$

$$= \frac{0}{1+1-0} = 0 \ \#$$

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$$\lim_{x \to \frac{1}{2}} \frac{\cos^2(\pi x)}{e^{2x} - 2e \cdot x} = \frac{0}{0}$$

$$\lim_{x \to 1/2} \frac{-2\pi \ \sin\pi x \cdot \cos\pi x}{2 e^{2x} - 2e}$$

$$\lim_{x \to 1/2} \frac{-\pi \ \sin(2\pi x)}{2 e^{2x} - 2e} = \frac{0}{0}$$

$$\lim_{x \to 1/2} \frac{-2\pi^2 \cos(2\pi x)}{4 e^{2x}}$$

$$= \frac{-2\pi^2(-1)}{4 \cdot e} = \frac{\pi^2}{2e}$$

$$\lim_{x \to 1} \frac{1 - x}{\cot\left(\frac{\pi x}{2}\right)} = \frac{0}{0}$$

$$\lim_{x \to 1} \frac{-1}{-\frac{\pi}{2} \csc^2\left(\frac{\pi x}{2}\right)} = \frac{2 \sin^2\left(\frac{\pi x}{2}\right)}{\pi}$$

$$= \frac{2}{\pi}$$

$$\lim_{x \to \infty} x^2 \cdot e^{-x} \qquad \infty \cdot 0$$

نحول ل صورة كسر

$$\lim_{x \to \infty} \frac{x^2}{e^x} = \frac{\infty}{\infty}$$

$$\lim_{x \to \infty} \frac{2x}{e^x} = \frac{\infty}{\infty} = \lim_{x \to \infty} \frac{2}{e^x} = \frac{2}{\infty}$$

$$= 0$$

$$\lim_{x \to 0} x \ln x = 0 \cdot \infty$$

$$\lim_{x \to 0} \frac{\ln x}{x^{-1}} = \lim_{x \to 0} \frac{1/x}{-x^{-2}}$$

$$= -\lim_{x \to 0} \frac{\cancel{x^2}}{\cancel{x}} = -\lim_{x \to 0} x = 0$$

$$\lim_{x \to \pi/2} \left(x - \frac{\pi}{2}\right) \cdot \tan x = 0 \cdot \infty$$

$$\lim_{x \to \pi/2} \frac{(x - \pi/2)}{\cot x} = \frac{0}{0}$$

$$\lim_{x \to \pi/2} \frac{1}{-\csc^2(x)}$$

$$= \lim_{x \to \pi/2} -\sin^2(x) = -1$$

اذا كان الناتج 0^{∞} ∞^{0} , 0^{0} 1^{∞}

نفرض المسألة $y =$

ثم ناخذ ln للطرفين و نحولها صورة كسرية

نطبق لوبيتال

ثم نرفع الناتج للاساس e

Ex

$$\lim_{x \to 0} x^{x} = 0^{0}$$

$$y = \lim_{x \to 0} x^{x} \qquad \ln y = \lim_{x \to 0} x \ln x$$

$$\ln y = \lim_{x \to 0} \frac{\ln x}{x^{-1}} = \lim_{x \to 0} \frac{1/x}{-x^{-2}} = \lim_{x \to 0} \frac{-x}{1} = 0$$

$$\ln y = 0 \qquad y = \lim_{x \to 0} x^{x} = e^{0} = 1$$

$$\lim_{x \to \pi/2} (\tan x)^{\cos x} \qquad \infty^{0}$$

الحل

$$y = \lim_{x \to \pi/2} (\tan x)^{\cos x}$$

$$\ln y = \lim_{x \to \pi/2} \cos x \ln \tan x$$

$$= \lim_{x \to \pi/2} \frac{\ln \tan x}{\sec x} = \frac{\infty}{\infty}$$

$$\lim_{x \to \pi/2} \frac{\frac{1}{\tan x} \cdot \cancel{\sec^2 x}}{\cancel{\sec x} \tan x} = \lim_{x \to \pi/2} \frac{\sec x}{\tan^2 x}$$

$$\lim_{x \to \frac{\pi}{2}} \frac{\cos^2 x \cdot \sec x}{\sin^2 x} = \lim_{x \to \pi/2} \frac{\cos x}{\sin^2 x} = 0$$

$$\ln y = 0 \qquad y = e^{0} = 1$$

$$\lim_{x \to 0} (\tan x)^{\cos x} = e^{0} = 1$$

$$\lim_{x\to 0} (\cot x)^{\frac{1}{\ln x}} \qquad \infty^{\frac{1}{\infty}} = \infty^0$$

$$y = \lim (\cot x)^{\frac{1}{\ln x}}$$

$$\ln y = \lim_{x\to 0} \frac{1}{\ln x} \ln \cot x$$

$$\lim_{x\to 0} \frac{\ln \cot x}{\ln x} = \frac{\infty}{\infty}$$

$$= \lim_{x\to 0} \frac{\frac{1}{\cot x}(-\operatorname{cosec}^2 x)}{1/x}$$

$$\lim_{x\to 0} \frac{-x \operatorname{cosec}^2 x}{\cot x} = \lim \frac{-x \cancel{\sin x}}{\sin^{\cancel{2}} x \cos x}$$

$$= \lim_{x\to 0} \frac{-2x}{2\sin x \cos x} = \lim_{x\to 0} \frac{-2x}{\sin 2x} = \frac{0}{0}$$

$$\lim_{x\to 0} \frac{-2}{2\cos(2x)} = \frac{-2}{2} = -1$$

$$y = e^{-1} = \lim (\cot x)^{\frac{1}{\ln x}} = e^{-1}$$ a

$$\lim_{x \to \pi/2} (\cos x)^{\frac{\pi}{2} - x}$$

$$y = \lim_{x \to \pi/2} (\cos x)^{\frac{\pi}{2} - x}$$

$$\ln y = \lim_{x \to \pi/2} \left(\frac{\pi}{2} - x\right) \ln \cos x$$

$$\lim_{x \to \pi/2} \frac{\ln \cos x}{\left(\frac{\pi}{2} - x\right)^{-1}}$$

$$\lim_{x \to \pi/2} \frac{\frac{1}{\cos x}(-\sin x)}{-\left(\frac{\pi}{2} - x\right)^{-2}(-1)}$$

$$\lim_{x \to \pi/2} \frac{-\tan x}{\left(\frac{\pi}{2} - x\right)^{-2}} = \lim_{x \to \pi/2} -\frac{\left(\frac{\pi}{2} - x\right)^2}{\cot x} = \frac{0}{0}$$

$$\lim_{x \to \pi/2} \frac{\cancel{-}2\left(\frac{\pi}{2} - x\right)(-1)}{\cancel{-}\operatorname{cosec}^2 x}$$

$$\lim_{x \to \pi/2} -2\left(\frac{\pi}{2} - x\right) \cdot \sin^2 x = 0$$

$$y = e^0 = 1 = \lim (\cos x)^{\frac{\pi}{2} - x}$$

$$\lim_{x \to 0} (\cos x)^{\frac{1}{x^2}} = 1^{\infty}$$

$$y = \lim_{x \to 0} (\cos x)^{\frac{1}{x^2}}$$

$$\ln y = \lim_{x \to 0} \frac{\ln \cos x}{x^2}$$

$$= \lim_{x \to 0} \frac{\frac{1}{\cos x}(-\sin x)}{2x} = -\lim_{x \to 0} \frac{\tan x}{2x}$$

$$= -\tfrac{1}{2}$$

$$\therefore y = e^{-1/2}$$

$$\lim_{x \to 0} (\cos x)^{\frac{1}{x^2}} = e^{-1/2}$$

$$\lim_{x \to 0} (1 + \sin^{-1} x)^{\frac{1}{x}}$$

$$y = \lim_{x \to 0} (1 + \sin^{-1} x)^{\frac{1}{x}}$$

$$\ln y = \lim_{x \to 0} \frac{\ln(1 + \sin^{-1} x)}{x} = \frac{0}{0}$$

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$$\ln y = \lim_{x \to 0} \frac{\frac{1}{(1+\sin^{-1}x)} \frac{1}{\sqrt{1-x^2}}}{1} = 1$$

$$\ln y = 1$$

$$y = e^{+1} = \lim (1+\sin^{-1}x)^{\frac{1}{x}} = e^{1}$$

$$\lim_{x \to 0} x^{\sin x}$$

$$y = \lim_{x \to 0} x^{\sin x}$$

$$\ln y = \lim (\sin x) \ln x$$

$$= \lim_{x \to 0} \frac{\ln x}{\operatorname{cosec} x} = \lim_{x \to 0} \frac{1/x}{-\operatorname{cosec} x \cot x}$$

$$= -\lim_{x \to 0} \frac{\sin x \tan x}{x} = \frac{0}{0}$$

$$-\lim \frac{\cos x \tan x + \sin x \sec^2 x}{1} = -1$$

$$y = e^{-1} \qquad \lim_{x \to 0} x^{\sin x} = e^{-1}$$

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$$\lim_{x\to 0} \left(\frac{1}{x}\right)^{\tan x}$$

نفرض

$$y = \lim_{x\to 0} \left(\frac{1}{x}\right)^{\tan x}$$

$$\therefore \ln y = \lim_{x\to 0} \tan x \ln\left(\frac{1}{x}\right)$$

$$= \lim_{x\to 0} \frac{\ln(1/x)}{\cot x} = \frac{\ln 1 - \ln x}{\cot x}$$

$$= \lim_{x\to 0} -\frac{\ln x}{\cot x} = \frac{\infty}{\infty}$$

نفاضل البسط لوحده ، المقام لوحده

$$= \lim_{x\to 0} \frac{-1/x}{-\operatorname{cosec}^2 x} = \lim_{x\to 0} \frac{\sin^2 x}{x}$$

$$\ln y = \lim_{x\to 0} \frac{2\sin x \cos x}{1}$$ نطبق لوبيتال مرة أخرى $= 0$

$$y = e^0 = 1 \quad \#$$