COLLEGE
Tanta Engineering

إعدادي ٢٠٢٠

الرياضيات

التفاضل

محاضرة المشتقات العليا- وقاعدة لايبنز

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Higher derivatives and leibnte's theren

y', $f'(x)$, D_x, $\frac{dy}{dx}$ 1^{st} deriv

y'', $f''(x)$, ----, $\frac{d^2y}{dx^2}$ 2^{nd} deriv

y''', $f'''(x)$, ----, $\frac{d^3y}{dx^3}$ 3^{rd} deriv

$y^{(4)}(x)$, $f^{(4)}(x)$, ---, $\frac{d^4y}{dx^4}$ 4^{th} derive

$$y = 3x^4 - 12x^3 + 5x^2 + 10$$

$y' = 12x^3 - 36x^2 + 10x$

$y'' = 36x^2 - 72x + 10$

$y''' = 72x - 72$

$y^{(4)} = 72$

$y^{(5)} = 0$

$$y = e^{ax}$$

$y' = a e^{ax}$

$y'' = a^2 e^{ax}$

$y''' = a^3 e^{ax}$

$y^{(n)} = a^n e^{ax}$

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$$y = \sin(ax+b)$$

$$y' = a\cos(ax+b)$$
$$= a\sin(ax+b+\frac{\pi}{2})$$

$$y'' = a^2(\cos(ax+b+\frac{\pi}{2})$$
$$= a^2\sin(ax+b+\frac{\pi}{2}\times 2)$$

$$y''' = a^3\sin(ax+3\frac{\pi}{2}+b)$$

$$y^{(n)} = a^n\sin(ax+n\frac{\pi}{2}+b)$$

$$y = \ln(ax+b)$$

$$y' = \frac{a}{ax+b} = a(ax+b)^{-1}$$

$$y'' = -a^2(ax+b)^{-2} = \frac{-a}{(ax+b)^2}$$

$$y''' = \frac{2a^3(-1)^2}{(ax+b)^3}$$

$$y^{(n)} = (-1)^{n-1}(n-1)!\,a^n(ax+b)^{-n}$$

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leibnts's theorem

$$y = uv$$

$$y' = u \cdot v' + v u'$$

$$y'' = u \cdot v'' + v' \cdot u' + v u'' + u' v'$$

$$= u \cdot v'' + 2u'v' + u'' \cdot v$$

$$y^{(n)} = (uv)^{(n)} = u^{(n)} v + n u^{(n-1)} v' + \frac{n(n-1)}{2!} u^{(n-2)} v'' + \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v''' + \cdots + u v^{(n)}$$

$$y = x^3 e^{2x}$$

$u = e^{2x}$ $\quad$ $v = x^3$

$u^{(n-3)}$ u^{n-3} $\cancel{2^{n-3}}$ e^{2x} $\quad$ $v' = 3x^2$

$u^{(n-2)} = u^{(n-2)} e^{2x}$ $\quad$ $v'' = 6x$

$u^{(n-1)} = 2^{n-1} e^{2x}$ $\quad$ $v''' = 6$

$u^{(n)} = 2^n e^{2x}$ $\quad$ $v^{(4)} = 0$

$$y^{(n)} = (x^3 e^{2x})^{(n)}$$

$$= x^3 (2^n e^{2x}) + n(3x^2) 2^{n-1} e^{2x} + \frac{n(n-1)}{2!} (6x)(2^{n-2} e^{2x}) + \frac{n(n-1)(n-2)}{3!} (6)(n^{n-3} e^{2x}) + 0$$

$$= 2^n x^3 e^{2x} + 3n x^2 2^{n-1} e^{2x} + n(n-1)(n-2)(2^{n-3}) e^{2x}$$

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$$y = \sin(2x+3)\ln(3x+2)$$

$u = \sin(2x+3)$

$u^{(n)} = 2^n(2x+3 + n\frac{\pi}{2})$

$v = \ln(3x+2)$

$v^{(n)} = \frac{(-1)^{n-1}(n-1)!\,3^n}{(3x+2)^n}$

$$y^{(n)} = [\sin(2x+3)\ln(3x+n)]^{(n)} = 2^n\sin(2x+3+n\frac{\pi}{2}).\ln(3x+2)$$

$$+n2^{n-1}\sin(2x+3+(n-1)\frac{\pi}{2})\left(\frac{3}{(3x+2)}\right)$$

$$+\frac{n(n-1)}{2!}2^{n-2}\sin(2x+3+(n-2)\frac{\pi}{2})\left(\frac{-3^2}{(3x+2)^2}\right)$$

$$+\ldots\ldots+\sin(2x+3)\left(\frac{(-1)^{n-1}(n-1)!\,3^n}{(3x+2)^n}\right)$$

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الامتحان

٣ أسئلة

Q1) find the domain of the fn $f(x) = ???$

Q1b) Prove that $(\ )^{-1} = \ln$ - - - - - -

Q2) Discus the continuety of the fn $f(x) = \{$

Q3) find y' $y =$ - - - - $y =$ - - - - $y =$ - - - - -

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