COLLEGE
Tanta Engineering

إعدادي 2020

الجبر

المصفوفات

سنتر فيوتشر

سنتر فيوتشر

Subject: إعدادي رياضة

Chapter: تابع مصفوفات ٣

Mob: 0112 3333 122

0109 3508 204

eigen value , and eigen vector

① اذا كانت A مصفوفة مربعة لايجاد eigenvalue

نطرح من القطر الرئيسي λ ونقل المحدد ونساويه بالصفر نحصل على قيم λ [هي eigenvalue]

ملاحظة اذا كانت المصفوفة 2X2 فاننا نحصل على قيمتين لـ λ

واذا كانت 3X3 نحصل على ثلاث قيم لـ λ

if $A_{2\times2} \longrightarrow \therefore$ A has two eigen value

if $A_{3\times3} \longrightarrow \therefore$ A has 3 eigen value

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Ex find an eigen value and eigen vector $A = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix}$

الحل

$$(A - \lambda I) = \begin{pmatrix} 1-\lambda & 1 \\ -2 & 4-\lambda \end{pmatrix}$$

نعتبر المصفوفة محدد ونقل المحدد ونساويه بالصفر

Chartristic equation المعادلة المميزة

$$(1-\lambda)(4-\lambda)+2=0$$

$$\lambda^2-5\lambda+6=0$$

$$(\lambda-2)(\lambda-3)=0$$

$\lambda=2$, $\lambda=3$ is eigen value

مجموع عناصر القطر الرئيسي = eigen value مجموع الـ ملاحظة

$$\text{tra } A = \sum \text{eigen value}$$

$$= \sum \text{عناصر القطر الرئيسي}$$

<u>eigen vector</u> ايجاد

$$(A-\lambda I)\underset{\substack{\uparrow \\ \text{eigen vector}}}{X}=0$$

<u>at $\lambda=2$</u>
$$\begin{bmatrix} -1 & 1 \\ -2 & 2 \end{bmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}=\begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$-x_1+x_2=0$ let $x_1=c$

$x_2=c$

$\therefore X_1 = \begin{bmatrix} c \\ c \end{bmatrix} \rightarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\underline{OR}$ $\left[\begin{array}{cc|c} -1 & 1 & 0 \\ -2 & 2 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right]$

حول مصفوفة المعاملات الى
gauss — Jorden

$\therefore$ eigen vector $\cancel{\begin{bmatrix} 0 \\ 0 \end{bmatrix}} + c_1 \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

افرض $c_1 = -1$

$X_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\underline{at\ \lambda = 3}$ نكرر نفس الحل عندما

$$\begin{bmatrix} -2 & 1 \\ -2 & 1 \end{bmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$-2y_1 + y_2 = 0 \qquad y_1 = C \qquad y_2 = 2C$

$X_2 = \begin{pmatrix} C \\ 2C \end{pmatrix} \xrightarrow[c=1]{\text{فرض}} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$\underline{OR}$ $\left[\begin{array}{cc|c} -2 & 1 & 0 \\ -2 & 1 & 0 \end{array}\right] \rightarrow \left[\begin{array}{cc|c} -2 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$

$$\rightarrow \left[\begin{array}{cc|c} 1 & -1/2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$\therefore X_2 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} + r_2 \begin{bmatrix} -\frac{1}{2} \\ -1 \end{bmatrix}$$

let $r_2 = -2$

$$X_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$\therefore$ eigen value $2, 3$

eigen vector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

ملحوظة اذا كانت قيم λ غير متساوية يبقى

eigen vector are linear independent

, the matrix is semi simple

لمعرفة نوعي eigen vector

dependent or independent

$[v_1, v_2]$ — $\neq 0$ → independent

$[v_1, v_2]$ — $= 0$ → linear dep

is $v_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $v_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

are dependent or linear independent

$$\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} = 1 \neq 0$$

$\therefore$ v_1, v_2 are linear independent

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if $A = \begin{bmatrix} 1 & 3 & 0 \\ 3 & -2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

Find an eigen value, and eigen vector

is eigen vector linear independent

or not, is the matrix semi simple?

why? ———

$$(A - \lambda I) = \begin{bmatrix} 1-\lambda & 3 & 0 \\ 3 & -2-\lambda & -1 \\ 0 & -1 & +1-\lambda \end{bmatrix}$$

$$(1-\lambda)\left[(-2-\lambda)(1-\lambda) - 1\right] - 3\left[3 - 3\lambda\right] = 0$$

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$$(1-\lambda)\left[\lambda^2+\lambda-3\right]+9(\lambda-1)=0$$

$$(1-\lambda)\left[\lambda^2+\lambda-3-9\right]=0$$

$$(1-\lambda)\left(\lambda^2+\lambda-12\right]=0$$

$$(1-\lambda)(\lambda-3)(\lambda+4)=0$$

$\therefore$ eigen value $\lambda=1$, $\lambda=3$ $\lambda=-4$

<u>at $\lambda=1$</u>

$$\left[\begin{array}{ccc|c} 0 & 3 & 0 & 0 \\ 3 & -1 & -1 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & -1/3 & -1/3 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right]$$

بدلت الصف الاول والثالث وقسمت على 3

$$\left[\begin{array}{ccc|c} 1 & -1/3 & -1/3 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -\frac{1}{3} & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$V_1 = C_1 \begin{bmatrix} -\frac{1}{3} \\ 0 \\ -1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}$$ <u>eigen Vector</u>

$\underline{\lambda = 3}$ $\begin{bmatrix} -2 & 3 & 0 \\ 3 & -5 & -1 \\ 0 & -1 & -2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$

$-2u_1 + 3u_2 = 0$, $-u_2 - 2u_3 = 0$

let $u_3 = c$

$u_2 = -2c$

$\therefore -2u_1 - 6c = 0$ $u_1 = -3c$

$\therefore u = \begin{bmatrix} -3c \\ -2c \\ c \end{bmatrix} \longrightarrow \begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$

$\underline{\text{at } \lambda = -4}$ $\left[\begin{array}{ccc|c} 5 & 3 & 0 & 0 \\ 3 & 2 & -1 & 0 \\ 0 & -1 & 5 & 0 \end{array}\right]$

$\longrightarrow \left[\begin{array}{ccc|c} 1 & 3/5 & 0 & 0 \\ 3 & 2 & -1 & 0 \\ 0 & -1 & 5 & 0 \end{array}\right] \longrightarrow \left(\begin{array}{ccc|c} 1 & 3/5 & 0 & 0 \\ 0 & +1/5 & -1 & 0 \\ 0 & -1 & 5 & 0 \end{array}\right)$

$5r_2 \left[\begin{array}{ccc|c} 1 & 3/5 & 0 & 0 \\ 0 & 1 & -5 & 0 \\ 0 & -1 & 5 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|c} 1 & 0 & 3 & 0 \\ 0 & 1 & -5 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

eigen vector $= c_3 \begin{bmatrix} 3 \\ -5 \\ -1 \end{bmatrix} \longrightarrow \begin{pmatrix} 3 \\ -5 \\ -1 \end{pmatrix}$

$\because$ eigen value, 1, 3, -4

eigen vector $\begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$, $\begin{pmatrix} 3 \\ 2 \\ -1 \end{pmatrix}$, $\begin{pmatrix} 3 \\ -5 \\ -1 \end{pmatrix}$

$\because$ eigen value قيم مختلفة

$\therefore$ eigen vector linear independent

$\therefore$ matrix is semi simple.

Find eigen value, eigen vector

$$A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$(A - \lambda I) = \begin{pmatrix} -\lambda & 1 \\ -1 & -\lambda \end{pmatrix}$$

$$\lambda^2 + 1 = 0 \qquad \lambda = \pm i$$

at $\lambda = i$ $\qquad \left[\begin{array}{cc|c} -i & 1 & 0 \\ -1 & -i & 0 \end{array}\right]$

$\therefore$ eigen vector $\begin{pmatrix} 1 \\ i \end{pmatrix}$.

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at $\lambda = -i$

$$\left[\begin{array}{cc|c} i & 1 & 0 \\ -1 & i & 0 \end{array}\right]$$

eigen vector $\begin{pmatrix} 1 \\ -i \end{pmatrix}$

$\therefore$ eigen vector $\begin{pmatrix} 1 \\ i \end{pmatrix}$, $\begin{pmatrix} 1 \\ -i \end{pmatrix}$

Find an eigen value, eigen vector

$$A = \begin{pmatrix} 6 & -1 \\ 5 & 4 \end{pmatrix}$$

$$(A - \lambda I) = \begin{pmatrix} 6-\lambda & -1 \\ 5 & 4-\lambda \end{pmatrix}$$

$$(6-\lambda)(4-\lambda) + 5 = 0$$

$$\lambda^2 - 10\lambda + 29 = 0$$

$$\lambda = 5 \pm 2i$$

at $\lambda = 5 + 2i$ $\left[\begin{array}{cc|c} 1-2i & -1 & 0 \\ 5 & -1-2i & 0 \end{array}\right]$

$\therefore$ eigen vector $\begin{bmatrix} 1 \\ 1-2i \end{bmatrix} = k_1$

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at $\lambda = 5 - 2i$

$$\left(\begin{array}{cc|c} 1+2i & -1 & 0 \\ 5 & -1+2i & 0 \end{array}\right)$$

eigen vector $= K_2 = \overline{K_1}$

$$= \begin{bmatrix} 1 \\ 1+2i \end{bmatrix}$$

ملحوظة

tr A $\rightarrow$ مجموع eigen value

= مجموع عناصر القطر الرئيسي

$|A| = (-1)^n($ حاصل ضرب eigen values

eigen value of A^n is $\lambda_1^n, \lambda_2^n, \lambda_3^n$

eigen value of A $\ni \lambda_1, \lambda_2, \lambda_3$ حيث

eigen value of A^{-1} is $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \frac{1}{\lambda_3}$

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Show that if A has eigen value λ, A^n has eigen value is λ^n and eigen value of A^{-1} is $\frac{1}{\lambda}$

$\because AX = \lambda x$

بالضرب في A

$A^2 X = \lambda A x$

$\because AX = \lambda x$

$\therefore A^2 X = \lambda^2 X$

$\therefore$ eigen value of A^2 is λ^2

بالضرب في A

$A^3 X = \lambda^2 A x$

$= \lambda^2 \cdot \lambda x = \lambda^3 X$

$\therefore$ eigen value of A^3 is λ^3

$\because AX = \lambda x$

بالضرب ف A^{-1}

$A^{-1} A X = \lambda A^{-1} X$

$X = \lambda A^{-1} X$

$\frac{1}{\lambda} x = A^{-1} X$

$\therefore$ eigen value of A^{-1} is $\frac{1}{\lambda}$

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if A matrix 3×3 has eigen value $3, -1, 2$ find

trace A, $|A|$, $|A^5|$

find eigen value for A^4, eigen value for A^{-1}

trace $A = 3 + 2 - 1 = 4$ #

$|A| = (-1)^3 (3)(2)(-1) = 6$ #

$|A^5| = |A|^5 = 6^5$ #

eigen value for A^4 is

$3^4, -1^4, 2^4 \rightarrow 81, 1, 16$

eigen value for A^{-1} is $\frac{1}{3}, \frac{1}{-1}, \frac{1}{2}$

$= \frac{1}{3}, \frac{1}{2}, -1$

<u>digonzaible matrix</u>

اذا كانت A مصفوفة مربعة

① لها eigen value λ_1, λ_2

② لها eigen vector v_1, v_2

$T = [v_1, v_2]$

T^{-1} = inverse of T

$$\boxed{D = T^{-1} A T}$$

$A^k = T D_\lambda^k T^{-1}$

$D_\lambda^k \Rightarrow$ (مصفوفة قطرية قطرها هو λ اس k) $= \begin{bmatrix} \lambda_1^k & 0 & 0 \\ 0 & \lambda_2^k & 0 \\ 0 & 0 & \lambda_3^k \end{bmatrix}$

Ex if A is matrix 2×2 and has eigen value $2, 3$ and eigen vector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Find A, find A^5

$$\because T = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}, \quad T^{-1} = \frac{1}{1}\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

$$A = T D_{\lambda} T^{-1}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 3 \\ 2 & 6 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -2 & 4 \end{pmatrix} \#$$

جيد

$$A^5 = T D_{\lambda}^{5} T^{-1}$$

$$\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} 32 & 0 \\ 0 & 243 \end{bmatrix}\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

$$A^5 = \begin{pmatrix} 32 & 243 \\ 32 & 486 \end{pmatrix} \begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$$

$$= \begin{bmatrix} 64-243 & -32+243 \\ 64-486 & -32+486 \end{bmatrix}$$

$$A^5 = \begin{bmatrix} -179 & 211 \\ -422 & 454 \end{bmatrix}$$

Convert A to digonZable matrix Find A^{10} if $A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

$$(A - \lambda I) = \begin{vmatrix} -\lambda & 1 \\ -1 & -\lambda \end{vmatrix}$$

$\lambda^2 + 1 = 0 \qquad \lambda = \pm i$

$\because$ eigen value تختلف

$\therefore$ eigen vector linear independent

$\therefore$ المصفوفه Semi Simple (digonZable)

$\underline{\lambda = i}$ $\left[\begin{array}{cc|c} -i & 1 & 0 \\ -1 & -i & 0 \end{array}\right]$ $k_1 = \begin{pmatrix} 1 \\ i \end{pmatrix}$

$k_2 = \begin{pmatrix} 1 \\ -i \end{pmatrix}$

eigen vector $\begin{pmatrix} 1 \\ i \end{pmatrix}, \begin{pmatrix} 1 \\ -i \end{pmatrix}$

$$T = \begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} \quad , \quad T^{-1} = \frac{1}{-2i}\begin{pmatrix} -i & -1 \\ -i & 1 \end{pmatrix}$$

$$T^{-1} = \frac{i}{2}\begin{pmatrix} -i & -1 \\ -i & 1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}$$

$$D = T^{-1} A T$$

$$= \frac{1}{2}\begin{bmatrix} 1 & -i \\ 1 & i \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix}$$

$$\frac{1}{2}\begin{bmatrix} i & 1 \\ -i & 1 \end{bmatrix}\begin{pmatrix} 1 & 1 \\ i & -i \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2i & 0 \\ 0 & -2i \end{pmatrix}$$

$$D = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}$$

مصفوفة قطرية
قطرها هو قيم $\underline{\lambda}$

$$A^{10} = T\, D_{\lambda}^{10}\, T^{-1}$$

$$\begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix}\begin{bmatrix} +i^{10} & 0 \\ 0 & -i^{10} \end{bmatrix}\frac{1}{2}\begin{pmatrix} 1 & -i \\ 1 & +i \end{pmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} 1 & 1 \\ i & -i \end{bmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix}$$

$$= \frac{1}{2}\begin{bmatrix} -1 & -1 \\ -i & i \end{bmatrix}\begin{pmatrix} 1 & -i \\ 1 & i \end{pmatrix} = \frac{1}{2}\begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix}$$

$$A^{10} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \quad \#$$

نظرية كايلي ـ هاملتون كل مصفوفة مربعة
تحقق المعادلة المميزة لها

if $A = \begin{pmatrix} -2 & 4 \\ -1 & 3 \end{pmatrix}$

Find A^5 by using Caely hamilton and find A^{-1}

$$\therefore (A - \lambda I) = \begin{bmatrix} -2-\lambda & 4 \\ -1 & 3-\lambda \end{bmatrix}$$

$$(-2-\lambda)(3-\lambda) + 4 = 0$$

$$\lambda^2 - \lambda - 2 = 0$$

نعوض عن كل λ بـ A

$$A^2 - A - 2I = 0$$

$$A^2 = A + 2I$$

بالضرب في A :

$$A^3 = A^2 + 2A$$

$$= A + 2I + 2A = 3A + 2I$$

$$A^3 = 3A + 2I$$

بالضرب في A :

$$A^4 = 3A^2 + 2A$$

$$= 3[A + 2I] + 2A = 5A + 6I$$

$$A^4 = 5A + 6I$$

بالضرب في A :

$$A^5 = 5A^2 + 6A = 5[A + 2I] + 6A$$

$$A^5 = 11A + 10I$$

$$\therefore A^5 = 11\begin{bmatrix} -2 & 4 \\ -1 & 3 \end{bmatrix} + \begin{pmatrix} 10 & 0 \\ 0 & 10 \end{pmatrix}$$

$$= \begin{pmatrix} -12 & 44 \\ -11 & 43 \end{pmatrix}$$

حل ثاني لإيجاد $\underline{A^5}$

نوجد

<u>eigenval</u> $\lambda = 2$ $\lambda = -1$

نوجد eigen vector

$\underline{\lambda = 2}$ $\left[\begin{array}{cc|c} -4 & 4 & 0 \\ -1 & -1 & 0 \end{array}\right] \longrightarrow V_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

$\underline{\lambda = -1}$ $\left[\begin{array}{cc|c} -1 & 4 & 0 \\ -1 & 4 & 0 \end{array}\right] \longrightarrow V_2 = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

$\therefore$ eigen vector $\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

$$T = \begin{pmatrix} 1 & 4 \\ 1 & 1 \end{pmatrix} \qquad T^{-1} = \frac{-1}{3}\begin{pmatrix} 1 & -4 \\ -1 & 1 \end{pmatrix}$$

$$T^{-1} = \frac{1}{3}\begin{pmatrix} -1 & 4 \\ 1 & -1 \end{pmatrix}$$

$$A^5 = T D_\lambda^5 T^{-1}$$

$$= \begin{pmatrix} 1 & 4 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 32 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 4 \\ 1 & -1 \end{pmatrix} \frac{1}{3}$$

$$= \frac{1}{3} \begin{bmatrix} 32 & -4 \\ 32 & -1 \end{bmatrix} \begin{pmatrix} -1 & 4 \\ 1 & -1 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} -36 & 132 \\ -33 & 129 \end{pmatrix}$$

$$= \begin{pmatrix} -12 & 44 \\ -11 & 33 \end{pmatrix}$$

Find an eigen value, eigen vector for

$$A = \begin{pmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{pmatrix}.$$

$$(A - \lambda I) = \begin{pmatrix} 5-\lambda & 4 & 2 \\ 4 & 5-\lambda & 2 \\ 2 & 2 & 2-\lambda \end{pmatrix}$$

$$(5-\lambda)\left[(5-\lambda)(2-\lambda) - 4\right] - 4\left[8 - 4\lambda - 4\right]$$

$$+ 2(8 - 16 + 2\lambda) =$$

$$(5-\lambda)(\lambda^2 - 7\lambda + 6) - 16(1-\lambda) - 4(1-\lambda)$$

$$(5-\lambda)(\lambda-1)(\lambda-6) - 20(1-\lambda) =$$

$$(\lambda-1)\left[(5-\lambda)(\lambda-6) + 20\right] =$$

$$(\lambda-1)\left[-\lambda^2 - 30 + 11\lambda + 20\right] =$$

$$(\lambda-1)(\lambda^2 - 11\lambda + 10) =$$

$$(\lambda-1)(\lambda-1)(\lambda-10) = 0$$

$\lambda = 1$
$\lambda = 1$
$\lambda = 10$

$\underline{\lambda = 1}$

$$\left[\begin{array}{ccc|c} 4 & 4 & 2 & 0 \\ 4 & 4 & 2 & 0 \\ 2 & 2 & 1 & 0 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & 1 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$V_1 = C_1 \begin{bmatrix} \frac{1}{2} \\ 0 \\ -1 \end{bmatrix} \qquad V_2 = C_2 \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$$

$$V_1 = \begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}, \qquad V_2 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

$\underline{\lambda = 10}$

$$\left[\begin{array}{ccc|c} -5 & 4 & 2 & 0 \\ 4 & -5 & 2 & 0 \\ 2 & 2 & -8 & 0 \end{array}\right]$$

$$\to \left[\begin{array}{ccc|c} 1 & 1 & -4 & 0 \\ 4 & -5 & 2 & 0 \\ -5 & 4 & 2 & 0 \end{array}\right]$$

بدلت r_1 ، r_3
وقسمت r_3 على 2

$$\to \left(\begin{array}{ccc|c} 1 & 1 & -4 & 0 \\ 0 & -9 & -18 & 0 \\ 0 & 9 & 18 & 0 \end{array}\right) \to \left(\begin{array}{ccc|c} 1 & 0 & -6 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

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تابع المصفوفات

Find an eigen value and eigen vector

of $A = \begin{pmatrix} 9 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 9 \end{pmatrix}$

$$(A - \lambda I) = \begin{bmatrix} 9-\lambda & 1 & 1 \\ 1 & 9-\lambda & 1 \\ 1 & 1 & 9-\lambda \end{bmatrix}$$

$$(9-\lambda)\left[(9-\lambda)^2 - 1\right] - \left[9-\lambda-1\right] + \left[1-9+\lambda\right] = 0$$

$$(9-\lambda)(\lambda^2 - 18\lambda + 80) + (2\lambda - 16) = 0$$

$$(9-\lambda)(\lambda-10)(\lambda-8) + 2(\lambda-8) = 0$$

$$(\lambda-8)\left[(9-\lambda)(\lambda-10) + 2\right] = 0$$

$$(\lambda-8)\left[-\lambda^2 - 90 + 19\lambda + 2\right] = 0$$

$$-(\lambda-8)(\lambda^2 - 19\lambda + 88) = 0$$

$$-(\lambda-8)(\lambda-8)(\lambda-11) = 0$$

$\lambda = 8$ $\qquad \lambda = 8 \qquad \lambda = 11$

$\underline{\lambda = 8}$

at $\lambda = 11$

$$\left[\begin{array}{ccc|c} -2 & 1 & 1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 0 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 1 & -2 & 1 & 0 \\ -2 & 1 & 1 & 0 \end{array}\right] \longrightarrow \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & -3 & +3 & 0 \\ 0 & 3 & 3 & 0 \end{array}\right)$$

$$\Rightarrow \left(\begin{array}{ccc|c} 1 & 1 & -2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 3 & 3 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$k_1 = c_1 \begin{pmatrix} -1 \\ -1 \\ -1 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

at $\lambda = 8$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{array}\right]$$

$$\rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

eigen vectors

$$k_2 = c_2 \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \qquad k_3 = c_3 \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$$

$\therefore$ eigen vectors $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ ©

$$V_3 = c_3 \begin{bmatrix} -6 \\ 2 \\ -1 \end{bmatrix} \rightarrow \begin{pmatrix} 6 \\ -2 \\ 1 \end{pmatrix}$$

eigen vector $\begin{pmatrix} 1 \\ 0 \\ -2 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 6 \\ -2 \\ 1 \end{pmatrix}$

eigen vector linear indepen لو

$$\begin{vmatrix} 1 & 1 & 6 \\ 0 & -1 & -2 \\ -2 & 0 & 1 \end{vmatrix} = -1 + 4 + 12 \neq 0$$

$\because$ eigen vector linear independent

matrix semi simple

Prove that $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ is an eigen vector

$$\text{of } A = \begin{pmatrix} 3 & 2 \\ 3 & -2 \end{pmatrix}$$

$$(A - \lambda I) v = 0$$

$$(A-\lambda I) = \begin{pmatrix} 3-\lambda & 2 \\ 3 & -2-\lambda \end{pmatrix}$$

$$(3-\lambda)(-2-\lambda) - 6 = 0$$

$$\lambda^2 - \lambda - 12 = 0$$

$$(\lambda-4)(\lambda+3) = 0$$

$\underline{\lambda = 4}$ $\begin{bmatrix} -1 & 2 \\ 3 & -6 \end{bmatrix} v_1 = 0$

$$v_1 = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \#$$

Find $\underline{a}$, $\underline{b}$ and eigenvalue of corresponding eigenvector $\begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix}$

$$\text{if } A = \begin{pmatrix} 1 & a & 4 \\ 0 & 2 & b \\ 1 & 3 & 1 \end{pmatrix}$$

$$(A - \lambda I)v = 0$$

$$\begin{bmatrix} 1-\lambda & a & 4 \\ 0 & 2-\lambda & b \\ 1 & 3 & 1-\lambda \end{bmatrix} \begin{bmatrix} 3 \\ 1 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 3-3\lambda + a + 16 \\ 2-\lambda + 4b \\ 3 + 3 + 4 - 4\lambda \end{bmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$10 - 4\lambda = 0$ $\qquad \lambda = 2.5$

$2 - \lambda + 4b = 0$ $\qquad b = \frac{1}{8}$

$-\frac{1}{2} + 4b = 0$

$19 + a - 3\lambda = 0$

$19 + a - 7.5 = 0$ $\qquad a = -11.5$

$a = -11.5$ $\qquad b = \frac{1}{8}$ $\qquad \underline{\underline{\lambda = 2.5}}$

c.