COLLEGE
Tanta Engineering

إعدادي 2020

الرياضيات

جبر - الحل الرقمي للمعادلات الغير خطية

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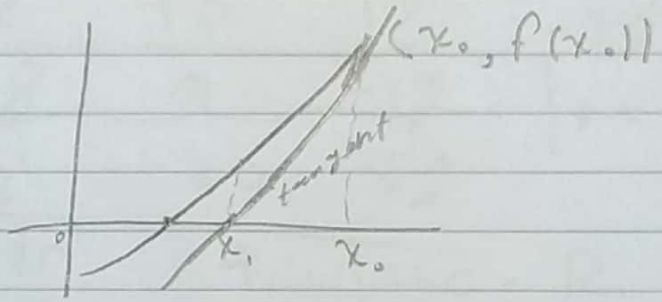
Numerical Solve for nonlinear eqns

Find a root for the eqn $f(x)=0$

1] Newton's Method

1 - Start with an intial root x_0.
2 - Improve this root using the tangent at x_0 to the curve of $f(x)$ as for lows



the eqn is:

$f(x) - f(x_0) = f'(x_0)(x - x_0)$, at $x = x_1$, $f(x) = 0 \Rightarrow$

$0 - f(x_0) = f'(x_0)(x_1 - x_0)$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

OR $$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, \ldots$$

2] Simple Iterative Method

1- start with x_0

2- Rewrite $f(x)=0$ as $x=\Phi(x)$ Provided

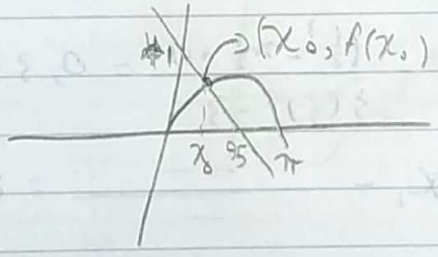
$$|\Phi'(x_0)|<1 \Rightarrow$$

condition for convergence

3- Improve x_0 from $x_{n+1}=\Phi(x_n)$, $n=1,2,3,\dots$

Ex:- Find a root for $f(x)=\sin x+2x-1=0$

$$\sin x=1-2x$$



$x_0=0,3$

$$x=\frac{1-\sin x}{2}$$

$$|\Phi'(x)|=|-\tfrac{1}{2}\cos(x)|$$

$$|\Phi'(x_0)|=|-\tfrac{1}{2}\cos(0,3)| \leq \tfrac{1}{2}<1$$

$$x_{n+1}=\tfrac{1}{2}(1-\sin x_n),\ n=0,1,2\dots$$

$n=0$ $\quad x_1=\frac{1}{2}(1-\sin 0,3)=0,35\dots$

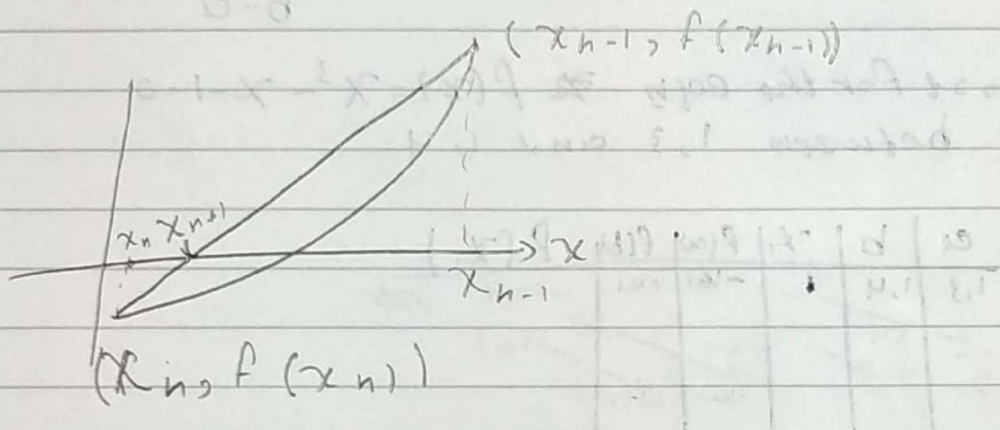
$x_2=0,3274945$

$x_3=0,3391618$

3] Secant Method

1- Start with $x_n, x_{n-1} \ni f(x_n) \times f(x_{n-1}) < 0$

2- Improve using secant eqn to get x_{n+1} as



$$\frac{f(x) - f(x_n)}{x - x_n} = m = \frac{f(x_{n-1}) - f(x_n)}{x_{n-1} - x_n}$$

at $x = x_{n+1} \rightarrow f(x) = 0 \Rightarrow$

$$x_{n+1} = x_n - \frac{f(x_n)}{m} \quad , n = 1, 2, 3, \ldots$$

4] False position Method

1- Start with $a, b \Rightarrow f(a) \cdot f(b) > 0$

2- Improve from

$$x_1 = a - \frac{f(a)}{m}, \quad m = \frac{f(b) - f(a)}{b - a}$$

Ex: find a root for the eqn $f(x) = x^3 - x - 1 = 0$ between 1,3 and 1,4

a	b	x_1	f(a)	f(b)	f(x_1)
1,3	1,4	.	-Val	+Val	-Val
1,4	.	.	-Val	+Val	
.	.		-Val	+Val