COLLEGE
Tanta Engineering

إعدادي 2020

تكامل

التكامل بالتعويض

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Integration

Types of integration

1) Integration by Sub.

2) Integration by Parts.

3) Integration by partial fraction.

Notes of Integration

- $\int (f(x))^n f'(x)\,dx = \frac{(f(x))^{n+1}}{n+1} + C$
- $\int \sqrt{f(x)} \cdot f'(x)\,dx = \frac{2}{3}(f(x))^{3/2} + C$
- $\int \frac{f'(x)}{f(x)} = \ln|f(x)| + C$

"Integration by sub"

1) $I = \int \frac{x^m}{(ax+b)^n}\,dx$

let $z = ax + b$

$\therefore x = \frac{z-b}{a}$

$\therefore dx = \frac{dz}{a}$

$I = \int \frac{(\frac{z-b}{a})^m}{z^n} \cdot \frac{dz}{a}$

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Ex: $I = \int \frac{x^5}{(2+3x)^4}\,dx$ let $z = 2+3x$

$\therefore x = \frac{z-2}{3}$

$\therefore dx = \frac{dz}{3}$

$I = \int \frac{(z-2)^5}{81(z)^4}\,dz = \frac{1}{81}\int \frac{(z-2)^5}{z^4}\,dz$

$= \frac{1}{81}\int \frac{z^5 - 6z^4 + 12z^3 - 8}{z^4}\,dz$

$= \frac{1}{81}\int \left(\frac{1}{z} - \frac{6}{z^2} + \frac{12}{z^3} - \frac{8}{z^4}\right)dz$

$= \frac{1}{81}\int \left(\frac{1}{z} - 6z^{-2} + 12z^{-3} - 8z^{-4}\right)dz$

$= \frac{1}{81}\left(\ln|z| + \frac{6}{z} - \frac{6}{z^2} + \frac{8}{3z^3} + C\right)$

$= \frac{1}{81}\left(\ln|2+3x| + \frac{6}{2+3x} - \frac{6}{(2-3x)^2} + \frac{8}{3(2-3x)^3}\right) + C$

2] $I = \int \frac{1}{x^m(ax+b)^n}\,dx$ let $z = \frac{ax+b}{x}$

$\therefore xz - ax = b$

$x(z-a) = b$

$x = \frac{b}{z-a}$

$\therefore dx = \frac{-b}{(z-a)^2}\,dz$

Ex: $I = \int \frac{dx}{x^3(1+x)^4}$ let $z = \frac{1+x}{x}$ $\therefore x = \frac{1}{z-1}$ $\therefore dx = \frac{-dz}{(z-1)^2}$

$I = -\int \frac{1}{\left(\frac{1}{z-1}\right)^3\left(1+\frac{1}{(z-1)}\right)^4}\cdot\frac{dz}{z-1} = -\int \frac{(z-1)^5}{z^4}$

$= \int \frac{z^5 - 5z^4 + 10z^3 - 10z^2 + 5z - 1}{z^4}\,dz = \cdots$

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3] $I = \int \frac{1}{x(ax^n+b)}\,dx$ let $z^{-1} = x^n$

$\frac{1}{z} = x^n \therefore x = z^{-\frac{1}{n}}$

$\therefore dx = \frac{-1}{n} z^{-\frac{1}{n}-1}\,dz$

Ex: $I = \int \frac{dx}{x(x^5+3)}$ let $x^5 = \frac{1}{z} \therefore x = z^{-\frac{1}{5}}$

$dx = \frac{-1}{5} z^{-\frac{6}{5}}\,dz$

$I = \int \frac{-dz}{5(3z+1)} = -\frac{1}{5}\int \frac{dz}{3z+1} = -\frac{1}{15}\ln|3z+1| + C$

$= -\frac{1}{15}\ln\left|\frac{3}{x^5}+1\right| + C$

"Some Examples..."

1) $I = \int \frac{e^x}{e^x-1}\,dx$ let $u = e^x$, $du = dx\, e^x$

$= \int \frac{du}{u-1} = \ln|u-1| + C = \ln|ex-1| + C$

2) $I = \int \frac{dx}{x(1-\ln x)}$ $u = \ln x \therefore du = \frac{dx}{x}$

$= \int \frac{du}{(1-u)} = -\ln|1-u| + C = -\ln|1-\ln x| + C$

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