سنتر فيوتشر

Subject: رياضة « اعدادي »

Chapter: النهايات

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النهايات

limits

$\frac{1}{0} = \infty$ $\qquad$ $\frac{1}{\infty} = 0$ ،

(رقم اصغر سرداهر) $^{\infty} = 0$

(رقم اكبر سرداهر) $^{\infty} = \infty$

التعويض المباشر

* $\lim_{x \to 0} 2x+3 = 3$

$\lim_{x \to \infty} \frac{3}{x+1} = 0$

. $\lim_{x \to \infty} \left(\frac{1}{2}\right)^x = 0$

$\lim_{x \to \infty} 2^{3x} = \infty$

$\lim_{x \to 0} \frac{2}{x} + 4 =$

$\infty + 4 = \infty$

$\lim_{x \to \infty}$

$\lim_{x \to 0} \frac{\sin \alpha x}{x} = \alpha$

$\tan x = \frac{\sin x}{\cos x}$

$\lim_{x \to 0} \frac{\tan \alpha x}{x} = \alpha$

$\frac{1}{\cos x} = \sec x$ (تا)

$\frac{1}{\sin x} = \operatorname{cosec} x$ (تا)

$\frac{1}{\tan x} = \cot x$ (تا)

①

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$\sin 2X = 2\sin X \cos X$

$\cos 2X = 2\cos^2 X - 1$

$= 1 - 2\sin^2 X = \cos^2 X - \sin^2 X$

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- $\sin^2 X + \cos^2 X = 1$

$1 + \tan^2 X = \sec^2 X$

$\cot^2 X + 1 = \operatorname{cosec}^2 X$

Find (evalute)

$\lim_{x \to 0} \frac{\sin 3X}{X} = 3$

$\lim_{x \to 0} \frac{\sin 2X}{5X} = 2/5$

$\lim_{x \to 0} \frac{\sin 2X^2}{X^2} = 2$

$\lim_{x \to 0} \frac{\tan 5X}{X} = 5$

$\lim_{x \to 0} \frac{\tan 4X}{\sin 3X} = \frac{4}{3}$

$\lim_{x \to 0} \frac{\sin^2 3X}{X^2}$

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$$\lim_{x \to 0} \left(\frac{\sin 3x}{x} \right)^2 = 9$$

$$\lim_{x \to 0} \frac{\sin 5x}{\sin 3x} = 5/3$$

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*

$$\lim_{x \to 0} \frac{1 - \cos 2x}{x^2}$$

$\because 1 - \cos 2x = 2 \sin^2 x$

$$\lim_{x \to 0} 2 \left(\frac{\sin^2 x}{x^2} \right) = 2 \lim_{x \to 0} \left(\frac{\sin x}{x} \right)^2 = 2$$

$$\lim_{x \to 0} \frac{1 - \cos x}{x}$$

$\because 1 - \cos x = 2 \sin^2 \frac{x}{2}$

$$\lim_{x \to 0} \frac{2 \sin^2 \frac{x}{2}}{x} = 2 \lim_{x \to 0} \frac{\sin \frac{1}{2} x}{x} \cdot \frac{\sin \frac{x}{2}}{2}$$

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$$= 2 \lim_{x \to 0} \frac{\sin \frac{1}{2}x}{x} \lim_{x \to 0} \sin \frac{1}{2}x$$

$$2 \cdot \left(\frac{1}{2}\right) \cdot 0 = \text{Zero}$$

$$\lim_{x \to 0} \frac{1 - \cos x}{x^2}$$

$$\lim_{x \to 0} \frac{2 \sin^2 \frac{x}{2}}{x^2} = 2 \lim_{x \to 0} \left(\frac{\sin \frac{x}{2}}{x}\right)^2$$

$$= 2 \left(\frac{1}{2}\right)^2 = \frac{1}{2}$$

$$\lim_{x \to 0} \frac{\sin[\sin(5x)]}{x}$$

$$\lim_{x \to 0} \frac{\sin[\sin(5x)]}{\sin(5x)} \cdot \frac{\sin 5x}{x}$$

$$1 \cdot * 5 = 5$$

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$\lim_{x \to \infty} x \sin\left(\frac{1}{x}\right)$

$0 \cdot \infty$ صيغة غير معرفة

$\frac{1}{x} \to \frac{1}{\infty}$

$\lim \frac{\sin(1/x)}{1/x} = 1$

$\frac{1}{x} \to \frac{1}{0}$

$\lim_{x \to \infty} x \tan\left(\frac{5}{x}\right)$

$\lim \frac{\tan(5/x)}{1/x} = 5$

$\frac{1}{x} \to \frac{1}{0}$

قاعدة السندويش

$g(x) \leq f(x) \leq h(x)$

$\lim h(x) = L$

$\lim g(x) = L$

$\therefore \lim f(x) = L$

$-1 \leq \cos x \leq 1$

$-1 \leq \sin x \leq 1$

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$$\lim_{x \to \infty} \frac{\sin x}{x} \quad \text{---} \quad (1)$$

$$\therefore \frac{-1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}$$

$$\lim_{x \to \infty} \frac{1}{x} = 0 \,, \qquad \lim_{x \to \infty} \frac{-1}{x} = 0$$

$$\therefore \lim_{x \to \infty} \frac{\sin x}{x} = 0$$

$$\lim_{x \to \infty} \frac{\cos(x^2+4)}{x} \quad \text{---} \quad (1)$$

$$\frac{-1}{x} \leq \frac{\cos(x^2+4)}{x} \leq \frac{1}{x}$$

$$\lim_{x \to \infty} \frac{1}{x} = 0 \,, \qquad \lim_{x \to \infty} \frac{-1}{x} = 0$$

$$\lim_{x \to \infty} \frac{\cos(x^2+4)}{x} = 0$$

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$\lim_{x \to \infty} x \sin\left(\frac{1}{x}\right)$ & $\lim_{x \to 0} x \sin\left(\frac{1}{x}\right)$

$\lim_{x \to \infty} x \sin(1/x)$

$\lim_{\frac{1}{x} \to 0} \frac{\sin(1/x)}{1/x} = 1$

$-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$

$-x \leq x \sin\left(\frac{1}{x}\right) \leq x$

$\lim_{x \to 0} x = 0, \quad \lim_{x \to 0} -x = 0$

$\therefore \lim_{x \to 0} x \sin\left(\frac{1}{x}\right) = 0$

$\lim_{x \to 2} \frac{\sin(x^2-4)}{x-2}$

$\lim_{x \to 2} \frac{\sin(x^2-4)}{(x-2)(x+2)} \cdot (x+2)$

$1 \cdot 4 = 4$ #

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$$\lim_{x \to 0} \frac{\sin\sqrt{x}}{x}$$

اذا كان

$$\lim_{x \to 0} \frac{\sin\sqrt{x}}{\sqrt{x}} \cdot \frac{1}{\sqrt{x}}$$

$$1 * \infty = \infty$$

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④ اذا كان الناتج $\frac{0}{0}$ أو $\frac{\infty}{\infty}$ خلال البسط والمقام وتختصر

$$\lim_{x \to 2} \frac{x^2 - 4}{x^2 - x - 2}$$

بالتعويض $\frac{0}{0}$

$$\lim_{x \to 2} \frac{(x-2)(x+2)}{(x-2)(x+1)} = \frac{4}{3}$$

$$\lim_{x \to 1} \frac{x^3 - 1}{x^2 - 1}$$

$$\lim_{x \to 1} \frac{(x-1)(x^2+x+1)}{(x-1)(x+1)} = 3/2$$

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اذا كان الناتج $\frac{\infty}{\infty}$ نقسم البسط والمقام على أكبر أس

$$\lim_{x \to \infty} \frac{x^3 + 4x + 7}{2x^3 + 5x^2 - 9}$$

بالتعويض $\frac{\infty}{\infty}$

$$\lim_{x \to \infty} \frac{1 + \frac{4}{x^2} + \frac{7}{x^3}}{2 + \frac{5}{x} - \frac{9}{x^3}} = \frac{1 + 0 + 0}{2 + 0 - 0} = \frac{1}{2}$$

$$\lim_{x \to \infty} \frac{2x + 8}{x^3 + 9x - 1}$$

بالتعويض $\frac{\infty}{\infty}$

$$\lim_{x \to \infty} \frac{\frac{2}{x^2} + \frac{8}{x^3}}{1 + \frac{9}{x^2} - \frac{1}{x^3}} = \frac{0 + 0}{1 + 0 - 0} = 0$$

درجة البسط أقل من درجة المقام ← الناتج = صفر

$$\lim_{x \to \infty} \frac{x^2 + 7}{x - 1}$$

اكبر أس $\div x^2$

$$\lim_{x \to \infty} \frac{1 + \frac{7}{x^2}}{\frac{1}{x} - \frac{1}{x^2}} = \frac{1 + 0}{0 - 0} = \frac{1}{0} = \infty$$

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$$\lim_{x\to\infty} \frac{x^3+7}{8x^3+9} \longrightarrow 3/8$$

$$\lim_{x\to\infty} \frac{2x+5}{x-3} = 2$$

$$\lim_{x\to\infty} \frac{x+3}{x^2+9} = 0 \qquad \lim_{x\to\infty} \frac{x^2+4}{x-1} = \infty$$

اذا كان الناتج $\infty - \infty$ نعوض ≠ الناتج

$$\lim_{x\to\infty} \sqrt{x+7} - \sqrt{2x+3}$$

بالطرق

$$\lim_{x\to\infty} \frac{\left(\sqrt{x+7} - \sqrt{2x+3}\right)\left(\sqrt{x+7} + \sqrt{2x+3}\right)}{\sqrt{x+7} + \sqrt{2x+3}}$$

$$\lim_{x\to\infty} \frac{x+7-(2x+3)}{\sqrt{x+7}+\sqrt{2x+3}} = \frac{4-x}{\sqrt{x+7}+\sqrt{2x+3}}$$

$$\lim \frac{4/x - 1}{\sqrt{\frac{1}{x} + 7/x^2} + \sqrt{\frac{2}{x} + \frac{3}{x^2}}} = \frac{-1}{0} = -\infty$$

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قاعدة اويلر [مهمة جدا]

$$\lim_{x \to a} [1+f]^{g}$$

نرى

$$\lim f = 0 \quad , \quad \lim g = \infty$$

$$\lim f \cdot g = C$$

$$\therefore \lim [1+f]^{g} = e^{C} \qquad e \approx 2.7$$

$$\lim_{x \to 0} [1+2x]^{\frac{1}{x}}$$

$$\lim_{x \to 0} 2x = 0 \quad , \quad \lim_{x \to 0} \frac{1}{x} = \infty$$

$$\lim_{x \to 0} 2x \cdot \frac{1}{x} = 2$$

$$\therefore \lim_{x \to 0} (1+2x)^{\frac{1}{x}} = e^{2}$$

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Ex

$$\lim_{x \to 0} \underbrace{[1 + \sin x]^{\frac{3}{x}}}_{\to}$$

$\lim_{x \to 0} \sin x = 0$, $\lim_{x \to 0} \frac{3}{x} = \infty$

$$\lim_{x \to 0} 3 \frac{\sin x}{x} = 3$$

$$\therefore \lim_{x \to 0} (1 + \sin x)^{3/x} = e^3$$

$$\lim_{x \to 0} \underbrace{[1 + \sin x]^{2 \cot x}}_{\to}$$

$\lim_{x \to 0} \sin x = 0$ $\lim_{x \to 0} 2 \cot x = 2 \lim_{x \to 0} \frac{\cos x}{\sin x} = \frac{2}{0} = \infty$

$$\lim_{x \to 0} \cancel{\sin x} \cdot \frac{2 \cos x}{\cancel{\sin x}} = 2$$

$$\lim_{x \to 0} (1 + \sin x)^{2 \cot x} = e^2$$

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Ex $\lim_{x \to \infty} \left(\frac{x+3}{x-1}\right)^{3x}$

$\lim \left[\frac{x-1+4}{x-1}\right]^{3x}$

$\lim_{x \to \infty} \left[\frac{x-1}{x-1} + \frac{4}{x-1}\right]^{3x}$

1

$\lim_{x \to \infty} \frac{4}{x-1} = 0$, $\lim_{x \to \infty} 3x = \infty$

$\lim_{x \to \infty} \frac{4(3x)}{x-1} = 12$

$\lim_{x \to \infty} \left(\frac{x+3}{x-1}\right)^{3x} = e^{12}$

$\lim \left(\frac{x^2-1}{x^2-3}\right)^{x}$

$\lim_{x \to \infty} \left[\frac{x-3}{x-3} + \frac{2}{x^2-3}\right]^{x}$

$\lim_{x \to \infty}$

1

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$$\lim_{x \to \infty} \frac{2x}{x^2 - 3} = 0$$

$$\therefore \lim_{x \to \infty} \left(\frac{x^2 - 1}{x^2 - 3} \right)^x = e^0 = 1$$

$$\lim_{x \to 0} (\cos x)^{\frac{1}{x^2}}$$

$$\lim_{x \to 0} \left[1 - 2\sin^2 \frac{x}{2} \right]^{\frac{1}{x^2}}$$

$$\lim_{x \to \infty} -2\sin^2 \frac{x}{2} = 0 \qquad \lim_{x \to 0} \frac{1}{x^2} = \infty$$

$$\lim_{x \to 0} \frac{-2\sin^2 \frac{x}{2}}{x^2} = -2 \lim_{x \to 0} \left(\frac{\sin \frac{1}{2}x}{x} \right)^2$$

$$= -2\left(\frac{1}{2}\right)^2 = -1/2$$

$$\lim_{x \to 0} (\cos x)^{\frac{1}{x^2}} = e^{-\frac{1}{2}}$$

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$$\lim_{x \to 0} \left(\sqrt{\cos x}\right)^{\frac{1}{x}}$$

$$\lim_{x \to 0} (\cos x)^{\frac{1}{2x}} = \lim_{x \to 0} \left[1 - 2\sin^2 \frac{x}{2}\right]^{\frac{1}{2x}}$$

$$\lim_{x \to 0} -2\sin^2 \frac{x}{2} = 0 \qquad \lim_{x \to 0} \frac{1}{2x} = \infty$$

$$\lim_{x \to 0} \frac{-2\sin^2 \frac{x}{2}}{2x} = \lim_{x \to 0} -\frac{\sin \frac{x}{2}}{x} \cdot \sin \frac{x}{2}$$

$$= \left(-\tfrac{1}{2}\right)(0) = 0$$

$$\therefore \lim_{x \to 0} \left(\sqrt{\cos x}\right)^{\frac{1}{x}} = e^0 = 1$$

$$\lim_{x \to 0} (\sec^3 x)^{\frac{1}{x^2}}$$

$$\lim_{x \to 0} (\sec^2 x)^{\frac{3}{2x^2}} = \lim_{x \to 0} \left(1 + \tan^2 x\right)^{\frac{3}{2x^2}}$$

$$\lim_{x \to 0} \tan^2 x = 0 \qquad \lim_{x \to 0} \frac{3}{2x^2} = \infty$$

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$$\lim_{x \to 0} \frac{3 \tan^2 x}{2 x^2} = \lim_{x \to 0} \frac{3}{2} \left(\frac{\tan x}{x}\right)^2 = 3/2$$

$$\lim_{x \to 0} (\sec^3 x)^{1/x^2} = e^{3/2}$$

$$\lim_{x \to \infty} \ln \left[\frac{x+3}{x-5} \right]$$

$$\lim_{x \to \infty} \ln \left[\frac{x+3}{x-5} \right]$$

$$\ln \lim_{x \to \infty} \left(\frac{x-5}{x-5} + \frac{8}{x-5} \right)$$

$$\ln \lim_{x \to \infty} \left[1 + \frac{8}{x-5} \right]$$

$$\lim_{x \to \infty} \frac{8x}{(x-5)} = 8$$

$$\therefore \lim_{x \to \infty} x \ln \left(\frac{x+3}{x-5} \right) = \ln e^8 = 8$$

$$\ln x^y = y \ln x$$

$$\ln \left(\frac{x}{y}\right) = \ln x - \ln y$$

$$\ln (xy) = \ln x + \ln y$$

$$\ln e^{\alpha} = \alpha$$

$$e^{\ln \alpha} = \alpha$$

$$\frac{\ln (x)}{\ln y} \neq \ln \left(\frac{x}{y}\right)$$

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h-w

$$\lim_{x \to \infty} \left[\frac{x+3}{x+2}\right]^x$$

$$\lim_{x \to 0} (1 + \tan x)^{\frac{1}{x}}$$

$$\lim_{x \to \frac{\pi}{2}} (1 + \cos x)^{2 \tan x}$$

$$\lim_{x \to 0} \left(1 + \frac{1 - \cos x}{x}\right)^{\frac{1}{x}}$$

$$\lim_{x \to \infty} (x+3) - \sqrt{x^2+4}$$

$$\lim_{x \to 0} \frac{x}{\sin \sqrt{x}}$$

$$\lim_{x \to 0} \frac{\sin^2 x}{\tan^2 3x}$$

$$\lim_{x \to \infty} \frac{\cos \sqrt{x}}{x^2+4}$$

$$\lim_{x \to \infty} (2x+3)^x$$

$$\lim_{x \to \infty} \left(\frac{-}{2}\right)^x$$

$$\lim_{x \to \infty} \sin\left(\frac{1}{x}\right)$$

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$$\lim_{x \to \pi} \frac{\sin 3x}{\sin 2x}$$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin 3x = \sin(x+2x) = \sin x \cos 2x + \cos x \sin 2x$$

$$\therefore \lim_{x \to \pi} \frac{\sin x \cos 2x + \cos x \sin 2x}{\sin 2x}$$

$$\lim_{x \to \pi} \cos x + \frac{\sin x \cos 2x}{\sin 2x}$$

$$-1$$

$$-1 + \lim_{x \to \pi} \frac{\sin x \cos 2x}{2 \sin x \cos x}$$

$$-1 + \frac{1}{-2} = -3/2$$

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