سنتر فيوتشر

Subject: رياضة "اعدادي"

Chapter: الاستنتاج الرياضي "٢"
تابع الاستنتاج

Mob: 0112 3333 122

0109 3508 204

Prove That

مجموع مكعبات ثلاث اعداد متتاليه تقبل القسمه على 9

$$n^3+(n+1)^3+(n+2)^3 \div 9$$

$n=1$ $\quad 1^3+2^3+3^3=36=9*4$

تقبل القسمه على 9

$n=k$ asume

$$k^3+\underline{(k+1)^3+(k+2)^3}=9f$$

required to prov

$(k+1)^3+(k+2)^3+(k+3)^3$ divisible by 9

$$\underline{(k+1)^3+(k+2)^3}+(k+3)^3$$

$$=9f-k^3+(k+3)^3 \quad \boxed{1}$$

$$= 9f - k^3 + \left[k^3 + 3(k^2)(3) + 3(k)(9) + 27\right]$$

$$9f - \cancel{k^3} + \cancel{k^3} + 9k^2 + 27k + 27]$$

$$= 9\left[f + k^2 + 3k + 3\right]$$

المقدار يقبل القسمة على 9

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

ملحوظة

Prove that

$x^{2n+1} - y^{2n+1}$ divisble by $x-y$

ط

$\underline{n=1}$ $\quad x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

يقبل القسمة على $x-y$

asume $x^{2k+1} - y^{2k+1}$ divisble by $x-y$

$$x^{2k+1} - y^{2k+1} = (x-y)\, f(x,y) \quad \boxed{2}$$

required $x^{2k+3} - y^{2k+3}$ divisible by $(x-y)$

$$x^{2k+3} - y^{2k+3} = x^2 \cdot x^{2k+1} - y^2 \cdot y^{2k+1}$$

$$= x^2 \left[\underline{x^{2k+1} - y^{2k+1}}\right] + x^2 y^{2k+1} - y^2 y^{2k+1}$$

$$= x^2 \left[(x-y) f(x,y)\right] + y^{2k+1} \left(x^2 - y^2\right)$$

$$= \underline{x^2 (x-y) f(x,y)} + (x-y)(x+y)\, y^{2k+1}$$

$$= (x-y)\left[x^2 f(x,y) + (x+y)\, y^{2k+1}\right]$$

∴ الطرف يقبل القسمة على $\underline{x-y}$

Prove that

$$2^n > n^2 \qquad n \geq 5$$

ط

$\underline{n=5}$ $\quad$ R.H.S $= 5^2 = 25$

L.H.S $= 2^5 = 32$

∴ العلاقه صحيحه عند $n=5$

asume relation is true at $n=k$

$$2^k > k^2$$

required to prove $\quad 2^{k+1} > (k+1)^2$

$\because \quad 2^k > k^2$ $\qquad$ بالضرب 2

$$2 \cdot 2^k > 2k^2$$

$$2^{k+1} > 2k^2 > (k+1)^2$$

$$2k^2 - (k+1)^2 = k^2 - 2k - 1$$

$\boxed{٤}$

لمعرفة المقدار موجب دائما

$2k-2>0$

علشان يكون موجب $\underline{k>1}$ ومن $k \geq 5$

$\therefore 2k^2 > (k+1)^2$, $2^{k+1} > 2k^2$

$\therefore 2^{k+1} > (k+1)^2$

Prove that

$n! > n^2$ لكل $n \geq 4$

$\underline{n=4}$

R.H.S $= 4^2 = 16$

L.H.S $= 4! = 24$

العلاقة صحيحة عند $n=4$

asume $k! > k^2 \longrightarrow (1)$

required $(k+1)! > (k+1)^2$

بضرب المعادلة (1) بالعدد $(k+1)$

$(k+1)k! > (k+1)k^2$ $\boxed{2}$

$(k+1)! > k^3+k^2 > (k+1)^2$

$\because k^3+k^2-(k+1)^2 = k^3-2k-1$

$\because 3k^2-2>0 \qquad k>\sqrt{2/3}$

$k>4$ معرّف

$\therefore (k+1)k^2 > (k+1)^2 \quad , \quad (k+1)! > (k+1)k^2$

$\therefore (k+1)! > (k+1)^2$

---

hw

$\frac{1}{1\cdot 3} + \frac{1}{3\cdot 5} + \frac{1}{5\cdot 7} \ldots$ to nth term

nth term

$\frac{1}{(2n-1)(2n+1)}$

$\frac{8}{3\times 5} - \frac{12}{5\times 7} + \frac{16}{7\times 9} - \ldots$ to nth term

$\boxed{7}$

$$\frac{4n+4}{(2n+1)(2n+3)}(-1)^{n-1}$$

مش محتاج في ن

Prove that

$$(1+x)^n \geq 1+nx$$

$x \geq -1$

Prove that

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}\cdots \text{ to } n\text{th term} = \frac{n}{2n+1}$$

Prove that

$$\sum_{r=1}^{n} r\, r! = (n+1)!-1$$

Prove that

$$1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}\cdots+\frac{1}{\sqrt{n}} \geq \sqrt{n}$$

$n \geq 1$

$\boxed{V}$

Prove that

$5^n - 3^n$ is divisible by 2

Prove that

$$|\sin nx| \leq |n \sin x|$$

Prove that

$4^n + 15n - 1$ divisible by 9

$\boxed{\wedge}$

Prove that

$$\frac{8}{3\cdot 5} - \frac{12}{5\cdot 7} + \frac{16}{7\cdot 9} \cdots\cdots + \frac{(-1)^{n+1}\,4\,(n+1)}{(2n+1)(2n+3)}$$

$$= \frac{1}{3} + \frac{(-1)^{n-1}}{2n+3}$$

$n=1$ $\quad R.H.S = \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$

$L.H.S = \frac{8}{3\times 5} = \frac{8}{15}$

العلاقة صحيحة عندما $n=1$

نفرض صحة العلاقة

$n=k$ $\quad n=k$

$$\frac{8}{3\cdot 5} - \frac{12}{5\cdot 7} \cdots + \frac{4(-1)^{k+1}(k+1)}{(2k+1)(2k+3)} = \frac{1}{3} + \frac{(-1)^{k-1}}{2k+3}$$

required المطلوب اثبات صحة العلاقة

$n=k+1$

$$\frac{8}{3\cdot 5} - \frac{12}{5\cdot 7} \cdots + \frac{4(-1)^{k+2}(k+2)}{(2k+3)(2k+5)} = \frac{1}{3} + \frac{(-1)^{k}}{2k+5}$$

L.H.S

$$\frac{8}{3\cdot 5} - \frac{12}{5\cdot 7} \qquad \frac{4(-1)^{k+1}(k+1)}{(2k+1)(2k+3)} + \frac{4(-1)^{k+2}(k+2)}{(2k+3)(2k+5)}$$

$$\frac{1}{3} + \frac{(-1)^{k-1}}{2k+3} + \frac{4(-1)^{k+2}(k+2)}{(2k+3)(2k+5)}$$

$$= \frac{1}{3} + \frac{(-1)^{k+2}}{(2k+3)}\left[\frac{4(k+2)}{2k+5} - 1\right]$$

$$\frac{1}{3} + \frac{(-1)^{k+2}}{2k+3}\left[\frac{4k+8-2k-5}{2k+5}\right]$$

$$\frac{1}{3} + \frac{(-1)^{k+2}}{\cancel{2k+3}}\left(\frac{\cancel{2k+3}}{2k+5}\right)$$

$$(-1)^{k+2} = (-1)^k \cancel{(-1)^2}$$

$$= \frac{1}{3} + \frac{(-1)^k}{2k+5}$$

$= R.H.S$

العلاقة صحيحة لجميع قيم n

10

Prove that

$$|\sin nx| \leq |n \sin x|$$

<u>n=1</u> R.H.S $= \sin x$

L.H.S $= \sin x$

<u>n=2</u> R.H.S $= |2\sin x|$

L.H.S $= |\sin 2x| \Rightarrow |2 \sin x \cos x|$

$|\cos x| \leq 1$ <u>ح ـ ف</u>

∴ العلاقة صحيحة عندما $n = 1, 2$

نفرض صحة العلاقة <u>$n = k$</u>

$$|\sin kx| \leq |k \sin x|$$

المطلوب اثبات صحة العلاقة $n = k+1$

<u>required</u> $|\sin (k+1)x| \leq |(k+1)\sin x|$

$\boxed{11}$

$$L.H.S = |\sin(kx+x)| = |\sin kx \cos x + \cos kx \sin x|$$

نحل

$$|x+y| \leq |x| + |y|$$

$$|\sin kx + x| \leq |\sin kx \cos x| + |\cos kx \sin x|$$

$$\leq |\sin kx| + |\sin x|$$

$$\leq |k \sin x| + |\sin x|$$

$$\leq |(k+1)\sin x|$$

∴ العلاقة صحيحة لجميع قيم n

Prove that

$$1^2 - 2^2 + 3^2 - 4^2 \cdots \cdots \underset{(2n-1)^2}{\uparrow} - (2n)^2 = -n(2n+1)$$

ك

$n=1$

$$R.H.S = -(1)(3) = -3$$

$$L.H.S = 1^2 - 2^2 = -3$$

العلاقة صحيحة عندما $n=1$

١٢

$\underline{n=k}$

$$(1^2-2^2)+(3^2-4^2)----+(2k-1)^2-(2k)^2$$
$$=-k(2k+1)$$

$\underline{\text{requried}}$ المطلوب إثبات : صحة العلاقة $n=k+1$

$$1^2-2^2+3^2-4^2---+(2k+1)^2-(2k+2)^2$$
$$=-(k+1)(2k+3)$$

L.H.S

$$=\underline{\underline{1^2-2^2+3^2-4^2--(2k-1)^2-(2k)^2}}+(2k+1)^2-(2k+2)^2$$

$$=-k(2k+1)+(2k+1)^2-(2k+2)^2$$

$$=-2k^2-k+\cancel{4k^2}+4k+1-\cancel{4k^2}-8k-4$$

$$=-2k^2-5k-3$$

$$=-(k+1)(2k+3)=R.H.S$$

العلاقة صحيحة لجميع قيم $\underline{\underline{n}}$

Prove that $3n^5+5n^3+7n$ divisible by 15

أثبت أن المقدار يقبل القسمة على 15

$\underline{n=1}$ $\quad 3+5+7=15$

العلاقة صحيحة عند $n=1$

$\underline{n=k}$

نفرض صحة العلاقة $n=k$

$$3k^5+5k^3+7k=15f$$

$\underline{\text{required}}$ $\quad n=k+1$

المطلوب إثبات صحة العلاقة $n=k+1$

$$3(k+1)^5+5(k+1)^3+7(k+1)$$

نثبت القسمة على 15

$$3(k+1)^5+5(k+1)^3+7=3\left[k^5+5k^4+10k^3+10k^2+5k+1\right]$$

$$+5(k^3+3k^2+3k+1)+7(k+1)$$

$\therefore 3(k+1)^5 + 5(k+1) + 7$

$= \underline{3k^5 + 5k^3 + 7k} + 14k + 15k^4 + 30k^2$

$+ 15k + 3 + 15k^2 + 15k + 5 + 7$

$= 15f + 15k^4 + 30k^3 + 45k^2 + 30k + 15$

$= 15[f + k^4 + 2k^3 + 3k^2 + 2k + 1]$

∴ المقدار يقبل القسمة على 15

---

prove that

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 - - - + x^n$

$\underline{n=1}$ $R.H.S = 1+x,$ $L.H.S = 1+x$

العلاقة صحيحة عند $n=1$

asume relation is true at $n=k$

$(1+x)^k = 1 + kx + \frac{k(k-1)}{2!}x^2 - - - + x^k$

required to prove

$(1+x)^{k+1} = 1 + (k+1)x + \frac{k(k+1)}{2!}x^2 - - - + x^{k+1}$

$$\because (1+x)^{k+1} = (1+x)(1+x)^k$$

$$= (1+x)\left[1 + kx + \frac{k(k-1)}{2!}x^2 \cdots + x^k\right]$$

$$= 1 + kx + \frac{k(k-1)}{2!}x^2 \cdots + x^k$$

$$+ x + kx^2 + \cdots + x^{k+1}$$

$$= 1 + (kx + x) + \left[\frac{k(k-1)}{2!} + k\right]x^2 \cdots + x^{k+1}$$

$$= 1 + (k+1)x + \left[\frac{k(k-1) + 2k}{2!}\right]x^2 + \cdots + x^{k+1}$$

$$= 1 + (k+1)x + \frac{k(k+1)}{2!}x^2 \cdots + x^{k+1}$$

$$= R.H.S$$

: العلاقة صحيحة لجميع n

Prove that

$$(1+nx) \leq (1+x)^n$$

at $n=1$ $\quad R.H.S = 1 + x$

$\quad L.H.S = 1 + x$

at $n=2$ $\quad R.H.S \quad (1+x)^2 = 1 + 2x + x^2$

$\quad L.H.S = 1 + 2x$

if $y = e^{ax}$

Prove that $\dfrac{d^n y}{d x^n} = a^n \cdot e^{ax}$

a

$\because y = e^{ax}$ $\qquad \therefore \dfrac{dy}{dx} = a e^{ax}$

at $n=1$ $\qquad \dfrac{dy}{dx} = a \cdot e^{ax}$

العلاقة صحيحة عند $n=1$

asume relation is true at $n=k$

$$\frac{d^k y}{d x^k} = a^k \cdot e^{ax}$$

required to Prove $\dfrac{d^{k+1} y}{d x^{k+1}} = a^{k+1} e^{ax}$

$$\because \frac{d^k y}{d x^k} = a^k \cdot e^{ax}$$

بالتفاضل بالنسبة لـ x

$$\frac{d^{k+1} y}{d x^{k+1}} = a^k \cdot a \cdot e^{ax} = a^{k+1} e^{ax}$$

: العلاقة صحيحة لجميع n

if $y = \dfrac{1}{ax+b}$

Show that

$$\frac{d^n y}{dx^n} = \frac{(-1)^n \, n! \, a^n}{(ax+b)^{n+1}}$$

<u>at $n=1$</u>

$$\frac{dy}{dx} = \frac{(-1)(1)\, a}{(ax+b)^2}$$

ممكن نفاضل

$$y = (ax+b)^{-1}$$

$$\frac{dy}{dx} = -(ax+b)^{-2} \cdot a = \frac{-a}{(ax+b)^2}$$

relation is true at $n=1$

assume relation is true at $n=k$

$$\frac{d^k y}{dx^k} = \frac{(-1)^k (k!)\, a^k}{(ax+b)^{k+1}} \quad (1)$$

1A

required to prove that

$$\frac{d^{k+1} y}{d x^{k+1}} = \frac{(-1)^{k+1} (k+1)! \, a^{k+1}}{(ax+b)^{k+2}}$$

نبفاضل الى دلسه ①

$$\frac{d^{k+1} y}{d x^{k+1}} = (-1)^k (k!) \, a^k (-k-1) (ax+b)^{-k-2} * a$$

$$= \frac{(-1)(-1)^k a^{k+1} \, k! \, (k+1)}{(ax+b)^{k+2}}$$

$$= \frac{(-1)^{k+1} a^{k+1} (k+1)!}{(ax+b)^{k+2}} = R.H.S$$

relation is true for all value f_n

Prove that

$$\sum_{r=1}^{n} \frac{1}{\sqrt{r}} > \sqrt{n} \qquad n > 1$$

at n=2 R.H.S $= \sqrt{2} = 1.4$

L.H.S $= \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} = 1.7$

العلاقة صحيحة عندما $n=2$

asume relation is true at $n=k$

$$\sum_{r=1}^{k} \frac{1}{\sqrt{r}} > \sqrt{k}$$

reqtired to prove

$$\sum_{r=1}^{k+1} \frac{1}{\sqrt{r}} > \sqrt{k+1}$$

$$\because \sum_{r=1}^{k} \frac{1}{\sqrt{r}} > \sqrt{k}$$

باضافة $\frac{1}{\sqrt{k+1}}$ للطرفين

$$\therefore \sum_{r=1}^{k} \frac{1}{\sqrt{r}} + \frac{1}{\sqrt{k+1}} > \sqrt{k} + \frac{1}{\sqrt{k+1}}$$

$$= \sum_{r=1}^{k+1} \frac{1}{\sqrt{r}} > \sqrt{k} + \frac{1}{\sqrt{k+1}}$$

$$\because \sqrt{k} + \frac{1}{\sqrt{k+1}} - \sqrt{k+1}$$

$$= \frac{\sqrt{k}\sqrt{k+1} + 1 - k - 1}{\sqrt{k+1}} = \frac{\sqrt{k}\sqrt{k+1} - k}{\sqrt{k+1}}$$

$$= \frac{\sqrt{k}\sqrt{k+1} - \sqrt{k}\sqrt{k}}{\sqrt{k+1}} = \frac{\sqrt{k}\left(\sqrt{k+1} - \sqrt{k}\right)}{\sqrt{k+1}} > 0$$

$$\therefore \sqrt{k} + \frac{1}{\sqrt{k+1}} > \sqrt{k+1}$$

$$\sum_{r=1}^{k+1} \frac{1}{\sqrt{r}} > \sqrt{k} + \frac{1}{\sqrt{k+1}}$$

$$\therefore \sum_{r=1}^{k+1} \frac{1}{\sqrt{r}} > \sqrt{k+1}$$

العلاقة صحيحة
لجميع n

Prove that

$$(1+x)^n \geqslant 1+nx$$

$\underline{n=1}$ R.HS $= 1+x$

L.H.S $= 1+x$

$\underline{n=2}$ R.HS $= \underline{1+2x}$

L.HS $= (1+x)^2$

$= \underline{x^2+2x+1}$

∴ العلاقة صحيحة عند n = 1, n = 2

asume relation true at n = k

$$(1+x)^k \geqslant 1+kx$$

requried to Prove

$$(1+x)^{k+1} \geqslant 1+(k+1)x$$

$$(1+x)^k \geq 1+kx$$

بالضربه $1+x$

$$(1+x)^{k+1} \geq (1+kx)(1+x)$$

$$\geq 1+kx+x+kx^2$$

$$\geq \underline{1+(k+1)x+kx^2}$$

$$\because 1+(k+1)x+kx^2 \geq 1+(k+1)x$$

$$\therefore (1+x)^{k+1} \geq 1+(k+1)x$$

Prove that

$$1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}} - - - - + \frac{1}{\sqrt{n}} \geq \sqrt{n}$$

$$n \geq 1$$

$$\frac{1}{1}+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\,-+\frac{1}{\sqrt{k}}+\frac{1}{\sqrt{k+1}}>$$

$$\sqrt{k}+\frac{1}{\sqrt{k+1}}$$

$$\therefore \sqrt{k}+\frac{1}{\sqrt{k+1}}-\sqrt{k+1}$$

$$=\frac{\sqrt{k}\sqrt{k+1}+\cancel{1}-(k+\cancel{1})}{\sqrt{k+1}}$$

$$=\frac{\sqrt{k}\sqrt{k+1}-k}{\sqrt{k+1}}=\frac{\sqrt{k}\left(\sqrt{k+1}-\sqrt{k}\right)}{\sqrt{k+1}}>0$$

$$\therefore \sqrt{k}+\frac{1}{\sqrt{k+1}}>\sqrt{k+1}$$

$$\frac{1}{1}+\frac{1}{\sqrt{2}}-\,-+\frac{1}{\sqrt{k+1}}>\sqrt{k}+\frac{1}{\sqrt{k+1}}$$

$$\therefore \frac{1}{1}+\frac{1}{\sqrt{2}}-\,-+\frac{1}{\sqrt{k+1}}>\sqrt{k+1}$$

n=1 R.H.S $= \sqrt{1} = 1$

L.H.S $= 1$

n=2 R.H.S $= \sqrt{2} = 1.41$

L.H.S $= 1 + \frac{1}{\sqrt{2}} = 1.707$

∴ العلاقة صحيحة عند n=1 , n=2

asume relation is true at n=k

$$\frac{1}{1} + \frac{1}{\sqrt{2}} - - - + \frac{1}{\sqrt{k}} \geq \sqrt{k}$$

required to prove

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} - - + \frac{1}{\sqrt{k+1}} \geq \sqrt{k+1}$$

~~LHS=~~

$$\frac{1}{1} + \frac{1}{\sqrt{2}} + - - - + \frac{1}{\sqrt{k}} \geq \sqrt{k}$$

بإضافة $\frac{1}{\sqrt{k+1}}$ لكل من الطرفين