Lecture 1

رياضيات
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- Fourier series (expansions)
- laplace transform
- inverse
- Applications

introduction

Note :-

$$2\sin a\cos b = \sin(a-b) + \sin(a+b)$$
$$2\cos a\cos b = \cos(a-b) + \cos(a+b)$$
$$2\sin a\sin b = \cos(a-b) - \cos(a+b)$$
$$2\cos^2\theta = 1 + \cos 2\theta$$
$$2\sin^2\theta = 1 - \cos 2\theta$$

* التجزيء

$$\int x^2 \cos ax \, dx$$

Note $\int u\,dv = uv - \int v\,du$

Put $u = x^2$, $dv = \cos ax\,dx$

x^2	$+$	$\cos ax\,dx$
$2x$	$-$	$\frac{1}{a}\sin ax$
2	$+$	$\frac{-1}{a^2}\cos ax$
0		$\frac{-1}{a^3}\sin ax$

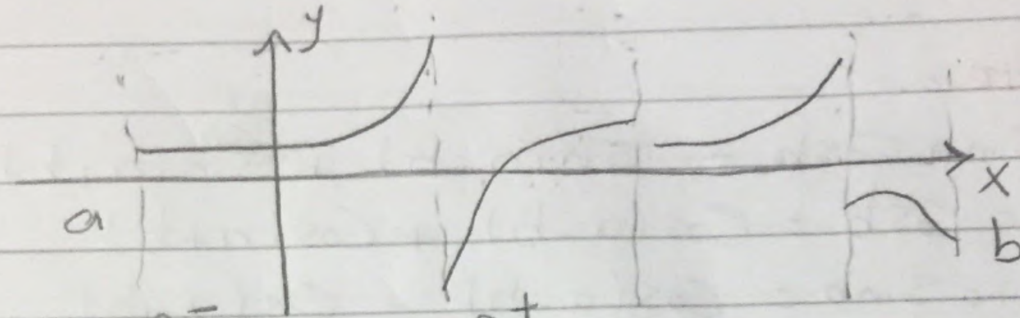
$$\therefore \int x^2\cos ax\,dx = \frac{x^2}{a}\sin ax + \frac{2x}{a^2}\cos ax - \frac{2}{a^3}\sin ax$$

* taylor's expansion

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n \qquad n = 0, 1, 2, \dots$$

$$f(x+h) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x_0)}{n!} h^n \qquad f^{(0)}(x_0) = f(x_0)$$

$f(x)$ is said to be a piecewise continuous f^n (a,b)



$f(x_0^-) = f(x_0^+)$. النهايتين موجودة

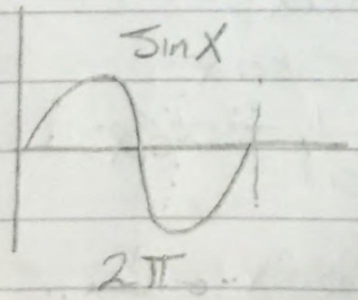
$f(x)$ is a periodic f^n of a period p when

$$f(x+p) = f(x)$$

$p > 0$, least number

like $\sin x$, $\cos x$, $\tan x$

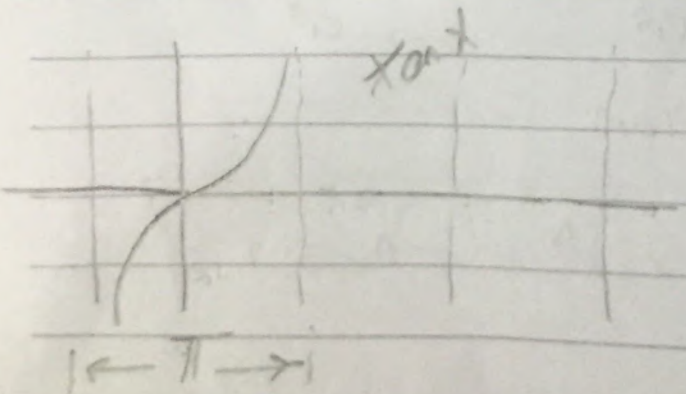
أقل عدد موجب $\leftarrow p$



$\therefore \sin(x+2\pi) = \sin x$

$\sin(x-2\pi) = \sin x$

$\sin(x+4\pi) = \sin x$



$\therefore \tan(x+\pi) = \tan x$

p

$\cos 2x \longrightarrow P = \frac{2\pi}{2} = \pi$

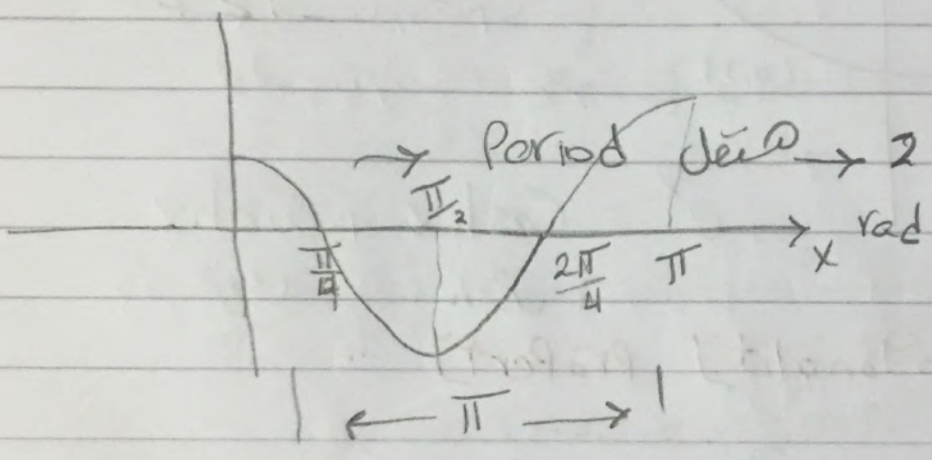
$\cos 3x \longrightarrow P = \frac{2\pi}{3}$

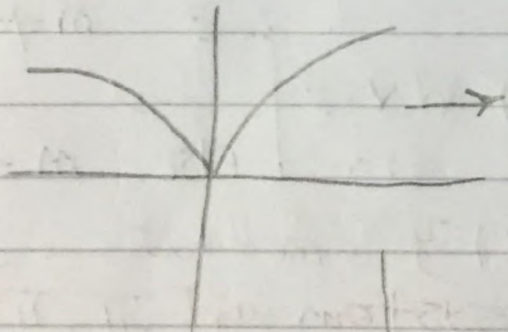
$\therefore \cos wx \longrightarrow P = \frac{2\pi}{w}$

$\therefore \cos w\left(x + \frac{2\pi}{w}\right) = \cos wx$

$\therefore f\left(x + \frac{2\pi}{w}\right) = f(x)$

Plot $\cos 2x$

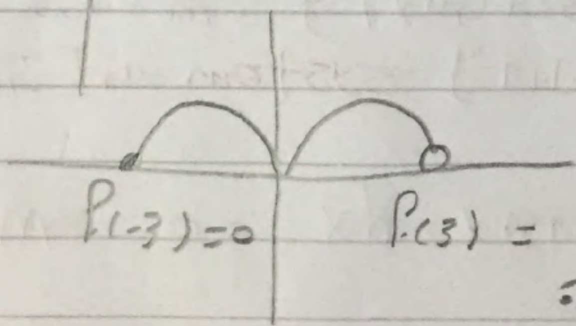




دالة زوجية لأنها متماثلة

$f(x) = +f(-x)$

For all $\in x$

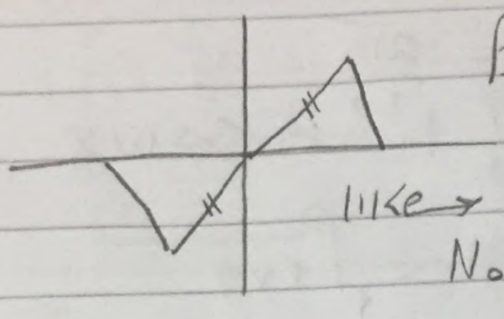


ليست دالة زوجية

Not even

Note $\rightarrow$ $f(x) = 3 + \frac{1}{x^2} + \cos x$

دالة زوجية لانه كل دالة من الدوال الثلاثة زوجية



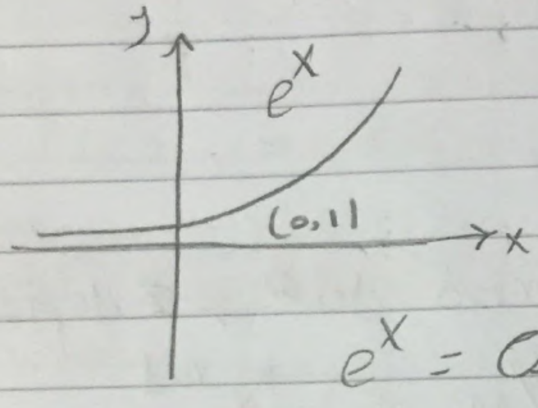
$f(x) = -f(-x)$

دالة فردية

like $\rightarrow$ $4x$, $\sin x$, $\tan x$

Note $x\cos x \rightarrow$ فردية

لانه x فردية



$e^x \leftarrow$ ليست زوجية

وليست فردية

$$e^x = \cosh x + \sinh x$$

زوجي فردي

* orthogonality property

$\{F_k(x)\}$, $k = 1, 2, \ldots$

if $\int_{-a}^{a} F_m(x) F_n(x)\, dx = \begin{cases} 0 & m \neq n \\ \text{else} & m = n \end{cases}$

Prove That $\{\sin(mx)\}$, $m = 1, 2, \ldots$
are orthogonality system over $[-\pi, \pi]$

[1] $\int_{-\pi}^{\pi} \sin mx \sin nx\, dx = 0 \quad m \neq n$

[2] $\int_{-\pi}^{\pi} (\sin nx)^2 \, dx \neq 0$

* proof $2\sin^2\theta = 1 - \cos 2\theta$

$$\int_{-\pi}^{\pi} \sin^2(nx) = \frac{1}{2}\int_{-\pi}^{\pi} [1 - \cos(2nx)] \, dx$$

Note $\longrightarrow \int_{-a}^{a} \text{odd } f^n = 0$

$$\int_{-a}^{a} \text{even } f^n = 2\int_{0}^{a} \text{even } f^n$$

$$\therefore \int_{-\pi}^{\pi} \sin^2(nx) = \int_{0}^{\pi} (1 - \cos(2nx)) \, dx$$

$$= x - \frac{\sin(2nx)}{2n} \Big|_{0}^{\pi} = \pi \neq 0$$

ex $\longrightarrow \int_{-\pi}^{\pi} \sin(nx) \sin(mx) \, dx$

$$= \frac{1}{2}\int_{-\pi}^{\pi} \cos(nx - mx) - \cos(nx + mx)$$

Note $\longrightarrow \sin n\pi = 0$

$\cos n\pi = (-1)^n$

$$= \int_{0}^{\pi} \cos(n-m)x \, dx - \int_{0}^{\pi} \cos(n+m)x$$

$$= 0$$

Fourier expansion :-

$f(x) \longrightarrow P = 2L$ دالة متصلة ودورية :

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{L} x\right) + \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{L} x\right)$$

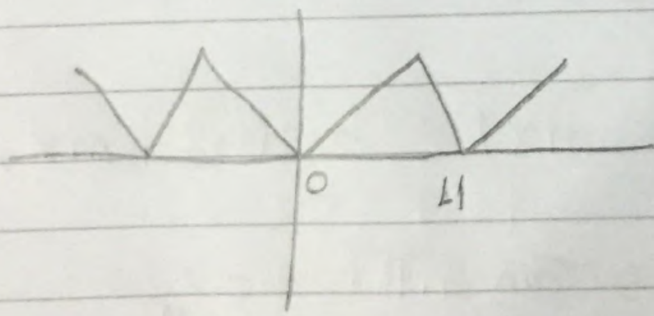
$$a_0 = \frac{1}{L} \int_{-L}^{+L} f(x)\, dx$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos\left(\frac{n\pi}{L} x\right) dx$$

$$b_n = \frac{1}{L} \int_{+h}^{h+2L} f(x) \sin\left(\frac{n\pi}{L} x\right) dx$$

أقل عدد $P = 4$

$\therefore L = 2$



احسب a_n , a_0

b_n

واكتب المفكوك