COLLEGE
Tanta Engineering

إعدادي 2020

الرياضيات

تفاضل - النهايات

م. أدهم أسامة

Section

Limits and continuity

the function ~~[illegible]~~ $f(x)$ has a limit at $x=a$ if

$$\lim_{x\to a^+} f(x) = \lim_{x\to a^-} f(x) = K$$

1] Characteristics of limits:

1] $\lim_{x\to a} (f(x) \pm g(x)) = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$

2] $\lim_{x\to a} (f(x) \cdot g(x)) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$

3] $\lim_{x\to a} \left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$

4] $\lim_{x\to a} [f(x)]^n = \left[\lim_{x\to a} f(x)\right]^n$

2] $\lim_{x\to 0} (1+f(x))^{g(x)}$:	3] $\lim_{x\to \infty} (1+f(x))^{g(x)}$:
if $\lim_{x\to 0} f(x) = 0$	if $\lim_{x\to \infty} f(x) = 0$
and $\lim_{x\to 0} g(x) = \infty$	and $\lim_{x\to \infty} g(x) = \infty$
then $\lim_{x\to 0} (1+f(x))^{g(x)} = e^K$	then $\lim_{x\to \infty} (1+f(x))^{g(x)} = e^K$
where $K = \lim_{x\to 0} (f(x) \cdot g(x))$	where $K = \lim_{x\to \infty} (f(x) \cdot g(x))$

1] $\lim_{x\to 0} \frac{\sin 3x}{\sin 2x} = \lim \frac{\frac{\sin 3x}{x}}{\frac{\sin 2x}{x}} = \frac{\lim_{x\to 0} \frac{\sin 3x}{x}}{\lim_{x\to 0} \frac{\sin 2x}{x}} = \frac{3}{2}$

2] $\lim_{x\to \pi} \frac{\sin 3x}{\sin 2x} = \lim_{x-\pi\to 0} \frac{\sin 3x}{\sin 2x} = \lim_{y\to 0} \frac{\sin 3(y+\pi)}{\sin 2(y+\pi)} = \lim_{y\to 0} \frac{\sin(3y-\pi)}{\sin(2y)}$, let $y = x-\pi$

$x = y+\pi$

$= -\lim \frac{\sin(3y)}{\sin(2y)} = \frac{-3}{2}$

3] $\lim_{x\to 0} \left[\frac{1}{x}(1+x-\cos x)\right]^{1/x}$

$= \lim_{x\to 0} \left[1 + \frac{1-\cos(x)}{x}\right]^{1/x}$

$\to \lim_{x\to 0} \frac{1-\cos x}{x} = \lim_{x\to 0} \frac{2\sin^2 \frac{x}{2}}{x}$

Note: $\cos x = 1 - 2\sin^2 \frac{x}{2}$

$= \lim_{x\to 0} \frac{\sin \frac{x}{2}}{\frac{x}{2}} \cdot \lim_{x\to 0} \sin \frac{x}{2}$

$= 1 \times 0 = 0$

$\to \lim_{x\to 0} \frac{1}{x} = \frac{1}{0} = \infty$

$\to \lim_{x\to 0} \left(\frac{1-\cos x}{x^2}\right) = \lim_{x\to 0} \frac{2\sin^2 \frac{x}{2}}{x^2} = 2 \cdot \lim \frac{\sin \frac{x}{2}}{x} \cdot \lim \frac{\sin \frac{x}{2}}{x} = 2 \times \frac{1}{2} \times \frac{1}{2}$

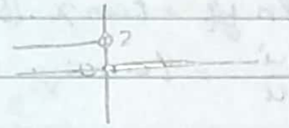
$\therefore \lim_{x\to 0} \left[\frac{1}{x}(1+x-\cos x)\right]^{1/x} = e^{1/2} = \sqrt{e}$

find Points of discontinuouty

1] $\frac{x}{x^2-1}$ $\therefore$ f is not continuous at $x = \{-1, 1\}$

2] $\frac{x-|x|}{x}$

$$f(x) = \begin{cases} \frac{x-x}{x} = 0 & x \geq 0 \\ \frac{x+x}{x} = 2 & x < 0 \end{cases}$$



$\therefore$ $f(x)$ isn't continuous at $x = 0$

3] $f(x) = \frac{1-x}{1-|x|}$

$$f(x) = \begin{cases} \frac{1-x}{1-x} = 1 & , x \geq 0 \\ \frac{1-x}{1+x} & , x < 0 \end{cases}$$

$f(0) = 1$, $\lim_{x \to 0^+} f(x) = 1$ $\lim_{x \to 0^-} \frac{1-x}{1+x} = 1$

$\therefore$ $f(x)$ is Continuous at $x = 0$, but is discontinuous at $x = -1$