

الميكانيكا

الحركة في المستوى

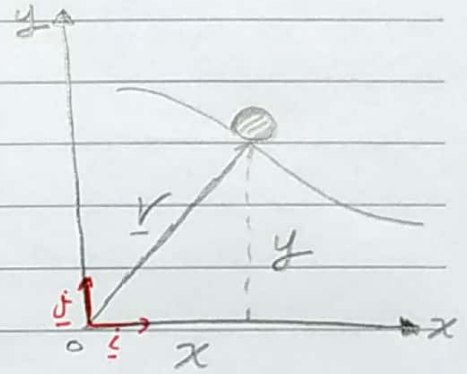
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$$\frac{dx}{dt} = \dot{x}$$

~~الحركة في المستوى~~ الحركة في المستوى

1- الإحداثيات الكارتيزية "x, y"



$$\underline{r} = x\underline{i} + y\underline{j}$$

$$\underline{v} = \frac{d\underline{r}}{dt} = \dot{x}\underline{i} + \dot{y}\underline{j}$$

v_x v_y

$$\underline{f} = \frac{d\underline{v}}{dt} = \ddot{x}\underline{i} + \ddot{y}\underline{j}$$

$f_x = \frac{dv_x}{dt}$ $f_y = \frac{dv_y}{dt}$

Ex: $\underline{f} = t^2\underline{i} + 2t\underline{j}$, $t=0$: $\underline{v}_0 = 2\underline{i} + 3\underline{j}$, $\underline{v}_0 = \underline{0}$

$$\therefore \underline{f} = \frac{d\underline{v}}{dt} = t^2\underline{i} + 2t\underline{j}$$

$$\int d\underline{v} = \int (t^2\underline{i} + 2t\underline{j}) dt$$

$$\underline{v} = \frac{t^3}{3}\underline{i} + t^2\underline{j} + \underline{C}_1$$

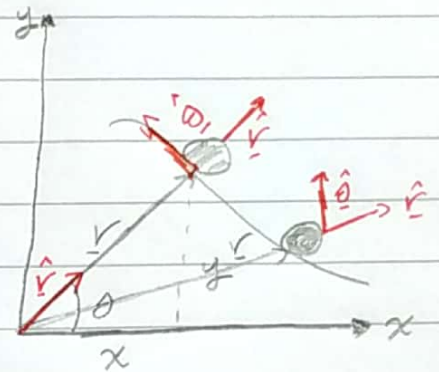
$$2\underline{i} + 3\underline{j} = 0\underline{i} + 0\underline{j} + \underline{C}_1$$

$$\underline{v} = \frac{t^3}{3}\underline{i} + t^2\underline{j} + 2\underline{i} + 3\underline{j}$$

$$\underline{v} = \left(\frac{t^3}{3} + 2\right)\underline{i} + (t^2 + 3)\underline{j}$$

٩- الإحداثيات القطبية

Polar coordinates



$$\underline{r} = x\underline{i} + y\underline{j}$$

$$\underline{r} = r \cos \theta \underline{i} + r \sin \theta \underline{j}$$

$$\underline{r} = r (\cos \theta \underline{i} + \sin \theta \underline{j})$$

$$\underline{r} = r \underline{\hat{r}}$$

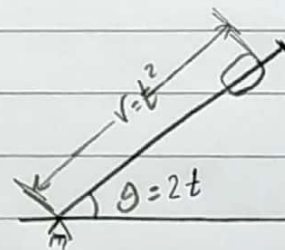
$$\underline{v} = \frac{d\underline{r}}{dt} = \dot{\underline{r}} \underline{\hat{r}} + r \frac{d\underline{\hat{r}}}{dt} = \dot{\underline{r}} \underline{\hat{r}} + r \dot{\theta} \underline{\hat{\theta}}$$

$$\underline{\hat{r}} = \cos \theta \underline{i} + \sin \theta \underline{j}$$

$$\frac{d\underline{\hat{r}}}{d\theta} \cdot \frac{d\theta}{dt} = \dot{\underline{\hat{r}}}$$

$$\underline{f} = \underbrace{(\ddot{r} - r\dot{\theta}^2)}_{f_r} \underline{\hat{r}} + \underbrace{(r\ddot{\theta} + 2\dot{r}\dot{\theta})}_{f_{\theta}} \underline{\hat{\theta}}$$

$$\frac{d\underline{\hat{r}}}{d\theta} = \dot{\theta} (-\sin \theta \underline{i} + \cos \theta \underline{j}) = \dot{\theta} \underline{\hat{\theta}}$$



$$r = t^2$$

$$\dot{r} = 2t$$

$$\ddot{r} = 2$$

$$\theta = 2t$$

$$\dot{\theta} = 2$$

$$\ddot{\theta} = 0$$

$$\underline{v} = 2t \underline{\hat{r}} + 2t^2 \underline{\hat{\theta}}$$