سنتر فيوتشر

Subject: رياضة إدارى

Chapter: تابع نظرية ذات الحدين

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Expand $\dfrac{3X}{(X-2)(2X+1)}$ and find coefficient

x^n , find condition for Convergance

$$\frac{3X}{(X-2)(2X+1)} = \frac{A}{X-2} + \frac{B}{2X+1}$$

$x = 2$ $\quad A = \frac{6}{5}$

$X = -\frac{1}{2}$ $\quad B = \dfrac{-3/2}{-5/2} = 3/5$

$$f(x) = \frac{6/5}{X-2} + \frac{3/5}{2X+1}$$

$$= \frac{-3}{5}\left[\frac{1}{1-x/2}\right] + \frac{3}{5}\left(\frac{1}{1+2X}\right)$$

$$= \frac{3}{5}\left[\frac{1}{1+2X} - \frac{1}{1-x/2}\right]$$

$|2x| < 1$, $\left|\frac{x}{2}\right| < 1$ شرط التقارب

$|X| < \frac{1}{2}$

$$f(x) = \frac{3}{5}\left[\left[1 - 2X + (2X)^2 - - - + (-2X)^n\right] - \left(1 + \frac{x}{2} - - + \left(\frac{x}{2}\right)^n\right)\right]$$

$\frac{3}{5}(-2)^n - \left(\frac{1}{2}\right)^n$ ① معامل X^n هو

Expand $\dfrac{8x+3}{(x-4)^2(x+3)}$

Find Coefficient x^7

$$\frac{8x+3}{(x-4)^2(x+3)} = \frac{A}{x+3} + \frac{B}{x-4} + \frac{C}{(x-4)^2}$$

$\underline{x=4}$ $\quad C = \dfrac{35}{7} = 5$

$\underline{x=-3}$ $\quad A = \dfrac{-21}{49} = -3/5$

$$\therefore 8x+3 = A(x-4)^2 + B(x-4)(x+3) + C(x+3)$$

$\underline{x=0}$

$$3 = 16A - 12B + 3C$$

$$3 = 16\left(\frac{-3}{5}\right) - 12B + 15$$

$$3 - \frac{27}{5} = -12B \qquad B = 1/5$$

$$\therefore \frac{8x+3}{(x-4)^2(x+3)} = \frac{-3/5}{x+3} + \frac{1/5}{x-4} + \frac{5}{(x-4)^2}$$

Condition for Convergance

$$= -\frac{1}{5}\left[\frac{1}{1+x/3}\right] - \frac{1}{20}\left(\frac{1}{1-x/4}\right) + \frac{5}{16}\left(\frac{1}{1-\frac{x}{4}}\right)^2$$

$$\left|\frac{x}{3}\right| < 1 \Rightarrow |x| < 3 \qquad \left|\frac{x}{4}\right| < 1 \Rightarrow x < 4$$

$\therefore$ Condition $|x| < 3$

المفكوك

$$-\frac{1}{5}\left[1 - \frac{x}{3} + \cdots + (-1)^n \left(\frac{x}{3}\right)^n\right]$$

$$-\frac{1}{20}\left[1 + \frac{x}{4} \cdots + \left(\frac{x}{4}\right)^n\right]$$

$$+\frac{5}{16}\left[1 + \frac{2x}{4} + \frac{3x^2}{16} \cdots + (n+1)\left(\frac{x}{4}\right)^n\right]$$

Coefficient x^n معامل x^n هو

$$-\frac{1}{5}\left[-\frac{1}{3}\right]^n - \frac{1}{20}\left(\frac{1}{4}\right)^n + \frac{5}{16}(n+1)\cdot\left(\frac{1}{4}\right)^n$$

Find coefficient x^n in the expansion

$$f(x) = \frac{5x + 1 + \frac{2}{x}}{(x+2)(x^2+1)}$$

ط

بالضرب x

$$\frac{5x^2 + x + 2}{x(x+2)(x^2+1)} = \frac{A}{x} + \frac{B}{x+2} + \frac{Cx+D}{x^2+1}$$

①

$x = 0$ $\quad A = 1$

$x = -2$ $\quad B = \dfrac{20}{(-2)(5)} = -2$

بالضرب ① في

$x(x+2)(x^2+1)$

$$5x^2 + x + 2 = A(x+2)(x^2+1) + Bx(x^2+1) + (Cx+D)(x)(x+2)$$

$$5x^2 + x + 2 = A[x^3 + 2x^2 + x + 1] + Bx^3 + Bx + Cx^3 + Dx^2 + 2Cx^2 + 2Dx$$

مقارنة معاملات x

x^3

$0 = A + B + C$ $\quad C = 1$

②

x^2 $\quad 5 = 2A + D + 2C$ $\quad D = 1$

$$f(x) = \frac{1}{x} + \frac{-2}{2+x} + (1+x)\frac{1}{1+x^2}$$

$$= \frac{1}{x} - \left[\frac{1}{1+\frac{x}{2}}\right] + (1+x)\left(\frac{1}{1+x^2}\right)$$

$$= \frac{1}{x} - \left[1 - \frac{x}{2} + \left(\frac{x}{2}\right)^2 + \cdots + \left(-\frac{x}{2}\right)^n\right]$$

$$+ (1+x)\left[1 - x^2 + x^4 \cdots (-1)^n x^{2n}\right]$$

معامل x^n هو $-\left(-\frac{1}{2}\right)^n$

معامل x^{2n+1} هو $(-1)^n$

" x^{2n} هو $(-1)^n$

دهل x^n — n زوجي

— n فردي

$\therefore$ <u>Coefficient</u> $x^{2n} = (-1)^n - \left(-\frac{1}{2}\right)^{2n}$

$x^{2n+1} = (-1)^n - \left(-\frac{1}{2}\right)^{2n+1}$

$\therefore$ معامل x^{2n} هو $(-1)^n - \left(\frac{1}{2}\right)^{2n}$

معامل x^{2n+1} هو $(-1)^n + \left(\frac{1}{2}\right)^{2n+1}$

0

Find Coefficient x^n in the expansion

$$\frac{1-x}{1+x+x^2}$$ ــــــــــط

$$\because \quad 1+x+x^2 = \frac{1-x^3}{1-x}$$

$$\frac{1-x}{1+x+x^2} = \frac{(1-x)(1-x)}{(1-x^3)} = (1-x)^2 \cdot \frac{1}{1-x^3}$$

$$= (1-2x+x^2)(1+x^3+x^6+\cdots+x^{3n}]$$

معامل x^{3n} هو 1

معامل x^{3n+1} هو -2

معامل x^{3n+2} هو 1

Find Coeffic x^n in the expansion

$$\frac{(1+x)^3}{(1-x+x^2)^2}$$ ــــــــــط

$$1-x+x^2 = \frac{1-(-x)^3}{1-(-x)} = \frac{1+x^3}{1+x}$$

7

$$f(x) = \frac{(1+x)^3}{(1+x^3)^2} = (1+x)^3 \cdot \frac{1}{(1+x^3)^2}$$

$$= \left[1 + 3\cancel{x} + 3x^2 + x^3\right]\left[1 - 2x^3 + \cdots (n+1)(-x^3)^n\right]$$

$$= (1 + 3x + 3x^2 + x^3)\left(1 - 2x^3 \cdots + (n+1)(-1)^n x^{3n}\right]$$

معامل X^{3n} هو $(-1)^n[1] = n(-1)^{n-1} + (n+1)(-1)^n$

معامل X^{3n+1} هو $3(n+1)(-1)^n$

معامل X^{3n+2} هو $3(n+1)(-1)^n$

<u>Expand</u> $\frac{1}{\sqrt{1+x}}$ Find Coefficient x^n

$$(1+x)^{-1/2} = 1 - \frac{1}{2}x + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}(x^2) + \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})}{3!}x^3$$

$$+ \frac{(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})(-\frac{7}{2}) \cdots (-)\frac{(2n-1)}{2}}{n!}x^n$$

$$\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)\left(-\frac{7}{2}\right)\cdots-\frac{(2n-1)}{2}}{n!}\,x^n$$

$$=\frac{(-1)^n\;1\cdot 2\; 3\cdot 4\; 5\; 6\cdots(2n-1)\,2n}{2^n\;n!\;\underset{2\cdot 1}{\cancel{2}}\;\underset{2\cdot 2}{\cancel{4}}\;\underset{2\cdot 3}{\cancel{6}}\cdots\underset{2\cdot n}{\cancel{2n}}}\,x^n$$

$$=\frac{(-1)^n\,(2n)!}{2^n\cdot n!\;2^n\;n!}\,x^n$$

$$=\frac{(-1)^n\,(2n)!}{2^{2n}\,(n!)^2}\,x^n$$

$\therefore$ معامل x^n هو

$$\frac{(-1)^n\,(2n)!}{2^{2n}\cdot(n!)^2}\;\#$$

Find Coff x^n in the expans $\frac{1}{\sqrt{1-x}}$

$$(1-x)^{-1/2} = 1 + \frac{1}{2}(x) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}x^2 + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3!}(-x^3)$$

$$+ \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})(-\frac{3}{2})\cdots(\frac{2n-3}{2})}{n!}x^n$$

معامل x^n هو:

$$\frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})(-\frac{5}{2})\cdots-(\frac{2n-3}{2})}{n!}$$

$$= \frac{(-1)^{n-1}(+1)(3)(5)\cdots(2n-3)}{2^n\, n!}$$

$$= \frac{(-1)^{n-1}}{n!\,2^n}\cdot\frac{1\cdot2\cdot3\cdot4\cdot5\cdots(2n-3)\overset{(2n-2)}{\cancel{(2n-1)}}}{2\cdot4\cdot6\cdots(2n-2)\cancel{(2n-1)}\cancel{(2n)}}$$

$$\frac{(-1)^{n-1}(2n-2)!}{n!\,2^n\,2^{n-1}(n-1)!}$$

$$= \frac{(-1)^{n-1}(2n-2)!}{2^{2n-1}\,n!\,(n-1)!} \quad \#$$

a

لو الفرق بين الاعداد الى اليمين لو الاعداد تتزايد

سالبة n

$$n = \pm \frac{\text{العدد الاول بعد الواحد}}{y}$$

$$Z = \pm \frac{y}{\text{العامل المشترك الموجود في المقام } (m)}$$

$$Z = \pm\ y/m$$

لو الحدود كلها موجبة او سالبة

Z سالبة

لو الحدود ~~كلها~~ تتذبذب

Z موجبة

$$\therefore (1+Z)^n$$

لتحديد الاشارات

البسط ارقام تتزايد n سالبة

لو الحدود كلها موجبة Z سالبة

لو الحدود متذبذبة Z موجبة

اذا تكرر الواحد في البسط

$n = \frac{1}{2}$

Find S

$$S = 1 + \frac{1}{4} + \frac{1\cdot 3}{4\cdot 8} + \frac{1\cdot 3\cdot 5}{4\cdot 8\cdot 12} + \cdots$$

$$S = 1 + \frac{1}{4} + \frac{\left(-\frac{1}{2}\right)\left(\frac{-3}{2}\right)}{2!\,4^2}(-2)^2 + \frac{\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)\left(\frac{-5}{2}\right)(-2)^3}{3!\,4^3}$$

$$1 + \frac{1}{4} + \frac{\left(-\frac{1}{2}\right)\left(\frac{-3}{2}\right)}{2!}\left(\frac{-2}{4}\right)^2 + \frac{\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)\left(\frac{-5}{2}\right)}{3!}\left(-\frac{2}{4}\right)^3$$

$$= 1 + \left(\frac{-1}{2}\right)\left(-\frac{2}{4}\right) + \cdots\cdots$$

$$= \left[1 + \left(\frac{-2}{4}\right)\right]^{-1/2} = \left(\frac{1}{2}\right)^{-1/2} = \sqrt{2} \quad \#$$

Find

$$S = \frac{1}{5} - \frac{1\cdot 4}{5\cdot 10} + \frac{1\cdot 4\cdot 7}{5\cdot 10\cdot 15} - \frac{1\cdot 4\cdot 7\cdot 10}{5\cdot 10\cdot 15\cdot 20}$$

$$= \frac{1}{5} - \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2!\,5^2}(-3)^2 + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)}{3!\,5^3}(-3)^3$$

11

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$$S = \frac{1}{5} - \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2!}\left(-\frac{3}{5}\right)^2 + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)}{3!}\left(-\frac{3}{5}\right)^3$$

$$= \frac{1}{5} - \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2!}\left(\frac{3}{5}\right)^2 - \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)}{3!}\left(\frac{3}{5}\right)^3 \cdots$$

$$= \frac{-\left(-\frac{1}{3}\right)\left(\frac{3}{5}\right)}{} - \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2!}\left(\frac{3}{5}\right)^2 - \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)}{3!}\left(\frac{3}{5}\right)^4$$

نطرح ١ ونجمع ١

$$1 - 1 - \left(-\frac{1}{3}\right)\left(\frac{3}{5}\right) - \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2!}\left(\frac{3}{5}\right)^2$$

$$= 1 - \left[1 + \left(-\frac{1}{3}\right)\left(\frac{3}{5}\right) + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2!}\left(\frac{3}{5}\right)^2 - - -\right]$$

$$= 1 - \left[\left(1 + \frac{3}{5}\right)^{-\frac{1}{3}}\right] = 1 - \left(\frac{8}{5}\right)^{-\frac{1}{3}}$$

$$= 1 - \sqrt[3]{5/8}$$

Find

$$S = 1 + \frac{4}{14} + \frac{4\cdot 7}{14\cdot 21} + \frac{4\cdot 7\cdot 10}{14\cdot 21\cdot 28} +$$

$$= 1 + \frac{4}{2!\,7} + \frac{4\cdot 7}{3!\,7^2} + \frac{4\cdot 7\cdot 10}{4!\,7^3} \cdots$$

$$= 7\left[\, 7 + \frac{4}{2!\,7^2} + \frac{4\cdot 7}{3!\,7^3} + \frac{4\cdot 7\cdot 10}{4!\,7^4} \cdots\right.$$

$$= 7\left[7 + \frac{1\cdot 4}{2!\,7^2} + \frac{1\cdot 4\cdot 7}{3!\,7^3} + \frac{1\cdot 4\cdot 7\cdot 10}{4!\,7^4} \cdots\right.$$

$$= 7\left[\, 7 + \frac{\left(\frac{-1}{3}\right)\left(-\frac{4}{3}\right)}{2!\,7^2}(-3)^2 + \frac{\left(\frac{-1}{3}\right)\left(-\frac{4}{3}\right)\left(-\frac{7}{3}\right)}{3!}(-3)^3 \cdots\right.$$

$$7\left[\frac{-1}{3}\left(\frac{-3}{7}\right) + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2!}\left(\frac{-3}{7}\right)^2 + \cdots \quad +1 \; -1\right.$$

$$= 7\left[1 + \left(-\tfrac{1}{3}\right)\left(\frac{-3}{7}\right) + \frac{\left(-\frac{1}{3}\right)\left(-\frac{4}{3}\right)}{2!}\left(\frac{-3}{7}\right)^2 \cdots \quad -1\right]$$

$$= 7\left[\left(1 - \frac{3}{7}\right)^{-1/3} - 1\right]$$

$$= 7\left[\left(\frac{4}{7}\right)^{-1/3} - 1\right] = 7\left[\sqrt[3]{7/4} - 1\right] \#$$

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Find

$$S = \frac{1}{3} + \frac{1\cdot 3}{3\cdot 6} + \frac{1\cdot 3\cdot 5}{3\cdot 6\cdot 9} + \cdots$$

حل

$$= \frac{1}{3} + \frac{(-1/2)(-3/2)}{2!\;3^2}(-2)^2 + \frac{(-1/2)(-\frac{3}{2})(-5/2)}{3!\;3^3}(-2)^3$$

$$= \left(\frac{-1}{2}\right)\left(\frac{-2}{3}\right) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}\left(\frac{-2}{3}\right)^2 + \cdots$$

نطرح ١ ونجمع ١

$$1 + (-1/2)\left(\frac{-2}{3}\right) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!}\left(\frac{-2}{3}\right)^2 \cdots \quad -1$$

$$= \left[(1 - 2/3)^{-1/2} - 1\right] = \left(\frac{1}{3}\right)^{-1/2} - 1 = \sqrt{3} - 1$$

Find S واجب

$$S = 1 - \frac{25}{64} + \frac{(25)(45)}{64\cdot 96} - \frac{(25)(45)(65)}{64\cdot 96\cdot 128}$$

$$S = 2 - \frac{1}{3} - \frac{1}{6} - \frac{5}{6\cdot 9} - \frac{5\cdot 7}{6\cdot 9\cdot 12} \cdots$$

ع

$$S = \frac{11}{6} + \frac{5\cdot 7}{6\cdot 12} + \frac{5\cdot 7\cdot 9}{6\cdot 12\cdot 18} + \cdots$$

· multinomial

$$\left[1 + a x + b y + c z\right]^{n}$$

لو عاوز معامل

$$x^{r_1}\, y^{r_2}\, z^{r_3}$$

حد

$$r_1 + r_2 + r_3 \leq n$$

$$L = n - \left[r_1 + r_2 + r_3\right]$$

الحل

$$\frac{n!}{r_1!\, r_2!\, r_3!\, L!} \cdot a^{r_1}\, b^{r_2}\, c^{r_3}$$

Find Cofficient $x^2 y^3 z^4$ in The expasion

$$\left[1 + \frac{1}{6} x + 3y - 2z\right]^{15}$$

الحل

$$L = 15 - (2+3+4) = 6$$

$$\therefore Coff = \frac{15!}{2!\, 3!\, 4!\, 6!} \left(\frac{1}{6}\right)^2 (3)^3 (-2)^4$$

Find coff $x^2 y^3 z^3$ in $\left[2 + 3x - y + \frac{1}{2} z\right]^{10}$

حل

$$2^{10}\left[1 + \frac{3x}{2} - \frac{y}{2} + \frac{1}{4} z\right]^{10}$$

$\therefore$ معامل $x^2 y^3 z^3$ هو

$$2^{10}\left[\frac{10!}{2!\, 3!\, 3!\, 2!} \left(\frac{3}{2}\right)^2 \left(-\frac{1}{2}\right)^3 \left(\frac{1}{4}\right)^3\right] \neq$$

(10)

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لاحظ لوين $n < r_1 + r_2 + r_3$

يكون المعامل = صفر

Find Coffic $x^3 y^4 z^7$

if $(1 + 3x + 5y + z)^9$

ك

معامل $x^3 y^4 z^7$ هو الصفر

Find Coff $x^6 \cdot y^5 z^8$ in the expan

$(1 + 2x^2 + y^5 + 3z^4)^7$

ك

let $x^2 = t$ $\quad y^5 = v,$ $\quad z^4 = u$

عدد معامل

$t^3 v \cdot u^2$ مفقود د

$(1 + 2t + v + 3u)^7$

هو

$$\frac{7!}{3!\ 1!\ 2!\ 1!} (2)^3 (1)^1 (3)^2$$

$$= \frac{7!}{12} (8)(9) \#$$

١٦