سنتر فيوتشر

Subject: رياضة "١ اعدادي"

Chapter: الاستنتاج الرياضي "٢"

تابع الاستنتاج

Mob: 0112 3333 122

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Prove that

مجموع مكعبات ثلاث اعداد متتالية تقبل القسمة على 9

$$n^3 + (n+1)^3 + (n+2)^3 \div 9$$

$n=1$ $\quad 1^3 + 2^3 + 3^3 = 36 = 9 \times 4$

تقبل القسمة على 9

$n=k$ asume

$$k^3 + \underline{\underline{(k+1)^3 + (k+2)^3}} = 9f$$

required to prov

$(k+1)^3 + (k+2)^3 + (k+3)^3$ divisble by 9

$$\underline{\underline{(k+1)^3 + (k+2)^3}} + (k+3)^3$$

$$= 9f - k^3 + (k+3)^3 \quad \boxed{1}$$

$$= 9f - k^3 + [k^3 + 3(k^2)(3) + 3(k)(9) + 27]$$

$$9f - \cancel{k^3} + \cancel{k^3} + 9k^2 + 27k + 27]$$

$$= 9[f + k^2 + 3k + 3]$$

∴ المقدار يقبل القسمة على 9

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$$

حفظ

Prove that

$x^{2n+1} - y^{2n+1}$ divisible by $x-y$

$\underline{n=1}$ $\quad x^3 - y^3 = (x-y)(x^2 + xy + y^2)$

يقبل القسمة على $x-y$

asume $\quad x^{2k+1} - y^{2k+1}$ divisible by $x-y$

$$x^{2k+1} - y^{2k+1} = (x-y) f(x,y) \quad \boxed{1}$$

required $x^{2k+3} - y^{2k+3}$ divisible by $(x-y)$

$$x^{2k+3} - y^{2k+3} = x^2 \cdot x^{2k+1} - y^2 \cdot y^{2k+1}$$

$$= x^2 \left[\underline{x^{2k+1} - y^{2k+1}} \right] + x^2 y^{2k+1} - y^2 y^{2k+1}$$

$$= x^2 \left[(x-y) f(x,y) \right] + y^{2k+1} \left(x^2 - y^2 \right)$$

$$= \underline{x^2 (x-y) f(x,y)} + (x-y)(x+y) y^{2k+1}$$

$$= (x-y) \left[x^2 f(x,y) + (x+y) y^{2k+1} \right]$$

∴ المقدار يقبل القسمة على $\underline{x-y}$

Prove that

$$2^n > n^2 \qquad n \geq 5$$

ط

n = 5 $\quad R.H.S = 5^2 = 25$

$L.H.S = 2^5 = 32$

∴ العلاقة صحيحة عند n = 5

asume relation is true at n = k

$$2^k > k^2$$

required to prove $\quad 2^{k+1} > (k+1)^2$

$\because \; 2^k > k^2$ $\qquad$ بالضرب 2

$$2 \cdot 2^k > 2k^2$$

$$2^{k+1} > 2k^2 \quad > \; (k+1)^2$$

$$2k^2 - (k+1)^2 = k^2 - 2k - 1$$

$\boxed{\varepsilon}$

لمعرفة المقدار موجب لأن k

$2k - 2 > 0$

علشان يكون موجبة $k > 1$ ، ده من $k \geq 5$

$\therefore 2k^2 > (k+1)^2$ ، $2^{k+1} > 2k^2$

$\therefore 2^{k+1} > (k+1)^2$

Prove that

$n! > n^2$ ، $n \geq 4$

$n = 4$ $R.H.S = 4^2 = 16$

$L.H.S = 4! = 24$

العلاقة صحيحة عند $n = 4$

asume $k! > k^2 \longrightarrow (1)$

requried $(k+1)! > (k+1)^2$

من العلاقة (1) بالضرب $(k+1)$

$(k+1)\,k! > (k+1)\,k^2$ $\boxed{0}$

$$(k+1)! > k^3+k^2 > (k+1)^2$$

$$\therefore k^3+k^2-(k+1)^2 = k^3-2k-1$$

$$\therefore 3k^2-2>0 \qquad k > \sqrt{2/3}$$

$k>4$ [illegible]

$$\therefore (k+1)k^2 > (k+1)^2 \;, \quad (k+1)! > (k+1)k^2$$

$$\therefore (k+1)! > (k+1)^2$$

[illegible]

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}+\frac{1}{5\cdot 7}\;\cdots\;\text{to nth term}$$

nth term

$$\frac{1}{(2n-1)(2n+1)}$$

$$\frac{8}{3\times 5}-\frac{12}{5\times 7}+\frac{16}{7\times 9}\;\cdots\;\text{to nth term}$$

$$\frac{4n+4}{(2n+1)(2n+3)}\,(-1)^{n-1}$$

[illegible]

Prove that

$$(1+x)^n \geq 1+nx$$

$x \geq -1$

Prove that

$$\frac{1}{1\cdot 3}+\frac{1}{3\cdot 5}\cdots \text{ to } n\text{th term} = \frac{n}{2n+1}$$

Prove that

$$\sum_{r=1}^{n} r\, r! = (n+1)!-1$$

Prove that

$$1+\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}\cdots+\frac{1}{\sqrt{n}} \geq \sqrt{n}$$

$n \geq 1$

$\boxed{V}$

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Prove that

$5^n - 3^n$ is divisible by 2

Prove that

$|\sin nx| \leq |n \sin x|$

Prove that

$4^n + 15n - 1$ divisible by 9

$\boxed{\wedge}$

Prove that

$$\frac{8}{3\cdot 5} - \frac{12}{5\cdot 7} + \frac{16}{7\cdot 9} \cdots\cdots + \frac{(-1)^{n+1}\, 4\,(n+1)}{(2n+1)(2n+3)}$$

$$= \frac{1}{3} + \frac{(-1)^{n-1}}{2n+3}$$

$n=1$ $\quad R.H.S = \frac{1}{3} + \frac{1}{5} = \frac{8}{15}$

$L.H.S = \frac{8}{3 * 5} = \frac{8}{15}$

العلاقة صحيحة عند $n=1$

$n=k$ $\quad$ نفرض صحة العلاقة $n=k$

$$\frac{8}{3\cdot 5} - \frac{12}{5\cdot 7} \cdots + \frac{4(-1)^{k+1}(k+1)}{(2k+1)(2k+3)} = \frac{1}{3} + \frac{(-1)^{k-1}}{2k+3}$$

requried $\quad$ المطلوب اثبات صحة العلاقة $n=k+1$

$$\frac{8}{3\cdot 5} - \frac{12}{5\cdot 7} \cdots + \frac{4(-1)^{k+2}(k+2)}{(2k+3)(2k+5)} = \frac{1}{3} + \frac{(-1)^{k}}{2k+5}$$

L.H.S

$$\frac{8}{3\cdot5} - \frac{12}{5\cdot7} \quad \frac{4(-1)^{k+1}(k+1)}{(2k+1)(2k+3)} + \frac{4(-1)^{k+2}(k+2)}{(2k+3)(2k+5)}$$

$$\frac{1}{3} + \frac{(-1)^{k-1}}{2k+3} + \frac{4(-1)^{k+2}(k+2)}{(2k+3)(2k+5)}$$

$$= \frac{1}{3} + \frac{(-1)^{k+2}}{(2k+3)}\left[\frac{4(k+2)}{2k+5} - 1\right]$$

$$\frac{1}{3} + \frac{(-1)^{k+2}}{2k+3}\left[\frac{4k+8-2k-5}{2k+5}\right]$$

$$\frac{1}{3} + \frac{(-1)^{k+2}}{\cancel{2k+3}}\left(\frac{\cancel{2k+3}}{2k+5}\right)$$

$$(-1)^{k+2} = (-1)^k \cancel{(-1)^2}$$

$$= \frac{1}{3} + \frac{(-1)^k}{2k+5}$$

$= R.H.S$

العلاقة صحيحة لجميع

$\underline{n}$

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Prove that

$$|\sin nx| \leq |n \sin x|$$

$\underline{n=1}$ $\quad R.H.S = \sin x$

$\quad L.H.S = \sin x$

$\underline{n=2}$ $\quad R.H.S = |2 \sin x|$

$\quad L.H.S = |\sin 2x| \Rightarrow |2 \sin x \cos x|$

$|\cos x| \leq 1$ $\quad$ لاحظ

∵ العلاقة صحيحة عند $n=1,2$

نفرض صحة العلاقة $\underline{n=k}$

$$|\sin kx| \leq |k \sin x|$$

المطلوب اثبات صحة العلاقة $n=k+1$

required $\quad |\sin (k+1)x| \leq |(k+1) \sin x|$

$L.H.S = |\sin(kx+x)| = |\sin kx \cos x + \cos kx \sin x|$

وحسب

$$|x+y| \leq |x| + |y|$$

$$|\sin kx + x| \leq |\sin kx \cos x| + |\cos kx \sin x|$$

$$\leq |\sin kx| + |\sin x|$$

$$\leq |k \sin x| + |\sin x|$$

$$\leq |(k+1) \sin x|$$

∴ العلاقة صحيحة لجميع قيم n

Prove that

$$1^2 - 2^2 + 3^2 - 4^2 - - - \ldots \underset{(2n-1)^2}{\uparrow} - (2n)^2 = -n(2n+1)$$

ط

$n=1$ $\quad R.H.S = -(1)(3) = -3$

$L.H.S = 1^2 - 2^2 = -3$

العلاقة صحيحة عندما $n=1$

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$\underline{n=k}$

$$(1^2-2^2)+(3^2-4^2)----+(2k-1)^2-(2k)^2$$
$$=-k(2k+1)$$

<u>required</u> المطلوب اثبات صحة العلاقة $n=k+1$

$$1^2-2^2+3^2-4^2---+(2k+1)^2-(2k+2)^2$$
$$=-(k+1)(2k+3)$$

L.H.S

$$=\underline{\underline{1^2-2^2+3^2-4^2--(2k-1)^2-(2k)^2}}+(2k+1)^2-(2k+2)^2$$

$$=-k(2k+1)+(2k+1)^2-(2k+2)^2$$

$$=-2k^2-k+\cancel{4k^2}+4k+1-\cancel{4k^2}-8k-4$$

$$=-2k^2-5k-3$$

$$=-(k+1)(2k+3)=R.H.S$$

العلاقة صحيحة لجميع قيم $\underline{\underline{n}}$

$\boxed{١٣}$

Prove that $3n^5+5n^3+7n$ divisible by 15

أثبت أن المقدار يقبل القسمة على 15

$n=1$ $\quad 3+5+7=15$

العلاقة صحيحة عند $n=1$

$n=k$

نفرض صحة العلاقة $n=k$

$$3k^5+5k^3+7k=15f$$

required $\quad n=k+1$

المطلوب إثبات صحة العلاقة $n=k+1$

$$3(k+1)^5+5(k+1)^3+7(k+1)$$

يقبل القسمة على 15

$$3(k+1)^5+5(k+1)^3+7=3[k^5+5k^4+10k^3+10k^2+5k+1]$$

$$+5(k^3+3k^2+3k+1)+7(k+1)$$

$\therefore 3(k+1)^5 + 5(k+1) + 7$

$= \underline{3k^7 + 5k^3 + 7k} + 14k + 15k^4 + 30k^2$

$+ 15k + 3 + 15k^2 + 15k + 5 + 7$

$= 15f + 15k^4 + 30k^3 + 45k^2 + 30k + 15$

$= 15[f + k^4 + 2k^3 + 3k^2 + 2k + 1]$

∴ المقدار يقبل القسمة على 15

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prove that

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 \text{ - - - } + x^n$

ت

$\underline{n=1}$ $R.H.S = 1+x,$ $L.H.S = 1+x$

العلاقة صحيحة عندما $n=1$

asume relation is true at $n=k$

$(1+x)^k = 1 + kx + \frac{k(k-1)}{2!}x^2 \text{ - - - } + x^k$

required to prove

$(1+x)^{k+1} = 1 + (k+1)x + \frac{k(k+1)}{2!}x^2 \text{ - - - } + x^{k+1}$

$\boxed{10}$

$$\because (1+x)^{k+1} = (1+x)(1+x)^k$$

$$= (1+x)\left[1 + kx + \frac{k(k-1)}{2!}x^2 - - - + x^k\right]$$

$$= 1 + kx + \frac{k(k-1)}{2!}x^2 - - - + x^k$$

$$+ x + kx^2 + - - - - - + x^{k+1}$$

$$= 1 + (kx + x) + \left[\frac{k(k-1)}{2!} + k\right]x^2 - - - - + x^{k+1}$$

$$= 1 + (k+1)x + \left[\frac{k(k-1) + 2k}{2!}\right]x^2 + - - - + x^{k+1}$$

$$= 1 + (k+1)x + \frac{k(k+1)}{2!}x^2 - - - + x^{k+1}$$

$$= R.H.S$$

∴ العلاقة صحيحة لجميع n

Prove that

$$(1+nx) \leq (1+x)^n$$

at n=1   R.H.S = $1+x$

L.H.S = $1+x$

at n=2   R.H.S $(1+x)^2 = 1 + 2x + x^2$

L.H.S = $1 + 2x$

if $y = e^{ax}$

prove that $\dfrac{d^n y}{d x^n} = a^n \cdot e^{ax}$

$\because y = e^{ax}$ $\quad \therefore \dfrac{dy}{dx} = a e^{ax}$

at $n=1$ $\quad \dfrac{dy}{dx} = a \cdot e^{ax}$

العلاقة صحيحة عند $n=1$

asume relation is true at $n=k$

$$\frac{d^k y}{d x^k} = a^k \cdot e^{ax}$$

required to prove $\dfrac{d^{k+1} y}{d x^{k+1}} = a^{k+1} e^{ax}$

$$\because \frac{d^k y}{d x^k} = a^k \cdot e^{ax}$$

بالتفاضل بالنسبة لـ x

$$\frac{d^{k+1} y}{d x^{k+1}} = a^k \cdot a \cdot e^{ax} = a^{k+1} e^{ax}$$

$\therefore$ العلاقة صحيحة لجميع n

if $y = \dfrac{1}{ax+b}$

Show that

$$\frac{d^n y}{dx^n} = \frac{(-1)^n\, n!\, a^n}{(ax+b)^{n+1}}$$

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at $n=1$ $\quad \dfrac{dy}{dx} = \dfrac{(-1)(1)a}{(ax+b)^2}$

دلك تفاضل

$$y = (ax+b)^{-1}$$

$$\frac{dy}{dx} = -(ax+b)^{-2} \cdot a = \frac{-a}{(ax+b)^2}$$

relation is true at $n=1$

assume relation is true at $n=k$

$$\frac{d^k y}{dx^k} = \frac{(-1)^k (k!)\, a^k}{(ax+b)^{k+1}} \quad (1)$$

requried to prove that

$$\frac{d^{k+1}y}{dx^{k+1}} = \frac{(-1)^{k+1}(k+1)!\, a^{k+1}}{(ax+b)^{k+2}}$$

بتفاضل المعادلة (2)

$$\frac{d^{k+1}y}{dx^{k+1}} = (-1)^k (k!)\, a^k (-k-1)(ax+b)^{-k-2} * a$$

$$= \frac{(-1)(-1)^k a^{k+1}\, k!\, (k+1)}{(ax+b)^{k+2}}$$

$$= \frac{(-1)^{k+1} a^{k+1} (k+1)!}{(ax+b)^{k+2}} = R.H.S$$

relation is true for all value of n

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واجب

Prove that

$$\sum_{r=1}^{n} \frac{1}{\sqrt{r}} > \sqrt{n} \qquad n > 1$$

at n = 2 $\quad R.H.S = \sqrt{2} = 1.4$

$$L.H.S = \frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} = 1.7$$

العلاقة صحيحة عندما $n = 2$

asume relation is true at $n = k$

$$\sum_{r=1}^{k} \frac{1}{\sqrt{r}} > \sqrt{k}$$

reqtied to prove

$$\sum_{r=1}^{k+1} \frac{1}{\sqrt{r}} > \sqrt{k+1}$$

$$\because \sum_{r=1}^{k} \frac{1}{\sqrt{r}} > \sqrt{k}$$

بإضافة $\frac{1}{\sqrt{k+1}}$ للطرفين

$$\therefore \sum_{r=1}^{k} \frac{1}{\sqrt{r}} + \frac{1}{\sqrt{k+1}} > \sqrt{k} + \frac{1}{\sqrt{k+1}}$$

$$= \sum_{r=1}^{k+1} \frac{1}{\sqrt{r}} > \sqrt{k} + \frac{1}{\sqrt{k+1}}$$

$$\because \sqrt{k} + \frac{1}{\sqrt{k+1}} - \sqrt{k+1}$$

$$= \frac{\sqrt{k}\sqrt{k+1} - 1 - k - 1}{\sqrt{k+1}} = \frac{\sqrt{k}\sqrt{k+1} - k}{\sqrt{k+1}}$$

$$= \frac{\sqrt{k}\sqrt{k+1} - \sqrt{k}\sqrt{k}}{\sqrt{k+1}} = \frac{\sqrt{k}\left(\sqrt{k+1} - \sqrt{k}\right)}{\sqrt{k+1}} > 0$$

$$\therefore \sqrt{k} + \frac{1}{\sqrt{k+1}} > \sqrt{k+1}$$

$$\sum_{r=1}^{k+1} \frac{1}{\sqrt{r}} > \sqrt{k} + \frac{1}{\sqrt{k+1}}$$

$$\therefore \sum_{r=1}^{k+1} \frac{1}{\sqrt{r}} > \sqrt{k+1}$$

العلاقة صحيحة
لجميع قيم n

Prove that

$$(1+x)^n \geq 1+nx$$

$n=1$

$R.HS = 1+x$

$L.H.S = 1+x$

$n=2$

$R.HS = \underline{1+2x}$

$L.HS = (1+x)^2$

$= \underline{x^2+2x+1}$

∴ العلاقة صحيحة عندما $n=1,\ n=2$

asume relation true at $n=k$

$$(1+x)^k \geq 1+kx$$

requried to Prove

$$(1+x)^{k+1} \geq 1+(k+1)x$$

$$(1+x)^k \geq 1 + kx$$

بالضرب $1 + x$

$$(1+x)^{k+1} \geq (1+kx)(1+x)$$

$$\geq 1 + kx + x + kx^2$$

$$\geq \underline{1 + (k+1)x + kx^2}$$

$$\because 1 + (k+1)x + kx^2 \geq 1 + (k+1)x$$

$$\therefore (1+x)^{k+1} \geq 1 + (k+1)x$$

Prove that

$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - - - - + \frac{1}{\sqrt{n}} \geq \sqrt{n}$$

$$n \geq 1$$

$$\frac{1}{1} + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} \cdots + \frac{1}{\sqrt{k}} + \frac{1}{\sqrt{k+1}} \geq \sqrt{k} + \frac{1}{\sqrt{k+1}}$$

$$\therefore \sqrt{k} + \frac{1}{\sqrt{k+1}} - \sqrt{k+1}$$

$$= \frac{\sqrt{k}\sqrt{k+1} + \cancel{1} - (k + \cancel{1})}{\sqrt{k+1}}$$

$$= \frac{\sqrt{k}\sqrt{k+1} - k}{\sqrt{k+1}} = \frac{\sqrt{k}\left[\sqrt{k+1} - \sqrt{k}\right]}{\sqrt{k+1}} > 0$$

$$\therefore \sqrt{k} + \frac{1}{\sqrt{k+1}} > \sqrt{k+1}$$

$$\frac{1}{1} + \frac{1}{\sqrt{2}} \cdots + \frac{1}{\sqrt{k+1}} > \sqrt{k} + \frac{1}{\sqrt{k+1}}$$

$$\therefore \frac{1}{1} + \frac{1}{\sqrt{2}} \cdots + \frac{1}{\sqrt{k+1}} > \sqrt{k+1} \;//$$

$\underline{n=1}$ $R.H.S = \sqrt{1} = 1$

$L.H.S = 1$

$\underline{n=2}$ $R.H.S = \sqrt{2} = 1.41$

$L.H.S = 1 + \frac{1}{\sqrt{2}} = 1.707$

∴ العلاقة صحيحة عند

$n=1, n=2$

asume relation is true at $n=k$

$$\frac{1}{1} + \frac{1}{\sqrt{2}} --- + \frac{1}{\sqrt{k}} \geq \sqrt{k}$$

required to prove

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} --- + \frac{1}{\sqrt{k+1}} \geq \sqrt{k+1}$$

~~L.H.S~~ =

$$\frac{1}{1} + \frac{1}{\sqrt{2}} + ---- + \frac{1}{\sqrt{k}} \geq \sqrt{k}$$

بإضافة $\frac{1}{\sqrt{k+1}}$ لكل من الطرفين

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