سنتر فيوتشر

Subject: إعدادي رياضه

Chapter: الاشتقاق

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التفاضل لاستقام

$$\sin x = \frac{1}{\csc x} \qquad \cos x = \frac{1}{\sec x}$$

$$\tan x = \frac{1}{\cot x}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\cot^2 x + 1 = \csc^2 x$$

$$\sin 2x = 2\sin x \cos x$$

$$\left.\begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x \\ &= 1 - 2\sin^2 x \\ &= 2\cos^2 x - 1 \end{aligned}\right\}$$

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

خواص اللوغاريتمات

$$\ln(xy) = \ln x + \ln y$$

$$\ln(x/y) = \ln x - \ln y$$

$$\ln x^n = n \ln x$$

$$\ln x = \log_e x$$

$$\ln e^x = x \qquad e^{\ln x} = x$$

$$\log_y x = \frac{\ln x}{\ln y}$$

$$\log_b x = y \longrightarrow x = b^y$$

$y = \text{Constant}$ $\qquad \frac{dy}{dx} = y' = 0$

$y = x^m$ $\qquad y' = m x^{m-1}$

$y = f \cdot g$ $\qquad y' = f \cdot g' + g f'$

$y = f / g$ $\qquad y' = \frac{g f' - g' f}{g^2}$

$y = (f)^m$ $\qquad y' = m (f)^{m-1} f'$

$y = \sqrt{f}$ $\qquad$ C $\qquad y' = \frac{1}{2\sqrt{f}} f'$

if $y = \ln(x^2+2e^{-3x})^7$

الحل

$$y = 7\ln(x^2+2e^{-3x})$$

$$y' = \frac{dy}{dx} = 7\left[\frac{2x-6e^{-3x}}{x^2+2e^{-3x}}\right]$$

if $y = \ln\sqrt{\frac{1+x^2}{1-x^2}}$

$$y = \frac{1}{2}\ln\left(\frac{1+x^2}{1-x^2}\right)$$

$$= \frac{1}{2}\left[\ln(1+x^2) - \ln(1-x^2)\right]$$

$$y' = \frac{1}{2}\left[\frac{2x}{1+x^2} - \frac{-2x}{1-x^2}\right]$$

$$y = \ln\left[\frac{e^{4x}\sqrt[3]{2x^3-1}}{(x^7+2)^5(3x^4+2x-1)^3}\right]$$

$$y = \cancel{\ln e}^{4x} + \ln(2x^3-1)^{1/3} - \ln(x^7+2)^5 - \ln(3x^4+2x-1)^3$$

Find 1st derv. الصبر الـشغل ٢، ط

$$y = x^4 + 2x^3 + \frac{2}{x^5} + \sqrt[3]{x^2+1}$$

$$y' = 4x^3 + 6x^2 - 10x^{-6} + \frac{1}{3}(x^2+1)^{-2/3}(2x)$$

if $\quad y = \dfrac{x^4+7}{2\sqrt{x+3}}$

$$y' = \frac{1}{2}\left[\frac{\sqrt{x+3}\cdot 4x^3 - (x^4+7)\cdot\frac{1}{2\sqrt{x+3}}}{x+3}\right]$$

$$y = (x^4+9)^{10}\cdot\sqrt{x^3-9}$$

$$y' = (x^4+9)^{10}\cdot\frac{1}{2\sqrt{x^3-9}}\cdot 3x^2 + \sqrt{x^3-9}\cdot(10)\underset{4x^3}{(x^4+9)}$$

$$y = \sqrt{x+\sqrt{x+\sqrt{x}}}$$

$$y' = \frac{1}{2\sqrt{x+\sqrt{x+\sqrt{x}}}}\left[1 + \frac{1}{2\sqrt{x+\sqrt{x}}}\left(1+\frac{1}{2\sqrt{x}}\right)\right]$$

if $y = \sqrt{x\sqrt{x\sqrt{x}}}$

حل

$$y = x^{\frac{1}{2}} \cdot x^{\frac{1}{4}} \cdot x^{\frac{1}{8}}$$

$$y = x^{7/8} \qquad y' = \frac{7}{8} x^{-\frac{1}{8}}$$

تفاضل الدالة الأسية واللوغاريتمية :

$$y = e^{u} \qquad y' = e^{u} \cdot u' \cancel{\ln e}$$

$$y = a^{u} \qquad y' = a^{u} \cdot u' \ln a$$

$$y = \ln u \qquad y' = \frac{1}{u} \cdot u'$$

$$y = \log_b x \qquad y' = \frac{1}{\ln b} \frac{u'}{u}$$

ثابت ← b

اذا كان b متغير

$$y = \log_b x = \frac{\ln x}{\ln b}$$

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if $y = e^{x^2} + 4 e^{-3x} + 8\sqrt{2 + e^{x}}$

$$y' = 2x e^{x^2} + (-12 e^{-3x}) + 8 \cdot \frac{1}{2\sqrt{2+e^{x}}} \cdot e^{x}$$

6

if

$$y = e^{x} + x^{e} + e$$

$$y' = e^{x} + e\,x^{e-1} + 0$$

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if $y = \sqrt[3]{1+e^{-x}} + \dfrac{e^{4x}+9}{2-e^{-3x}}$

$$y' = \frac{1}{3}\left(1+e^{-x}\right)^{-2/3}\cdot\left(-e^{-x}\right) + \frac{4\left(2-e^{-3x}\right)e^{4x} - \left(9+e^{4x}\right)3e^{-3x}}{\left[2-e^{-3x}\right]^{2}}$$

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if

$$y = e^{e^{x}}$$

$$y' = e^{e^{x}} \cdot e^{x}$$

$$y = 4^{x^{3}} + 3x^{4}$$

$$y' = 4^{x^{3}} \cdot 3x^{2} \ln 4 + 12x^{3}$$

نفسه   مشتقة الأس   س b ln

$$y = 2^{-3x} + 3^{-2x}$$

$$y' = 2^{-3x}(-3)\ln 2 + 3^{-2x}(-2)\ln 3$$

if $y = \dfrac{2^{3x}}{3^{2x}}$ Find y'

$$y = \left(\frac{2^3}{3^2}\right)^x \qquad y = \left(\frac{8}{9}\right)^x$$

$$\therefore y' = \left(\frac{8}{9}\right)^x \cdot 1 \cdot \ln(8/9)$$

$$y = \ln(x^2 + 8x + 1)$$

$$y' = \frac{1}{x^2 + 8x + 1}[2x + 8]$$

$$y = \ln(2^{3x} + e^{-4x})$$

$$y' = \frac{1}{2^{3x} + e^{-4x}}\left[2^{3x} \cdot 3\ln 2 - 4e^{-4x}\right]$$

4

$$y = 4x + \frac{1}{3}\ln(2x^3-1) - 5\ln(x^7+2) - 3\ln(3x^4+2x-1)$$

$$y' = 4 + \frac{1}{3}\left[\frac{6x^2}{2x^3-1}\right] - 5\left[\frac{7x^6}{x^7+2}\right] - 3\left(\frac{12x^3+2}{3x^4+2x-1}\right)$$

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if $y = e^{-4\ln x}$

$$y = e^{\ln x^{-4}} = x^{-4}$$

$$y' = -4x^{-5}$$

$$y = \log_3(x^2+1)$$

$$y = \log_3(x^2+1) = \frac{\ln(x^2+1)}{\ln 3}$$

$$y' = \frac{1}{\ln 3}\left[\frac{2x}{x^2+1}\right]$$

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if $y = \log_x(x^2+8x)$

$$y = \frac{\ln(x^2+8x)}{\ln x}$$

$$y' = \frac{(\ln x)\left[\frac{2x+8}{x^2+8x}\right] - \ln(x^2+8x) \cdot \frac{1}{x}}{(\ln x)^2}$$

التفاضل الضمني

$$y^3 + 2x^4 = 0$$

$$3y^2 \cdot \frac{dy}{dx} + 8x^3 = 0$$

$$e^{x+y^2} + \ln(2x+7) + \sqrt{1+y^3} = 0$$

$$e^{x+y^2}\left[1+2yy'\right] + \frac{2}{2x+7} + \frac{3y^2 \cdot y'}{2\sqrt{1+y^3}}]$$

$$y \ln x + x \ln y = 5$$

$$y \cdot \frac{1}{x} + \ln x \cdot y' + \ln y + x \cdot \frac{y'}{y} = 0$$

$$\frac{y}{x} + \ln y + y' \ln x + \frac{x}{y} y' = 0$$

9

if

$$x^3 + 3xy^2 + 3x^3y + y^3 = 1$$

Find $\dfrac{dy}{dx}$

$$\cancel{3}x^2 + \cancel{3}y^2 + \overset{2}{\cancel{6}}xy\cdot y' + \overset{3}{\cancel{9}}x^2y + \cancel{3}x^2y' + \cancel{3}y^2y' = 0$$

$$(x^2 + y^2 + 3x^2y) + (2xyy' + 3x^2y' + y^2y') = 0$$

$$y' = -\frac{x^2 + y^2 + 3x^2y}{2xy + 3x^2 + y^2} \quad \#$$

تفاضل دالة ^ دالة

① نأخذ ln للطرفين

$$y = x^{x^2}$$

$$\ln y = \ln x^{x^2}$$

$$\ln y = x^2 \ln x$$

$$\frac{y'}{y} = x^2\frac{1}{x} + 2x\ln x$$

بالضرب $\times\, y$ (١٠)

$$y' = y\left[x + 2x\ln x\right]$$

$$= x^{x^2}\left[x + 2x\ln x\right]$$

<u>if</u> $y = (x^2+4)^{3e^{2x}}$ Find $\frac{dy}{dx}$

حل

$$\ln y = 3e^{2x}\ln(x^2+4)$$

$$\frac{1}{y}y' = 3e^{2x}\left(\frac{2x}{x^2+4}\right) + \ln(x^2+4)\cdot 6\cdot e^{2x}$$

$$y' = (x^2+4)^{3e^{2x}}\left[\frac{6x\,e^{2x}}{x^2+4} + 6\cdot e^{2x}\ln(x^2+4)\right]$$

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if $x^y + y^x = 5$

Find $\frac{dy}{dx}$ ——— الحل

$$u + v = 5 \qquad u = x^y$$

$$u' + v' = 0 \qquad v = y^x$$

$$u = x^{y}$$

ناخذ ln للطرفين

$$\ln u = y \ln x$$

$$\frac{u'}{u} = y\,\frac{1}{x} + y' \ln x$$

بالضرب u

$$u' = u\left[\frac{y}{x} + y' \ln x\right]$$

$$= x^{y}\left[\frac{y}{x} + y' \ln x\right] \longrightarrow ②$$

$$\because v = y^{x}$$

$$\ln v = x \ln y$$

$$\frac{v'}{v} = \ln y + x \cdot \frac{y'}{y}$$

$$v' = v\left[\ln y + \frac{x y'}{y}\right]$$

$$v' = y^{x}\left[\ln y + \frac{x}{y} y'\right] \longrightarrow ③$$

بالتعويض ①

$$x^{y}\left[\frac{y}{x} + y' \ln x\right] + y^{x}\left[\ln y + \frac{x}{y} y'\right] = 0$$

#

if $y = e^{-4x} + \ln(x^2+9) + x^{\sqrt{x}}$

الحل

$u = x^{\sqrt{x}}$ $\quad\quad \ln u = \sqrt{x} \ln x$

$$\frac{u'}{u} = \sqrt{x}\,\frac{1}{x} + \ln x \cdot \frac{1}{2\sqrt{x}}$$

$$u' = x^{\sqrt{x}}\left[\frac{\sqrt{x}}{x} + \frac{\ln x}{2\sqrt{x}}\right]$$

$$\therefore \frac{dy}{dx} = e^{-4x}(-4) + \frac{2x}{x^2+9} + u'$$

تفاضل حاجات مضروبة ببعضها وصعبة ناخد ln للطرفين

$$y = \frac{\sqrt[3]{2x-1} \cdot (x^4+9)^7}{2e^{-3x}(x^3+5)^{10}}$$

ط

$$\ln y = \ln(2x-1)^{1/3} + \ln(x^2+9)^7 - \ln(2e^{-3x}) - \ln(x^3+5)^{10}$$

$$\ln y = \frac{1}{3}\ln(2x-1) + 7\ln(x^2+9) - \ln 2 - \cancel{\ln e^{-3x}}$$
$$-10\ln(x^3+5)$$

$$\frac{y'}{y} = \frac{1}{3}\left(\frac{2}{2x-1}\right) + 7\left(\frac{2x}{x^2+9}\right) + 3 - 10\left(\frac{3x^2}{x^3+5}\right)$$

$$y' = y\left[\qquad \right]$$

if $y = (x^4+9)^5 (2x-1)^{10} (x^3+1)^{5/2}$

Find $\frac{dy}{dx}$ ——— ط

$$\ln y = 5\ln(x^4+9) + 10\ln(2x-1) + \frac{5}{2}\ln(x^3+1)$$

$$\frac{y'}{y} = 5\left[\frac{4x^3}{x^4+9}\right] + 10\left(\frac{2}{2x-1}\right) + \frac{5}{2}\left(\frac{3x^2}{x^3+1}\right)$$

$$\therefore y' = y\left(\frac{20x^3}{x^4+9} + \frac{20}{2x-1} + \frac{15x^2}{2(x^3+1)}\right)$$

ثم عوض عن قيمة y

تفاضل الدوال المثلثية

$y = \sin u$ $\quad\quad$ $y' = u' \cos u$

$y = \cos u \longrightarrow y' = -u' \sin u$

$y = \tan u$ $\quad\quad$ $y' = u' \sec^2 u$

$y = \cot u \longrightarrow y' = -u' \csc^2 u$

$y = \sec u$ $\quad\quad$ $y' = u' \sec u \cdot \tan u$

$y = \csc u \longrightarrow y' = -u' \csc u \cdot \cot u$

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if $y = \sin(3x) + \cos x^2 + e^{\tan\sqrt{x}}$

ح

$$\frac{dy}{dx} = 3\cos 3x - 2x \sin x^2 + e^{\tan\sqrt{x}} \cdot \sec^2\sqrt{x}\,\frac{1}{2\sqrt{x}}$$

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if $y = \cancel{\sin^3 x} + \sin x^3 + \sin 3x$

ح $\quad (\sin x)^3$

$$\frac{dy}{dx} = 3(\sin x)^2 \cdot \cos x + 3x^2 \cos x^3 + 3\cos 3x$$

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if $y = \sin^2(2x^2+1)^2$

Find $\frac{dy}{dx}$

$$y = \left[\sin(2x^2+1)^2\right]^2$$

$$y' = 2\left[\sin(2x^2+1)^2\right] \cdot \cos(2x^2+1)^2 \cdot (2x^2+1) \cdot 4x$$

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if $y = \dfrac{\cos x}{1-\cos x}$

$$\frac{dy}{dx} = \frac{(1-\cos x)(-\sin x) - \cos x \cdot \sin x}{(1-\cos x)^2}$$

$$= \frac{-\sin x + \cancel{\sin x \cos x} - \cancel{\sin x \cos x}}{(1-\cos x)^2}$$

$$\frac{dy}{dx} = \frac{-\sin x}{(1-\cos x)^2} = \frac{-2\sin x/2 \, \cos x/2}{\left[2\sin^2\frac{x}{2}\right]^2}$$

$$= -\frac{\cos x/2}{2\sin^3(x/2)\, \cancel{\cos}}$$

if $y = \ln\sqrt{\dfrac{1+\sin x}{1-\sin x}}$

Prove that $\dfrac{dy}{dx} = \sec x$

$$y = \frac{1}{2}\left[\ln(1+\sin x) - \ln(1-\sin x)\right]$$

$$y' = \frac{1}{2}\left[\frac{\cos x}{1+\sin x} + \frac{\cos x}{1-\sin x}\right]$$

$$= \frac{\cos x}{2}\left[\frac{1-\cancel{\sin x}+1+\cancel{\sin x}}{(1-\sin x)(1+\sin x)}\right] = \frac{\cancel{2}\cos x}{\cancel{2}(1-\sin^2 x)} =$$

$$= \frac{\cancel{\cos x}}{\cos^{\cancel{2}} x} = \frac{1}{\cos x} = \sec x$$

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if $y = \left[\sec 2x + \tan 2x\right]^3$

Prove that $\dfrac{dy}{dx} = 6y\sec 2x$

$$\frac{dy}{dx} = 3\left[\sec 2x + \tan 2x\right]^2 \cdot \left[2\sec^2 2x + 2\sec 2x\tan 2x\right]$$

$$= 6\sec 2x\left[\sec 2x + \tan 2x\right]^2 (\sec 2x + \tan 2x)$$

$$= 6\sec 2x(\sec 2x + \tan 2x)^3 = 6y\sec 2x \ \#$$

if $y = 2^{\tan x} + \sin(\ln x) + \operatorname{cosec}^3(e^{-x})$

Find y'

$$y' = 2^{\tan x} \cdot \ln 2 \sec^2 x + \cos(\ln x) \frac{1}{x}$$

$$+ 3\operatorname{cosec}^2(e^{-x})\left[-\operatorname{cosec}(e^{-x}) \cdot \cot(e^{-x})\right] \cdot (-e^{-x})$$

---

if

$$y = \ln(\sec x + \tan x) + (\sin x)^{\cos x} + e^{\sec\sqrt{x}}$$

$$y' = \frac{1}{\sec x + \tan x}\left[\sec x \tan x + \sec^2 x\right] + \frac{e^{\sec\sqrt{x}} \sec\sqrt{x} \tan\sqrt{x}}{2\sqrt{x}}$$

$$+ u'$$

$\because u = (\sin x)^{\cos x}$

$$\ln u = \cos x \ln(\sin x)$$

$$\frac{u'}{u} = -\sin x \ln \sin x + \cos x \cdot \frac{\cos x}{\sin x}$$

$$u' = u\left[-\sin x \ln \sin x + \frac{\cos^2 x}{\sin x}\right]$$

$$\sinh x = \frac{e^{x} - e^{-x}}{2} \quad , \quad \cosh x = \frac{e^{x} + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x}$$ قانون $$\coth x = \frac{1}{\tanh x}$$

$$\operatorname{sech} x = \frac{1}{\cosh x} \qquad \operatorname{cosech} x = \frac{1}{\sinh x}$$

Prove that: $\cosh x - \sinh x = e^{-x}$

- $\cosh x + \sinh x = e^{x}$
- $\cosh^2 x - \sinh^2 x = 1$
- $1 - \tanh^2 x = \operatorname{sech}^2$
- $\coth^2 x - 1 = \operatorname{sech}^2 x$

قانون

$$\because \cosh x - \sinh x = \frac{e^{x} + e^{-x}}{2} - \frac{e^{x} - e^{-x}}{2}$$

$$= \frac{\cancel{e^{x}} + e^{-x} \cancel{- e^{x}} + e^{-x}}{2} = \frac{2e^{-x}}{2} = e^{-x}$$

$$\therefore \boxed{\cosh x - \sinh x = e^{-x}}$$

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$$\cosh x + \sinh x = \frac{e^{x} + e^{-x}}{2} + \frac{e^{x} - e^{-x}}{2}$$

$$= \frac{e^{x} + \cancel{e^{-x}} + e^{x} - \cancel{e^{-x}}}{2} = \frac{\cancel{2} e^{x}}{\cancel{2}}$$

$$\boxed{\cosh x + \sinh x = e^{x}}$$

$$\cosh^2 x - \sinh^2 x = (\cosh x - \sinh x)(\cosh x + \sinh x)$$

$$= e^{-x} \cdot e^{x} = 1$$

$$\boxed{\cosh^2 x - \sinh^2 x = 1}$$

$\because \cosh^2 x - \sinh^2 x = 1$

بالقسمة على $\cosh^2 x$

$$1 - \frac{\sinh^2 x}{\cosh^2 x} = \frac{1}{\cosh^2 x}$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x \quad \#$$

$\because \cosh^2 x - \sinh^2 x = 1$

بالقسمة على $\sinh^2 x$

$$\coth^2 x - 1 = \operatorname{cosech}^2 x \quad \#$$

تفاضل الدوال الزائدية

$y = \sinh u \qquad y' = u' \cosh u$

$y = \cosh u \qquad y' = u' \sinh u$

$y = \tanh u \qquad y' = u' \operatorname{sech}^2 u$

$y = \coth u \longrightarrow y' = -u' \operatorname{cosech}^2 u$

$y = \operatorname{sech} u \longrightarrow y' = -u' \operatorname{sech} u \cdot \tanh u$

$y = \operatorname{cosech} u \longrightarrow y' = -u' \operatorname{cosech} u \cdot \coth u$

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if $y = \sinh(\sqrt{x}) + \cosh(e^{-3x}) + \sin(\tanh x)$

$$\frac{dy}{dx} = \frac{\cosh\sqrt{x}}{2\sqrt{x}} + \sinh(e^{-3x}) \cdot (-3e^{-3x}) + \cos(\tanh x) \cdot (\operatorname{sech}^2 x)$$

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if $y = \left(\frac{\cosh x + \sinh x}{\cosh x - \sinh x}\right)^4$

$\because y = \left(\frac{e^{x}}{e^{-x}}\right)^4 = e^{8x} \qquad y' = 8e^{8x}$

if $y = \operatorname{Cosech}(x^2) + \operatorname{Sech}(\sqrt{x})$

$$\frac{dy}{dx} = -2x \operatorname{Cosech}(x^2) \cdot \operatorname{Coth}(x^2) - \frac{\operatorname{Sech}\sqrt{x}\,\tanh\sqrt{x}}{2\sqrt{x}}$$

if $y = e^{\tanh x} + 2^{x \cdot \operatorname{Cosh} x^3}$

$$\frac{dy}{dx} = e^{\tanh x} \cdot \operatorname{Sech}^2 x + 2^{x \operatorname{Cosh} x^3} \ln 2 \left[\cosh x^3 + 3x^{2} \cdot x \sinh x^3\right]$$

$$y = \frac{x}{(1+x^2)^2}$$

$$y(1+x^2)^2 = x$$

$$y'(1+x^2)^2 + 2y(1+x^2) \cdot 2x = 1$$

$$y'(1+x^2)^2 + 4xy(1+x^2) = 1 \quad \#$$