COLLEGE
Tanta Engineering

إعدادي 2020

الجبر

المحاضرة الأولى - الكسور الجزئية

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Partial fractions

Dif: The function $P_m(x) = a_m x^m + a_{m-1} x^{m-1} + \dots + a_1 x + a_0$ is called a plonomial of degree m, as $a_m \neq 0$

The function $Q(x) = \dfrac{P_m(x)}{q_m(x)}$ → numerator, → rational function, → denominator

$$\frac{2}{x+1} - \frac{3}{x+2} = \frac{2(x+2)-3(x+1)}{(x+1)(x+2)} = \frac{1-x}{(x+1)(x+2)}$$

To put any fraction in the form of partial follow these steps:

1- Make sure that the degree of the numerator is ~~higher~~ lower then the degree of the denominator

2- Analize $q_n(x)$. The roots of "q" are one of four cases

Case 1: Simple roots of 1st degree.

$$Q(x) = \frac{P_m(x)}{(\dots)(x-a)} = \dots\dots + \frac{a_1}{x-a}$$

$$a_1 = \lim_{x \to a} (x-a)\,Q(x) = \frac{P_m(x)}{(\dots\dots)}$$

Ex: factorize $\dfrac{x^2+1}{(x+2)(x-1)}$

$$\begin{array}{r|l} & 1 \\ x^2+x-2 & x^2+1 \\ & x^2+x-2 \\ \hline & -x+3 \end{array}$$

$$Q(x) = 1 + \frac{3-x}{x^2+x-2} = 1 + \frac{3-x}{(x-1)(x+2)} = 1 + \frac{a}{x-1} - \frac{b}{x+2}$$

$$a = \lim_{x \to 1} \frac{3-x}{x+2} = \frac{2}{3}$$

$$b = \lim_{x \to -2} \frac{3-x}{x-1} = \frac{5}{-3}$$

$$Q(x) = 1 + \frac{2}{3(x-1)} + \frac{5}{3(x+2)}$$ #

Case 2: Rebeated real roots.

$$Q(x) = \frac{P_m(x)}{(x-a)^k} = \dots + \frac{a_k}{(x-a)^k} + \frac{a_{k-1}}{(x-a)^{k-1}} \dots + \frac{a_1}{(x-a)}$$

where: $a_k \to$ By direct method

$a_{k-1} \dots a_1 \to$ By one of two methods

1- By assuming different values for x in both sides.

2- Compairing the cooffeicients of boath sides.

Ex: factorize $f(x) = \frac{x+1}{(x+2)(x-1)^3}$

$$f(x) = \frac{a}{(x+2)} + \frac{b}{(x-1)^3} + \frac{c}{(x-1)^2} + \frac{d}{x-1}$$

$$\frac{x+1}{(x+2)(x-1)^3} = \frac{a(x-1)^3 + b(x+2) + (x+2)(x-1) + d(x+2)(x-1)^2}{(x+2)(x-1)^3}$$

$$a = \lim_{x \to -2} \frac{x+1}{(x-1)^3} = \frac{-1}{-27} = \frac{1}{27}$$

$$b = \lim_{x \to 1} \frac{x+1}{x+2} = \frac{2}{3}$$

Put $x = -1$

$$0 = -8 \times \frac{1}{27} + \frac{2}{3} - 2c + 4d$$
$$0 = \frac{-8}{27} + \frac{2}{3} - 2c + 4d \longrightarrow ①$$

put $x = 0$

$$1 = -\frac{1}{27} - \frac{2}{3} - 2c - 2d \longrightarrow ②$$

$d = \dots \qquad c = \dots$

Case 3 : Distencet complex root.

$$Q(x) = \frac{P_n(x)}{ax^2 + bx + c} = \frac{ax + b_1}{ax^2 + bx + c}$$

Case 4 : Repeated complex root.

$$Q(x) = \frac{P_n(x)}{(ax^2 + bx + c)^k} = \frac{a_1 x + b_1 x}{(ax^2 + bx + c)^k}$$

Ex: $f(x) = \dfrac{x+2}{(x-1)(x-2)^2 (x^2+4)}$

$$f(x) = \frac{x+2}{(x-1)(x-2)^2 (x^2+4)}$$

$$= \frac{a}{x-1} + \frac{b}{(x-2)^2} + \frac{c}{x-2} + \frac{dx+e}{(x^2+4)}$$

$a = \frac{3}{5} \qquad b = \frac{4}{8}$

$$x+2 = a(x-2)^2(x^2+4) + b(x-1)(x^2+4) + c(x-1)(x-2)(x^2+4)$$
$$+ (x-1)(x-2)^2(dx+e)$$

Put: $x^2 + 4 = 0$

$x^2 = -4$

$$x + 2 = (dx + e)(16 + 4x)$$
$$= 16dx + 16e - 16d + 4ex$$

③

C.O x : $1 = 16d + 4e$

: $2 = 16e - 16d$

$d: \dots$ $e = \dots$

ثم نعود للمعادلة ستار للحصول على C

Ex: $f(x) = \dfrac{x^2+3}{(x^2+1)(x^2+2)(x^2+3)(x^2+4)\cdots}$

حول كل x^2 إلى y

Ex: $f(x) = \dfrac{3x^4+1}{x^2+2x}$ $\times x$

$= \dfrac{3+x}{x^3+2x^2}$

$= \dfrac{a}{x^2} + \dfrac{b}{x} + \dfrac{c}{x+2}$

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