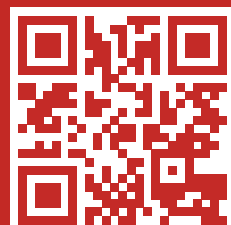


إعدادي 2020

# الرياضيات

جبر - الحل الرقمي للمعادلات الغير خطية  
م. أدهم أسامة

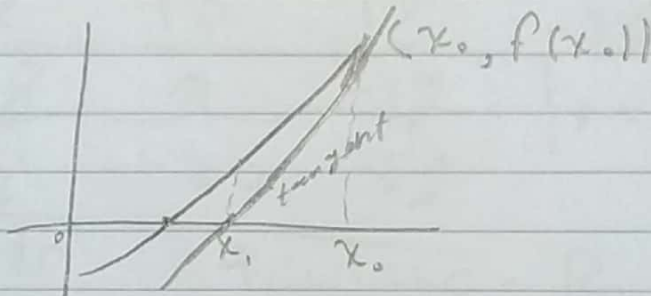


## Numerical Solve for nonlinear eqns

Find a root for the eqn  $f(x)=0$

### 1] Newton's Method

- 1 - Start with an initial root  $x_0$ .
- 2 - Improve this root using the tangent at  $x_0$  to the curve of  $f(x)$  as for lows



the eqn is:

$$f(x) - f(x_0) = f'(x_0)(x - x_0), \text{ at } x = x_1, f(x) = 0 \Rightarrow$$

$$0 - f(x_0) = f'(x_0)(x_1 - x_0)$$

$$\therefore x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

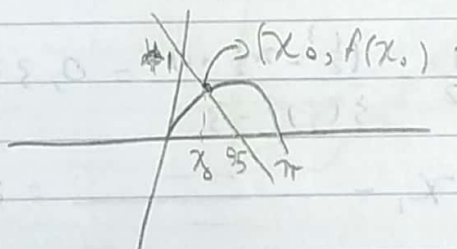
$$\text{OR } x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0, 1, 2, \dots$$

## 2] Simple Iterative Method

1. Start with  $x_0$
2. Rewrite  $f(x) = 0$  as  $x = \Phi(x)$  Provided  
 $|\Phi'(x_0)| < 1 \Rightarrow$   
 Condition for Convergence
3. Improve  $x_0$  from  $x_{n+1} = \Phi(x_n)$ ,  $n=1, 2, 3, \dots$

Ex:- Find a root for  $f(x) = \sin x + 2x - 1 = 0$

$$\sin x = 1 - 2x$$



$$x_0 = 0,3$$

نقطة التقاطع  
الذي نريده

$$x = \frac{1 - \sin x}{2}$$

$$|\Phi'(x)| = |1 - \frac{1}{2} \cos(x)|$$

$$|\Phi'(x_0)| = |1 - \frac{1}{2} \cos(0,3)| \leq \frac{1}{2} < 1$$

$$x_{n+1} = \frac{1}{2} (1 - \sin x_n), n=0, 1, 2, \dots$$

$$n=0 \quad x_1 = \frac{1}{2} (1 - \sin 0,3) = 0,35$$

$$x_2 = 0,3274945$$

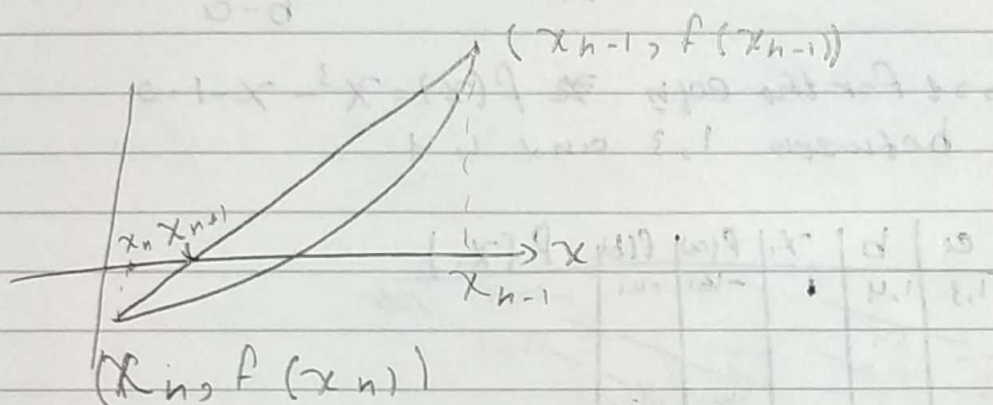
$$x_3 = 0,3391618$$



### 3] Secant Method

1- Start with  $x_n, x_{n-1} \ni f(x_n) \times f(x_{n-1}) < 0$

2- Improve using secant eqn to get  $x_{n+1}$  as



$$\frac{f(x) - f(x_n)}{x - x_n} = m = \frac{f(x_{n-1}) - f(x_n)}{x_{n-1} - x_n}$$

$$\text{at } x = x_{n+1} \rightarrow f(x) = 0 \Rightarrow$$

$$x_{n+1} = x_n - \frac{f(x_n)}{m}, \quad n = 1, 2, 3, \dots$$

#### 4] False position Method

1- Start with  $a, b \ni f(a) \cdot f(b) > 0$

2- Improve from

$$x_1 = a - \frac{f(a)}{m}, \quad m = \frac{f(b) - f(a)}{b - a}$$

Ex: Find a root for the eqn  $f(x) = x^3 - x - 1 = 0$   
between 1,3 and 1,4

a	b	$x_1$	$f(a)$	$f(b)$	$f(x_1)$
1,3	1,4		-1,727	+1,648	-1,041
	1,4		-1,727	+1,648	+1,041
			-1,727	+1,648	

