

الجبر

المحاضرة الثالثة - نظرية ذات الحدين
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Binomial theorem

$$(a+b)^n = a^n + C_1^n a^{n-1} b + C_2^n a^{n-2} b^2 + C_3^n a^{n-3} b^3 + \dots + C_n^n a^1 b^n$$

$n = \text{integer \#}$

$$C_r^n = \binom{n}{r} = \frac{n!}{r! (n-r)!}$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!} x^2 + \frac{n(n-1)(n-2)}{3!} x^3 + \dots, |x| < 1, n = \text{integer \#}$$

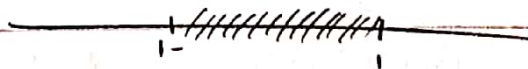
$$f(x) = (1+x)^{-3}, \quad x \ll, \quad x \gg$$

$x \ll$
very small

$$f(x) = (1+x)^{-3} = 1 + (-3)(x) + \frac{(-3)(-4)}{2!} + \frac{(-3)(-4)(-5)}{3!} + \dots$$

$$|x| < 1$$

$$-1 < x < 1$$

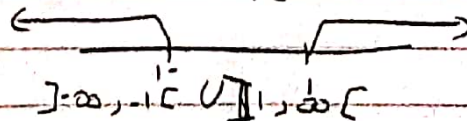


$x \ll$
very large

$$f(x) = x^{-3} \left(1 + \frac{1}{x}\right)^{-3} = x^{-3} \left[1 + (-3)\left(\frac{1}{x}\right) + \frac{(-3)(-4)}{2!} + \dots\right]$$

$$\frac{1}{x} < 1 \Rightarrow |x| > 1$$

$$1 \rightarrow x > 1$$



find and approximate a value for $f(x) = \frac{\sqrt{1+2x}}{\sqrt[3]{1+3x}}$
neglecting x^3 and ~~all other things~~ the following

$$f(x) = (1+2x)^{\frac{1}{2}} \cdot (1+3x)^{-\frac{1}{3}}$$

$$= \left(1 + \frac{1}{2} \times 2x + \frac{1}{2} \times \frac{-1}{2} \times 4x^2\right) \cdot \left(1 + \frac{1}{3} \times 3x + \frac{-1}{3} \times \frac{-4}{3} \times 9x^2\right)$$

$$\approx 1 + \frac{x^2}{2}$$

نكسر هنا

$$\sqrt[3]{\frac{41}{25}} \cdot \frac{\sqrt{2}}{\sqrt[3]{3}}$$

$$\sqrt[3]{\frac{41}{25}} \times \frac{5}{5} = \frac{1}{5} \sqrt[3]{205} = \frac{1}{5} \sqrt[3]{216-11} = \frac{1}{5} \sqrt[3]{6^3-11} = \frac{6}{5} \times \left(1 - \frac{11}{216}\right)^{\frac{1}{3}}$$

نظروا

$$\frac{\sqrt{2}}{\sqrt[3]{3}} = \left(\left(\frac{\sqrt{2}}{\sqrt[3]{3}} \right)^6 \right)^{\frac{1}{6}}$$

أرقام البسط جديده n سالبة

الاشارة يتبدل بـ n سالبة و x موجبة

لوا ارقام يتزايد في الارقام من واحد يعني ان n كسر مقامه هو الفرق

Ex 13.

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$$S = 1 + \frac{1}{3} + \frac{1 \cdot 3}{3 \cdot 6} + \frac{1 \cdot 3 \cdot 5}{3 \cdot 6 \cdot 9}$$

$$1 + \left(\frac{-1}{2}\right)\left(\frac{-2}{3}\right) + \frac{\left(\frac{-1}{2}\right)\left(\frac{-2}{3}\right)}{1 \cdot 2} \left(\frac{-2}{3}\right)^2 + \frac{\left(\frac{-1}{2}\right)\left(\frac{-3}{2}\right)\left(\frac{-5}{2}\right)}{1 \cdot 2 \cdot 3} \left(\frac{-2}{3}\right)^3 + \dots$$

$$n = -\frac{1}{2} \quad x = \frac{-2}{3}$$

١- الأرقام يتزايد n سالبة

٢- الأشارات كلها موجبة $\therefore x$ سالبة

٣- قسمت على ثابت المتكافؤ ونزبت فيه

٤- أخذنا مسدود (المقام)

$$(1+x)^n = \left(1 - \frac{2}{3}\right)^{-\frac{1}{2}} = \sqrt{3}$$

$$S = \frac{11}{6} + \frac{5 \cdot 7}{6 \cdot 12} + \frac{5 \cdot 7 \cdot 9}{6 \cdot 12 \cdot 18}$$

$$S = 1 + \left(\frac{-5}{2}\right)\left(\frac{-2}{6}\right) + \frac{\left(\frac{-5}{2}\right)\left(\frac{-7}{2}\right)}{1 \cdot 2} \left(\frac{-2}{6}\right)^2 + \frac{\left(\frac{-5}{2}\right)\left(\frac{-7}{2}\right)\left(\frac{-9}{2}\right)}{1 \cdot 2 \cdot 3} \left(\frac{-2}{6}\right)^3 + \dots$$

$$n = -\frac{5}{2}$$

$$x = \frac{-2}{6}$$

$$S = \left(1 - \frac{1}{3}\right)^{-\frac{5}{2}}$$

$$(1-x)^{-1} = 1 + x + x^2 + x^3 + \dots + x^r$$

$$(1+x)^{-1} = 1 - x + x^2 - x^3 + \dots + (-1)^r (x)^r$$

$$(1+x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \dots + (r+1)x^r$$

$$(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \dots + (-1)^r (r+1)x^r$$

$$|x| < 1$$

the coefficient of x^n in $f(x) = \frac{1-x}{1+x+x^2}$

$$\frac{1-x}{1+x+x^2} \times \frac{1-x}{1-x} = \frac{(1-x)^2}{(1-x^3)^2} = (1-2x+x^2)(1-x^3)^{-1}$$

$$= (1-2x+x^2)(1+x^3+x^6+x^9+\dots)$$

اكتب المصطلحات

$$C.O. x^{3n} = 1$$

$$C.O. x^{3n+1} = -2$$

$$C.O. x^{3n+2} = 1$$

$$f(x) = \frac{1-x}{(1+x-2x^2)}$$

$$= \frac{1-x}{(1-x)(2x+1)} = \frac{A}{1+2x} + \frac{B}{1-x}$$

Partial Fraction