سنتر فيوتشر

Subject: رياضة "اعدادي"

Chapter: النهايات

Mob: 0112 3333 122

0109 3508 204

النهايات

limits

$$\frac{1}{0} = \infty \qquad \text{و} \qquad \frac{1}{\infty} = 0$$

$$(\text{رقم أصغر من الواحد})^{\infty} = \text{صفر}$$

$$(\text{رقم أكبر من الواحد})^{\infty} = \infty$$

التعويض المباشر

* $$\lim_{x \to 0} 2x+3 = 3$$

$$\lim_{x \to \infty} \frac{3}{x+1} = 0$$

$$\lim_{x \to \infty} \left(\frac{1}{2}\right)^{x} = 0$$

$$\lim_{x \to 0} \frac{2}{x} + 4 = \infty + 4 = \infty$$

$$\lim_{x \to \infty} 2^{3x} = \infty$$

$$\lim_{x \to 0} \frac{\sin \alpha x}{x} = \alpha$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\lim_{x \to 0} \frac{\tan \alpha x}{x} = \alpha$$

$$\frac{1}{\cos x} = \overset{\text{قا}}{\sec x}$$

$$\frac{1}{\sin x} = \overset{\text{قتا}}{\operatorname{cosec} x}$$

$$\frac{1}{\tan x} = \overset{\text{ظتا}}{\cot x}$$

تذكير

$\sin 2x = 2 \sin x \cos x$

$\cos 2x = 2\cos^2 x - 1$

$= 1 - 2\sin^2 x = \cos^2 x - \sin^2 x$

- $\sin^2 x + \cos^2 x = 1$

$1 + \tan^2 x = \sec^2 x$

$\cot^2 x + 1 = \csc^2 x$

Find (evalute)

$\lim\limits_{x \to 0} \frac{\sin 3x}{x} = 3$

$\lim\limits_{x \to 0} \frac{\sin 2x}{5x} = 2/5$

$\lim\limits_{x \to 0} \frac{\sin 2x^2}{x^2} = 2$

$\lim\limits_{x \to 0} \frac{\tan 5x}{x} = 5$

$\lim\limits_{x \to 0} \frac{\tan 4x}{\sin 3x} = \frac{4}{3}$

$\lim\limits_{x \to 0} \frac{\sin^2 3\cancel{x^2}}{x^2}$

$$\lim_{x\to 0}\left(\frac{\sin 3x}{x}\right)^2 = 9$$

$$\lim_{x\to 0}\frac{\sin 5x}{\sin 3x} = 5/3$$

*

$$\lim_{x\to 0}\frac{1-\cos 2x}{x^2}$$

حل

$$\because 1-\cos 2x = 2\sin^2(x)$$

$$\lim_{x\to 0} 2\frac{\sin^2 x}{x^2} = 2\lim_{x\to 0}\left(\frac{\sin x}{x}\right)^2 = 2$$

$$\lim_{x\to 0}\frac{1-\cos x}{x}$$

حل

$$\because 1-\cos x = 2\sin^2\frac{x}{2}$$

$$\lim_{x\to 0} 2\frac{\sin^2\frac{x}{2}}{x} = 2\lim_{x\to 0}\frac{\sin\frac{1}{2}x}{x}\cdot\sin\frac{x}{2}$$

$$= 2 \lim_{x \to 0} \frac{\sin \frac{1}{2} x}{x} \lim_{x \to 0} \sin \frac{1}{2} x$$

$$2 \cdot \left(\frac{1}{2}\right) \cdot 0 = \text{Zero}$$

$$\lim_{x \to 0} \frac{1 - \cos 5x}{x^2}$$

b

$$\lim_{x \to 0} \frac{2 \sin^2 \frac{x}{2}}{x^2} = 2 \lim_{x \to 0} \left(\frac{\sin \frac{x}{2}}{x} \right)^2$$

$$= 2 \left(\frac{1}{2}\right)^2 = \frac{1}{2}$$

$$\lim_{x \to 0} \frac{\sin[\sin(5x)]}{x}$$

b

$$\lim_{x \to 0} \frac{\sin[\sin(5x)]}{\sin(5x)} \cdot \frac{\sin 5x}{x}$$

$$1 * 5 = 5$$

$$\lim_{x \to \infty} x \sin\left(\frac{1}{x}\right)$$

$0 \cdot \infty$

كمية غير معرفة

$$\lim_{\frac{1}{x} \to \frac{1}{\infty} = 0} \frac{\sin(1/x)}{1/x} = 1$$

$$\lim_{x \to \infty} x \tan\left(\frac{5}{x}\right)$$

$$\lim_{\frac{1}{x} \to \frac{1}{\infty} = 0} \frac{\tan(5/x)}{1/x} = 5$$

قاعدة السندوتش

$$g(x) \leq f(x) \leq h(x)$$

$$\lim h(x) = L$$

$$\lim g(x) = L$$

$$\therefore \lim f(x) = L$$

$$-1 \leq \cos x \leq 1$$

$$-1 \leq \sin x \leq 1$$

$$\lim_{x \to \infty} \frac{\sin x}{x}$$

الحل

$$\because \frac{-1}{x} \le \frac{\sin x}{x} \le \frac{1}{x}$$

$$\lim_{x \to \infty} \frac{1}{x} = 0 \quad , \quad \lim_{x \to \infty} \frac{-1}{x} = 0$$

$$\therefore \lim_{x \to \infty} \frac{\sin x}{x} = 0$$

$$\lim_{x \to \infty} \frac{\cos(x^2+4)}{x}$$

الحل

$$\frac{-1}{x} \le \frac{\cos(x^2+4)}{x} \le \frac{1}{x}$$

$$\lim_{x \to \infty} \frac{1}{x} = 0 \quad , \quad \lim_{x \to \infty} \frac{-1}{x} = 0$$

$$\lim_{x \to \infty} \frac{\cos(x^2+4)}{x} = 0$$

$$\lim_{x \to \infty} x \sin\left(\frac{1}{x}\right) \quad , \quad \lim_{x \to 0} x \sin\left(\frac{1}{x}\right)$$

$$\lim_{x \to \infty} x \sin(1/x)$$

$$\lim_{\frac{1}{x} \to 0} \frac{\sin(1/x)}{1/x} = 1$$

$$-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$$

$$-x \leq x \sin\left(\frac{1}{x}\right) \leq x$$

$$\lim_{x \to 0} x = 0 \quad , \quad \lim_{x \to 0} -x = 0$$

$$\therefore \lim_{x \to 0} x \sin\left(\frac{1}{x}\right) = 0$$

$$\lim_{x \to 2} \frac{\sin(x^2-4)}{x-2}$$

$$\lim_{x \to 2} \frac{\sin(x^2-4)}{\cancel{(x-2)}(x+2)} \cdot (x+2)$$

$$1 * 4 = 4 \ \#$$

$$\lim_{x \to 0} \frac{\sin\sqrt{x}}{x}$$

الحل

$$\lim_{x \to 0} \frac{\sin\sqrt{x}}{\sqrt{x}} \cdot \frac{1}{\sqrt{x}}$$

$$1 * \infty = \infty$$

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② اذا كان الناتج $\frac{0}{0}$ أو ~~$\frac{\infty}{\infty}$~~ نحلل البسط والمقام ونختصر

$$\lim_{x \to 2} \frac{x^2 - 4}{x^2 - x - 2}$$

الحل

بالتعويض $\frac{0}{0}$

$$\lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x+1)} = \frac{4}{3} \#$$

$$\lim_{x \to 1} \frac{x^3 - 1}{x^2 - 1}$$

الحل

$$\lim_{x \to 1} \frac{\cancel{(x-1)}(x^2 + x + 1)}{\cancel{(x-1)}(x+1)} = 3/2$$

إذا كان الناتج $\frac{\infty}{\infty}$ نقسم البسط والمقام على أكبر أس

$$\lim_{x \to \infty} \frac{x^3 + 4x + 7}{2x^3 + 5x^2 - 9}$$

بالتعويض $\frac{\infty}{\infty}$

$$\lim_{x \to \infty} \frac{1 + \frac{4}{x^2} + \frac{7}{x^3}}{2 + \frac{5}{x} - \frac{9}{x^3}} = \frac{1 + 0 + 0}{2 + 0 - 0} = \frac{1}{2}$$

$$\lim_{x \to \infty} \frac{2x + 8}{x^3 + 9x - 1}$$

بالتعويض $\frac{\infty}{\infty}$

$$\lim_{x \to \infty} \frac{\frac{2}{x^2} + 8/x^3}{1 + \frac{9}{x^2} - \frac{1}{x^3}} = \frac{0 + 0}{1 + 0 - 0} = 0$$

درجة البسط أقل من درجة المقام ← الناتج = صفر

$$\lim_{x \to \infty} \frac{x^2 + 7}{x - 1}$$ الحل $\div x^2$

$$\lim_{x \to \infty} \frac{1 + 7/x^2}{\frac{1}{x} - \frac{1}{x^2}} = \frac{1 + 0}{0 - 0} = \frac{1}{0} = \infty$$

$$\lim_{x \to \infty} \frac{3x^3 + 7}{8x^3 + 9} \longrightarrow 3/8$$

$$\lim_{x \to \infty} \frac{2x + 5}{x - 3} = 2$$

$$\lim_{x \to \infty} \frac{x + 3}{x^2 + 9} = 0 \qquad \lim_{x \to \infty} \frac{x^2 + 4}{x - 1} = \infty$$

اذا كان الناتج $\infty - \infty$ نضرب $\times$ المرافق

$$\lim_{x \to \infty} \sqrt{x + 7} - \sqrt{2x + 3}$$

$$\lim_{x \to \infty} \frac{\left(\sqrt{x + 7} - \sqrt{2x + 3}\right)\left(\sqrt{x + 7} + \sqrt{2x + 3}\right)}{\sqrt{x + 7} + \sqrt{2x + 3}}$$

$$\lim_{x \to \infty} \frac{x + 7 - (2x + 3)}{\sqrt{x + 7} + \sqrt{2x + 3}} = \frac{4 - x}{\sqrt{x + 7} + \sqrt{2x + 3}}$$

$$\lim \frac{4/x - 1}{\sqrt{\frac{1}{x} + 7/x^2} + \sqrt{\frac{2}{x} + \frac{3}{x^2}}} = \frac{-1}{0} = -\infty$$

قاعدة اويلر [مهمة جداً]

$$\lim_{x \to a} [1+f]^{g}$$

شرط

$$\lim f = 0 \quad , \quad \lim g = \infty$$

$$\lim f \cdot g = C$$

$$\therefore \lim [1+f]^{g} = e^{C}$$

$e = 2.7$

$$\lim_{x \to 0} [1+2x]^{\frac{1}{x}}$$

ط

$$\lim_{x \to 0} 2x = 0 \quad , \quad \lim_{x \to 0} \frac{1}{x} = \infty$$

$$\lim_{x \to 0} 2x \cdot \frac{1}{x} = 2$$

$$\therefore \lim_{x \to 0} (1+2x)^{\frac{1}{x}} = e^{2}$$

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Ex

$$\lim_{x \to 0} [1 + \sin x]^{\frac{3}{x}}$$

$$\lim_{x \to 0} \sin x = 0 \quad , \quad \lim_{x \to 0} \frac{3}{x} = \infty$$

$$\lim_{x \to 0} 3 \frac{\sin x}{x} = 3$$

$$\therefore \lim_{x \to 0} (1 + \sin x)^{3/x} = e^{3}$$

$$\lim_{x \to 0} [1 + \sin x]^{2 \cot x}$$

$$\lim_{x \to 0} \sin x = 0 \qquad \lim_{x \to 0} 2 \cot x = 2 \lim_{x \to 0} \frac{\cos x}{\sin x} = \frac{2}{0} = \infty$$

$$\lim_{x \to 0} \cancel{\sin x} \cdot \frac{2 \cos x}{\cancel{\sin x}} = 2$$

$$\lim_{x \to 0} (1 + \sin x)^{2 \cot x} = e^{2}$$

Ex

$$\lim_{x \to \infty} \left(\frac{x+3}{x-1}\right)^{3x}$$

$$\lim \left[\frac{x-1+4}{x-1}\right]^{3x}$$

$$\lim_{x \to \infty} \left[\cancel{\frac{x-1}{x-1}}_{1} + \frac{4}{x-1}\right]^{3x}$$

$$\lim_{x \to \infty} \frac{4}{x-1} = 0 \;, \quad \lim_{x \to \infty} 3x = \infty$$

$$\lim_{x \to \infty} \frac{4(3x)}{x-1} = 12$$

$$\lim_{x \to \infty} \left(\frac{x+3}{x-1}\right)^{3x} = e^{12}$$

$$\lim_{x \to \infty} \left(\frac{x^2-1}{x^2-3}\right)^{x}$$

$$\lim_{x \to \infty} \left(\cancel{\frac{x-3}{x-3}}_{1} + \frac{2}{x^2-3}\right)^{x}$$

$$\lim_{x \to \infty} \frac{2x}{x^2 - 3} = 0$$

$$\therefore \lim_{x \to \infty} \left(\frac{x^2 - 1}{x^2 - 3} \right)^x = e^0 = 1$$

مثال

$$\lim_{x \to 0} (\cos x)^{\frac{1}{x^2}}$$

الحل

$$\lim_{x \to 0} \left[1 - 2 \sin^2 \frac{x}{2} \right]^{\frac{1}{x^2}}$$

$$\lim_{x \to 0} -2 \sin^2 \frac{x}{2} = 0 \qquad \lim_{x \to 0} \frac{1}{x^2} = \infty$$

$$\lim_{x \to 0} \frac{-2 \sin^2 \frac{x}{2}}{x^2} = -2 \lim_{x \to 0} \left(\frac{\sin \frac{1}{2} x}{x} \right)^2$$

$$= -2 \left(\frac{1}{2} \right)^2 = -\frac{1}{2}$$

$$\lim_{x \to 0} (\cos x)^{\frac{1}{x^2}} = e^{-\frac{1}{2}}$$

$$\lim_{x \to 0} \left(\sqrt{\cos x}\right)^{\frac{1}{x}}$$

$$\lim_{x \to 0} (\cos x)^{\frac{1}{2x}} = \lim_{x \to 0} \left[1 - 2\sin^2 \frac{x}{2}\right]^{\frac{1}{2x}}$$

$$\lim_{x \to 0} -2\sin^2 \frac{x}{2} = 0 \qquad \lim_{x \to 0} \frac{1}{2x} = \infty$$

$$\lim_{x \to 0} \frac{-\cancel{2}\sin^{\cancel{2}} \frac{x}{2}}{\cancel{2}x} = \lim_{x \to 0} -\frac{\sin \frac{x}{2}}{x} \cdot \sin\frac{x}{2}$$

$$= \left(-\tfrac{1}{2}\right)(0) = 0$$

$$\therefore \lim_{x \to 0} \left(\sqrt{\cos x}\right)^{\frac{1}{x}} = e^0 = 1$$

$$\lim_{x \to 0} \left(\sec^3 x\right)^{\frac{1}{x^2}}$$

$$\lim_{x \to 0} \left(\sec^2 x\right)^{\frac{3}{2x^2}} = \lim_{x \to 0} \left(1 + \tan^2 x\right)^{\frac{3}{2x^2}}$$

$$\lim_{x \to 0} \tan^2 x = 0 \qquad \lim_{x \to 0} \frac{3}{2x^2} = \infty$$

$$\lim_{x\to 0} \frac{3\tan^2 x}{2x^2} = \lim_{x\to 0} \frac{3}{2}\left(\frac{\tan x}{x}\right)^2 = 3/2$$

$$\lim_{x\to 0} (\sec^3 x)^{1/x^2} = e^{3/2}$$

$$\lim_{x\to\infty} \cancel{x} \ln\left[\frac{x+3}{x-5}\right]^{\cancel{x}}$$

$$\lim_{x\to\infty} \ln\left[\frac{x+3}{x-5}\right]^{\cancel{x}}$$

$\ln x^y = y\ln x$

$\ln\left(\frac{x}{y}\right) = \ln x - \ln y$

$\cancel{x}\ln(xy) = \ln x + \ln y$

$\ln e^{\alpha} = \alpha$

$e^{\ln\alpha} = \alpha$

$\frac{\ln(x)}{\ln y} \neq \ln\left(\frac{x}{y}\right)$

$$\ln \lim_{x\to\infty} \left(\frac{\cancel{x-5}}{\cancel{x-5}} + \frac{8}{x-5}\right)$$

$$\ln \lim_{x\to\infty} \left[1 + \frac{8}{x-5}\right]^{\cancel{x}}$$

$$\lim_{x\to\infty} \frac{8x}{\cancel{x}(x-5)} = 8$$

$$\therefore \lim_{x\to\infty} x \ln\left(\frac{x+3}{x-5}\right) = \cancel{\ln} e^{8} = 8$$

h-w

$$\lim_{x \to \infty} \left[\frac{x+3}{x+2}\right]^{x}$$

$$\lim_{x \to 0} (1 + \tan x)^{\frac{1}{x}}$$

$$\lim_{x \to \frac{\pi}{2}} (1 + \cos x)^{2\tan x}$$

$$\lim_{x \to 0} \left(1 + \frac{1-\cos x}{x}\right]^{\frac{1}{x}}$$

$$\lim_{x \to \infty} (x+3) - \sqrt{x^2+4}$$

$$\lim_{x \to 0} \frac{x}{\sin\sqrt{x}} \quad , \quad \lim_{x \to 0} \frac{\sin^2 x}{\tan^2 3x}$$

$$\lim_{x \to \infty} \frac{\cos\sqrt{x}}{x^2+4} \qquad \lim_{x \to \infty} (2x+3)^{x}$$

$$\lim_{x \to \infty} 2^{-x} \qquad \lim_{x \to \infty} \sin(1/x)$$

$$\lim_{x \to \pi} \frac{\sin 3x}{\sin 2x}$$

$$\sin(x+y) = \sin x \cos y + \cos x \sin y$$

$$\sin 2x = 2 \sin x \cos x$$

$$\sin 3x = \sin(x+2x) = \sin x \cos 2x + \cos x \sin 2x$$

$$\therefore \lim_{x \to \pi} \frac{\sin x \cos 2x + \cancel{\cos x \sin 2x}}{\cancel{\sin 2x}}$$

$$\lim_{x \to \pi} \underbrace{\cancel{\cos x}}_{-1} + \frac{\sin x \cos 2x}{\sin 2x}$$

$$-1 + \lim_{x \to \pi} \frac{\cancel{\sin x} \cos 2x}{2 \cancel{\sin x} \cos x}$$

$$-1 + \frac{1}{-2} = -3/2$$