Dynamics,
Study

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1 CONTENTS

2 INTRODUCTION

In Dynamic study we try to make a model for the machine, a good model that would describe the machine in the transient actions, a model is which relates outputs to inputs through number of parameters

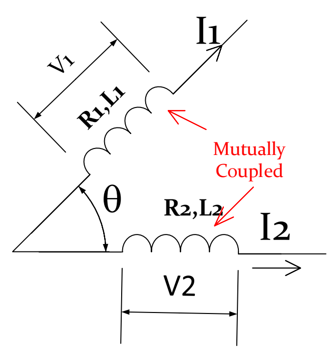
$$E(t) = Ri(t) + L\frac{di(t)}{dt}$$

Example : DC Machine model in steady state is as follows

$$V_a = R_a i_a + E \qquad \& \quad T = kI_a I_f$$

3 GENERALIZED MODEL OF VOLTAGE & TORQUE EQUATIONS OF TWO WINDING MACHINE

Assume each coil has currents i_1, i_2 due to voltages v_1, v_2



Note all symbols are represented in ***small letters*** as we focus in time domain (transient)

3.1 Voltage equation

Note we represent differential d/dt by letter "p"

$$v_1 = R_1 i_1 + e_1$$
$$e_1 = \frac{d\lambda_1}{dt}$$
$$v_1 = R_1 i_1 + \frac{d}{dt}\lambda_1$$

$$\boldsymbol{v_1 = R_1 i_1 + p\lambda_1}$$

$$\boldsymbol{v_2 = R_2 i_2 + p\lambda_2}$$

Now we just want to substitute λ and differentiate

Flux Linkage "λ" consists of

1- **Self Linking :** between the coil itself (Magnetizing + Leakage)

$$L_m = \frac{N_1^2}{R_m} \quad (Magnetizing)$$
$$L_l = \frac{N_1^2}{R_l} \quad (\,leakage)$$

2- **Mutual Linking :** between the coil and another coil (Mutual)

$$M = \frac{N_1 N_2}{R_m}$$

Hence,

$$\lambda_1 = N_1 \phi_1 = N_1 \left(\frac{N_1 i_1}{R_l} + \frac{N_1 i_1}{R_m} + \frac{N_2 i_2}{R_m} \right)$$
$$= L_{l_1} I_1 + L_{m_1} I_1 + M I_2$$

Let's say $\boldsymbol{L_1}$ ***includes*** $\boldsymbol{L_{l_1}} \& \boldsymbol{L_{m_1}}$

$$\lambda_1 = L_1 I_1 + M I_2$$
$$\lambda_2 = L_2 I_2 + M I_1$$

So the voltage equations are

$$v_1 = R_1 i_1 + \frac{d}{dt}(L_1 i_1 + M i_2)$$
$$v_2 = R_2 i_2 + \frac{d}{dt}(\,L_2 i_2 + M i_1)$$

$$\boldsymbol{v_1 = R_1 i_1 + L_1 \frac{di_1}{dt} + i_1 \frac{dL_1}{dt} + M \frac{di_2}{dt} + i_2 \frac{dM}{dt}}$$

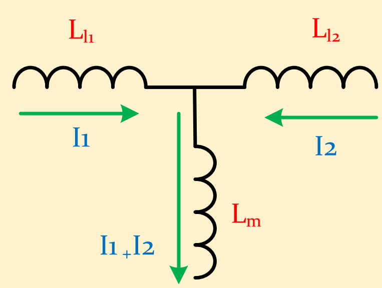
$$\boldsymbol{v_2 = R_2 i_2 + L_2 \frac{di_2}{dt} + i_2 \frac{dL_2}{dt} + M \frac{di_1}{dt} + i_1 \frac{dM}{dt}}$$

<u>Note:</u> we can represent M by Lm
But

$$M = \frac{N_1 N_2}{R}$$
$$M * \frac{N_1}{N_2} = \frac{N_1 N_2}{R} * \frac{N_1}{N_2} = \frac{N_1^2}{R_m} = L_m$$
$$\boldsymbol{\therefore M = L_m * \frac{N_2}{N_1}}$$

So,

$$M I_2 = L_m \left(\frac{N_2}{N_1} \, I_2 \right) = L_m I_2'$$



3.2 Instantaneous Input Power

$$
\begin{aligned}
\boldsymbol{P_{in}} &= \boldsymbol{v_1 i_1 + v_2 i_2} \\
&= \left(\boldsymbol{R_1 i_1^2 + R_2 i_2^2}\right) \\
&+ \left(\boldsymbol{i_1^2 \frac{dL_1}{dt} + i_2^2 \frac{dL_2}{dt}}\right) \\
&+ \left(\boldsymbol{L_1 i_1 \frac{di_1}{dt} + L_2 i_2 \frac{di_2}{dt}}\right) \\
&+ \left(\boldsymbol{M i_1 \frac{di_2}{dt} + M i_2 \frac{di_1}{dt}}\right) \\
&+ \boldsymbol{2\, i_1 i_2 \frac{dM}{dt}}
\end{aligned}
$$

<u>NOTE</u> : for <u>cylindrical</u> rotor inductance won't change with rotation, while with <u>salient</u> pole rotor inductance changes with rotation , and Inductance is always constant when stationary

3.2.1 At Stationary condition

$$L_1 \,\&\, L_2 \,\&\, M \text{ are constants}$$

$$\therefore \frac{dL_1}{dt} = 0 \;, \frac{dL_2}{dt} = 0 \;, \frac{dM}{dt} = 0$$

$$\therefore P_{in} = P_{cu} + \frac{d}{dt}(W_{stored})$$

$$\boldsymbol{P_{in} = i_1^2 R_1 + i^2 R_2 \quad + \quad \frac{d}{dt}\left(\frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M\, i_1 i_2\right)}$$

3.2.2 At Rotation condition

$$P_{in} = P_{cu} + \frac{d}{dt}(W_{stored}) + P_{em}$$

$$\boldsymbol{P_{in} = i_1^2 R_1 + i^2 R_2 \quad + \quad \frac{d}{dt}\left(\frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M\, i_1 i_2\right) + \left(\frac{1}{2} i_1^2 \frac{dL_1}{d\theta} + \frac{1}{2} i_2^2 \frac{dL_2}{d\theta} + i_1 i_2 \frac{dM}{d\theta}\right) \frac{\overbrace{\mathrm{d}\theta}^{\omega}}{\mathrm{dt}}}$$

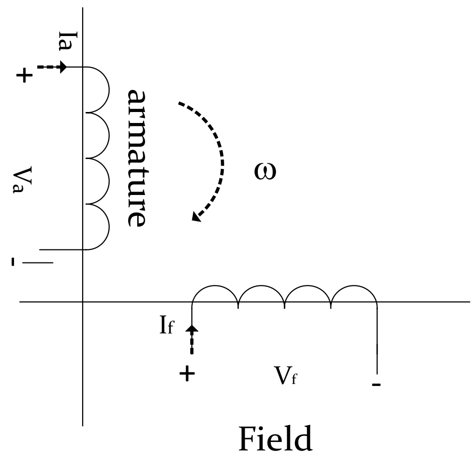
3.3 Electromagnetic developed Torque

$$\boldsymbol{T_{em} = \frac{P_{em}}{\omega} = \frac{1}{2} i_1^2 \frac{dL_1}{d\theta} + \frac{1}{2} i_2^2 \frac{dL_2}{d\theta} + i_1 i_2 \frac{dM}{d\theta}}$$

4 MODEL FOR DC MOTOR

4.1 General Seperately Excited DC Machine Equations

Orthogonal Model of DC Machine



Types of Transient in DC Machine

1- Electrical Transient (v , i , λ)
2- Electromechanical Transient ($v, i, \lambda, M, \omega_r$)
3- Mechanical Transient (M, ω_r)

<u>NOTE</u> we determine mode of operation in DC machine through current direction into the machine then it's motoring mode , if current is out from the machine then it's generating power

Consists of :

1- Voltage Equation
2- Flux Linkage Relations
3- Electromagnetic Developed Torque Expression
4- Mechanical Equation (Relates between em-developed torque and Load)

4.1.1 Voltage Equation

For DC Machine (Cylindrical Rotor (armature) , Salient stationary Stator (Field))

$$v_a = R_a i_a + p\lambda_a \quad , \qquad \lambda_a = L_a i_a + M i_f$$

$$v_f = R_f i_f + p\,\lambda_f \quad , \qquad \lambda_f = v_f i_f + M i_a$$

4.1.1.1 Armature Coil

$$v_a = i_a R_a + L_a \frac{di_a}{dt} + i_a \frac{dL_a}{dt} + M \frac{di_f}{dt} + i_f \frac{dM}{dt}$$

But,

$i_a \frac{dL_a}{dt} = 0$	as, L_a is const (field path is const cuz of brushes)
$M \frac{di_f}{dt} = 0$	as, angle is 90 ° between field winding and equivalent armature winding so M=0

Hence,

$$\boldsymbol{v_a = i_a R_a + L_a \frac{di_a}{dt} + i_f \frac{dM}{dt}}$$

$$\boldsymbol{v_a = i_a R_a + L_a \frac{di_a}{dt} + i_f M_{Max} \omega_r}$$

$$\boldsymbol{v_a = i_a R_a + L_a \frac{di_a}{dt} + \frac{P}{2} i_f M_{Max} \omega_m}$$

Note : $\omega_r = \frac{P}{2}\omega_m$

4.1.1.2 Field Coil

$$v_f = R_f i_f + L_f \frac{di_f}{dt} + i_f \frac{dL_f}{dt} + M \frac{di_a}{dt} + i_a \frac{dM}{dt}$$

But,

$i_f \frac{L_f}{dt} = 0$	As Rotor is Cylindrical then $\frac{dL_f}{dt} = 0$
$M \frac{di_a}{dt} = 0$	as, angle is 90 ° between field winding and equivalent armature winding so M=0
$i_a \frac{dM}{dt}$	As $\frac{dM}{dt} = \frac{DM}{d\theta} * \frac{d\theta}{dt}$ and Field winding is stationary so $\frac{d\theta}{dt} = 0$

$$\boldsymbol{v_f = R_f i_f + L_f \frac{di_f}{dt}}$$

<u>Note</u> **for armature** $i_f \frac{dM}{dt} \neq 0$ **beacause armature winding is rotating so** $\frac{d\theta}{dt} \neq 0$

<u>NOTE</u> we said that $M = M_{Max}\, cos\theta \;\; so \frac{dM}{dt} = -M_{Max} \sin\theta \;\; for\; \theta = 90 \therefore \frac{dM}{dt} = -M$

$$i_f \frac{dM}{dt} = i_f \frac{dM}{d\theta}\frac{d\theta}{dt} = i_f(-M_{Max})\omega$$

$$\frac{dM}{dt}\; is + ve\;\; because\; \omega\; is\; in\; the\; direction\; of\; reaction$$

<u>Note</u> the "M" here is just rotational inductance only there is no transformer or static component to it

<u>Voltage equation at Steady State</u> ($\frac{d}{dt} i = 0$)

$$\boldsymbol{V_a = r_a i_a + \frac{P}{2} M_{Max} i_f\, \omega_m = r_a i_a + K\, \omega_m}$$
$$\boldsymbol{V_f = r_f i_f}$$

<u>Note</u> ac effect Lec3 P.2

4.1.2 Electromagnetic Developed Torque

$$P_{in} = v_a i_a + v_f i_f = \left(r_a i_a^2 + r_f i_F^2 \right) \quad \rightarrow \; P_{cu}$$
$$+ \left(L_a i_a \frac{di_a}{dt} + L_f i_f \frac{di_f}{dt} \right) \rightarrow \frac{d}{dt(W_{stored})}$$
$$+ \left(\frac{P}{2} M_{Max} i_a i_f\, \omega \right) \rightarrow electromechanical$$

Electromagnetic Developed Torque

$$\boldsymbol{T_{em} = \frac{P_{em}}{\omega_m} = \frac{P}{2} M\, i_f i_a}$$

4.1.3 Mechanical Equation

$$\boldsymbol{T_{em} = T_L + J \frac{d\omega_m}{dt} + B\, \omega_m}$$

<u>Note</u> $R \rightarrow B\; and\; L \rightarrow J$

4.2 Dynamic Model Building for Seperately Excited DC Motor

As for now the equation of SE DC Motor has

	NonLinear
input	v_a, v_f, T_L
States	i_a, i_f, ω
Order	3^{rd}

NOTE

up till now we can't use Laplace to solve this system as it's nonlinear because it has some states multiplied by each other,
to use it, we have to assume some state to be constant (as a const speed generator w_m or const excitation vf) or by using Linearization method

SE DC MOTOR EQUATION SET

$$v_a = i_a r_a + L_a \frac{di_a}{dt} + i_f M_{Max} \omega_r$$

$$v_f = r_f i_f + L_f \frac{di_f}{dt}$$

$$T_{em} = \frac{P}{2} M\, i_f i_a = T_L + J \frac{d\omega_m}{dt} + B\,\omega_m$$

4.2.1 Methods to make it Linear

4.2.1.1 Under constant excitation (i_f, v_f const)

	Linear
input	v_a, T_L
States	i_a, ω
Order	2^{nd}

4.2.1.1.1 Using state space to represent the system

$$\frac{di_a}{dt} = \frac{1}{L_a}\left[v_a - r_a i_a - k_f \omega_m \right]$$
$$\frac{d\omega_m}{dt} = \frac{1}{J}\left[k_f\, i_a - B\omega_m - T_L \right]$$

4.2.1.1.2 Using matrix form of state space

$$\dot{x} = Ax + Bu$$

$$\frac{d}{dt}\begin{bmatrix} i_a \\ \omega_m \end{bmatrix} = \begin{bmatrix} -\frac{r_a}{L_a} & -\frac{k_f}{L_a} \\ \frac{k_f}{J} & -\frac{B}{J} \end{bmatrix}\begin{bmatrix} i_a \\ \omega_m \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix}\begin{bmatrix} v_a \\ T_L \end{bmatrix}$$

4.2.1.1.3 Using Laplace transform

<u>NOTE</u> to use Laplace it's easy make [i to I(s)] and make [di/dt to S I(s)]

$$\boldsymbol{V_a(s)} = r_a I_a(s) + L_a\, S I_a(s) + k_f \omega_m(s) = \boldsymbol{(r_a + L_a S) I_a(s) + k_f \omega_m(s)} \rightarrow (\mathbf{1})$$

$$\boldsymbol{T_{em}} = k_f I_a(s) = T_L(s) + B\omega_m(s) + J\, S\, \omega_m(s) = \boldsymbol{(B + JS)\omega_m(s) + T_L(s)} \rightarrow (\mathbf{2})$$

So set of Equations become

Get an I(s) expression from (2) then substitute in (1) so we get

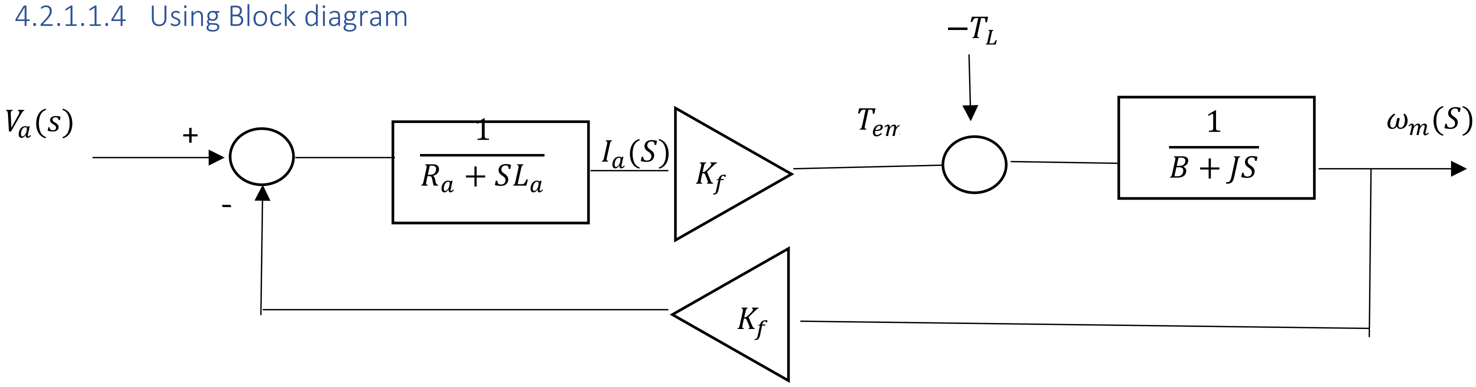
$$\boldsymbol{V_a(s) = \frac{R_a + L_a S}{k_f}[\, T_L(s) \;+ (B + JS)\omega(s)\,] + k_f \omega(s)}$$

$$\boldsymbol{\omega(s) = \frac{k_F V_a(s) - (R_a + L_a S) T_L(s)}{(R_a + L_a S)(B + JS) + k_f^2}}$$

$$\boldsymbol{I_a(s) = \frac{(B + JS) V_a(s) + T_L(s) k_f}{(R_a + L_a S)(B + JS) + k_f^2}}$$

T_L

4.2.1.1.4 Using Block diagram



<u>Note</u> it's an <u>open loop diagram</u> for it to be closed loop the output must follow a reference

4.2.1.1.5 Using Transfer Function

We can say that the speed ω_m is affected by two factors

1- Load Torque $\frac{\Delta\omega_m(s)}{T_L(s)}$
2- Armature Voltage $\frac{\Delta\omega_m(s)}{\Delta v_a(s)}$

4.2.1.2 Linearization Model

Condition : small disturbance, perturbation or excursion (x changes by Δx)

Math Principle Proof Lec4 p.4

To do linearization to the system we replace each variable Ax to $A\Delta x$ from steady state equations and for xy it's $x_0\Delta y + y_o\Delta x$

	Linear
input	v_a, v_f, T_L
States	$i_a\,, i_f, \omega$
Order	3^{rd}

$$\Delta v_a = R_a\Delta i_a + L_a\frac{d}{dt}\Delta i_a + M\left(I_{f_0}\Delta\omega_m + \omega_{m_o}\,\Delta i_f\,\right)$$

$$\Delta v_f = R_f\Delta i_f + L_f\frac{d}{dt}\,\Delta i_f$$

$$\Delta T_e = M\left(I_{f_o}\Delta i_a + I_{a_o}\Delta i_f\right) = \Delta T_L + J\frac{d}{dt}\,\Delta\omega_m + R_a\Delta\omega_m$$

NOTE we use "p" or "S" in laplace to represent the d/dt

4.2.1.2.1 Using laplace transform

Stability : that for a bounded input the system will have a bounded output we can test system stability by applying step change if it maintains stability then we can decide that it's stable for that value

$$\Delta V_a(s) = (\,R_a + L_aS)\,\Delta I_a(s) + M\left(\,I_{f_o}\Delta\omega_m(s) + \omega_{m_o}\,\Delta I_f(s)\right)$$

$$\Delta V_f(s) = \left(R_f +\; L_fS\right)\Delta I_f(s)$$

$$\Delta T_e(s) = M\left(I_{f_0}\Delta I_a(s) + I_{a_o}\Delta I_f(s)\right) = \Delta T_L(s) + (B + JS)\Delta\omega_m(s)$$

4.2.1.2.2 In Matrix form

Because system is linearized it's in the form

$$U_{in} = A\,X_{out}$$

$$\begin{bmatrix} \Delta V_a(s) \\ \Delta V_f(s) \\ \Delta T_L(s) \end{bmatrix} = \begin{bmatrix} R_a + L_a S & M\omega_{m_0} & MI_{f_0} \\ 0 & R_f + L_f S & 0 \\ MI_{f_0} & MI_{a_0} & -(B + JS) \end{bmatrix} \begin{bmatrix} \Delta I_a(s) \\ \Delta I_f(s) \\ \Delta \omega_m \end{bmatrix}$$

<u>NOTE</u> if question didn't ask you to use Laplace then replace "s" with "p" in the matrix and use small letter v_a

$$\frac{\Delta\omega(s)}{\Delta V_f(s)} = \frac{M[(R_a + L_a S)I_{a_o} - \omega_{r_o} I_{f_o} M]}{[JS(R_a + L_a S) + M^2 {I_{f_o}}^2](R_f + L_f S)}$$

4.3 Dynamic Model for Shunt DC Motor

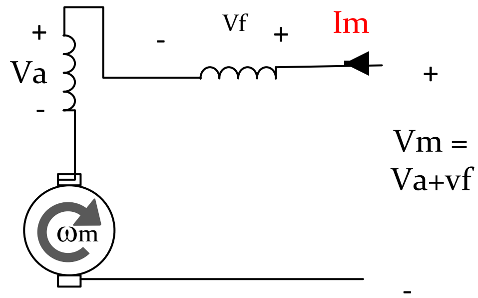
In shunt we won't have v_f

	<u>NonLinear</u>
input	v_a, T_L
States	i_a, i_f, ω
Order	2^{nd}

Same Equations

وطالما الدكتور مكلمش عنه كتير ف المحاضره فمش هيكلم عنه كتير ف الامتحان ولا ايه 😉

4.4 Dynamic Model for Series DC Motor



	NonLinear
input	v_m, T_L
States	i_m, ω_m
Order	2^{nd}

$$v_a + v_F = v_m$$

$$i_a = i_f = i_m$$

$$v_m = v_a + v_f = R_m i_m + L_m \frac{di_m}{dt} + M\, i_m \omega_m$$

$$T_{em} = M\, i_m^2 = T_L + J\frac{d\omega_m}{dt} + B\omega_m$$

Note it's not linear because of the i_m^2

4.4.1 Linearized model for DC Series Motor

	Linear
input	v_m, T_L
States	i_m, ω_m
Order	2^{nd}

$$\Delta v_m = R_m \Delta I_m + L_m \frac{d}{dt}\Delta i_m + M\left(I_{m_0}\Delta\omega_m + \omega_{m_0}\Delta i_m\right)$$

$$\Delta T_e = 2I_{m_0}\,\Delta i_m * M = \Delta T_L + J\frac{d}{dt}\Delta\omega_m + B\Delta\omega_m$$

4.4.1.1 Using laplace transform

$$\Delta v_m(s) = \left(R_m + L_m S + M\omega_{m_o}\right)\Delta i_m(S) + MI_{m_o}\Delta\omega_m(s)$$

$$\Delta T_{em}(s) = 2MI_{m_o}\Delta I_m(s) = \Delta T_L(s) + (B + JS)\Delta\omega_m(s)$$

4.4.1.2 In matrix form

$$\begin{bmatrix}\Delta v_m(s)\\ \Delta T_L(s)\end{bmatrix} = \begin{bmatrix}R_m + L_m S + M\omega_{m_o} & MI_{m_o}\\ 2MI_{m_o} & -(B + Js)\end{bmatrix}\begin{bmatrix}\Delta I_m(s)\\ \Delta\omega_m(s)\end{bmatrix}$$

$$V_a(s) = R\,I(s) + M\left(I_o\omega(s) + \omega_o I(s)\right)$$
$$2\,M\,I_0 I(s) = T_L(s) + (B + JS)\omega(s)$$

$$\text{for const } \mathrm{T_L} \therefore \quad \Delta\mathrm{T_L}(\mathrm{s}) = 0$$

$$I(s) = \frac{B + JS}{2MI_o}\,\omega(s)$$

$$V_a(s) = \omega(s)\left[R\frac{(B + JS)}{2MI_o} + M\,I_o + M\omega_o\,\frac{B + JS}{2MI_o}\right]$$

$$\omega(s) = \frac{V_a(s)}{\left[\frac{R(B + JS)}{2MI_o} + M\,I_o + \frac{M\omega_o(B + JS)}{2MI_o}\right]} = \frac{V_a(s)}{\left[\frac{(R + M\omega_o)(B + JS)}{2MI_o} + M\,I_o\right]}$$

$$\boldsymbol{\omega(s) = \frac{2MI_oV_a(s)}{[(R + M\omega_o)(B + JS) + 2M^2\,I_o^2\,]}}$$

5 DYNAMIC MODEL OF A SE DC GENERATOR

	NonLinear
input	v_f, T_e
States	$\omega_m, i_f . i_a$
Order	3^{rd}

we can represent this system either by putting $SAY\ T_e = -M_{af} i_f i_a$ in motor equations

$$T_m = T_e + B\omega + J\frac{d\omega}{dt}$$

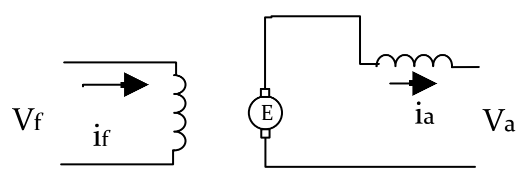
Or

Assume the correct current path

$$v_a = -R_a i_a - L_a \frac{di_a}{dt} + M\ i_f\ \omega_r$$

$$v_f = R_f i_f + L_f \frac{di_f}{dt}$$

$$T_e = -M\ i_f i_a$$



It's a non linear equation system

Transient	→	Motor	Assume elect. Const so the <u>it's mech slow response</u>
	→	Generator	Assume mech. Const so <u>it's electromagnetic fast response</u>

5.1 Linearizing By assuming const speed

$$\boldsymbol{assume\ \omega\ const}$$

	Linear
input	v_f
States	$i_f . i_a$
Order	2^{nd}

It's practically invalid assumption (because when we load the machine speed decreases then increases which is called swing) but we use this assumption as a very strong prime mover

$$\boldsymbol{Voltage\ Equation:\ v_a = -R_a i_a - L_a \frac{di_a}{dt} + M\, i_f\, \omega_r}$$

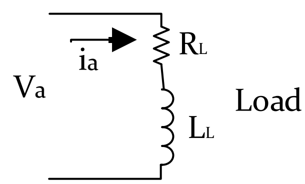
$$\boldsymbol{Field\ Equation:\ v_f = R_f i_f + L_f \frac{di_f}{dt}}$$

But

$$\boldsymbol{Load\ Equation:\ v_a = i_a R_L + L_L \frac{di_a}{dt}}$$

$$\therefore R_L i_a + L_L \frac{di_a}{dt} = -R_a i_a - L_a \frac{di_a}{dt} + M\, i_f\, \omega_r$$

$$put\ \ (L_a + L_L) = L_t$$
$$(R_a + R_L\,) = R_t$$



For const speed put $K_\omega = M\omega$

$$\boldsymbol{K_\omega i_f = R_t i_a + L_t \frac{di_a}{dt}}$$

$$\boldsymbol{v_f = R_f i_f + L_f \frac{di_f}{dt}}$$

5.1.1 Differential Equations

$$\boldsymbol{system\ (\,input\ v_f\ \ and\ \ output\ \ i_a, i_f)}$$

$$\boldsymbol{\frac{di_a}{dt} = -\frac{R_t}{L_t} i_a + \frac{K_\omega}{L_t}\, i_F + 0\, v_f}$$

$$\boldsymbol{\frac{di_f}{dt} = -\frac{R_f}{L_f} i_f + \frac{1}{L_f} v_f}$$

5.1.2 State space representation

$$\begin{bmatrix} \dot{i_a} \\ \dot{i_f} \end{bmatrix} = \begin{bmatrix} -\dfrac{R_t}{L_t} & \dfrac{K_\omega}{L_t} \\ 0 & -\dfrac{R_f}{L_f} \end{bmatrix} \begin{bmatrix} i_a \\ i_f \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \dfrac{1}{L_f} \end{bmatrix} \begin{bmatrix} 0 \\ v_f \end{bmatrix}$$

$$T_m = M i_f i_a + B\,\omega + J\frac{d\omega}{dt}$$

NOTES

1- **Response can be either**
overdamped : Roots are Real, different
Underdamped : Roots are Imaginary,conj.

2- **System can be stable but has a bad response and it can be improved through closed loop control by changing the gains**

3- $\tau_e = \frac{L_a}{R_a}$, $\tau_m = \frac{J}{B}$

4- **If τ_a is very small we can neglect effect of armature response**

5- $i_a(s) = \frac{V_a(s)}{R_a}$ $and\ V_f = R_f I_f$

6- **At no load assume** $E = v_a = M\, I_f \omega = K_\omega I_f$ **(Ia=0)**

7- v_a is not input nor output $v_a = Ri + L\frac{di}{dt}$ if we do transient for ia do same for va

5.1.3 Transfer Functions

$$Armature\ Current\ T.F.\ \frac{I_a}{V_f}$$

$$Armature\ Voltage\ T.F.\frac{v_a}{v_f}$$

Example at lec 6 p 4

5.1.3.1 Armature Current T.F. $\frac{I_a}{V_f}$

Using laplace

$$K_\omega I_f(s) = (R + LS)I_a(s) \to (1)$$
$$V_f(s) = \left(R_f + L_f S\right)I_f(s) \to (2)$$

From (1)

$$\frac{I_a(s)}{I_f(s)} = \frac{K_\omega}{R + LS}$$

From (2)

$$\frac{I_f(s)}{v_f(s)} = \frac{1}{R_f + L_f S}$$

$$\therefore \frac{I_a(s)}{I_f(s)} * \frac{I_f(S)}{V_f(s)} = \boldsymbol{\frac{I_a}{V_f} = \frac{k_\omega}{(R + LS)\left(R_f + L_f S\right)}}$$

5.1.3.2 Armature Voltage T.F. $\frac{v_a}{v_f}$

From load side

$$\because v_a(s) = (R_L + L_L S)I_a(s)$$

$$\therefore I_a(s) = \frac{v_a(s)}{R_L + L_L(s)}$$

$$\therefore \boldsymbol{\frac{v_a(s)}{v_f(s)} = \frac{k_\omega(R_L + L_L S)}{(R + LS)\left(R_f + L_f S\right)}}$$

6 DC MACHINE SHEETS

Sheet 2

1- The Parameters of a dc shunt machine are $\boldsymbol{R_f = 240\ \Omega, L_{ff} = 120\ H, L_{Af} = 1.8\ H, r_a = 0.6\ \Omega, L_{AA} = 0}$ .The load torque is 5 N.m and $\boldsymbol{V_a = V_f = 240\ V}$. Calculate the steady-state rotor speed.

Given :

$$R_f = 240\ \Omega\ , L_f = 120\ H \qquad , M = 1.8\ H \quad , R_a = 0.6\ \Omega, \qquad L_a = 0$$

Solution :

$$\boldsymbol{v_a = i_a R_a + L_a \frac{di_a}{dt} + \frac{P}{2} i_f M_{Max} \omega_m}$$

$$v_a = i_a R_a + L_a \frac{di_a}{dt} + \frac{P}{2}\left(\frac{v_F}{R_f}\right) M_{Max}\omega_m$$

$$240 = 0.6 i_a + 0 + 1 * \frac{240}{240} * 1.8 * \omega_m \rightarrow (1)$$

$$T_{em} = \frac{P}{2}\ M\ i_f i_a = T_L + j\frac{d\omega}{dt} + B\ \omega_m$$

$$at\ steady\ state\ \ j\ and\ B\ terms\ are\ zero$$

$$1 * 1.8 * \frac{240}{240} * i_a = 5$$

$$\therefore i_a = 2.7777\ A$$

$$\therefore in\ (1)\ \boldsymbol{\omega_m = 132.407\ rad/s}$$

2- The power input to a dc shunt motor during rated-load conditions is 100W. The rotor speed is 2000 rpm and the armature voltage is 100V. The armature resistance is 2 $\boldsymbol{\Omega}$ and $\boldsymbol{R_f = 200\ \Omega}$. Calculate the no-load rotor speed.

Given :

$$shunt\ (v_a = v_f), P_{in} = 100\ W\ , \omega_m = 209.439 \frac{rad}{s}\ , v_a = 100V\ , R_a = 2\Omega, R_f = 200\Omega$$

Solution

$$P_{in} = v_a i_a + \left(\frac{v_a^2}{r_f}\right)$$

$$100 = 100 * i_a + (\frac{100^2}{200})$$

$$i_a = 0.5A$$

$$v_a = i_a r_a + 0 + k\ i_f \omega_m$$

$$100 = 0.5 * 2\ + \frac{p}{2} M * \frac{1}{2} * 209.437$$

$$M * \frac{p}{2} = k =\ 0.94539$$

$$at\ no\ load\ i_a =\ 0$$

$$v_a = k\ i_f \omega_{nl} \rightarrow\ \ \omega_{nl} = \frac{v_a}{k * i_f} = \frac{v_a}{k\left(\frac{v_a}{r_f}\right)} = \frac{r_f}{k} = \frac{200}{0.94539} = 211.55\ rad/s$$

Note : we could say kf= P/2*M*if = 0.4725

3- A permanent-magnet dc motor has the following parameters : $r_a = 8\Omega$ and $k_v = 0.01\ V.\frac{s}{rad}$ .The shaft load Torque is approximated as $T_L = K\omega_r$ where $K = 5 \times 10^{-6}\ N.m.s$ The applied voltage is $6V$ and $B_m = 0$ calculate the steady state rotor speed ω_r in rad/s

Given

$$no\ v_f, i_f\ , r_a = 8\Omega, k_v = 0.01, K_t = 5 \times 10^{-6}, v_a = 6V$$

Solution

$$\boldsymbol{v_a = i_a r_a + k_v \omega_m}$$

$$6 = i_a * 8 + 0.01 * \omega_m$$

$$\boldsymbol{T_{em} = k_v i_a = K * \omega_m}$$

$$0.01 i_a = 5 * 10^{-6} * \omega_m$$

$$solving\ equations$$
$$i_a = 0.214\ A$$
$$\omega_m = 428.5714\ rad/s$$

4- A 250-V 600rpm 200 hp dc shunt motor is delivering rated horsepower at rated speed $R_f = 12\ \Omega, M = 0.18H\ and\ r_a = 0.012\ \Omega$

(a) Calculate the terminal voltage that must be applied to this machine to satisfy this load condition

(b) Calculate the full-load ohmic losses and determine the efficiency

Given

$$rated:\ v_a = v_f = 250V, \omega_m = 62.831\frac{rad}{s}, P_o = 149200\ watt$$
$$R_f = 12\Omega\ , M = 0.18H\ , R_a = 0.012\ \Omega$$

$$(A)$$

$$\frac{P}{2}\ M i_a i_f = 1 * M\ i_a \frac{v_f}{R_f} = T_L = \frac{P_o}{\omega_m}$$
$$0.18 * i_a * \frac{v_a}{12} = \frac{149200}{62.831}$$
$$v_a i_a = 158308.266$$
$$v_a = 0.012 * i_a + 0.18 * \frac{v_a}{12} * 62.831$$
$$\therefore\ I_a = 871.218\ A\ \ (huge\ !!!\ )$$
$$\boldsymbol{v_a = 181.70896\ V}$$

(B)

$$\boldsymbol{P_{cu_{fl}} = i_a^2 * r_a + \frac{v_a^2}{r_f}} = 871.218^2 * 0.012 + \frac{181.811^2}{12} = 11862.085\ watt$$
$$\eta = \frac{P_o}{P_{cu_{fl}} + P_o} * 100\% = \frac{149200}{149200 + 11862.085} * 100 = \boldsymbol{92.638}\ \%$$

5- The parameters of a dc shunt machine are $r_a = 10\ \Omega, R_f = 50\Omega\ and\ M = 0.5\ H$ neglect B_m and $V_a = V_f = 25\ V$ calculate

(a) The steady state stall torque

(b) The no-load speed

(c) The steady-state rotor speed with $T_L = 3.75 \times 10^{-3}\ \omega_r$

Given :

$$R_a = 10\ \Omega, R_f = 50\Omega\ , M = 0.5\ H\ , \qquad V_a = V_f = 25\ V$$

Solution

(A)

$$\boldsymbol{T_{Stall} = T_{starting}\ (\omega_m = 0\ )}$$

$$v_a = i_a r_A + k i_f \omega_m$$

$$i_a = \frac{25}{10} = 2.5\ A$$

$$\frac{P}{2} * M * i_a i_f = T_L + 0 + 0 = 0.5 * i_a * \left(\frac{25}{50}\right)$$

$$\therefore T_L = 0.25 i_a = 0.25 * 2.5 = \ 0.625\ N.m$$

(B)

At no load ia =0

$$v_a = \frac{P}{2} M \left(\frac{v_a}{r_f}\right) \omega_m = 0.5 * \frac{25}{50} * \omega_{m_{nl}}$$

$$\omega_{m_{nl}} = \frac{25}{0.25} = 100\ rad/s$$

(c)

$$25 = 10 i_a + 0.5 * 0.5 * \omega_m$$

$$0.5 * 0.5 * i_a = 3.75 * 10^{-3} \omega_m$$

$$\therefore \omega_m = 62.5 \frac{rad}{s}$$

6- A permanent-magnet dc motor is driven by a mechanical source at 3820 rpm. The measured open-circuit armature voltage is 7V. The mechanical source is disconnected and a 12V electric source is connected to the armature. With zero load torque, $I_A = 0.1\ A$ and $\omega_r = 650\frac{rad}{s}$ calculate k_v, B_m, r_a

Given

As generator $V = 7V\ at\ \omega_r = 400.03\frac{rad}{s}$ as motor $v_a = 12\ v, i_a = 0.1a\ at\ \omega_r = 650$

Solution

$$\boldsymbol{as\ generator}$$

$$\boldsymbol{V = k_v\omega_r}$$

$$\therefore k_v = \frac{V}{\omega_r} = 0.01749\ \ V.\frac{s}{rad}$$

As motor

$$v_a = i_a r_a + k_v\omega_m = 12 = 0.1 * r_a + 0.01749 * 650$$

$$r_a = 6.315\ \Omega$$

$$B_m * \omega_m = k_v i_a$$

$$B_m = \frac{0.01749 * 0.1}{650} = 2.69 * 10^{-6}\ \ (very\ very\ small\ value!!)$$

7- Express the maximum steady-state power output of a dc shunt motor($P_{out} = T_l\omega_r$) if the field current i_f and armature voltage v_a are held constant. Let $B_m = 0$

$$\because v_a = R_a i_a + k\frac{v_a}{R_f}\omega_m$$

$$v_a\left(1 - \frac{k\omega_m}{R_f}\right) = R_a i_a$$

$$\boldsymbol{i_a = \frac{v_a}{R_a}\left(1 - \frac{k\omega_m}{R_f}\right)}$$

$$\because k i_a \frac{v_a}{r_f} = T_L$$

$$T_L = \frac{kv_a}{R_f} i_a = \frac{kv_a}{R_f}\boldsymbol{\frac{v_a}{R_a}\left(1 - \frac{k\omega_m}{R_f}\right)}$$

$$P_o = T_L * \omega_m = \frac{kv_A}{R_f}\left[\frac{v_A}{R_a}\omega_m - \frac{v_a k}{R_a R_f}\omega_m^2\right]$$

$$\frac{\partial P_o}{\partial \omega_m} = 0$$

$$\frac{v_a}{R_a} - \frac{2v_a k}{R_a R_f}\omega_m$$

$$\boldsymbol{\omega_m = \frac{R_f}{2K}}$$

$$\boldsymbol{P_{o_{Max}} = \frac{v_a^2}{4R_a}}$$

8- The parameters of a 5-hp dc shunt machine are $r_a = 0.6\ \Omega, L_a = 0.012H, R_f = 120\ \Omega, L_f = 120H, M = 1.8H\ and\ V_a = V_f = 240V$. Calculate the steady-state rotor speed ω_r for $I_t = 0$

Given

$$P = 3730\ watt, r_a = 0.6\ \Omega, L_a = 0.012H, R_f = 120\Omega, L_f = 120H, M = 1.8H\ and\ V_a = V_f = 240V, I_t = 0$$

Solution

$$let\ \boldsymbol{i_t = i_a + i_f = 0} \rightarrow \boldsymbol{i_a} = -\frac{\boldsymbol{v_a}}{\boldsymbol{R_f}} = -\frac{240}{120} = -2\ A$$

$$v_a = R_a i_a + M i_f \omega_r$$

$$\therefore \omega_r = \frac{v_a - R_a i_a}{M i_f} = \frac{240 - 0.6 * -2}{1.8 * 2} = 67\ rad/s$$

Sheet 3

1- A separately excited DC motor is rated at 1150 $r.p.m, 240\ V, 3\ HP, R_a = 1.43\ \Omega,$ $L_A = 10.4\ mH, J = 0.068\ , k_v = 1.8\frac{V}{rad/sec}\ , B = 0$, Determine the speed and the armature current at no load as a function of time after the motor is energized by applying the rated voltage at t=0 assuming the motor is at rest for t<0.

Given:

Sep,($1150\ r.p.m, 240\ V, 3\ HP$) $\quad k_v = 1.8$

$$R_a = 1.43\ \Omega, \qquad L_A = 10.4\ mH$$

$$J = 0.068\ , \qquad B = 0$$

Required: $i_a(t)\ , \omega(t)\ \ at\ no\ load$ when $v_a = 240\ \ at\ t = 0\ (sudden)$

Solution:

$$we\ know\ that$$

$$V_a(s) = (R + LS)I_a(s) + k_f\omega(s)$$
$$k_fI_a(s) = (B + JS)\omega(s) + T_L(s)$$
$$but\ T_L = 0\ (no\ load\ )and\ B = 0\ (given)$$
$$so,$$

$$I_a(s) = \frac{JS}{k_f}\ \omega(s)$$

$$V_a(s) = (\ (R + LS)\frac{JS}{k_f} + k_f)\ \omega(s)$$

$$\boldsymbol{\omega(s) = \frac{V_a(s)k_f}{(R + LS)JS + k_f^2}}$$

$$\boldsymbol{I_a(S) = \frac{JS\ V_a(s)}{(R + LS)JS + k_f^2}}$$

The n Numerically

$$\omega(s) = \frac{V_a(s)k_f}{(R + LS)JS + k_f^2} = \frac{240}{S}\frac{1.8}{(1.43 + 10.4 * 10^{-3} * S)0.068 * S + (1.8)^2} = \frac{610860}{S(S^2 + 137.5S + 4581.4)}$$
$$= \frac{610860}{S(S + 56.7)(S + 80.8)}$$

$$\omega(s) = \frac{133.3}{s} + -\frac{447.03}{s + 56.7} + \frac{313.69}{s + 80.8}$$

$$L^{-1} \rightarrow \omega(t) = 133.3 - 447.03\, e^{-56.7t} + 313.7\, e^{-80.8t}$$

$$i_a(s) = \frac{JS}{k_f}\omega(s)$$

$$i_a(s) = \frac{23076.9}{(S + 56.7)(s + 80.8)}$$

$$i_a(s) = -\frac{957.55}{s + 56.7} + \frac{957.5}{s + 0.8}$$

$$i_a(t) = -957.55\, e^{-56.7t} + 957.5\, e^{-80.8t}$$

2- A separately excited DC motor $has\ R_a = 0.4\ \Omega, L_a = 0, and\ k_v = 2V/rad/sec.$

Motor is connected to a load whose torque is proportional to the speed (T $\propto \omega$). $J = J_{motor} + J_{Load} = 2.5\ kg.m2\ and\ B = B_{motor} + B_{Load} = 0.25\ kg.m/s$

The field current is maintained constant at its rated value. A voltage of 200 V is suddenly applied across the motor armature terminals.

a. Obtain an expression for the motor speed as a function of time.

b. Determine the steady state speed.

c. Determine the induced e.m.f. and the developed torque as a function of time.

Given

$$R_a = 0.4\ \Omega, L_a = 0, and\ k_v = 2$$

$$\mathrm{T} \propto \omega\ (compresser)$$

$$J = J_{motor} + J_{Load} = 2.5\ kg.m2\ and\ B = B_{motor} + B_{Load} = 0.25\ kg.m/s$$

$$I_f\ constant\ at\ rated\ value\ , V_a = 200\ V\ (sudden)$$

Required $\omega(t), \omega_{SS}, E(t)\ and\ T_e(t)$

Solution

a)

$$for\ constant\ excitation\ and\ T_l \propto \omega$$

$$\boldsymbol{v_A = r_a i_a + L\frac{di_a}{dt} + k_f \omega}$$

$$T_{em} = k_F i_a = T_L + B_m\omega + J_m\frac{d\omega}{dt}$$

$$\because T_L \propto \omega\ \therefore\ T_L = B_L\omega + J_L\frac{d\omega}{dt}$$

$$T_{em} = k_F i_a = B_L\omega + J_L\frac{d\omega}{dt} + B_m\omega + J_m\frac{d\omega}{dt}$$

$$\boldsymbol{T_{em} = B\ \omega + J\frac{d\omega}{dt}}$$

Laplace

$$V_a(s) = (R)I_a + k_f\ \omega(s)$$

$$k_f\ I_a(s) = (B + JS)\omega(s)$$

$$I_a = \frac{B + JS}{k_f}\ \omega(s)$$

$$V_a(s) = \omega(s)\left[\frac{R(B+JS)}{k_f} + k_f\right]$$

$$\omega(s) = \frac{k_f V_a(s)}{R(B+JS) + k_f^2} = \frac{2 * (200/S)}{0.4(0.25 + 2.5S) + (2)^2} = \frac{400}{S(S+4.1)}$$

$$\omega(s) = \frac{97.56}{s} + \frac{-97.56}{s+4.1}$$

$$\omega(t) = 97.56 - 97.56\, e^{-4.1t} \quad rad/s$$

$$\boldsymbol{\omega_{SS} = \omega(\infty) = 97.56\ rad\,/sec}$$

$$\boldsymbol{E = k_f\, \omega_s = 2 * 97.56 = 195.12\ V}$$

$$T_e(s) = (B+JS)\,\omega(s) = \frac{400(2.5s + 0.25)}{S(S+4.1)}$$

$$T(s) = \frac{24.4}{S} + \frac{975.6}{s+4.1}$$

$$\boldsymbol{T_e(t) = 24.4 + 975.6\, e^{-4.1t}}$$

3- A separately excited DC motor has the following parameters:

$$R_a = 0.5\,\Omega \qquad L_a = 0 \quad B = 0 \quad J = 2.5\,Kg.m$$

The motor generates an open circuit armature voltage of 220 Vat 2000 r.p.m. and with field current of 1 A. The motor drives a constant load of 25 N.m with the field current constant at 1A. If the motor terminals are suddenly connected to a 220 V DC source. Determine the speed and armature current as a function of time.

Given

$$Sep\ R_a = 0.5\,\Omega \qquad L_a = 0 \quad B = 0 \qquad J = 2.5\,Kg.m$$

$$V_a = 220\,V \quad 2000\,rpm \quad I_f = 1\,A \quad open\ circuit$$

$$T_L = 25\,N.m\ \ constant\ field\ current$$

Required $\omega_r(t)\ and\ i_a(t)$

Solution

$$\underline{at\ no\ load\ (I_a = 0)}$$

$$V_a = k_f\omega$$

$$220 = k_f\frac{2\pi * 2000}{60}$$

$$k_f = 1.05\frac{V}{rad/s}$$

At constant I_f

$$v_a(s) = RI_a(s) + k_f\omega(s)$$

$$k_fI_a(s) = JS\,\omega(s)$$

$$I_a = \frac{JS}{k_f}\,\omega(s)$$

NOTE Although he gave us a value for T_L but we put $T_L = 0$ in laplace equation because it's constant and in Laplace we study changes only

As when we put $V_a\ we\ put\ \frac{220}{s}(step\ change)$

$$\omega(s) = \frac{k_f V_a(s)}{R_a J S + k_f^2}$$

$$\omega(s) = \frac{1.05 * \frac{220}{s}}{1.25s + 1.1025}$$

$$\omega(s) = \frac{184.8}{s(s+0.882)} = \frac{209.5}{s} + -\frac{209.5}{s+0.882}$$

$$\therefore \omega(t) = 209.5 - 209.5\, e^{-0.882t}$$

$$I_a(s) = \left(\frac{J}{K_f}\right)\frac{184.8}{s+0.882} = \frac{440}{s+0.882}$$

$$i_a(t) = 440\, e^{-0.882t}$$

$\omega(t) = -12.85 + 12.85\, e^{-0.882t}$

$\omega_{s.s} = 196.65$

4- A 230 V DC series motor, running at 150 rad/sec. and takes 20 A from the supply. The armature and field resistances are Ra + R_{fs}= 1 Ω. The total moment of inertia J= 5.4 Kg.m2 and B=0. 2 N.m.sec./rad.

a. Calculate the rotational mutual inductance M_{af} and the load torque.

b. If the supply voltage is suddenly reduced to 220 V with the load torque remaining constant, find the speed as a function of time.

Given

$\omega = 150\frac{rad}{s}, I_a = 20A, R = 1\,\Omega, J = 5.4, B = 0.2, I = I_f = I_a$

$$v_a = IR + MI\,\omega$$

$$230 = 20 * 1 + M * 20 * 150$$

$$M = 0.07$$

$$M\,I^2 = T_L + B\omega$$
$$0.07 * 20^2 = T_L + 0.2 * 150$$

$$T_L = -2\ N.m$$

Which means it's a series generator

For R=0.1 and B = 0.02

M = 0.07 and T_L = 25 N.M

-b-

Change = final – initial = 220- 230 = -10 V

$$\therefore V_a(s) = -\frac{10}{s}$$

$$for\ series$$

$$v_a = Ri + Mi\,\omega$$
$$T = i^2M = T_L + B\omega + j\frac{d\omega}{dt}$$

$$laplace$$

$V_a(s) = R\, I(s) + M\big(I_o \omega(s) + \omega_o I(s)\big)$
$2\, M\, I_0 I(s) = T_L(s) + (B + JS)\omega(s)$
$I(s) = \dfrac{B + JS}{2MI_o}\ \omega(s)$
$V_a(s) = \omega(s)\left[\dfrac{R(B + JS)}{2MI_o} + M\, I_o + \omega_o\ \dfrac{B + JS}{2MI_o}\right]$
$\omega(s) = \dfrac{V_a(s)}{\left[\dfrac{R(B + JS)}{2MI_o} + M\, I_o + \dfrac{\omega_o(B + JS)}{2MI_o}\right]} = \dfrac{V_a(s)}{\left[\dfrac{(R + M\omega_o)(B + JS)}{2MI_o} + M\, I_o\right]}$

$$\boldsymbol{\omega(s) = \frac{2MI_oV_a(s)}{[(R + M\omega_o)(B + JS) + 2M^2\, I_o^2\,]}}$$

By subs

$$\boldsymbol{\omega(s) = -\frac{10}{s}\,\frac{2 * 0.07 * 20}{(1 + 0.07 * 150)(0.02 + 5.45S) + 2 * (0.07 * 20)^2}}$$
$$\boldsymbol{\omega(s) = -\frac{28}{s(62.1s + 0.23 + 3.92)}}$$

$\boldsymbol{\omega(s) = -6.\dfrac{73}{s} + \dfrac{6.73}{s + 0.0668}}$
$\boldsymbol{so, \Delta\omega(t) = -6.73 + 6.73\, e^{-0.0668t}}$
$\boldsymbol{\therefore\ \omega(t) = \Delta\omega(t) + \omega_o}$
$\boldsymbol{\omega(t) = 150 - 6.73 + 6.73\, e^{-0.0668t} = 143.27 + 6.73\, e^{-0.0668t}}$

$$\boldsymbol{\omega_{SS} = 143.27}$$

Note ,Speed decreased with voltage

5- A separately excited dc generator has the following parameters:

$R_a = 0.25\,\Omega\,, L_{aa} = 0.02\,H\,, R_f = 100\,\Omega,\ L_{ff} = 25H,\ K = 100\,V/A$ **at rated speed.**

Solve the following cases:

a. The generator is driven at rated speed and the field voltage V_f =200 V is suddenly applied to the field winding. Determine the armature generated voltage as a function of time and the steady state voltage.

b. The generator is driven at rated speed and a load consisting of R_L= 1 Ω and L_L= 0.15 H in series is connected to the armature terminals. A field voltage V_f =200 V is suddenly applied to the field winding. Determine the armature current as a function of time.

Given :

$$R_a = 0.25\,\Omega, L_a = 0.02\,H\,, R_f = 100\,\Omega, L_f = 25\,H, k_f = 100V/A$$

Solution :

(a) Assume no load

$$V_f = 200\,V\ suddenly\ applied\,, req. V_a(t)\ and\ V_{SS}$$

- *TIME DOMAIN SOLUTION* -

$$v_a = -R_a i_a - L_a \frac{di_a}{dt} + M\,i_f \omega_r$$
$$v_f = R_f i_f + L_f \frac{di_f}{dt}$$

$$\boldsymbol{i_f(t) = I_{f_0}\,e^{-\frac{t}{\tau}} + I_{f_{final}}\left(1 - e^{-\frac{t}{\tau}}\right)}$$

$$i_f(t) = I_f - I_f e^{-\frac{t}{\tau}} + I_{f_0}\,e^{-\frac{t}{\tau}}$$

$$I_{f_o} = 0$$

$$I_f = \frac{V_f}{R_f} = \frac{200}{100} = 2\,A$$
$$\therefore \tau_f = \frac{L_f}{R_f} = \frac{25}{100} = 0.25\,s$$

At no load I_a =0

$$\omega const\ (rated\ speed)$$

$$V_a(s) = k_f i_f(t) = 100\,(\,2 - 2e^{-4t}) = 200 - 200\,e^{-4t}$$
$$V_{A_{SS}} = 200\,V$$

- *S DOMAIN SOLUTION –*

$$I_f(s) = \frac{V_f(s)}{R_f + L_f S}$$
$$V_a(s) = k_f\,I_f(s)$$
$$V_a(s) = \frac{k_f V_f(s)}{R_f + L_f S}$$
$$v_f(s) = \frac{200}{s}$$
$$V_a(s) = \frac{20000}{s(25s + 100)} = \frac{800}{s(s+4)}$$

(b)

$R_L = 1\,\Omega\; and\; L_L = 0.15\,H$
$V_f = 200\,V\;\; sudden$
$i_A(t)\; req.$

Then $R_t = 0.25 + 1 = 1.25\,\Omega$

$L_t = 0.02 + 0.15 = 0.17\,H$

$$R_L i_a + L_l\frac{di_a}{dt} = \;-R_a i_a - L_a\frac{di_a}{dt} + k_f i_f$$
$$v_f = R_f i_F + L_f\frac{di_F}{dt}$$
$$(R_t + L_t S)I_a(s) = k_f i_f(s)$$
$$i_f(S) = \frac{v_f(s)}{R_f + L_f S}$$
$$(R_t + L_t S)I_a(s) = \;\frac{k_f v_f(s)}{R_f + L_f S}$$
$$I_a(s) = \frac{k_f v_f(s)}{(R_t + L_t S)(R_f + L_f S)}$$
$$v_f = \frac{200}{s}$$

$$i_a(s) = \frac{100 * 200}{s(1.25 + 0.17s)(100 + 25s)}$$

$$I_A(s) = \frac{160}{s} + \frac{32.5}{0.17 + 1.25s} + -\frac{8775}{25s + 100}$$

$$\boldsymbol{i_a(s) = 160 + 191.17\, e^{-7.35t} - 351.18\, e^{-4t}}$$

Sheet Lec DC. Machines Dynamics

1- A dc PM motor has the following data: $V_a = 110\ V$, $I_a = 10\ A$, $R_a = 0.5\ \Omega$, $N_m = 1200\ rpm, P = 2\ pole\ pairs, \tau_e = 2\ ms$. For a step increase in voltage of 10 V, determine the speed response for the total moment of inertia of J=0.005 Kg.m2 and J=0.05 Kg.m2 for a constant rated torque.

Given

$$V_a = 110\ v\ , I_A = 10\ A\ , R_a = 0.5\ \Omega\ , \mathrm{N_m} = 1200\ rpm\ , P = 2\ pole$$
$$\tau_e = 2\ ms\ , \Delta v = 10$$

$$\tau_e = \frac{L_a}{R_A} \rightarrow L_A = 2 * 0.5 = 1\ H$$

$$V_a = R_a i_a + L_a \frac{di_a}{dt} + k_f \omega$$

At SS

$$V_a = 110 = 0.5 * 10 + k_f \left(1200 * \frac{2\pi}{60}\right)$$
$$so\ k_f = 0.83556$$
$$k_f i_a = T_L + B\omega + J\frac{d\omega}{dt}$$

Laplace

1- J=0.005 (UNDERDAMPED) $J \downarrow\downarrow$

$$V_a(s) = (R + LS)I_a(s) + k_f\ \omega(s)$$
$$k_f I_a(s) = T_L(s) + (\ B + JS)\omega(s)$$

$$subs\ and\ T_L = 0$$

$$\boldsymbol{\omega(s) = \frac{k_F V_a(s)}{(R_a + L_a S)(JS) + k_f^2} = \frac{10}{s}\ \frac{0.8355}{(0.5 + 1S)(0.005S) + (0.8355)^2}}$$

$$\boldsymbol{S_{1,2} = -250 \pm 277.99j}$$

$$\omega(s) = \frac{11.968}{s} + \frac{11.968s + 5984}{(s + 250)^2 + 77132.1}$$

$$= \frac{11.968}{s} + \frac{11.968s}{(s+250)^2 + 77132.1} + \frac{\left(\left(\frac{5984}{277.7}\right) + 277.7\right)}{(s+250)^2 + 77132.1}$$

$$\omega_m(t) = 11.968 + 11.968\sin(277.72t)\,e^{-250t} - 10.74\cos(277.7) + 21.54\cos(277.71\,t)\,e^{-250t} + 40\pi(\omega_{SS})$$

1- J= 0.05

$$\omega_m(s) = \frac{8.3556}{s(s+29.68)(s+70.31)}$$

$$= \frac{8.3556}{s} + -\frac{0.0063}{s+29.68} + \frac{0.00004}{s+470.31}$$

$$\omega_m(t) = 8.3556 - 0.0006 + e^{-29.68t} + 0.00004\,e^{-470.31t}$$

2- Consider a separately-excited dc generator with the following data:

$R_a = 0.1\ \Omega, \mathrm{R_f} = 1\ \Omega, \mathrm{L_a} = 5\ \mathrm{mH}, \mathrm{L_f} = 0.5\ \mathrm{H}, \mathrm{V_a} = 200\ \mathrm{V}, \mathrm{I_a} = 100\ \mathrm{A}, \mathrm{I_f} = 5\ \mathrm{A}, \mathrm{Nm} = 1500\ \mathrm{rpm}$, operates at <u>constant speed</u>. Determine generator transfer function T.F and voltage and current variations as a function of time for a step increase of 20% in field voltage.

<u>Given:</u>

$R_a = 0.1\ \Omega, \mathrm{R_f} = 1\ \Omega, \mathrm{L_a} = 5\ \mathrm{mH}, \mathrm{L_f} = 0.5\ \mathrm{H}, \mathrm{V_a} = 200\ \mathrm{V}, \mathrm{I_a} = 100\ \mathrm{A}, \mathrm{I_f} = 5\ \mathrm{A}$,
$\mathrm{Nm} = 1500\ \mathrm{rpm}, \omega_m = 157.079 const$ [assuming Resistive Load]

<u>Solution</u>

QUICK

LONG

$$V_f = I_f R_f = 5\ volt$$

$$R_L = \frac{V_a}{I_a} = \frac{200}{100} = 2\Omega$$

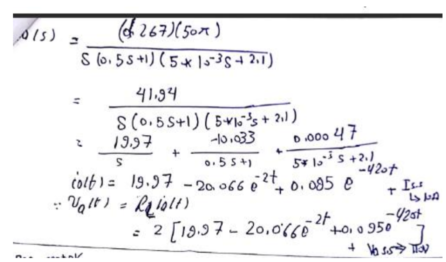
$$at\ S.S\ v_a = -R_a I_a + k I_f \omega_m$$

$$200 = -0.1 * 100 + k * 5 * 157.079$$
$$k = 0.267\ \ so\ k_\omega = 41.94$$

Use equation (but be exact) or deduce it

Change of voltage = $0.2 * 5 = 1\ V$

$$I_a(s) = V_f(s) \frac{k_\omega}{(R)(R_f + L_f S)} = \frac{20}{s} * \frac{41.94}{(2.1 + 5 * 10^{-3} S)(1 + 0.5S)} =$$
$$i(t) = 20 - 20.1914\ e^{-2t} + 0.095 e^{-420t}$$



$$\boldsymbol{v_a(t) = R_L i_a(t) = 40 - 40.1914\ e^{-2t} + 0.19\ e^{-420t}}$$

NOTE : we can ignore the e^{-420t} Term as it's very small (represents effect of armature transient)

3- Find the speed/field winding voltage T.F at rated torque of a dc-separately excited motor with the following data:

$$I_{f0} = 5\,A, \quad R_f = 1\,\Omega, L_f = 0.05\,H, I_{a0} = 100\,A, R_a = 0.1\,\Omega, L_a = 5\,mH, V_{a0} = 210\,V,$$
$$J = 1\,Kg.m^2, N_m = 1200\,rpm, B = 0.$$

Required :

$$\frac{\omega(s)}{V_f(s)}$$

Solution :

$$Linearization\ then\ Laplacation$$

$$\boldsymbol{\Delta V_a(s) = (R_a + L_a S)\,\Delta I_a(s) + M\left(I_{f_o}\Delta\omega_m(s) + \omega_{m_o}\,\Delta I_f(s)\right)}$$

$$\boldsymbol{\Delta V_f(s) = \left(R_f + L_f S\right)\Delta I_f(s)}$$

$$\boldsymbol{\Delta T_e(s) = M\left(I_{f_o}\Delta I_a(s) + I_{a_o}\Delta I_f(s)\right) = \Delta T_L(s) + (B + JS)\Delta\omega_m(s)}$$

$$so,$$

$$\Delta I_f(s) = \frac{\Delta V_f}{R_f + L_f s}$$

$$\Delta V_a(s) = (R_a + L_a s)\Delta I_a(s) + \boldsymbol{M}\left(\boldsymbol{I_{f_o}\Delta\omega_m(s) + \omega_{m_o}}\,\frac{\Delta V_f}{R_f + L_f s}\right)$$

$$\therefore I_a(s) = \frac{\Delta V_a(s) - \boldsymbol{M}\left(\boldsymbol{I_{f_o}\Delta\omega_m(s) + \omega_{m_o}}\,\frac{\Delta V_f}{R_f + L_f s}\right)}{\boldsymbol{R_a + L_a S}}$$

$$but$$

$$\Delta V_a(s) = 0\ and\ \Delta T_L(s) = 0 \quad \%\%\ they\ didn't\ change$$

$$by\ more\ simplification$$

$$\frac{\Delta\omega(s)}{\Delta V_f(s)} = \frac{M[(R_a + L_a S)I_{a_o} - \omega_{r_o} I_{f_o} M]}{[JS(R_a + L_a S) + M^2 I_{f_o}^2](R_f + L_f S)}$$

$$\frac{\Delta\omega(s)}{\Delta V_f(s)} = \frac{31.8(0.1 + 5 * 10^{-3}s) - 63.538}{s(0.1 + 5 * 10^{-3}s)(1 + 0.05s) + 2.53(1 + 0.05s)}$$

4- A dc separately-excited motor operates at no-load. $R_f = 120\,\Omega, L_f = 0.7\,H, M = 1.1\,H$, $J = 0.08\,Kg.m2$ .Determine the field current and motor speed as a function of time when the field is suddenly reduced from 240 V to 100 V.

As it's 1st order differential equation we can solve it in the time domain easily and no need to use Laplace

LONG SOLUTION Using <u>Linearization then laplacation</u>

$$V_f = R_f i_f + L_f \frac{di_F}{dt}$$

Then solve it with Laplace et cetera

SHORT SOLUTION Time Domain

$$I_f(t) = I_{f_o} e^{-\frac{t}{\tau}} + I_{f_{final}} \left(1 - e^{-\frac{t}{\tau}}\right)$$

$$i_f(t) = I_{f_{final}} + \left(I_o - I_{final}\right)e^{-\frac{t}{\tau}}$$

$$as\ V_f\ changes\ from\ 240\ to\ 100$$

$$i_F\ changes\ from\quad \frac{240}{120} = 2A\ \ to \frac{100}{120} = 0.833\ A$$

$$and\ \tau = \frac{L_f}{R_f} = \frac{0.7}{120}$$

$$\boldsymbol{I_f(t) = 0.833 + 1.167\, e^{-171.28\, t}}$$

At no load

$$put\ \ i_a = 0$$

$$E = V_a = 240\ , E = M I_f(t)\omega_m(t)$$

$$\omega_m(t) = \frac{240}{M\left(i_f(t)\right)} = \frac{240}{M(\boldsymbol{0.833 + 1.167\, e^{-171.28\, t}}\ )}$$

5- A sep-excited dc generator running at 1433 rpm .

$$R_a = 0.1\,\Omega, L_a = 0.5\,mH, R_f = 80\,\Omega, L_f = 40\,H, \qquad M = 0.8\,H.$$

a) The field excited and armature windings are open. Find va(t) if a constant voltage of 160 V is suddenly impressed across the field terminals.

b) Find ia(t) if the armature is initially short-circuited.

c) If the armature voltage, Va is attained steady-state value in part (a) and the armature is suddenly connected to a load of (1.1 Ω ,1.7 mH). Determine the armature current ia(t) and armature terminal voltage va(t).

d) Find Te(t) and the average value of electromagnetic torque in (c)

e) In (c), if Ia has attained steady-state and additional resistance of 0.4 Ω is inserted in series in the load circuit, find the armature current after this change.

a) Given

Open arm. , step $V_f = 0 \rightarrow 160$

So

$$I_f(t) = I_{f\,final}\left(1 - e^{-\frac{t}{\tau}}\right) = \frac{160}{80}\left(1 - e^{-t*\frac{80}{40}}\right)$$

$$\boldsymbol{v_a(t) = M\,\omega_m\,i_f(t) = 0.8 * 1433 * \frac{2\pi}{60} * \frac{160}{80}\left(1 - e^{-t*\frac{80}{40}}\right)}$$

b) Given

Armature is SC , find i_a(t)

LONG SOLUTION EXACT

$$R_L = 0$$

$$0 = -R_a i_a - L_a\frac{di_a}{dt} + M\,i_f\omega$$

$$(R_a + L_a S)I_a(s) = M\,i_f(s)\omega$$

$$\therefore I_f(s) = \frac{R_a + L_a S}{M\omega}\,I_a(s)$$

$$V_f(s) = (R_f + L_f S)I_f(s)$$

$$\therefore V_f(s) = \frac{(R_f + L_f s)(R_a + L_a s)}{M\omega}\,I_a(s)$$

$$I_a(s) = \frac{M\omega V_f(s)}{(R_f + L_f S)(R_a + L_a S)}$$

$$I_a(s) = \frac{(0.8 * 150.06 * 160)}{s(80 + 40s)(0.1 + 0.5 * 10^{-3}s)}$$

$$I_a(s) = \frac{2400.96}{s} + -\frac{97008.48}{405s + 80} + \frac{0.0121}{0.05 * 10^{-3}s + 0.1}$$

$$\boldsymbol{i_a(t) = 2400.96 + 24.25e^{-200t} - 97008.48e^{-2t}}$$

$$LEC\ says: I_{a_{sc}} = 2400 - 24.14e^{-2t} + 14.5e^{-333.33t}$$

SHORT SOLUTION

$$i_f = \frac{V_f}{R_f} = \frac{160}{80} = 2\ A\ , \tau = \frac{L_f}{R_f} = \frac{40}{80} = \frac{1}{2}$$

$$i_f(t) = 2(1 - e^{-2t})$$

$$v_a(t) = i_f M\omega = 240.1010(1 - e^{-2t})$$

$$I_{a_{final}} = \frac{240}{R_a + R_L} = \frac{240.1010(1 - e^{-2t})}{0.1} = 2400$$

$$\tau_e = \frac{(L_a + L_L)}{R_a + R_L} = \frac{0.5 * 10^{-3}}{0.1} = \frac{1}{200}$$

$$2400(1 - e^{-200t})$$

c) Given

$$v_f = 160\ V\ then\ added\ load\ (1.1\ \Omega, 1.7\ \text{mH}).\ \text{getI}_\text{a}(t) and\ V_a(t)$$

$$i_f = \frac{V_f}{R_f} = \frac{160}{80}$$

$$v_a(t) = i_f M\omega = 240.1010$$

$$I_{a_{final}} = \frac{240}{R_a + R_L} = \frac{240}{0.1 + 1.1} = 200$$

$$\tau_e = \frac{(L_a + L_L)}{R_a + R_L} = \frac{(1.7 + 0.5) * 10^{-3}}{0.1 + 1.1} = \frac{11}{6000}$$

$$\boldsymbol{i_a(t) = 200\left(1 - e^{-\frac{6000}{11}t}\right)}$$

$$\boldsymbol{v_A(T) = R_L i_a + L\frac{di_a}{dt} = 1.1 * 200\left(1 - e^{-\frac{6000}{11}t}\right) + (200 * 1.7 * 10^{-3} * 545.45)e^{-545.4t}}$$

d) Given

$$T_e = M i_f i_a = 0.8 * 2 * \mathbf{200}\left(\mathbf{1} - \boldsymbol{e}^{-\frac{\mathbf{6000}}{\mathbf{11}}\boldsymbol{t}}\right)$$

$$average = 0.8 * 2 * \mathbf{200}$$

e) $R_{L_{add}} = 0.4$ on SS of (c) [NOTE HERE THERE IS INITIAL $i_a(t) = i_o e^{-\frac{t}{\tau}} + i_f\left(1 - e^{-\frac{t}{\tau}}\right)$

$$I_{a_{final}} = \frac{240}{0.1 + 1.1 + 0.4} = 150$$

$$\tau = \frac{(1.7 + 0.5) * 10^{-3}}{0.1 + 1.1 + 0.4} = \frac{11}{8000}$$

$$\dot{I}_{a_{new}} = 150 + 50\, e^{-800t}$$

7 DYNAMIC MODEL OF 3ϕ INDUCTION MOTOR

In induction motor parameters from output to input are

$$T_m \; from \; T_e \; from \; \lambda \; from \; v$$

- It has 3ph stator and 3ph rotor (6 voltage equations)
- It's model can be put in 4 sets of equations
  a) Voltage Equations (V)
  b) Flux Linkage Equations (λ)
  c) Electromagnetic Induced Torque (T_e)
  d) Mechanical Equations(T_m)

7.1 For ABC (natural) Machine Axis

7.1.1 Voltage Equations

7.1.1.1 Stator Voltage Equations

$$V_{as} = R_{as} i_{as} + \frac{d}{dt} \lambda_{as}$$

$$V_{bs} = R_{bs} i_{bs} + \frac{d}{dt} \lambda_{bs}$$

$$V_{cs} = R_{cs} i_{as} + \frac{d}{dt} \lambda_{cs}$$

7.1.1.2 Rotor Voltage Equations

$$V_{ar} = R_{ar} i_{ar} + \frac{d}{dt} \lambda_{ar}$$

$$V_{br} = R_{br} i_{br} + \frac{d}{dt} \lambda_{br}$$

$$V_{cr} = R_{cr} i_{cr} + \frac{d}{dt} \lambda_{cr}$$

7.1.1.3 In Matrix Form

$$[V_{abc,s}] = [r_{abc,s}][i_{abc,s}] + P[\lambda_{abc,s}]$$

$$\begin{bmatrix} V_{as} \\ V_{bs} \\ V_{cs} \end{bmatrix} = \begin{bmatrix} r_{as} & 0 & 0 \\ 0 & r_{bs} & 0 \\ 0 & 0 & r_{cs} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + P \begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix}$$

If $r_{as} = r_{bs} = r_{cs}$

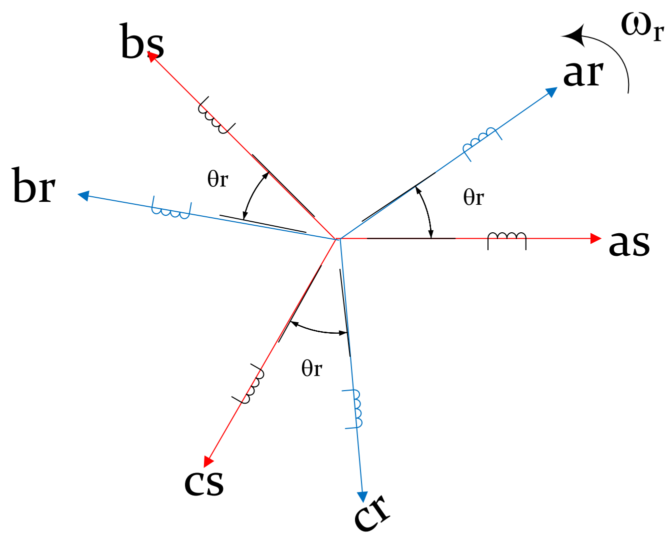
Then,

$$V_{abc,s} = r_s i_{abc,s} + P\ \lambda_{abc,s}$$
$$similllary\ , V_{abc,r} = r_r i_{abc,r} + P\ \lambda_{abc,r}$$

7.1.2 Flux Linkage Equations

λ for each coil consists of

1- Self between the winding it self
2- Mutual between it and 2 other stator
3- Mutual between it and 3 other rotor



So λ consists of 6 terms

7.1.2.1 e.g. For statror phase a

$$\lambda_{abcs} = L_{abc,s-abc,s} i_{abc,s} + L_{abc,s-abc,r}\ i_{abc,r}$$

$$\lambda_{as} = L_{as-as} i_{as} + L_{as-bs} i_{bs} + L_{as-cs} i_{cs} + L_{as-ar} i_{ar} + L_{as-br} i_{br} + L_{as-cr} i_{cr}$$

7.1.2.2 In matrix form

$$\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix} = \begin{bmatrix} L_{as-as} & L_{as-bs} & L_{as-cs} \\ L_{bs-as} & L_{bs-bs} & L_{bs-cs} \\ L_{cs-as} & L_{cs-bs} & L_{cs-cs} \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + \begin{bmatrix} L_{as-ar} & L_{as-br} & L_{as-cr} \\ L_{bs-ar} & L_{bs-br} & L_{bs-cr} \\ L_{cs-ar} & L_{cs-br} & L_{cs-cr} \end{bmatrix} \begin{bmatrix} i_{ar} \\ i_{br} \\ i_{cr} \end{bmatrix}$$

Remember Self inductance = Magnetizing + Leakage
Mutual = Mmax cosθ

7.1.2.3 Simplifications

At balanced 3ph

7.1.2.3.1 All selves are the same

$$\boldsymbol{L_{ss}} = L_{as-as} = L_{bs-bs} = L_{cs-cs} = L_{ss} = L_{ms} + L_{ls} = \boldsymbol{N_{ph}^2 P_g} + \boldsymbol{L_{ls}} = N_{ph}^2 P_g + N_{ph}^2 P_L$$

7.1.2.3.2 Mutual between same (Stator windings) are same

All angles between same member are 120 degrees

$$L_{bs-as} = L_{as-bs} = L_{cs-as} = \cdots = L_{(cs-bs)} = L_{ms}\cos(120) = -0.5\, L_{ms} = -0.5\, N_{ph}^2 P_g$$

7.1.2.3.3 Mutual with other (Rotor) changes

$$L_{as-ar} = L_{sr}\cos\theta_r$$

$$L_{as-br} = L_{sr}\cos(\theta_r + \frac{2\pi}{3})$$

$$L_{as-cr} = L_{sr}\cos(\theta_r - \frac{2\pi}{3})$$

Stator to Rotor

$$[L_{abc,s} - L_{abc,r}] = L_{sr}\begin{bmatrix} \cos\theta_r & \cos(\theta_r + \frac{2\pi}{3}) & L_{sr}\cos(\theta_r - \frac{2\pi}{3}) \\ \cos(\theta_r - \frac{2\pi}{3}) & \cos\theta_r & \cos(\theta_r + \frac{2\pi}{3}) \\ \cos(\theta_r + \frac{2\pi}{3}) & \cos(\theta_r - \frac{2\pi}{3}) & \cos\theta_r \end{bmatrix}$$

Rotor to Stator (Transpose)

$$[L_{abc,r} - L_{abc,s}] = L_{sr}\begin{bmatrix} \cos\theta_r & \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r + \frac{2\pi}{3}) \\ \cos(\theta_r + \frac{2\pi}{3}) & \cos\theta_r & \cos(\theta_r - \frac{2\pi}{3}) \\ \cos(\theta_r - \frac{2\pi}{3}) & \cos(\theta_r + \frac{2\pi}{3}) & \cos\theta_r \end{bmatrix}$$

NOTE $[L_{abc,r} - L_{abc,s}] = [L_{abc,s} - L_{abc,r}]^T$

So,

$$\begin{bmatrix}\lambda_{as}\\ \lambda_{bs}\\ \lambda_{cs}\end{bmatrix} = \begin{bmatrix}L_{ms}+L_{ls} & -0.5\,L_{ms} & -0.5\,L_{ms}\\ -0.5\,L_{ms} & L_{ms}+L_{ls} & -0.5\,L_{ms}\\ -0.5\,L_{ms} & -0.5\,L_{ms} & L_{ms}+L_{ls}\end{bmatrix}\begin{bmatrix}i_{as}\\ i_{bs}\\ i_{cs}\end{bmatrix} + L_{sr}\begin{bmatrix}\cos\theta_r & \cos(\theta_r+\frac{2\pi}{3}) & \cos(\theta_r-\frac{2\pi}{3})\\ \cos(\theta_r-\frac{2\pi}{3}) & \cos\theta_r & \cos(\theta_r+\frac{2\pi}{3})\\ \cos(\theta_r+\frac{2\pi}{3}) & \cos(\theta_r-\frac{2\pi}{3}) & \cos\theta_r\end{bmatrix}\begin{bmatrix}i_{ar}\\ i_{br}\\ i_{cr}\end{bmatrix}$$

Example for voltage equation :

$$\begin{aligned}V_{as} = r_S i_{as} + \frac{d}{dt}\,[\,(L_{ls}+L_{ns}\,)i_{as} - 0.5L_{ms}i_{bs} - 0.5L_{ms}i_{cs} + L_{sr}\cos\theta_r\, i_{ar} + L_{sr}\cos\left(\theta_r+\frac{2\pi}{3}\right)i_{br}\\ + L_{sr}\cos\left(\theta_r-\frac{2\pi}{3}\right)i_{cr}\end{aligned}$$

7.1.3 Electromagnetic Developed Torque

$$T_e = \frac{P_{in}}{\omega_m} = \frac{iv}{\omega_m}$$

$$T_{em} = i_{abc,s}^{T} \frac{\partial}{\partial \theta_m} \left[L_{abc,s-abc,r}\right] i_{abc,r}$$

$$= \frac{P}{2}\, i_{abc,s}^{T} \frac{\partial}{\partial \theta_e} \left[L_{abc,s-abc,r}\right] i_{abc,r}$$

Note if we partial to θ_e we must multiply by P

7.1.4 Mechanical Equation

$$T_{em} = T_L + B\,\omega_m + J\frac{d\omega_m}{dt}$$

7.1.5 Disadvantages

1- Time Variant Mutual Inductance between rotor and stator windings

2- Mutually coupled stator and rotor self inductances

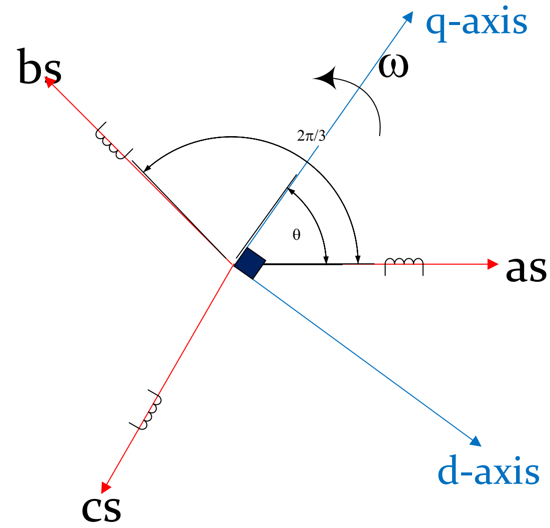
3- Complex system of 7th order (Hard to solve)

7.1.6 Advantages

1- Actual Model of the machine that describes it in the natural axes

2- Can Model unbalanced and fault conditions accurately

7.2 QDO- Axis Model



As we've seen in abc model it's too complex to solve so we use an equivalent <u>2-phase system</u>

So convert it into a <u>5th order time invariant</u> system

7.2.1 Reference Frame Theory

We try to put both stator and rotor to the same refrence

1- Rotor refer to $\omega_s = 0$
2- Stator refer to ω_r
3- Both to common reference ($0 \; or \; \omega_s \; or \; \omega_r$)

So both stator and rotor are at the same speed which will **solve the problem of Time variant mutual inductance between them**

7.2.2 Principle of qdo-axis (Mathematical Approach)

d- direct axis

q- perpendicular axis

both rotate at speed of ω

from graph

$$V_a = V_q \cos\theta + V_d \sin\theta + V_o$$

$$V_b = V_q \cos\left(\theta - \frac{2\pi}{3}\right) + V_d \sin\left(\theta - \frac{2\pi}{3}\right) + V_o$$

$$V_c = V_q \cos\left(\theta + \frac{2\pi}{3}\right) + V_d \sin\left(\theta + \frac{2\pi}{3}\right) + V_o$$

<u>NOTE</u> we added terms V_o just to make matrix 3x3 , $V_o = 0$ at balanced cases

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta & 1 \\ \cos(\theta - \frac{2\pi}{3}) & \sin(\theta - \frac{2\pi}{3}) & 1 \\ \cos(\theta + \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) & 1 \end{bmatrix} \begin{bmatrix} V_q \\ V_d \\ V_o \end{bmatrix}$$

But , Actually we need qdo from abc

So take the inverse

$$\begin{bmatrix} V_q \\ V_d \\ V_o \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos\theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \sin\theta & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

7.2.3 Principle of qdo-axis (Machine Approach)

It’s that the AT is the same at both axes systems

$$AT_{3-phase} = AT_{2-phase}$$

$$AT_{abc} = AT_{qdo}$$

$$\frac{NI}{3} = \frac{NI}{2}$$

$$\frac{N}{2}\, i_q = \frac{N}{3}\,[\, i_a \cos\theta + i_b \cos\left(\theta - \frac{2\pi}{3}\right) + i_c \cos\left(\theta + \frac{2\pi}{3}\right)]$$

$$\therefore i_q = \frac{2}{3}\, I_{3\phi}$$

$$\textit{so we get to the same matrix } (\, SOBHAN\ ALLAH)$$

$$\begin{bmatrix} i_q \\ i_d \\ i_o \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos\theta & \cos(\theta - \frac{2\pi}{3}) & \cos(\theta + \frac{2\pi}{3}) \\ \sin\theta & \sin(\theta - \frac{2\pi}{3}) & \sin(\theta + \frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

7.2.4 Transformation Matrix

$$[A_{qdo}] = K[A_{abc}]$$

$$K = \frac{2}{3}\begin{bmatrix} \cos\theta & \cos\left(\theta - \frac{2\pi}{3}\right) & \cos\left(\theta + \frac{2\pi}{3}\right) \\ \sin\theta & \sin\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$K^{-1} = \begin{bmatrix} \cos\theta & \sin\theta & 1 \\ \cos\left(\theta - \frac{2\pi}{3}\right) & \sin\left(\theta - \frac{2\pi}{3}\right) & 1 \\ \cos\left(\theta + \frac{2\pi}{3}\right) & \sin\left(\theta + \frac{2\pi}{3}\right) & 1 \end{bmatrix}$$

$$KK^{-1} = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

7.2.5 Speed References

Remember

$$\theta = \omega t$$

$$stator\ speed\ , \theta_s = 0 * t = 0$$

$$Rotor\ Speed\ \theta_r = \omega_r t$$

We refer both rotor and stator to

$\omega = 0$	Stationary reference frame
$\omega = \omega_r$	Rotating reference frame
$\omega = \omega_e$	Synchronous reference frame
$\omega = \omega$	Arbitrary reference frame

7.2.6 3-phase voltage supply

7.2.6.1 Stator voltage supply

$$V_{as} = V_m \cos(\omega_e t + \phi)$$

$$V_{bs} = V_m \cos(\omega_e t - \frac{2\pi}{3} + \phi)$$

$$V_{cs} = V_m \cos(\omega_e t + \frac{2\pi}{3} + \phi)$$

NOTE(A) WE USED ω_e to make synchronous in the gap from the frequency of the supply

7.2.6.2 Qdo-axis voltage supply in stationary reference frame

NOTE(B) WE USED $\theta = 0$ to make synchronous in the gap from the frequency of the supply
BECAUSE

Stator/Rotor refered to a QDO that rotate at ω_r

So Supply Must be refered to QDO that is $\theta = 0$

So that relative speed between them is ω_r
NOTE : we can swap !!

To put supply in qdo refrence frame we $\times K$

For $\theta = 0$

$$K_{\theta=0} = \frac{2}{3}\begin{bmatrix} \mathbf{1} & -\frac{\mathbf{1}}{\mathbf{2}} & -\frac{\mathbf{1}}{\mathbf{2}} \\ \mathbf{0} & -\frac{\sqrt{\mathbf{3}}}{\mathbf{2}} & \frac{\sqrt{\mathbf{3}}}{\mathbf{2}} \\ \frac{\mathbf{1}}{\mathbf{2}} & \frac{\mathbf{1}}{\mathbf{2}} & \frac{\mathbf{1}}{\mathbf{2}} \end{bmatrix}$$

$$\begin{bmatrix} V_{qs} \\ V_{ds} \\ V_{os} \end{bmatrix} = \frac{2}{3}\begin{bmatrix} \mathbf{1} & -\frac{\mathbf{1}}{\mathbf{2}} & -\frac{\mathbf{1}}{\mathbf{2}} \\ \mathbf{0} & -\frac{\sqrt{\mathbf{3}}}{\mathbf{2}} & \frac{\sqrt{\mathbf{3}}}{\mathbf{2}} \\ \frac{\mathbf{1}}{\mathbf{2}} & \frac{\mathbf{1}}{\mathbf{2}} & \frac{\mathbf{1}}{\mathbf{2}} \end{bmatrix}\begin{bmatrix} V_{as} \\ V_{bs} \\ V_{cs} \end{bmatrix}$$

$$V_{qs}^{s} = \frac{2}{3}\left[V_a - \frac{1}{2}V_b - \frac{1}{2}V_c \right] = \frac{2}{3}\left[V_a - \frac{1}{2}(V_{bs} + V_{cs}) \right]$$

$$V_{ds}^{s} = \frac{2}{3}\left[-\frac{\sqrt{3}}{2}V_b + \frac{\sqrt{3}}{2}V_c \right]$$

If system balanced

$$V_q = \frac{2}{3}\left[V_{as} + \frac{1}{2}V_{as} \right] = V_{as} = V_m\cos(\omega t + \phi)$$

$$V_{os} = V_m\cos(\omega_e t + \phi)$$

Let's subs from 3ϕ into it

$$V_{ds} = \frac{1}{\sqrt{3}}\left[-V_m\cos\left(\omega_e t - \frac{2\pi}{3} + \phi\right) + V_m\cos\left(\omega_e t + \frac{2\pi}{3} + \phi\right) \right]$$

$$V_{ds} = \frac{2}{\sqrt{3}}\left[-V_m\sin(\omega_e t + \phi)\sin\left(\frac{2\pi}{3}\right) \right]$$

$$V_{ds} = -\frac{2V_m}{\sqrt{3}} \sin\left(\frac{2\pi}{3}\right) \sin(\omega_e t + \phi) = -\frac{2V_m}{\sqrt{3}}\frac{\sqrt{3}}{2} \sin(\omega_e t + \phi)$$

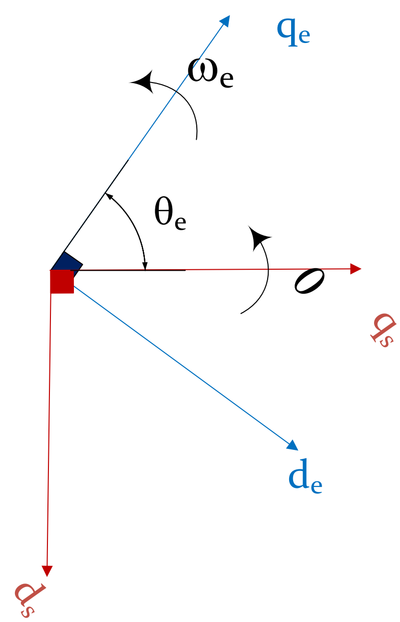
$$\boldsymbol{V_d^s = -V_m \sin(\omega_e t + \phi)}$$

So to get V_{qs} it's perpendicular

$$\boldsymbol{V_q^s = V_m \cos(\omega_e t + \phi)}$$

7.2.6.3 Qdo-axis voltage supply in synchronous reference frame

We find a relation to transfer between stationary reference frame to synch. Frame.



$$\begin{bmatrix} V_q^e \\ V_d^e \end{bmatrix} = \begin{bmatrix} \cos\theta_e & -\sin\theta_e \\ \sin\theta_e & \cos\theta_e \end{bmatrix} \begin{bmatrix} V_q^s \\ V_d^s \end{bmatrix}$$

$$V_q^e = V_q^s \cos\theta_e - V_d^s \sin\theta_e$$

$$V_q^e = V_m \cos(\omega_e t + \phi)\cos\theta_e + V_m \sin(\omega_e t + \phi)\sin\theta_e$$

Then ,

$$V_q^e = V_m \cos(\omega_e t + \phi - \theta_e)$$

$$but, \theta_e = \omega_e t$$

$$V_q^e = V_m \cos\phi$$

$$V_d^e = -V_m \sin\phi$$

<u>NOTE</u> It must be constant because we referred it, they are 2 balanced dc ref, frame and DC deals with max value

7.2.6.4 Qdo-axis voltage supply in arbitrary reference frame

$$\begin{bmatrix} V_q^e \\ V_d^e \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} V_q^s \\ V_d^s \end{bmatrix}$$

Arbitrary frame $\theta = \omega t$

$$V_{qs} = V_m \cos(\omega_e t - \theta + \phi)$$

$$V_{qs} = V_m \cos\big((\omega_e - \omega)t + \phi\big)$$

$$V_{ds} = -V_m \sin\big((\omega_e - \omega)t + \phi\big)$$

7.2.7 Resistive Element

$$v = ri$$

$$V_{abc} = [r] i_{abc}$$

$$V_{abc} = \begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix} i_{abc}$$

$$i_{abc} = k^{-1}\, i_{qdo}$$

$$[r]_{inqdo} = k[r]$$

So they cancel each other

$$V_{qdo} = k\,[r] * k^{-1}\, i_{qdo}$$

$$\begin{bmatrix} V_q \\ V_d \\ V_o \end{bmatrix} = \begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ 0 & 0 & r \end{bmatrix} \begin{bmatrix} i_q \\ i_d \\ i_o \end{bmatrix}$$

NOTE [r] didn't change by changing the axes

7.2.8 Flux Linkage

$$V_{abc} = P\,\lambda_{abc}$$

$$V_{qdo} = (P\,\lambda_{abc}\quad) \times k$$

$$V_{qdo} = k\,P\left(k^{-1}\lambda_{qdo}\right)$$

$$V_{qdo} = k\left(k^{-1}\,P\,\lambda_{qdo} + \lambda_{qdo}\,P\,k^{-1}\right)$$

$$V_{qdo} = \overbrace{kk^{-1}}^{\text{Unity}}\,P\lambda_{qdo} + kPk^{-1}\lambda_{qdo}$$

$$V_{qdo} = P\,\lambda_{qdo} + \omega\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\lambda_{qdo}$$

$$\therefore V_q = P\lambda_q + \omega\lambda_d$$

$$V_d = P\lambda_d - \omega\lambda_q$$

$$V_o = P\lambda_o$$

$$\boxed{\boldsymbol{\omega\lambda\ represent\ speed\ voltage}}$$

7.2.9 Self-Inductance

$$\lambda_{abc,s-abc,s} = L_{abc,s-abc,s}\,i_{abc,s}$$

$$\lambda_{qdo,s-qdo,s} = L_{abc,s-abc,s}\,i_{abc,s} \quad \times k$$

$$\lambda_{qdo,s-qdo,s} = k\,L_{abc,s-abc,s}\,k^{-1} \quad i_{qdo,s}$$

$$\lambda_{qdo,s-qdo,s} = \begin{bmatrix} L_{ls} + \frac{3}{2}L_{ms} & 0 & 0 \\ 0 & L_{ls} + \frac{3}{2}l_{ms} & 0 \\ 0 & 0 & L_{ls} \end{bmatrix}\begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{os} \end{bmatrix}$$

In qdo-axis

System-mutually decouples

So,

$$\because\ k\ L_{abc,s-abc,s}\ k^{-1} \quad = \begin{bmatrix} L_{ls}+\frac{3}{2}L_{ms} & 0 & 0 \\ 0 & L_{ls}+\frac{3}{2}l_{ms} & 0 \\ 0 & 0 & L_{ls} \end{bmatrix}$$

also

$$k\ L_{abc,r-abc,r}\ k^{-1} \quad = \begin{bmatrix} L_{lr}+\frac{3}{2}L_{mr} & 0 & 0 \\ 0 & L_{lr}+\frac{3}{2}l_{mr} & 0 \\ 0 & 0 & L_{lr} \end{bmatrix}$$

Mutually decoupled self-inductances

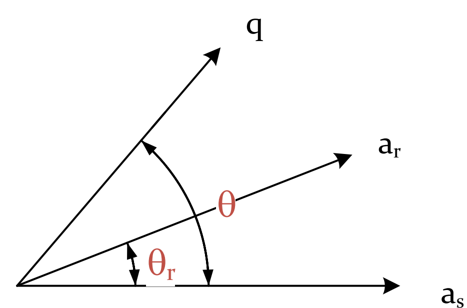
$$\lambda_{abc,s} = L_{abc,s-abc,s}\ i_{abc,s}\ + L_{abc,s-abc,r}\ i_{abc,r}$$

$$\lambda_{qdo,s} = L_{abc,s-abc,s}\ i_{abc,s}\ + L_{abc,s-abc,r}\ i_{abc,r} \qquad \times k_s$$

$$\lambda_{qdo,s}\ = k_s\left[L_{abc,s-abc,s}\right]k_s^{-1}\ i_{qdo,s}\ +\ k_s\left[L_{abc,s-abc,r}\right]k_r^{-1}\ i_{qdo,r}$$

Note the s and r

$$\boldsymbol{k_r = \frac{2}{3}\begin{bmatrix} \cos(\theta-\theta_r) & \cos(\theta-\theta_r-\frac{2\pi}{3}) & \cos(\theta-\theta_r+\frac{2\pi}{3}) \\ \sin(\theta-\theta_r) & \sin(\theta-\theta_r-\frac{2\pi}{3}) & \sin(\theta-\theta_r+\frac{2\pi}{3}) \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}}$$



$$K_s \rightarrow \theta$$
$$K_r \rightarrow \theta-\theta_r$$

but,

$$L_{abc,s-abc,r} = L_{sr}\begin{bmatrix} \cos\theta_r & \cos\left(\theta_r + \frac{2\pi}{3}\right) & L_{sr}\cos\left(\theta_r - \frac{2\pi}{3}\right) \\ \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\theta_r & \cos\left(\theta_r + \frac{2\pi}{3}\right) \\ L_{sr}\cos\left(\theta_r + \frac{2\pi}{3}\right) & \cos\left(\theta_r - \frac{2\pi}{3}\right) & \cos\theta_r \end{bmatrix}$$

$$k_s\left[L_{abc,s-abc,r}\right]k_r^{-1} = \begin{bmatrix} \frac{3}{2}L_{sr} & 0 & 0 \\ 0 & \frac{3}{2}L_{sr} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Time invariant Mutual inductance

7.2.10 QDO Pros and Cons

Pros:

1- Mutually decoupled system
2- Time invariant mutual inductance between stator and rotor
3- Reduction of system order 5th order

Cons :

1- Doesn't deal with actual values
2- Can't solve fault analysis

NOTES :

Add equations and matrices when you mention the comparison

Say the full sentence

7.2.11 Equivalent Circuit modelling

7.2.11.1 From flux linkage relations

Stator	$\boldsymbol{\lambda_{qs} = \left(L_{ls} + \frac{3}{2}L_{ms}\right) i_{qs} + \frac{3}{2}\, L_{sr} i_{qr}}$ $\boldsymbol{\lambda_{ds} = \left(L_{ls} + \frac{3}{2}L_{ms}\right) i_{ds} + \frac{3}{2}\, L_{sr} i_{dr}}$ $\boldsymbol{\lambda_{os} = L_{ls} i_{os}}$
Rotor	$\boldsymbol{\lambda_{qr} = \frac{3}{2}L_{sr}\, i_{qs} + \left(L_{lr} + \frac{3}{2}L_{mr}\right) i_{qr}}$ $\boldsymbol{\lambda_{qr} = \frac{3}{2}L_{sr}\, i_{ds} + \left(L_{lr} + \frac{3}{2}L_{mr}\right) i_{dr}}$ $\boldsymbol{\lambda_{or} = L_{lr}\, i_{or}}$

7.2.11.2 Referring

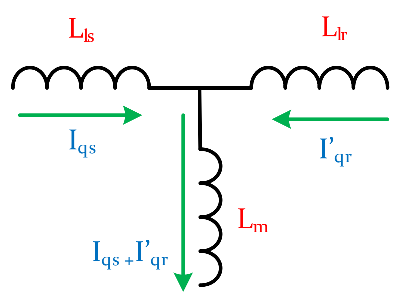
$$\boldsymbol{Let\ L_m = \frac{3}{2} L_{ms}}$$

$$L_{ms} = N_{ph_s}^2 P_g$$

$$\frac{3}{2} L_{sr} = \frac{3}{2} N_{ph_s} N_{ph_r}\, P_g * \frac{N_{ph_s}}{N_{ph_s}} = \frac{3}{2}\, L_{ms} \frac{N_{ph_r}}{N_{ph_s}}$$

$$\therefore \frac{3}{2} L_{sr} = L_m \frac{N_{ph_r}}{N_{ph_s}}$$

$$\lambda_{qs} = (L_{ls} + L_m) i_{qs} + L_m \overbrace{\frac{\mathrm{N_{ph_r}}}{\mathrm{N_{ph_s}}}\ \mathrm{i_{q_r}}}^{\text{referred}}$$



So , referring to stator

$$\boldsymbol{\lambda_{qs} = L_{ls}\, i_{qs} + L_m\left(i_{qs} + i'_{qr}\right)}$$

$$\boldsymbol{\lambda_{ds} = L_{ls}\, i_{ds} + L_m(i_{ds} + i'_{dr})}$$

Stator	$\boldsymbol{\lambda_{qs} = L_{ls}\, i_{qs} + L_m\left(i_{qs} + i'_{qr}\right)}$ $\boldsymbol{\lambda_{ds} = L_{ls}\, i_{ds} + L_m(i_{ds} + i'_{dr})}$
Rotor	$\boldsymbol{\lambda'_{qr} = L'_{lr}\, i'_{qr} + L_m\left(i_{qs} + i'_{qr}\right)}$ $\boldsymbol{\lambda'_{dr} = L'_{lr}\, i'_{dr} + L_m(i_{ds} + i'_{dr})}$

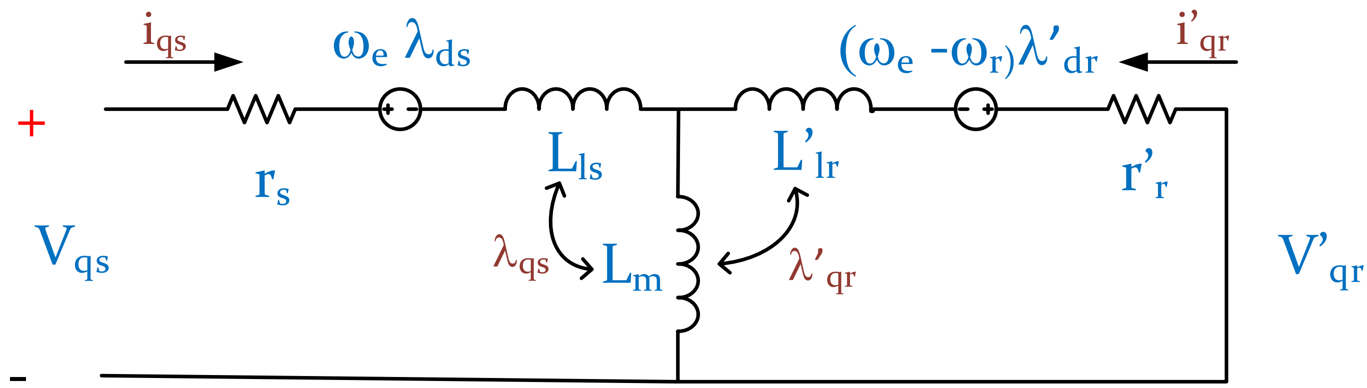
7.2.11.3 Dynamic Equivalent Circuit Equations

$$V_{qs} = r_s i_{qs} + p\,\lambda_{qs} + \omega\lambda_{ds}$$

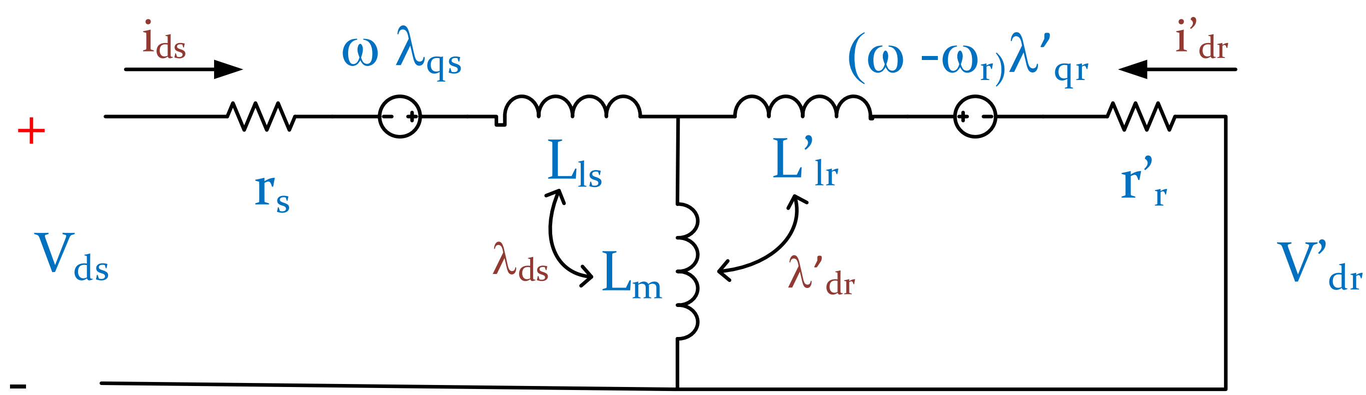
$$V_{ds} = r_s i_{ds} + p\,\lambda_{ds} - \omega\lambda_{qs}$$

$$V'_{qr} = \overbrace{0}^{cage} = r'_r\, i'_{qr} + p\,\lambda'_{qr} + (\omega - \omega_r)\lambda'_{dr}$$

$$V'_{dr} = \overbrace{0}^{cage} = r'_r i'_{dr} + p\,\lambda'_{dr} - (\omega - \omega_r)\lambda'_{qr}$$



Q- axis equivalent circuit



D- axis equivalent circuit

7.2.12 QDO system in per-unit

Stator	$\boldsymbol{\lambda_{qs} = L_s i_{qs} + L_m i'_{qr}}$ $\boldsymbol{\lambda_{ds} = L_s\, i_{ds} + L_m i'_{dr}}$
Rotor	$\boldsymbol{\lambda'_{qr} = L'_r\, i'_{qr} + L_m i_{qs}}$ $\boldsymbol{\lambda'_{dr} = L'_r i'_{dr} + L_m i_{ds}}$

7.2.12.1 Flux linkage relations in matrix form

$$\begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda'_{qr} \\ \lambda'_{dr} \end{bmatrix} = \begin{bmatrix} L_s & 0 & L_m & 0 \\ 0 & L_s & 0 & L_m \\ L_m & 0 & L'_r & 0 \\ 0 & L_m & 0 & L'_r \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i'_{qr} \\ i'_{dr} \end{bmatrix}$$

7.2.12.2 Reactance relations in matrix form

To model flux linkage in per-unit we need to use reactance $(X = \omega L)$ so $L = \frac{X}{\omega_e}$

$$\omega_e \lambda = \psi \qquad \rightarrow \quad \lambda = \frac{\psi}{\omega_e}$$

So, by multiplying above matrix by ω_e (we can get $\lambda \rightarrow \psi$ & $L \rightarrow X$)

$$\begin{bmatrix} \psi_{qs} \\ \psi_{ds} \\ \psi'_{qr} \\ \psi'_{dr} \end{bmatrix} = \begin{bmatrix} X_s & 0 & X_m & 0 \\ 0 & X_s & 0 & X_m \\ X_m & 0 & X'_r & 0 \\ 0 & X_m & 0 & X'_r \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i'_{qr} \\ i'_{dr} \end{bmatrix}$$

NOTE:

ψ is called flux linkage per-unit or per-second

7.2.12.3 Voltage equations in terms of flux linkage per-unit

$$V_{qs} = r_s i_{qs} + \frac{1}{\omega_e} P\, \psi_{qs} + \frac{\omega}{\omega_e} \psi_{ds}$$

$$V_{ds} = r_s i_{ds} + \frac{1}{\omega_e} P\, \psi_{ds} - \frac{\omega}{\omega_e} \psi_{qs}$$

$$V_{qr} = r_r i_{qr} + \frac{1}{\omega_e} P\, \psi_{qr} + \frac{\omega - \omega_r}{\omega_e} \psi_{dr}$$

$$V_{dr} = r_r i_{dr} + \frac{1}{\omega_e} P\, \psi_{dr} - \frac{\omega - \omega_r}{\omega_e} \psi_{qr}$$

all rotor quantities are referred to the stator

NOTE :

Instead of adding V_r', i_r', … to refer rotor to stator we just add a line
"all rotor quantities are referred to the stator"

7.2.12.4 Flux Linkage equations in terms of flux linkage per-unit

$$\psi_{qs} = X_{ls} i_{qs} + X_m\left(i_{qs} + i_{qr}\right)$$
$$\psi_{ds} = X_{ls} i_{ds} + X_m(i_{ds} + i_{dr})$$

$$\psi_{qr} = X_{lr} i_{qr} + X_m\left(i_{qs} + i_{qr}\right)$$
$$\psi_{dr} = X_{lr} i_{dr} + X_m(i_{ds} + i_{dr})$$

all rotor quantities are referred to the stator

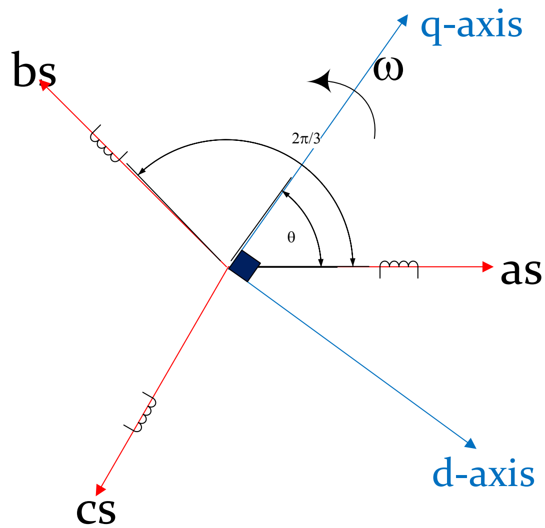
7.2.13 Power in terms of QD-axis variables

Starting from abc-axis

$$P = v_{as} i_{as} + v_{bs} i_{bs} + v_{cs} i_{cs} \quad + \quad v_{ar} i_{ar} + v_{br} i_{br} + v_{cr} i_{cr}$$

NOTE

$$v_{as}(t) * i_{as}(t) \quad gives\ average\ value\ \frac{3}{2} V_m I_m = 3\ vi$$



$v_{as} = v_{qs}\cos\theta + v_{ds}\sin\theta + v_o$ $i_{as} = iq_s\cos\theta + i_{ds}\sin\theta + i_o$ $v_{bs} = v_{qs}\cos\left(\theta - \frac{2\pi}{3}\right) + v_{ds}\sin\left(\theta - \frac{2\pi}{3}\right) + v_o$ $i_{bs} = iq_s\cos\left(\theta - \frac{2\pi}{3}\right) + i_{ds}\sin\left(\theta - \frac{2\pi}{3}\right) + i_o$ $v_{cs} = v_{qs}\cos\left(\theta + \frac{2\pi}{3}\right) + v_{ds}\sin\left(\theta + \frac{2\pi}{3}\right) + v_o$ $i_{cs} = iq_s\cos\left(\theta + \frac{2\pi}{3}\right) + i_{ds}\sin\left(\theta + \frac{2\pi}{3}\right) + i_o$	$v_{ar} = v_{qr}\cos\theta - \theta_r + v_{dr}\sin\theta - \theta_r + v_o$ $i_{ar} = iq_r\cos\theta - \theta_r + i_{dr}\sin\theta - \theta_r + i_o$ $v_{br} = v_{qr}\cos\left(\theta - \theta_r - \frac{2\pi}{3}\right) + v_{dr}\sin\left(\theta - \theta_r - \frac{2\pi}{3}\right) + v_o$ $i_{br} = iq_r\cos\left(\theta - \theta_r - \frac{2\pi}{3}\right) + i_{dr}\sin\left(\theta - \theta_r - \frac{2\pi}{3}\right) + i_o$ $v_{cr} = v_{qr}\cos\left(\theta - \theta_r + \frac{2\pi}{3}\right) + v_{dr}\sin\left(\theta - \theta_r + \frac{2\pi}{3}\right) + v_o$ $i_{cr} = iq_r\cos\left(\theta - \theta_r + \frac{2\pi}{3}\right) + i_{dr}\sin\left(\theta - \theta_r + \frac{2\pi}{3}\right) + i_o$

So,

$$v_{as}i_{as} = v_{qs}i_{qs}\cos^2\theta + v_{qs}i_{ds}\cos\theta\sin\theta + v_{qs}i_o\cos\theta + v_{ds}i_{qs}\cos\theta\sin\theta$$
$$+v_{ds}i_{ds}\sin^2\theta + v_o i_{qs}\cos\theta + v_o i_{ds}\sin\theta + v_o i_o + v_{ds}i_o\sin\theta$$

$$
\begin{aligned}
\boldsymbol{v_{as} i_{as}} = \;& \\
& \boldsymbol{v_{qs} \cos\theta \;\left(i_{qs}\cos\theta + i_{ds}\sin\theta + i_o\right)} \\
& \boldsymbol{+ v_{ds}\sin\theta \;\left(i_{qs}\cos\theta + i_{ds}\sin\theta + i_o\right)} \\
& \boldsymbol{+ v_o \qquad \left(i_{qs}\cos\theta + i_{ds}\sin\theta + i_o\right)}
\end{aligned}
$$

$$
\begin{aligned}
\boldsymbol{P} &= \boldsymbol{\frac{3}{2}\left[\, v_{qs}i_{qs} + v_{ds}i_{ds} + 2\, v_{o_s}\, i_{o_s}\right]} \\
&\quad \boldsymbol{+\frac{3}{2}\left[\, v_{qr}i_{qr} + v_{dr}i_{dr} + 2\, v_{or}i_{or}\,\right]}
\end{aligned}
$$

At balanced condition $2v_{or}i_{or} = 0$

$$\boldsymbol{v_{qs}i_{qs} = r_s i_{qs}^2 + i_{qs}p\,\lambda_{qs} + \omega i_{qs}\lambda_{ds}}$$

$$\boldsymbol{v_{ds}i_{ds} = r_s i_{ds}^2 + i_{ds}p\lambda_{ds} - \omega i_{ds}\lambda_{qs}}$$

$$\boldsymbol{v_{qr}i_{qr} = r_r i_{qr}^2 + i_{qr}p\,\lambda_{qr} + (\omega - \omega_r)i_{qr}\lambda_{dr}}$$

$$\boldsymbol{v_{dr}i_{dr} = r_r i_{dr}^2 + i_{dr}p\,\lambda_{dr} - (\omega - \omega_r)i_{dr}\lambda_{qr}}$$

all rotor quantities are referred to the stator

$$
\begin{aligned}
P_{in} &= \frac{3}{2}\left[\, r_s i_{qs}^2 + r_s i_{ds}^2 + r_r i_{qs}^2 + r_r i_{ds}^2\right] \qquad \rightarrow P_{cu} \\
&\quad + \frac{3}{2}\;\left[\, i_{qs}p\lambda_{qs} + i_{ds}p\lambda_{ds} + i_{qr}p\lambda_{qr} + i_{dr}p\lambda_{dr}\right] \rightarrow \frac{d}{dt}(W_{stored}) \\
&\quad + \frac{3}{2}\;\left[\,\omega\left(i_{qs}\lambda_{ds} - i_{ds}\lambda_{qs}\right) + (\omega - \omega_r)\left(i_{qr}\lambda_{dr} - i_{dr}\lambda_{qr}\right)\right] \rightarrow electromech.power
\end{aligned}
$$

$$\boldsymbol{P_{mech} = \frac{3}{2}\;\left[\,\omega\left(i_{qs}\lambda_{ds} - i_{ds}\lambda_{qs}\right) + (\omega - \omega_r)\left(i_{qr}\lambda_{dr} - i_{dr}\lambda_{qr}\right)\right]}$$

$$\boldsymbol{\lambda_{qs} = L_s i_{qs} + L_m i'_{qr}} \qquad\qquad \boldsymbol{\lambda'_{qr} = L'_r\, i'_{qr} + L_m i_{qs}}$$

$$\boldsymbol{\lambda_{ds} = L_s\, i_{ds} + L_m i'_{dr}} \qquad\qquad \boldsymbol{\lambda'_{dr} = L'_r i'_{dr} + L_m i_{ds}}$$

Substitute from λs $into$ P_{mech}

$$P_{mech} = \frac{3}{2}\left[\, \omega L_m\left(i_{qs}i'_{dr} - i_{ds}i'_{qr}\right) + (\omega - \omega_r)L_m\left(i_{ds}i'_{qr} - i_{qs}i'_{dr}\right)\right]$$

$$\text{But } \left(i_{qs}i_{dr} - i_{ds}i_{qr}\right) = -\left(i_{ds}i_{qr} - i_{qs}i_{dr}\right)$$

$$\boldsymbol{P_{mech} = \frac{3}{2}\, \omega_r\, L_m\left(i_{qs}i'_{dr} - i_{ds}i'_{qr}\,\right)}$$

7.2.14 Electromechanical Eqn

$$T_{em} = \frac{P_{em}}{\omega_m} = \frac{3}{2}\frac{\omega_r}{\omega_m}\left(\, \lambda_{ds}i_{qs} - \lambda_{qs}i_{ds}\,\right)$$

$$T_{em} = \frac{3}{2}\,\frac{\omega_r}{\omega_m}\, L_m\left(\, i_{dr}i_{qs} - i_{ds}i_{qr}\right)$$

$$T_{em} = T_L + B\omega_m + J\frac{d\omega_m}{dt} = \frac{3}{2}\frac{P}{2}\,\left(\lambda_{ds}\, i_{qs} - \lambda_{qs}i_{ds}\right)$$

In terms of flux linkage per sec

$$\boldsymbol{T_{em} = \frac{3}{2}\frac{P}{2}\frac{1}{\omega_e}\,\left(\,\psi_{ds}\, i_{qs} - \psi_{qs}i_{ds}\right)}$$

7.2.15 Per-Unit Model

We intend to put

$$v \ in \ terms \ of \ \psi$$
$$\psi \ in \ terms \ of \ X_l, X_m$$

7.2.15.1 Base Voltage "abc" to "qdo"

$$V_{\underset{abc}{base}} = V_{rms}$$
$$V_{b_{qd}} = V_{max}$$

$$so,$$
$$V_{b_{qd}} = \sqrt{2}\, V_{b_{abc}}$$

$$i_{b_{qd}} = \sqrt{2}\, i_{b_{abc}}$$

$$P_{b_{abc}} = 3\, V_{b_{abc}}\, i_{b_{abc}}$$
$$P_{b_{qd}} = 3 \frac{V_{b_{qd}}}{\sqrt{2}} \frac{i_{b_{qd}}}{\sqrt{2}}$$

$$\therefore P_{b_{qd}} = \frac{3}{2} V_{b_{qd}}\, i_{b_{qd}}$$

$$T_b = \frac{P_{b_{qd}}}{\omega_{b_m}} = \frac{P_{b_{qd}}}{\frac{2}{P}\omega_{b_e}}$$

$$\boldsymbol{T_{b_{qd}} = \frac{3}{2}\frac{P}{2}\frac{1}{\omega_e} V_{b_{qd}}\, i_{b_{qd}} = \frac{P_b}{\frac{2}{P}\omega_b}}$$

7.2.15.2 MODEL in Per-unit

7.2.15.2.1 Voltage Equations

$$V_{qs} = r_s\, i_{qs} + \frac{1}{\omega_e}\, p\, \psi_{qs} + \frac{\omega}{\omega_e}\, \psi_{ds}$$

$$V_{ds} = r_s\, i_{ds} + \frac{1}{\omega_e}\, p\, \psi_{ds} - \frac{\omega}{\omega_e}\, \psi_{qs}$$

$$V_{qr} = r_r\, i_{qr} + \frac{1}{\omega_e}\, p\, \psi_{qr} + \frac{\omega - \omega_r}{\omega_e}\, \psi_{dr}$$

$$V_{dr} = r_r\, i_{dr} + \frac{1}{\omega_e}\, p\, \psi_{dr} - \frac{\omega - \omega_r}{\omega_e}\, \psi_{qr}$$

7.2.15.2.2 Flux Linkage Equations

$$\psi_{qs} = X_s i_{qs} + X_m i_{qr}$$

$$\psi_{ds} = X_s i_{ds} + X_m i_{dr}$$

$$\psi_{qr} = X_r i_{qr} + X_m i_{ds}$$

7.2.15.2.3 Electromagnetic Torque

$$T_{em} = \frac{3}{2}\frac{P}{2}\frac{1}{\omega_e}\left(\psi_{ds} i_{qs} - \psi_{qs} i_{ds}\right)$$

Divide by base

$$T_{em_{pu}} = \frac{actual}{base} = \frac{\frac{3}{2}\frac{P}{2}\frac{1}{\omega_e}\left(\psi_{ds} i_{qs} - \psi_{qs} i_{ds}\right)}{\boldsymbol{\frac{3}{2}\frac{P}{2}\frac{1}{\omega_e} V_{b_{qd}}\, i_{b_{qd}}}} = \frac{\left(\psi_{ds} i_{qs} - \psi_{qs} i_{ds}\right)}{V_{b_{qd}}\, i_{b_{qd}}}$$

$$\boldsymbol{T_{em} = \left(\psi_{ds} i_{qs} - \psi_{qs} i_{ds}\right)}$$

7.2.15.2.4 Mechanical Equation

$$\boldsymbol{T_{em} = T_L + B\omega + J\frac{d\omega_m}{dt}}$$

$$T_j = J\frac{d\omega_m}{dt} = J\frac{2}{P}\frac{d}{dt}\left(\frac{\omega_r}{\omega_b} * \omega_b\right)$$

Divide by Tbase

$$T_{j_{pu}} = \frac{J\frac{2}{P}\frac{d}{dt}\left(\frac{\omega_r}{\omega_b} * \omega_b\right)}{\frac{P_b}{\frac{2}{P}\omega_b}} = \frac{J\left(\frac{2}{p}\omega_b\right)^2}{P_{base}}\frac{d}{dt}\left(\frac{\omega_r}{\omega_b}\right) = 2\left(\frac{\frac{1}{2}J\left(\frac{2}{P}\omega_b\right)^2}{P_b}\right)\frac{d}{dt}\left(\frac{\omega_r}{\omega_b}\right)$$

$$H = \left(\frac{\frac{1}{2}J\left(\frac{2}{P}\omega_b\right)^2}{P_b}\right) = \frac{K_E}{P_b} \rightarrow inertia\ constant$$

$$\boldsymbol{T_{j_{pu}} = 2H\frac{d}{dt}\left(\frac{\omega_r}{\omega_b}\right)}$$

$$\boldsymbol{T_{em} = T_L + B\omega + 2H\frac{d}{dt}\left(\frac{\omega_r}{\omega_b}\right)}$$

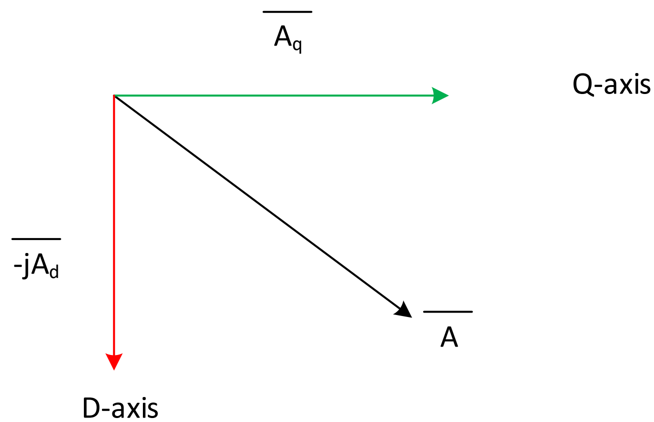
$$T_{em_{pu}} = T_{L_{pu}} + 2H\ P\ \omega_r\ + T_{friction}$$

$$P\omega_r = \frac{1}{2H}(T_{em} - T_L)$$

7.2.16 Space Vector Model

Any vector can be put in two ortho. Axes (direct – quadrant)

$$\bar{A} = A_q - j\,A_d$$
$$\bar{V} = V_q - j\,V_d$$
$$\bar{I} = I_q - j\,I_d$$
$$\bar{\lambda} = \lambda_q - j\,\lambda_d$$



7.2.16.1 Space Vector Model for voltages

7.2.16.1.1 stator

$$V_{qs} = r_s i_{qs} + p\,\lambda_{qs} + \omega\,\lambda_{ds}$$
$$V_{ds} = r_s\,i_{ds} + p\,\lambda_{ds} - \omega\lambda_{qs} \qquad (*\,j)$$

Then subtract

$$V_{qs} - jV_{ds} = r_s\left(i_{qs} - j\,i_{ds}\right) + p\left(\lambda_{qs} - j\,\lambda_{ds}\right) + \omega\left(\lambda_{ds} + j\,\lambda_{qs}\right)$$
$$= r_s\left(i_{qs} - j\,i_{ds}\right) + p\left(\lambda_{qs} - j\,\lambda_{ds}\right) + j\omega(\lambda_{qs} - j\,\lambda_{ds})$$

$$\boldsymbol{\overline{V_s} = r_s\,\overline{\imath_s} + p\,\overline{\lambda_s} + j\omega\,\overline{\lambda_s}}$$

7.2.16.1.2 rotor

$$V_{qr} = r_r i_{qr} + p\,\lambda_{qr} + (\omega - \omega_r)\,\lambda_{dr}$$
$$V_{dr} = r_r\,i_{dr} + p\,\lambda_{dr} - (\omega - \omega_r)\,\lambda_{qr} \qquad (*\,j)$$

$$V_{qr} - jV_{dr} = r_r\left(i_{qr} - j\,i_{dr}\right) + p\left(\lambda_{qr} - j\,\lambda_{dr}\right) + (\omega - \omega_r)\left(\lambda_{dr} + j\,\lambda_{qr}\right)$$

$$\boldsymbol{\overline{V_r} = r_r\,\overline{\imath_r} + p\,\overline{\lambda_r} + j\,(\omega - \omega_r)\overline{\lambda_r}}$$

7.2.17 Steady State Condition

Here in this section we will try to get to the SS equations of IM that we studied before , can we ?

RL circuits in steady state (rate of change is constant)

Meaning there is a change but it's rate now is constant , in Dynamic its rate changes

$$v = (R + jXL\,)I$$

$$\frac{d}{dt}\omega \; in\; dynamic \rightarrow J\omega \; in\; steady\; state$$

But it's modelling will depend on reference frame as follows

Supply frequency is $(\omega_e - \omega)$

Reference Frame	
Stationary	$P \equiv j\omega_e$
Rotor	$P \equiv J\,(\omega_e - \omega_r)$
Synch.	$P \;\equiv 0$
arbitrary	$P \equiv J\,(\omega_e - \omega)$

All reference frames must eventually give the same SS equations

From above table in

$$\overline{\boldsymbol{V_s}} = \boldsymbol{r_s}\,\overline{\boldsymbol{\imath_s}} + \boldsymbol{p}\,\overline{\boldsymbol{\lambda_s}} \;+ \boldsymbol{j\omega}\,\overline{\boldsymbol{\lambda_s}}$$

$$\overline{\boldsymbol{V_r}} = \boldsymbol{r_r}\,\overline{\boldsymbol{\imath_r}} + \boldsymbol{p}\,\overline{\boldsymbol{\lambda_r}} + \boldsymbol{j}\,(\boldsymbol{\omega} - \boldsymbol{\omega_r})\overline{\boldsymbol{\lambda_r}}$$

7.2.17.1 For stationary frame

$$v_s = r_s i_s + j\,\omega_e\,\lambda_s$$

$$v_r = r_r i_r + j\,(\omega_e - \omega_r)\lambda_r$$

$$\overbrace{S}^{\text{slip}} * \omega_e = (\omega_e - \omega_r)$$

$$\frac{v_r}{s} = \frac{r_r}{s}\,i_r + j\,\omega_e\,\lambda_r$$

$$for\; cage,\;\; 0 \;= \frac{r_r}{s}\,i_r + j\,\omega_e\,\lambda_r$$

7.2.17.2 For rotor reference frame

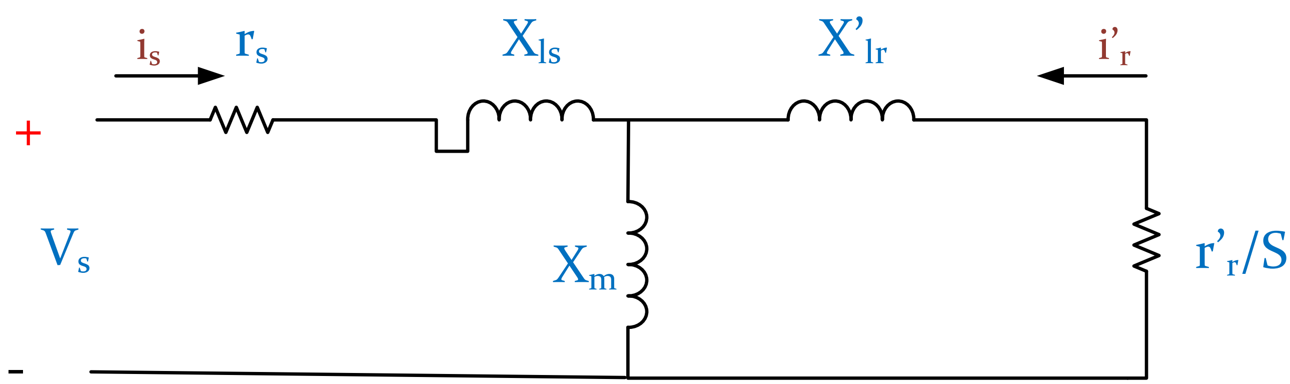
$$v_s = r_s i_s + j\,\omega_e\,\lambda_s$$
$$v_r = r_r i_r + j\,S\,\omega_e \lambda_r$$

For steady state

$$v_s = r_s i_s + j\,\omega_e\,\lambda_s$$

$$\frac{v_r}{s} = \frac{r_r}{s}\,i_r + j\,\omega_e\,\lambda_r$$

NOTE : we got the same equations no matter what reference we are at



Steady State equivalent circuit

8 DYNAMIC MODELLING OF SYNCHRONOUS MACHINE

Salient	Cylindrical
$Poles \uparrow \ -N_s \downarrow$ slow (hydraulic – wind)	$Poles \downarrow \ -N_s \uparrow$ Turbo (Gas – Steam)

Salient is the general case, then we get cylindrical

8.1.1 Assumptions In Modelling

- Ignore saturation
- Ignore slot effect
- Consider emf is a sin wave
- We must take eddy current effect as there's synchr.

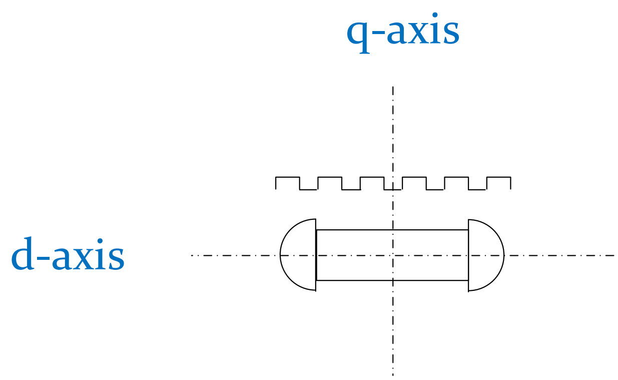
8.1.2 Construction of the machine

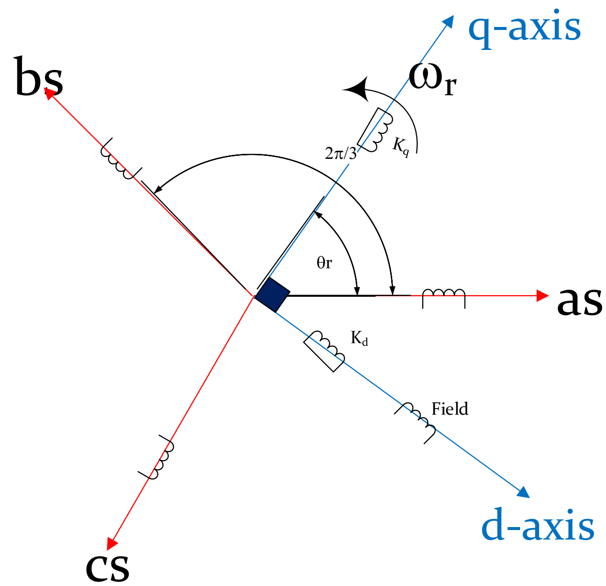
1- 3ϕ stator winding (Armature)

2- Rotor winding (Field)

Rotor have more 2 winding

- Compensating (Damper) Winding (more)
- Eddy Current Effect (less)


q-axis
d-axis


q-axis
bs
ωr
2π/3
Kq
θr
as
Kd
Field
d-axis
cs

9 DYNAMIC MODELLING OF SYNCHRONOUS MACHINE IN NATURAL MACHINE AXES

Windings : as , bs , cs, f, kd, kq

9.1 Voltage Equations

9.1.1 Stator

$$v_s = r_s i_s + p\,\lambda_s$$

$$\begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix} = \begin{bmatrix} r_s & 0 & 0 \\ 0 & r_S & 0 \\ 0 & 0 & r_s \end{bmatrix} \begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + p \begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix}$$

9.1.2 Rotor

$$v_r = r_r i_r + p\,\lambda_r$$

$$\begin{bmatrix} v_f \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_r & 0 & 0 \\ 0 & r_r & 0 \\ 0 & 0 & r_r \end{bmatrix} \begin{bmatrix} i_f \\ i_{kd} \\ i_{kq} \end{bmatrix} + p \begin{bmatrix} \lambda_f \\ \lambda_{kd} \\ \lambda_{kq} \end{bmatrix}$$

9.2 Flux Linkage Relations

$$\lambda_s = L_{ss} i_s + L_{sr} i_r$$

$$\lambda_r = L_{rs} i_s + L_{rr} i_r$$

$$L_{ss} = \begin{bmatrix} L_{as-as} & L_{as-bs} & L_{as-cs} \\ L_{bs-as} & L_{bs-bs} & L_{bs-cs} \\ L_{cs-as} & L_{cs-bs} & L_{cs-cs} \end{bmatrix}$$

Due to saliency self-inductance is time variant

$$L_{rr} = \begin{bmatrix} L_{ff} & L_{f-kd} & 0 \\ L_{kd-f} & L_{kd-kd} & 0 \\ 0 & 0 & L_{kq-kq} \end{bmatrix}$$

NOTE :

L_r **is constant because there is no mutual between Lf,kd and the kq as it's perpendicular**

$$L_{sr} = \begin{bmatrix} L_{as-f} \sin\theta_r & L_{as-kd} \sin\theta_r & L_{as-kq} \cos\theta_r \\ L_{bs-f} \sin\left(\theta_r - \frac{2\pi}{3}\right) & L_{bs-kd} \sin\left(\theta_r - \frac{2\pi}{3}\right) & L_{bs-kq} \cos\left(\theta_r - \frac{2\pi}{3}\right) \\ L_{cs-f} \sin\left(\theta_r + \frac{2\pi}{3}\right) & L_{cs-kd} \sin\left(\theta_r + \frac{2\pi}{3}\right) & L_{cs-kq} \cos\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix}$$

Time variant mutual inductance between stator and rotor

9.2.1 Stator self-inductance

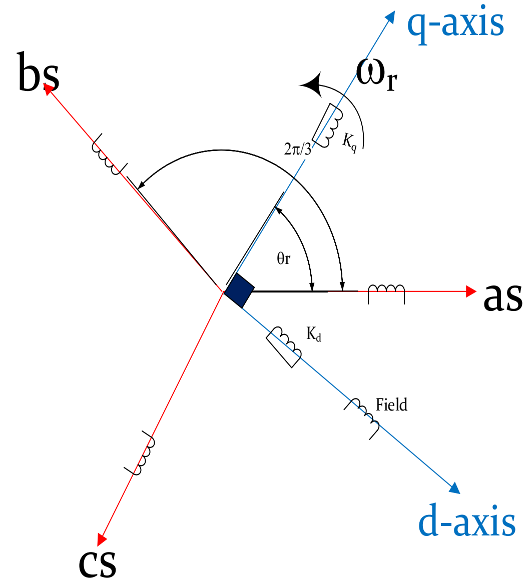
-In this section we study magnetizing component only , Leakage component is to be added afterwards

-Two types ((1) self winding and itself a-a & (2) self winding and another phase a-b)

9.2.1.1 self winding and itself a-a

for phase (a)

it's mmf we will divide into 2 directions



$$F_a \rightarrow AT \rightarrow NI$$
$$P \rightarrow Permenance$$

$$\phi_{da} = F_a \sin\theta_r \; P_d$$

$$\phi_{qa} = F_a \cos\theta_r \; P_q$$

As

$$\phi = \frac{NI}{R} = NIP$$

$$\lambda = N\phi$$

Flux linkage of phase (a) due to current I_a

$$\lambda_{aa} = N_{ph,s}\left[\,\phi_{da}\sin\theta_r + \phi_{qa}\cos\theta_r\,\right]$$

$$\lambda_{aa} = N_{phs}F_a\left[P_d\sin^2\,\theta_r + P_q\cos^2\theta_r\,\right]$$

$$\text{As } F_a = N_{phs}\, i_a$$

$$but\ L = \frac{\lambda}{i}$$

$$\therefore L_{aa} = N_{phs}^2\left[P_d\sin^2\,\theta_r + P_q\cos^2\theta_r\,\right]$$

$$\sin^2\theta_r = \frac{1}{2}(1-\cos 2\theta_r)$$

$$\cos^2\theta_r = \frac{1}{2}(1+\cos 2\theta_r)$$

After mathematical things

$$L_{aa} = N_{phs}^2 \left[\frac{P_d + P_q}{2} - \frac{P_d - P_q}{2} \cos 2\theta_r\right]$$

$$L_{aa} = L_o - L_{ms} \cos 2\theta_r$$

As,

$$L_o = N_{phs}^2 \left(\frac{P_d + P_q}{2}\right)$$

$$L_{ms} = N_{phs}^2 \left(\frac{P_d - P_q}{2}\right)$$

Same,

$$L_{aa} = L_o - L_{ms} \cos 2\theta_r$$

$$L_{bb} = L_o - L_{ms} \cos 2\left(\theta_r - \frac{2\pi}{3}\right)$$

$$L_{cc} = L_o - L_{ms} \cos 2\left(\theta_r + \frac{2\pi}{3}\right)$$

REMEMBER : L_{ls} is to be added to all of above after wards

9.2.1.2 Self-winding and another phase a-b , c-a

9.2.1.2.1 Effect of a on b

$$\lambda_{ba} = N_{phs}\left[\, \phi_{da} \sin\left(\theta_r - \frac{2\pi}{3}\right) + \phi_{qa} \cos\left(\theta_r - \frac{2\pi}{3}\right)\right]$$

As,

$$\phi_{da} = F_a \sin\theta_r \;\; P_d$$

$$\phi_{qa} = F_a \cos\theta_r \;\; P_q$$

$$\boxed{\lambda_{ca} = N_{phs}\left[\, \phi_{da} \sin\left(\theta_r + \frac{2\pi}{3}\right) + \phi_{qa} \cos\left(\theta_r + \frac{2\pi}{3}\right)\right]}$$

9.2.1.2.2 Effect of b on a

$$\lambda_{ab} = N_{phs}[\, \phi_{db} \sin\theta_r + \phi_{qb} \cos\theta_r]$$

as,

$$\phi_{db} = F_b \sin\left(\theta_r - \frac{2\pi}{3}\right) \; P_d$$

$$\phi_{qa} = F_b \cos\left(\theta_r - \frac{2\pi}{3}\right) \; P_q$$

NOTE BIENNE

$$if \; F_a = F_b \;\; \therefore \;\; \lambda_{ab} = \lambda_{ba}$$

$$\lambda_{ba} = N_{phs}\, F_a \left[\, P_d \sin\theta_r \sin\left(\theta_r - \frac{2\pi}{3}\right) + P_q \cos\theta_r \cos\left(\theta_r - \frac{2\pi}{3}\right)\right]$$

$$\sin A \sin B \; = \frac{1}{2}\,(\cos(A - B) - \cos(A + B))$$

$$\cos A \cos B \; = \frac{1}{2}\cos(A - B) + \frac{1}{2}\cos(A + B)$$

After mathematical things

$$\lambda_{ba} = N_{phs}F_a\left[-\frac{P_d+P_q}{4}-\frac{P_d-P_q}{2}\cos\left(2\theta_r-\frac{2\pi}{3}\right)\right]$$

$$F_a = N_{phs}\, i_a\,, L = \frac{\lambda}{i}$$

$$L_{ba} = N_{phs}^2\left[-\frac{P_d+P_q}{4}-\frac{P_d-P_q}{2}\cos\left(2\theta_r-\frac{2\pi}{3}\right)\right]$$

$$L_{ba} = -\frac{L_o}{2}-L_m\cos\left(2\theta_r-\frac{2\pi}{3}\right)$$

$$L_{ab} = L_{ba}$$

9.2.1.3 So to sum up

$$\boldsymbol{L_{ab} = L_{ba} = -\frac{L_o}{2}-L_{ms}\cos\left(2\theta_r-\frac{2\pi}{3}\right)}$$

$$\boldsymbol{L_{bc} = L_{cb} = -\frac{L_o}{2}-L_{ms}\cos(2\theta_r)} \qquad \boldsymbol{(\,by\ putting\ \theta_r = (\theta_r-\frac{2\pi}{3})}$$

$$\boldsymbol{L_{ac} = L_{ca} = -\frac{L_o}{2}-L_{ms}\cos\left(2\theta_r+\frac{2\pi}{3}\right)} \qquad \boldsymbol{(\,by\ putting\ \theta_r = (\theta_r+\frac{2\pi}{3})}$$

$$\boldsymbol{L_{ss} = \begin{bmatrix} L_o - L_{ms}\cos 2\theta_r + L_{ls} & -\frac{L_o}{2}-L_{ms}\cos\left(2\theta_r-\frac{2\pi}{3}\right) & -\frac{L_o}{2}-L_{ms}\cos\left(2\theta_r+\frac{2\pi}{3}\right) \\ -\frac{L_o}{2}-L_{ms}\cos\left(2\theta_r-\frac{2\pi}{3}\right) & L_o - L_{ms}\cos 2\left(\theta_r-\frac{2\pi}{3}\right)+L_{ls} & -\frac{L_o}{2}-L_{ms}\cos(2\theta_r) \\ -\frac{L_o}{2}-L_{ms}\cos\left(2\theta_r+\frac{2\pi}{3}\right) & -\frac{L_o}{2}-L_{ms}\cos(2\theta_r) & L_o - L_{ms}\cos 2\left(\theta_r+\frac{2\pi}{3}\right)+L_{ls} \end{bmatrix}}$$

9.2.2 Choosing the frame

1- To get rid off Time Variant **Mutually Coupled**

- We can refer to **any frame** and we will get rid off it

2- To get rid off Time Variant **Self-Inductance** of salient Pole

- Only **Rotor frame** will get rid off it

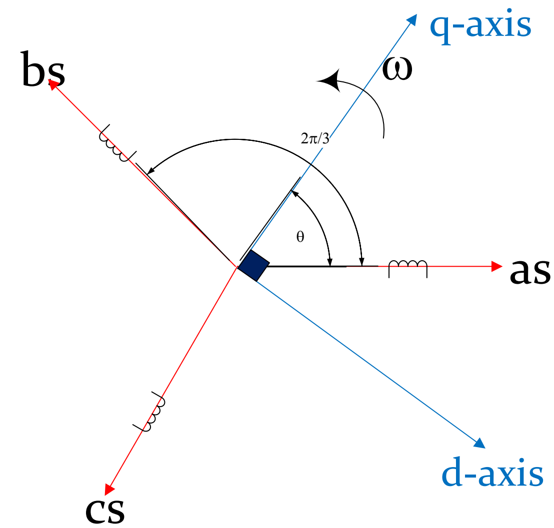
3- For a cylindrical rotor synchronous

- We are free to choose any frame as simple as Induction Machine

4- Any Problem with machine (saliency – unbalanced source)

- We put the frame on the **CAUSE** of the problem (salient rotor -> rotor) (unbalance -> supply)

10 SALIENT-POLE SYNCHRONOUS MACHINE IN QDO- AXIS ROTOR REFERENCE FRAME



$$K_s * V_{\underset{s}{abc}} = V_{\underset{s}{qdo}}$$

$$K_s = \frac{2}{3}\begin{bmatrix} \cos\boldsymbol{\theta_r} & \cos\left(\boldsymbol{\theta_r} - \frac{\mathbf{2\pi}}{\mathbf{3}}\right) & \cos\left(\boldsymbol{\theta_r} + \frac{\mathbf{2\pi}}{\mathbf{3}}\right) \\ \sin\boldsymbol{\theta_r} & \sin\left(\boldsymbol{\theta_r} - \frac{\mathbf{2\pi}}{\mathbf{3}}\right) & \sin\left(\boldsymbol{\theta_r} + \frac{\mathbf{2\pi}}{\mathbf{3}}\right) \\ \frac{\mathbf{1}}{\mathbf{2}} & \frac{\mathbf{1}}{\mathbf{2}} & \frac{\mathbf{1}}{\mathbf{2}} \end{bmatrix}$$

$$K_r = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

NOTE

-K_d, K_q (compensating & damper windings only have effect in starting and disturbance) in these situations machine acts in <u>induction mode</u>

-In <u>synchronous mode</u> we don't use these equations

10.1.1 FIRST : Remember ABC model

10.1.1.1 STATOR abc

$$v_s = r_s i_s + p\,\lambda_s$$

$$\begin{bmatrix} v_{as} \\ v_{bs} \\ v_{cs} \end{bmatrix} = \begin{bmatrix} r_s & 0 & 0 \\ 0 & r_s & 0 \\ 0 & 0 & r_s \end{bmatrix}\begin{bmatrix} i_{as} \\ i_{bs} \\ i_{cs} \end{bmatrix} + p\begin{bmatrix} \lambda_{as} \\ \lambda_{bs} \\ \lambda_{cs} \end{bmatrix}$$

10.1.1.2 ROTOR abc

$$v_f = r_r i_f + p\,\lambda_f$$

$$\begin{bmatrix} v_f \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_r & 0 & 0 \\ 0 & r_r & 0 \\ 0 & 0 & r_r \end{bmatrix}\begin{bmatrix} i_f \\ i_{kd} \\ i_{kq} \end{bmatrix} + p\begin{bmatrix} \lambda_f \\ \lambda_{kd} \\ \lambda_{kq} \end{bmatrix}$$

10.1.1.3 Flux Linkage

10.1.1.3.1 Stator Self Inductance

$$\lambda_s = L_{ss} i_s + L_{sr} i_r$$
$$\lambda_r = L_{rs} i_s + L_{rr} i_r$$
$$L_{ss} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix}$$

$$L_{aa} = L_{ls} + (L_o - L_{ms} \cos 2\theta_r)$$
$$L_{bb} = L_{ls} + \left(L_o - L_{ms} \cos 2\left(\theta_r - \frac{2\pi}{3}\right) \right)$$
$$L_{cc} = L_{ls} + \left(L_o - L_{ms} \cos 2\left(\theta_r + \frac{2\pi}{3}\right) \right)$$

$$L_{ab} = L_{ba} = -\frac{L_o}{2} - L_{ms} \cos\left(2\theta_r - \frac{2\pi}{3}\right)$$
$$L_{bc} = L_{cb} = -\frac{L_o}{2} - L_{ms} \cos(2\theta_r)$$
$$L_{ac} = L_{ca} = -\frac{L_o}{2} - L_{ms} \cos\left(2\theta_r + \frac{2\pi}{3}\right)$$

As, $L_o = N_{ph}^2 \frac{P_d + P_q}{2}$, $L_{ms} = N_{ph}^2 \frac{P_d - P_q}{2}$

Stator Self Inductance Lss has 2 Problems 1- Coupled 2- Time Variant

10.1.1.3.2 Rotor Self Inductance

$$L_{rr} = \begin{bmatrix} L_{ff} & L_{f-kd} & 0 \\ L_{kd-f} & L_{kd-kd} & 0 \\ 0 & 0 & L_{kq-kq} \end{bmatrix}$$

$$L_{ff} = L_{lf} + L_{mf}$$
$$L_{kd-kd} = L_{lkd} + L_{m_{kd}}$$
$$L_{kq-kq} = L_{l_{kq}} + L_{m_{kq}}$$

10.1.1.3.3 Stator Rotor Mutual Inductance

$$L_{sr} = \begin{bmatrix} L_{as-f} \sin\theta_r & L_{as-kd} \sin\theta_r & L_{as-kq} \cos\theta_r \\ L_{bs-f} \sin\left(\theta_r - \frac{2\pi}{3}\right) & L_{bs-kd} \sin\left(\theta_r - \frac{2\pi}{3}\right) & L_{bs-kq} \cos\left(\theta_r - \frac{2\pi}{3}\right) \\ L_{cs-f} \sin\left(\theta_r + \frac{2\pi}{3}\right) & L_{cs-kd} \sin\left(\theta_r + \frac{2\pi}{3}\right) & L_{cs-kq} \cos\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix}$$

Time variant mutual inductance between stator and rotor

10.1.2 TRANSFORMATION (To rotor reference frame)[PARK's EQUATIONS]

10.1.3 Stator QDOs

$$V_{\underset{s}{abs}} = r_s\, i_{\underset{s}{abc}} + p\, \lambda_{\underset{s}{abc}} \;\; (*\, k_s)$$

$$V_{\underset{s}{qdo}} = r_s i_{\underset{s}{qdo}} + p\, \lambda_{\underset{s}{qdo}} + \omega_r \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \lambda_{\underset{s}{qdo}}$$

$$\begin{bmatrix} V_{qs} \\ V_{ds} \\ V_{os} \end{bmatrix} = \begin{bmatrix} r_s & 0 & 0 \\ 0 & r_s & 0 \\ 0 & 0 & r_s \end{bmatrix} \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{os} \end{bmatrix} + \; p \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{os} \end{bmatrix} + \omega \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{os} \end{bmatrix}$$

$$V_{qs} = r_s i_{qs} + p\, \lambda_{qs} + \omega_r\, \lambda_{ds}$$
$$V_{ds} = r_s i_{ds} + p\, \lambda_{ds} - \omega_r \lambda_{qs}$$
$$V_{os} = r_s i_{os} + p\, \lambda_{os}$$

10.1.4 Rotor QDOs (same)

$$V_f = r_r i_f + p\, \lambda_f$$

$$V_{kd} = 0 = r_r i_{kd} + p\, \lambda_{kd}$$

$$V_{kq} = 0 = r_r i_{kq} + p\, \lambda_{kq}$$

10.1.5 Flux Linkage QDOs

$$\lambda_{abc,s} = L_{abc,s-abc,s}\, i_{abc,s} + L_{abc,s-fdq,r}\, i_{fdq,r} \qquad \times k_s$$

$$\lambda_{qdo,s} = K_s\, L_{ss}\, K_s^{-1}\, i_{\underset{s}{qdo}} + K_s\, L_{sr}\, K_r^{-1}\, i_f$$

simplicty please

$$L_{abc,s-abc,s} \equiv L_{ss}$$

10.1.5.1 Stator self

$$L_{aa} = L_{ls} + (\, L_o - L_{ms} \cos 2\theta_r \,)$$
$$L_{bb} = L_{ls} + \left(L_o - L_{ms} \cos 2 \left(\theta_r - \frac{2\pi}{3} \right) \right)$$
$$L_{cc} = L_{ls} + \left(L_o - L_{ms} \cos 2 \left(\theta_r + \frac{2\pi}{3} \right) \right)$$

$$L_{ss} = \begin{bmatrix} L_{aa} & L_{ab} & L_{ac} \\ L_{ba} & L_{bb} & L_{bc} \\ L_{ca} & L_{cb} & L_{cc} \end{bmatrix}$$

$$k_s\, L_{ss}\, k_s^{-1} = \begin{bmatrix} L_{ls} + \frac{3}{2}(L_o - L_{ms}) & 0 & 0 \\ 0 & L_{ls} + \frac{3}{2}(L_o + L_{ms}) & 0 \\ 0 & 0 & L_{ls} \end{bmatrix}$$

Therefore

$$\lambda_{qs} = \left[L_{ls} + \frac{3}{2}(L_o - L_{ms}) \right] i_{qs} + L_{s-k_q} i_{kq}$$

$$\lambda_{ds} = \left[L_{ls} + \frac{3}{2}(L_o + L_{ms}) \right] i_{ds} + L_{s-f} i_f + L_{s-kd}\, i_{kd}$$

$$\lambda_{os} = L_{ls} i_{os} + 0$$

Problem of time variant self-inductance solved

10.1.5.2 Rotor Self

$$L_{rr} = k_r\, L_{rr}\, k_r^{-1} = I\, L_{rr}\, I = L_{rr}$$

$$L_{rr} = \begin{bmatrix} L_{ff} & L_{f-kd} & 0 \\ L_{kd-f} & L_{kd-kd} & 0 \\ 0 & 0 & L_{kq-kq} \end{bmatrix}$$

$$L_{ff} = L_{lf} + L_{mf}$$
$$L_{kd-kd} = L_{lkd} + L_{m_{kd}}$$
$$L_{kq-kq} - L_{l_{kq}} + L_{m_{kq}}$$

10.1.5.3 Stator-Rotor

$$L_{sr} = \begin{bmatrix} L_{as-f} \sin\theta_r & L_{as-kd} \sin\theta_r & L_{as-kq} \cos\theta_r \\ L_{bs-f} \sin\left(\theta_r - \frac{2\pi}{3}\right) & L_{bs-kd} \sin\left(\theta_r - \frac{2\pi}{3}\right) & L_{bs-kq} \cos\left(\theta_r - \frac{2\pi}{3}\right) \\ L_{cs-f} \sin\left(\theta_r + \frac{2\pi}{3}\right) & L_{cs-kd} \sin\left(\theta_r + \frac{2\pi}{3}\right) & L_{cs-kq} \cos\left(\theta_r + \frac{2\pi}{3}\right) \end{bmatrix}$$

$$K_s\, L_{sr} K_r^{-1} = \begin{bmatrix} 0 & 0 & L_{s-kq} \\ L_{s-f} & L_{s-kd} & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$q \text{ axis is } \perp F\,, K_d \quad \rightarrow 0$$
$$d \text{ axis is } \perp \ K_q \quad \rightarrow 0$$
$$q \text{ and } d \text{ are } 90° \quad \rightarrow \text{all } 0 \text{ are } 0$$

Rotor-Stator

$$K_r L_{rs} K_s^{-1} = \begin{bmatrix} 0 & \frac{3}{2} L_{s-f} & 0 \\ 0 & \frac{3}{2}\, L_{s-kd} & 0 \\ \frac{3}{2} L_{s-kq} & 0 & 0 \end{bmatrix}$$

NOTE :

it's $L_{rs} = \frac{3}{2} * (L_{sr})^T$

$\frac{3}{2}$ *because* K_s *has* $\frac{2}{3}$ *but* K_r *is unity* (I)

NOW IT'S MUTUALLY TIME INVARIANT

10.1.5.4 Final Flux Linkage Relations

$$\lambda_{qs} = \left(L_{ls} + \frac{3}{2}(L_o - L_{ms})\right) i_{qs} + L_{s-kq}\, i_{kq}$$

$$\lambda_{ds} = \left(L_{ls} + \frac{3}{2}(L_o + L_{ms})\right) i_{ds} + L_{s-f} i_f + L_{s-kd} i_{kd}$$

$$\lambda_{os} = L_{ls} i_{os} + 0$$

$$\lambda_f = L_{ff} i_f + L_{f-kd} i_{kd} + \frac{3}{2} L_{s-f} i_{ds}$$

$$\lambda_{kd} = L_{kd-f} i_f + L_{kd-kd} i_{kd} + \frac{3}{2}\, L_{s-kd} i_{ds}$$

$$\lambda_{kq} = L_{kq-kq} i_{kq} + \frac{3}{2}\, L_{s-kq} i_{qs}$$

NOTE :

$$\lambda_{qs} \rightarrow has\ only\ what's\ on\ q\ axis\ (kq)$$
$$\lambda_{ds} \rightarrow has\ only\ what's\ on\ d\ axis\ (f, kd)$$

10.1.6 Referring to get to Equivalent Circuit

10.1.6.1 The term $\frac{3}{2}(L_o \pm L_{ms})$

FIRST

$$\boldsymbol{\frac{3}{2}(L_o - L_{ms})} = \frac{3}{2}\ N_{phs}^2\left(\frac{P_d + P_q}{2} - \frac{(P_d - P_q)}{2}\right)$$

$$say\ L_{mq} = \frac{3}{2}\ N_{phs}^2 P_q$$

$$\boldsymbol{\therefore L_{ls} + \frac{3}{2}\ (L_o - L_{ms}) = L_{qs} = L_{ls} + L_{mq}}$$

SECOND

$$\boldsymbol{\frac{3}{2}(L_o + L_{ms})} = \frac{3}{2}\ N_{phs}^2 P_d = L_{md}$$

$$\boldsymbol{\therefore\ L_{ls} + \frac{3}{2}(L_o + L_{ms}) = L_{ls} + L_{md} = L_{ds}}$$

10.1.6.2 Flux linkage equations into Lmd and Lmq

$$\lambda_{qs} = \left(L_{ls} + \frac{3}{2}(L_o - L_{ms})\right) i_{qs} + L_{s-kq}\ i_{kq}$$

$$\lambda_{qs} = (L_{ls} + L_{mq}) i_{qs} + N_{phs} N_{ph_{kq}} P_q\ i_{kq}$$

$$\lambda_{qs} = (L_{ls} + L_{mq}) i_{qs} + \frac{3}{2}\frac{2}{3}\ N_{phs}\ (\frac{N_{phs}}{N_{phs}}) N_{ph_{kq}} P_q\ i_{kq}$$

$$\lambda_{qs} = (L_{ls} + L_{mq}) i_{qs} + L_{mq}\ \ \frac{2}{3}\ (\frac{N_{ph_{kq}}}{N_{phs}})\ i_{kq}$$

$$\lambda_{qs} = (L_{ls} + L_{mq}) i_{qs} + L_{mq}\ i'_{kq}$$

$$\boldsymbol{\lambda_{qs} = L_{ls} i_{qs} + L_{mq}(i_{qs} + i'_{kq})}$$

$$\lambda_{ds} = (L_{ls} + L_{md})i_{ds} + L_{s-f}i_f + L_{s-kd}i_{kd}$$

by same ways

$$L_{sf} = \frac{\mathbf{3}}{\mathbf{2}}\frac{2}{3}\ \boldsymbol{N_{phs}}\, N_{phf}\, \boldsymbol{P_d}\, I_f \frac{\boldsymbol{N_{phs}}}{N_{phs}}$$

$$L_{s-kd} = \frac{\mathbf{3}}{\mathbf{2}}\frac{2}{3}\ \boldsymbol{N_{phs}}\, N_{phkd}\, \boldsymbol{P_d}\, I_{kd} \frac{\boldsymbol{N_{phs}}}{N_{phs}}$$

$$\lambda_{ds} = (L_{ls} + L_{md})i_{ds} + L_{md}i_f \frac{2}{3}\frac{N_{phf}}{N_{phs}} + L_{md}i_{kd}\frac{2}{3}\frac{N_{ph_{kd}}}{N_{ph_s}}$$

$$\boldsymbol{\lambda_{ds} = L_{ls}i_{ds} + L_{md}\left(i_{ds} + i_f' + i_{kd}'\right)}$$

$$\lambda_f = \left(\, L_{lf} + L_{mf}\right)i_f + L_{f-kd}\ i_{kd} + \frac{3}{2}L_{sf}i_{ds}$$

$$but\ \frac{\mathbf{3}}{\mathbf{2}}\boldsymbol{L_{sf}} = \frac{3}{2}N_{phs}N_{phf}\ P_d i_{ds}\frac{N_{phs}}{N_{phs}}$$

$$\frac{3}{2}N_{phs}^2\ P_d i_{ds}\left(\frac{N_{phf}}{N_{phs}}\right) = L_{md}i_{ds}\left(\frac{N_{phf}}{N_{phs}}\right)$$

$$\lambda_f \frac{N_{phs}}{N_{phf}} = \left(L_{lf} + L_{mf}\right)\frac{N_{phs}}{N_{phf}}i_f + L_{f-kd}\frac{N_{phs}}{N_{phf}}\ i_{kd} + L_{md}i_{ds}$$

By subs as above

$$\lambda_f' = L_{lf}'\ i_f' + L_{md}\left(\, i_f' + i_{kd}'\ + i_{ds}\right)$$

10.1.6.3 Flux Linkage Equations

$$\boldsymbol{\lambda_{qs} = L_{ls} i_{qs} + L_{mq}\left(i_{qs} + i'_{kq}\right)}$$

$$\boldsymbol{\lambda_{ds} = L_{ls} i_{ds} + L_{md}\left(\, i'_{f} + i'_{kd} \; + i_{ds}\right)}$$

$$\boldsymbol{\lambda'_{f} = L'_{lf}\, i'_{f} + L_{md}\,\left(\, i'_{f} + i'_{kd} \; + i_{ds}\right)}$$

$$\boldsymbol{\lambda'_{kd} = L'_{lkd}\, i'_{f} + L_{md}\,\left(\, i'_{f} + i'_{kd} \; + i_{ds}\right)}$$

$$\boldsymbol{\lambda'_{kq} = L'_{lkq}\, i'_{kq} + L_{mq}\,\left(i_{qs} + i'_{kq}\right)}$$

10.1.6.4 Voltage Equation to get equivalent circuit

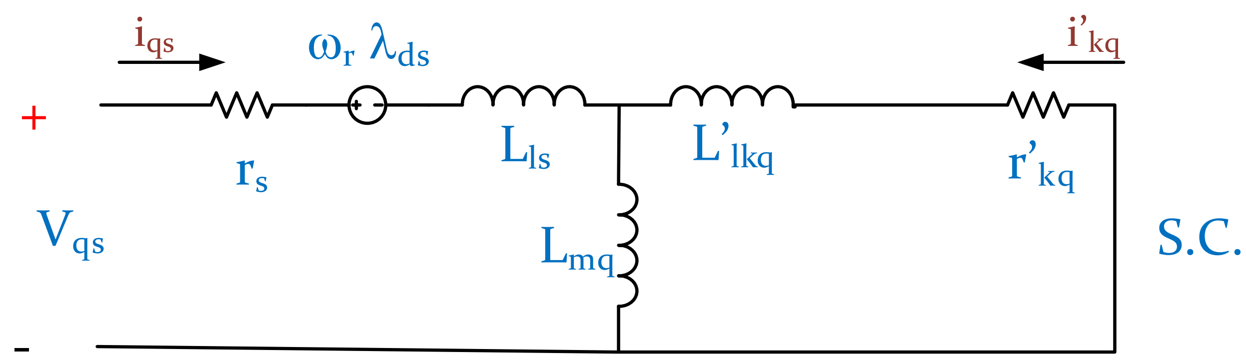
$$V_{qs} = r_s i_{qs} + p\,\lambda_{qs} + \omega_r\,\lambda_{ds}$$
$$V_{ds} = r_s i_{ds} + p\,\lambda_{ds} - \omega_r \lambda_{qs}$$

$$V'_f = r'_r i'_f + p\,\lambda'_f$$

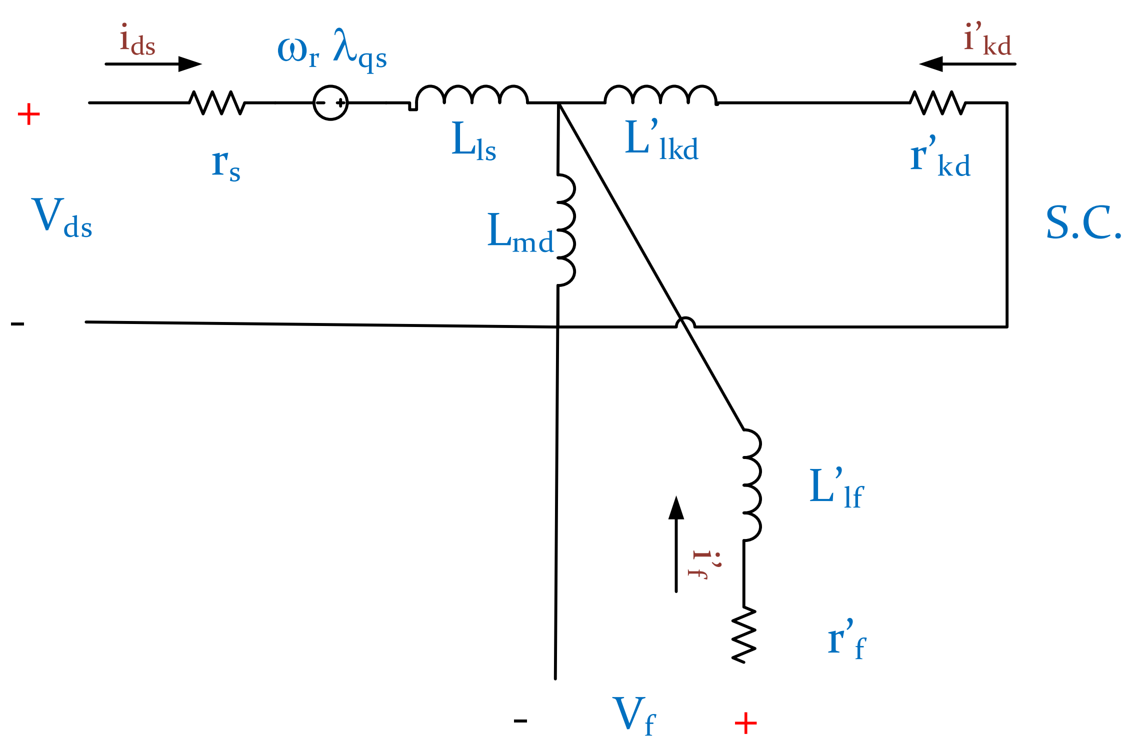
$$V'_{kd} = 0 = r'_{kd} i'_{kd} + p\,\lambda'_{kd}$$

$$V'_{kq} = 0 = r'_{kq} i'_{kq} + p\,\lambda'_{kq}$$

10.1.7 Equivalent Circuit in qdo axis



Q- axis equivalent circuit



D- axis equivalent circuit

10.2 FINAL MODEL

10.2.1 Voltage Equations

$$V_{qs} = r_s i_{qs} + p\,\lambda_{qs} + \omega_r\,\lambda_{ds}$$
$$V_{ds} = r_s i_{ds} + p\,\lambda_{ds} - \omega_r \lambda_{qs}$$
$$V_{os} = r_s i_{os} + p\,\lambda_{os}$$

$$V_f = r_r i_f + p\,\lambda_f$$

$$V_{kd} = 0 = r_r i_{kd} + p\,\lambda_{kd}$$

$$V_{kq} = 0 = r_r i_{kq} + p\,\lambda_{kq}$$

10.2.2 Flux Linkage

$$\boldsymbol{\lambda_{qs} = L_{ls} i_{qs} + L_{mq}(i_{qs} + i'_{kq})}$$

$$\boldsymbol{\lambda_{ds} = L_{ls} i_{ds} + L_{md}(\, i'_f + i'_{kd}\; + i_{ds})}$$

$$\boldsymbol{\lambda'_f = L'_{lf}\, i'_f + L_{md}\,(\, i'_f + i'_{kd}\; + i_{ds})}$$

$$\boldsymbol{\lambda'_{kd} = L'_{lkd}\, i'_f + L_{md}\,(\, i'_f + i'_{kd}\; + i_{ds})}$$

$$\boldsymbol{\lambda'_{kq} = L'_{lkq}\, i'_{kq} + L_{mq}\,(i_{qs} + i'_{kq})}$$

$$\boldsymbol{\lambda_{os} = L_{ls} i_{os}}$$

10.2.3 Electromagnetic Developed Torque

$$P_{in} = v_{as} i_{as} + v_{bs} i_{bs} + v_{cs} i_{cs} + v_f i_f$$
$$= \frac{3}{2}\,(v_{qs} i_{qs} + v_{ds} i_{ds} + 2\, v_o i_o) + v_f i_f$$

at balanced condition $v_o i_o = 0$

$$P_{in} = \frac{3}{2}\left[\, r_s\,(i_{qs}^2 + i_{ds}^2) + i_{qs} p\,\lambda_{qs} + i_{ds} p\,\lambda_{ds} + \boldsymbol{\omega_r(\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds})}\right] + r_f i_f^2 + i_f\, p\,\lambda_f$$

$$\frac{3}{2}\, r_s\,(i_{qs}^2 + i_{ds}^2) + r_f i_f^2 \;\; \rightarrow P_{cu}$$
$$\frac{3}{2}(\, i_{qs} p\,\lambda_{qs} + i_{ds} p\,\lambda_{ds}) + \; i_f\, p\,\lambda_f \rightarrow \frac{d}{dt} W_{stored}$$
$$\frac{3}{2}\, \boldsymbol{\omega_r(\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds})} \rightarrow \;\; \boldsymbol{P_{dev} = P_{mech}}$$

So,

$$T_{em} = \frac{P_{mech}}{\omega_m} = \frac{3}{2}\frac{\boldsymbol{P}}{\boldsymbol{2}}(\boldsymbol{\lambda_{ds}i_{qs}} - \boldsymbol{\lambda_{qs}i_{ds}})$$

With saliency

$$\boldsymbol{T_{em} = \frac{3}{2}\frac{P}{2}\ (L_{ds} - L_{qs})i_{ds}i_{qs} + L_{md}I_fI_{qs} + L_{md}i_{kd}i_{qs} - L_{mq}i_{kq}i_{ds}]}$$

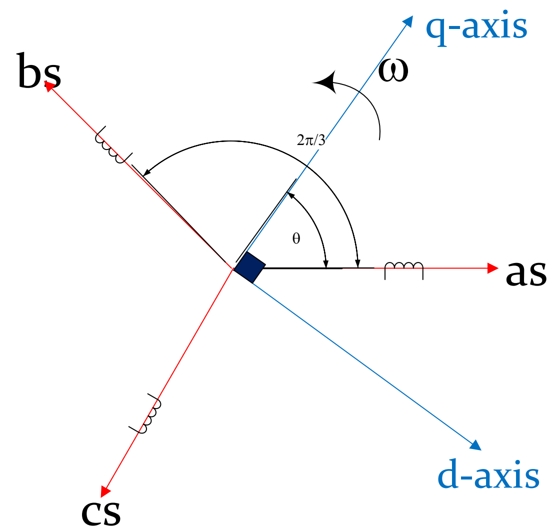
10.2.4 Mechanical Equation

$$T_{em} = T_L + B\omega_m + J\frac{d\omega}{dt}$$

10.2.5 Power Angle Equation

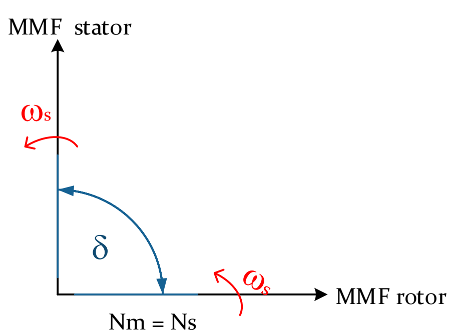
$$\delta : angle\ between\ poles\ of\ stator\ and\ rotor$$

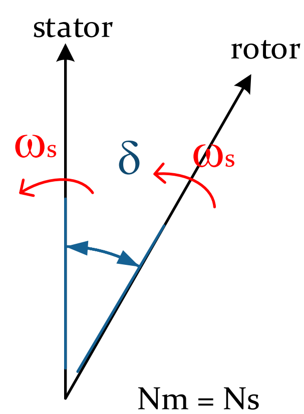
When we increase Load , N_m decreases instantaneously,

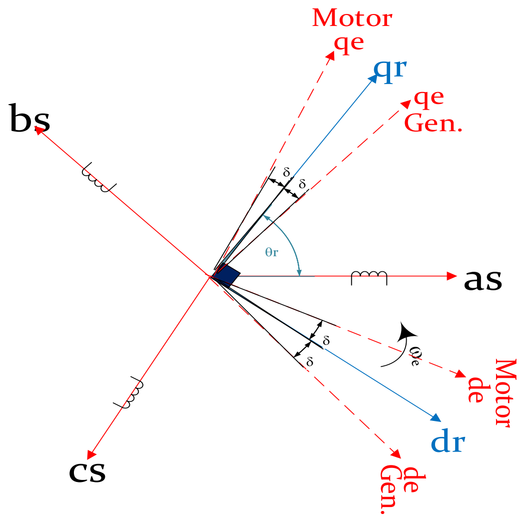


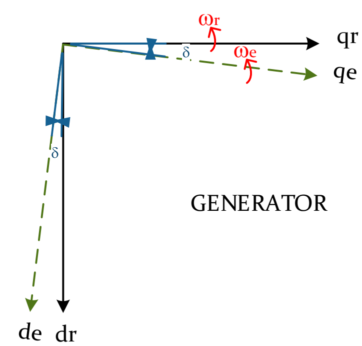
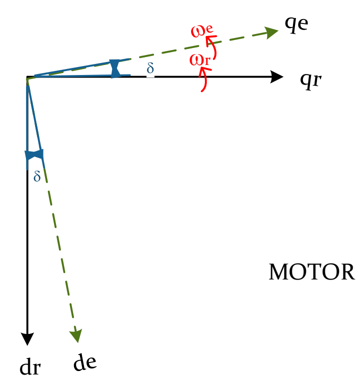
$$\boldsymbol{P^2\delta = p\omega = \frac{1}{J}[\,T_e - T_L - B\omega_m]}$$

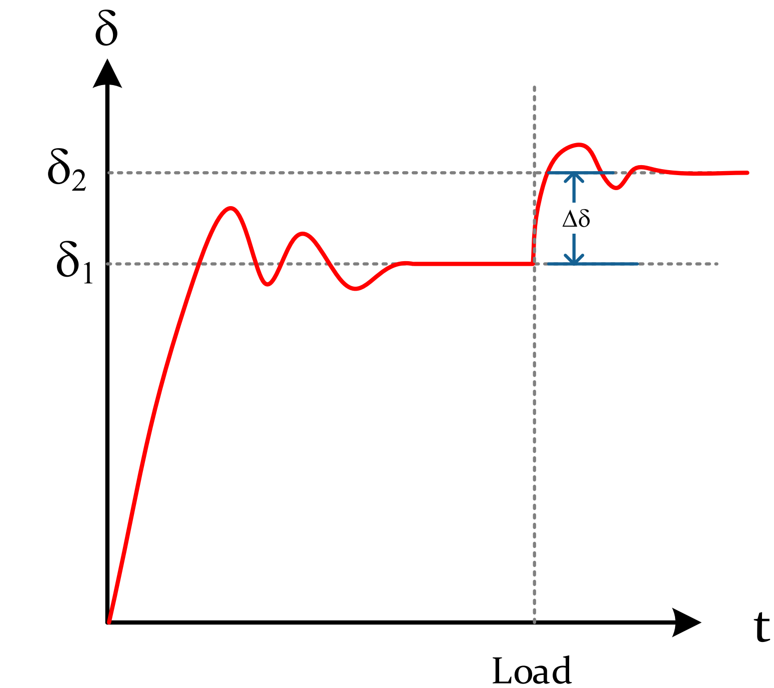
$$\boldsymbol{\delta = \left(\int (\omega_e - \omega)dt\ \right) + \theta_e(0) - \theta_r(0)}$$

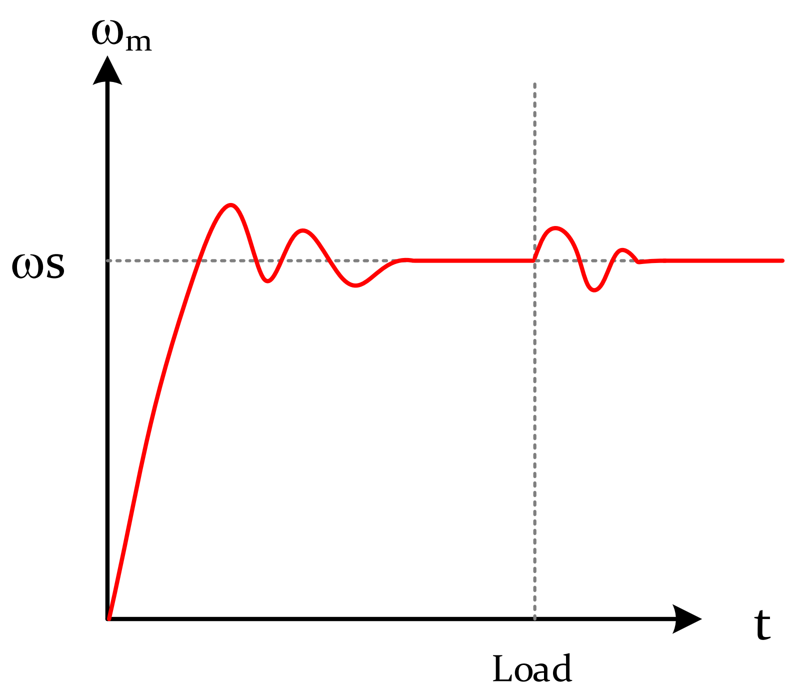






For Generator	$P\delta = \omega_r - \omega_e$	 GENERATOR **if grid connected it must has δ above qe so the power flow becomes from the generator to grid**
For motor	$P\delta = \omega_e - \omega_r$	 MOTOR


δ
δ2
δ1
Δδ
t
Load


ωm
ωs
t
Load

10.3 Steady-State Condition

$$P = J(\omega_e - \omega_r)$$

So, at steady state

$$\omega_r = \omega_e$$

$$P \equiv 0$$

10.3.1 For Synchronous mode operation

$$P \equiv 0$$

$$\omega_r = \omega_e$$

$$i_{kq}, i_{kd} \equiv 0$$

So,

$$V_{qs} = R_s I_{qs} + \omega_e \lambda_{ds}$$
$$V_{ds} = R_s I_{ds} - \omega_e \lambda_{qs}$$

$$\lambda_{qs} = L_{ls} i_{qs} + L_{mq} i_{qs}$$
$$\lambda_{ds} = L_{ls} i_{ds} + L_{md}\left(I_{ds} + i_f'\right)$$

But,

$$\lambda_{qs} = L_{sq} i_{qs}$$

$$\lambda_{ds} = L_{sd} i_{ds} + L_{md} I_f$$

Then,

$$V_{qs} = R_s I_{qs} + \omega_e\left(L_{sd} i_{ds} + L_{md} I_f\right)$$
$$V_{ds} = R_s I_{ds} - \omega_e\left(L_{qs} I_{qs}\right)$$

Let X= wL

$$V_{qs} = r_s i_{qs} + X_{ds} i_{ds} + X_m I_f$$
$$V_{ds} = r_s i_{ds} - X_{qs} i_{qs}$$

$$\boldsymbol{V_{qs} = E_f + r_s i_{qs} + X_{ds} i_{ds}}$$
$$\boldsymbol{V_{ds} = r_s i_{ds} - X_q\, i_{qs}}$$

$$\therefore T_{em_{ss}} = \frac{3}{2}\frac{P}{2}\frac{1}{\omega_e}\left[\left(X_{ds} - X_{qs}\right) I_{ds} I_{qs} + \overbrace{\mathrm{X_{md} I_f}}^{\mathrm{E\_f}}\, I_{qs}\right]$$

10.3.2 In space vector form

$$V_{qs} - jV_{ds} \rightarrow \overline{V_s}$$

$$V_{qs} = r_s i_{qs} + p\,\lambda_{qs} + \omega_r\,\lambda_{ds}$$
$$V_{ds} = r_s i_{ds} + p\,\lambda_{ds} - \omega_r \lambda_{qs}$$

$$V_{qs} - j\,V_{ds} = \ r_s i_{qs} + p\,\lambda_{qs} + \omega_r\,\lambda_{ds} - \mathrm{j}\left(r_s i_{ds} + p\,\lambda_{ds} - \omega_r \lambda_{qs}\right)$$

$$V_{qs} - j\,V_{ds} = \ r_s i_{qs} + p\,\lambda_{qs} + \omega_r\,\lambda_{ds} - \mathrm{jr_s i_{ds}} - \mathrm{jp\lambda_{ds}} + \mathrm{j}\ \omega_\mathrm{r}\lambda_\mathrm{qs}$$
$$\overline{V_s} = r_s(i_{qs} - j\,i_{ds}) + p(\lambda_{qs} - j\,\lambda_{ds}) \ + j\,\omega_r(\lambda_{qs} - j\,\lambda_{ds})$$

so,

$$\overline{V_s} = r_s \overline{I_s} + p\,\overline{\lambda_s} \ + j\,\omega_r \overline{\lambda_s}$$

For steady state $p\overline{\lambda_s} = 0$ & $\omega_r = \omega_e$

$$\overline{V_s} = r_s \overline{I_s} \ + j\,\omega_e \overline{\lambda_s}$$

$$\overline{V_s} = r_s \overline{I_s} \ + j\,\omega_e(\lambda_{qs} - j\,\lambda_{ds})$$

$$\overline{V_s} = r_s \overline{I_s} \ + j\,\omega_e\left(\,L_{qs} I_{qs} - j\left(\,L_{ds} I_{ds} + L_{md} I_f\right)\right)$$

<u>As for Cylindrical</u> $(L_{qs} = L_{ds} = L_s)$

$$\overline{V_s} = r_s \overline{I_s} \ + j\,\omega_e\left(\,L_s I_{qs} - j\left(\,L_s I_{ds} + L_{md} I_f\right)\right)$$
$$\overline{V_s} = r_s \overline{I_s} \ + j\,\omega_e\left(\,L_s I_{qs} - j\,L_s I_{ds} - j\,L_{md} I_f\right)$$
$$\overline{V_s} = r_s \overline{I_s} \ + j\,\omega_e\left(\,L_s(I_{qs} - j\,I_{ds}) - j\,L_{md} I_f\right)$$
$$\overline{V_s} = r_s \overline{I_s} + j\omega_e L_s \overline{I_s} \ + j\,\omega_e\left(-j\,L_{md} I_f\right)$$
$$\overline{V_s} = (r_s + j\omega_e L_s)\overline{I_s} \ + \overbrace{\omega_e L_{md} I_f}^{\mathrm{Ef}}$$

$$\overline{V_s} = (r_s + j\mathrm{X_s})\overline{I_s} \ + \mathrm{E_f}$$

<u>As for Salient Pole</u>

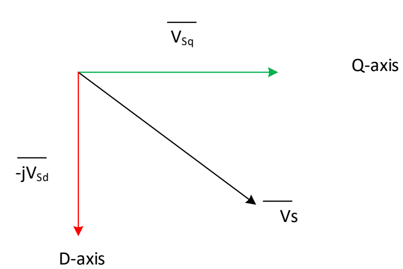
$$\overline{V_s} = r_s \overline{I_s} \ + j\,\omega_e\left(\,L_{qs} I_{qs} - j\left(\,L_{ds} I_{ds} + L_{md} I_f\right)\right)$$
$$\overline{V_s} = r_s \overline{I_s} \ + j\,\omega_e L_{qs} I_{qs} + \omega_e L_{ds} I_{ds} + \omega_e L_{md} I_f$$

We will add and subtract term $\omega_e L_{qs} I_{ds}$

$$\overline{V_s} = r_s \overline{I_s} \ + j\,\omega_e L_{qs}(I_{qs} - jI_{ds})$$
$$-\omega_e L_{qs} I_{ds}$$
$$+\omega_e L_{ds} I_{ds}$$
$$+\omega_e L_{md} I_f$$

So,

$$\begin{aligned} V_s = (r_s + j\omega_e L_{qs})\overline{I_s} \quad & \rightarrow as\ Cylind \\ +\omega_e L_{md} I_f \quad & \rightarrow as\ Cylind\ (E_f) \\ +\boldsymbol{\omega_e(L_{ds} - L_{qs})\overline{I_{ds}}} & \rightarrow saliency\ Source \end{aligned}$$



10.4 Motor to Generator

In question you might be asked either to determine the difference or rewrite the equations

<u>The difference (3 DIFFERENCES)</u>

$$(1) i_{qs} = -i_{qs}$$
$$i_{ds} = -i_{ds}$$
$$(2) T_{mech} = T_{em} + B\omega_m + J\frac{d\omega}{dt}$$
$$(3)\ \boldsymbol{P\delta = \omega_r - \omega_e}$$

<u>The Equations</u>

10.4.1 Voltage Equations

$$V_{qs} = -r_s i_{qs} + p\,\lambda_{qs} + \omega_r\,\lambda_{ds}$$
$$V_{ds} = -r_s i_{ds} + p\,\lambda_{ds} - \omega_r \lambda_{qs}$$

$$V_f = r_r i_f + p\,\lambda_f$$

$$V_{kd} = 0 = r_r i_{kd} + p\,\lambda_{kd}$$

$$V_{kq} = 0 = r_r i_{kq} + p\,\lambda_{kq}$$

10.4.2 Flux Linkage

$$\boldsymbol{\lambda_{qs} = -L_{ls} i_{qs} + L_{mq}\left(-i_{qs} + i'_{kq}\right)}$$

$$\boldsymbol{\lambda_{ds} = -L_{ls} i_{ds} + L_{md}\left(\, i'_f + i'_{kd} - i_{ds}\right)}$$

$$\boldsymbol{\lambda'_f = L'_{lf}\, i'_f + L_{md}\,\left(\, i'_f + i'_{kd} - i_{ds}\right)}$$

$$\boldsymbol{\lambda'_{kd} = L'_{lkd}\, i'_f + L_{md}\,\left(\, i'_f + i'_{kd} - i_{ds}\right)}$$

$$\boldsymbol{\lambda'_{kq} = L'_{lkq}\, i'_{kq} + L_{mq}\,\left(-i_{qs} + i'_{kq}\right)}$$

<u>For cylindrical</u>

$$\boldsymbol{\lambda_{qs} = -L_s i_{qs} + L_m i_{kq}}$$

$$\boldsymbol{\lambda_{ds} = -L_s i_{ds} + L_m (i_f + i_{kd})}$$

$$\boldsymbol{(leakage)\lambda_{qs} = -L_{ls} i_{qs} + L_{mq}\left(-i_{qs} + i'_{kq}\right)}$$

Or $\boldsymbol{(self)\lambda_{qs} = -L_{qs} i_{qs} + L_{mq} i_{kq}}$

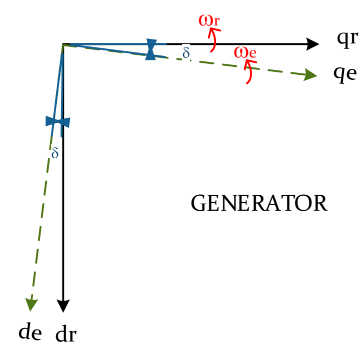
10.4.3 Electromagnetic Developed Torque

$$T_{em} = \frac{P_{mech}}{\omega_m} = \frac{3}{2}\frac{\boldsymbol{P}}{\boldsymbol{2}}\left(\boldsymbol{\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}}\right)$$

10.4.4 Mechanical Equation

$$T_{mech} = T_{em} + B\omega_m + J\frac{d\omega}{dt}$$

10.4.5 Power Angle Equation

For Generator	$\boldsymbol{P\delta = \omega_r - \omega_e}$	

10.5 Salient to Cylindrical

<u>Difference</u>

$$L_{qs} = L_{ds} = L_s = L_{ls} + L_m$$

$$L_{mq} = L_{md} = L_m$$

- Now It has same as induction machine ,
- Can be solved in any reference frame
- But has a time variant mutual inductance between stator and rotor

<u>So, edit</u>

$$L_{mq} = L_{md} = L_m$$

Salient motor	Cylindrical motor
$\boldsymbol{\lambda_{qs} = L_{ls}i_{qs} + L_{mq}(i_{qs} + i'_{kq})}$ $\boldsymbol{\lambda_{ds} = L_{ls}i_{ds} + L_{md}(i'_f + i'_{kd} + i_{ds})}$ $\boldsymbol{\lambda'_f = L'_{lf}\, i'_f + L_{md}\,(i'_f + i'_{kd} + i_{ds})}$ $\boldsymbol{\lambda'_{kd} = L'_{lkd}\, i'_f + L_{md}\,(i'_f + i'_{kd} + i_{ds})}$ $\boldsymbol{\lambda'_{kq} = L'_{lkq}\, i'_{kq} + L_{mq}\,(i_{qs} + i'_{kq})}$ $\boldsymbol{\lambda_{os} = L_{ls}i_{os}}$ $\boldsymbol{\lambda_{qs} = L_{s_q}i_{qs} + L_{mq}i_{kq}}$ $\boldsymbol{\lambda_{ds} = L_{s_d}i_{ds} + L_{md}(i_f + i_{kd})}$	$\boldsymbol{\lambda_{qs} = L_{ls}i_{qs} + L_m(i_{qs} + i'_{kq})}$ $\boldsymbol{\lambda_{ds} = L_{ls}i_{ds} + L_m(i'_f + i'_{kd} + i_{ds})}$ $\boldsymbol{\lambda'_f = L'_{lf}\, i'_f + L_m\,(i'_f + i'_{kd} + i_{ds})}$ $\boldsymbol{\lambda'_{kd} = L'_{lkd}\, i'_f + L_m\,(i'_f + i'_{kd} + i_{ds})}$ $\boldsymbol{\lambda'_{kq} = L'_{lkq}\, i'_{kq} + L_m\,(i_{qs} + i'_{kq})}$ $\boldsymbol{\lambda_{os} = L_{ls}i_{os}}$ $\boldsymbol{\lambda_{qs} = L_s i_{qs} + L_m i_{kq}}$ $\boldsymbol{\lambda_{ds} = L_s i_{ds} + L_m(i_f + i_{kd})}$

10.6 To without damped or compensating winding (i.e. Steady State)

At steady state or it was designed without it

<u>Difference</u>

$$delete\ any\ \ _{kd}\ or\ \ _{kq}$$

<u>So, edit</u>

$$L_{mq} = L_{md} = L_m$$

Salient motor	No damper motor
$\boldsymbol{\lambda_{qs} = L_{ls}i_{qs} + L_{mq}(i_{qs} + i'_{kq})}$ $\boldsymbol{\lambda_{ds} = L_{ls}i_{ds} + L_{md}(i'_f + i'_{kd} + i_{ds})}$ $\boldsymbol{\lambda'_f = L'_{lf}\, i'_f + L_{md}\,(i'_f + i'_{kd} + i_{ds})}$ $\boldsymbol{\lambda'_{kd} = L'_{lkd}\, i'_f + L_{md}\,(i'_f + i'_{kd} + i_{ds})}$ $\boldsymbol{\lambda'_{kq} = L'_{lkq}\, i'_{kq} + L_{mq}\,(i_{qs} + i'_{kq})}$ $\boldsymbol{\lambda_{os} = L_{ls}i_{os}}$	$\boldsymbol{\lambda_{qs} = L_{ls}i_{qs} + L_{mq}(i_{qs})}$ $\boldsymbol{\lambda_{ds} = L_{ls}i_{ds} + L_{md}(i'_f + i_{ds})}$ $\boldsymbol{\lambda'_f = L'_{lf}\, i'_f + L_{md}\,(i'_f + i_{ds})}$ $\boldsymbol{\lambda_{os} = L_{ls}i_{os}}$

10.7 Synchronous Reluctance machine

Reluctance Machine is a SM must be with salient pole but Without excitation

Difference

Eliminate Field Equations and Components (v_f , I_f, λ_f)

Salient Synchronous motor	Salient Reluctance motor
$V_{qs} = r_s i_{qs} + p\,\lambda_{qs} + \omega_r\,\lambda_{ds}$ $V_{ds} = r_s i_{ds} + p\,\lambda_{ds} - \omega_r \lambda_{qs}$ $V_{os} = r_s i_{os} + p\,\lambda_{os}$ $V_f = r_r i_f + p\,\lambda_f$ $V_{kd} = 0 = r_r i_{kd} + p\,\lambda_{kd}$ $V_{kq} = 0 = r_r i_{kq} + p\,\lambda_{kq}$ $\boldsymbol{\lambda_{qs} = L_{ls} i_{qs} + L_{mq}(i_{qs} + i'_{kq})}$ $\boldsymbol{\lambda_{ds} = L_{ls} i_{ds} + L_{md}(i'_f + i'_{kd} + i_{ds})}$ $\boldsymbol{\lambda'_f = L'_{lf}\, i'_f + L_{md}(i'_f + i'_{kd} + i_{ds})}$ $\boldsymbol{\lambda'_{kd} = L'_{lkd}\, i'_{kd} + L_{md}(i'_f + i'_{kd} + i_{ds})}$ $\boldsymbol{\lambda'_{kq} = L'_{lkq}\, i'_{kq} + L_{mq}(i_{qs} + i'_{kq})}$ $\boldsymbol{\lambda_{os} = L_{ls} i_{os}}$	$V_{qs} = r_s i_{qs} + p\,\lambda_{qs} + \omega_r\,\lambda_{ds}$ $V_{ds} = r_s i_{ds} + p\,\lambda_{ds} - \omega_r \lambda_{qs}$ $V_{os} = r_s i_{os} + p\,\lambda_{os}$ $V_{kd} = 0 = r_r i_{kd} + p\,\lambda_{kd}$ $V_{kq} = 0 = r_r i_{kq} + p\,\lambda_{kq}$ $\boldsymbol{\lambda_{qs} = L_{ls} i_{qs} + L_{mq}(i_{qs} + i'_{kq})}$ $\boldsymbol{\lambda_{ds} = L_{ls} i_{ds} + L_{md}(i'_{kd} + i_{ds})}$ $\boldsymbol{\lambda'_{kd} = L'_{lkd}\, i'_{kd} + L_{md}(i'_{kd} + i_{ds})}$ $\boldsymbol{\lambda'_{kq} = L'_{lkq}\, i'_{kq} + L_{mq}(i_{qs} + i'_{kq})}$ $\boldsymbol{\lambda_{os} = L_{ls} i_{os}}$

10.8 T_{em} to include saliency effect

<u>Torque on Synchronous machine has 3 sources</u>

1- Excitation
2- Induction Effect
3- Saliency

$$T_{em} = \frac{3}{2}\frac{P}{2}\left(\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}\right)$$

$$\lambda_{qs} = L_{qs} i_{qs} + L_{mq}\, i_{kq}$$
$$\lambda_{ds} = L_{ds} i_{ds} + L_{md} i_{kd} + L_{md} I_f$$

$$so,$$

$$T_{em} = \frac{3}{2}\frac{P}{2}\left(L_{ds} i_{ds i_{qs}} + L_{md} i_{kd} i_{qs} + L_{md} I_f i_{qs} - (L_{qs} i_{qs} i_{ds} + L_{mq}\, i_{kq} i_{ds})\right)$$
$$T_{em} = \frac{3}{2}\frac{P}{2}\ \left(L_{ds} - L_{qs}\right) i_{ds} i_{qs} + L_{md} I_f I_{qs} + L_{md} i_{kd} i_{qs} - L_{mq} i_{kq} i_{ds}]$$

Electromagnetic Torque = salient component Torque + Field Component Torque + Induction Torque

$$\boldsymbol{Saliency\ Torque = \frac{3}{2}\frac{P}{2}\left(L_{ds} - L_{qs}\right) i_{ds}\, i_{qs}}$$

$$\boldsymbol{Field\ Torque = \frac{3}{2}\frac{P}{2}\ \left(L_{md}\, I_f i_{qs}\right)}$$

$$\boldsymbol{Induction\ Torque = \frac{3}{2}\frac{P}{2}\left(L_{md} i_{kd} i_{qs} - L_{mq} i_{kq}\, i_{ds}\right)}$$

10.8.1 General Expression of Electromagnetic developed torque

$$\boldsymbol{T_{em} = \frac{3}{2}\frac{P}{2}\frac{1}{\omega_e}\left[\ \left(X_{ds} - X_{qs}\right) i_{ds} i_{qs} + E_f\, I_{qs} + X_{md}\, i_{kd} i_{qs} - \ X_{mq} i_{kq} i_{ds}\right]}$$

10.9 PM Synchronous Machine

- Eliminate field voltage equation V_f *deleted*
- $\lambda_f \rightarrow L_m I_f \rightarrow \boldsymbol{\lambda_m}$ ***constant*** !! (*it will just add* λ_m *in the D direction*)

Salient Synchronous motor	PM S motor
$V_{qs} = r_s i_{qs} + p\,\lambda_{qs} + \omega_r\,\lambda_{ds}$ $V_{ds} = r_s i_{ds} + p\,\lambda_{ds} - \omega_r \lambda_{qs}$ $V_{os} = r_s i_{os} + p\,\lambda_{os}$	$V_{qs} = r_s i_{qs} + p\,\lambda_{qs} + \omega_r\,\lambda_{ds}$ $V_{ds} = r_s i_{ds} + p\,\lambda_{ds} - \omega_r \lambda_{qs}$ $V_{os} = r_s i_{os} + p\,\lambda_{os}$
$V_f = r_r i_f + p\,\lambda_f$ $V_{kd} = 0 = r_r i_{kd} + p\,\lambda_{kd}$ $V_{kq} = 0 = r_r i_{kq} + p\,\lambda_{kq}$	$V_{kd} = 0 = r_r i_{kd} + p\,\lambda_{kd}$ $V_{kq} = 0 = r_r i_{kq} + p\,\lambda_{kq}$
$\boldsymbol{\lambda_{qs} = L_{ls} i_{qs} + L_{mq}(i_{qs} + i'_{kq})}$ $\boldsymbol{\lambda_{ds} = L_{ls} i_{ds} + L_{md}(i'_f + i'_{kd} + i_{ds})}$ $\boldsymbol{\lambda'_f = L'_{lf}\, i'_f + L_{md}\,(i'_f + i'_{kd} + i_{ds})}$ $\boldsymbol{\lambda'_{kd} = L'_{lkd}\, i'_f + L_{md}\,(i'_f + i'_{kd} + i_{ds})}$ $\boldsymbol{\lambda'_{kq} = L'_{lkq}\, i'_{kq} + L_{mq}\,(i_{qs} + i'_{kq})}$ $\boldsymbol{\lambda_{os} = L_{ls} i_{os}}$	$\boldsymbol{\lambda_{qs} = L_{ls} i_{qs} + L_{mq}(i_{qs} + i'_{kq})}$ $\boldsymbol{\lambda_{ds} = L_{ls} i_{ds} + L_{md}(i'_{kd} + i_{ds}) + \lambda_m}$ $\boldsymbol{\lambda'_f = \lambda_m}$ $\boldsymbol{\lambda'_{kd} = L'_{lkd}\, i'_{kd} + L_{md}\,(i'_{kd} + i_{ds}) + \lambda_m}$ $\boldsymbol{\lambda'_{kq} = L'_{lkq}\, i'_{kq} + L_{mq}\,(i_{qs} + i'_{kq})}$ $\boldsymbol{\lambda_{os} = L_{ls} i_{os}}$
$T_{em} = \frac{3}{2}\frac{P}{2}\ \left(L_{ds} - L_{qs}\right) i_{ds} i_{qs} + L_{md} I_f I_{qs} + L_{md} i_{kd} i_{qs} - L_{mq} i_{kq} i_{ds}]$	$T_{em} = \frac{3}{2}\frac{P}{2}\ \left(L_{ds} - L_{qs}\right) i_{ds} i_{qs} + \lambda_m I_{qs} + L_{md} i_{kd} i_{qs} - L_{mq} i_{kq} i_{ds}]$

10.10 Conversion summary from salient model

To Cylindrical	Cancel salient component	$L_{md} = L_{mq} = L_m$ $L_{qs} = L_{ds} = L_s$
To Steady State	Cancel induction component	$I_{kq} = 0 \ , I_{kd} \ = 0$ $p\lambda = \ 0$ $\omega_r = \omega_e$
To Reluctance	Cancel excitation component	$v_f = 0 \, , i_f = 0$
To PM excited	Make field linkage constant	$L_m I_f = \lambda_m = \lambda_f$ $v_f = 0$
To Generator	3 conditions	1- Iq = -Iq and Id= Id 2- Tmech = Tem +B+J 3- Pd = wr-we