

إعدادي ٢٠٢٠

الرياضيات

التفاضل

محاضرة المشتقات العليا-
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Higher Derivatives and Leibniz's theorem

$$y', f'(x), D_x, \frac{dy}{dx} \quad 1^{\text{st}} \text{ deriv}$$

$$y'', f''(x), \dots, \frac{d^2 y}{dx^2} \quad 2^{\text{nd}} \text{ deriv}$$

$$y''', f'''(x), \dots, \frac{d^3 y}{dx^3} \quad 3^{\text{rd}} \text{ deriv}$$

$$y^{(4)}(x), f^{(4)}(x), \dots, \frac{d^4 y}{dx^4} \quad 4^{\text{th}} \text{ derive}$$

$$y = 3x^4 - 12x^3 + 5x^2 + 10$$

$$y' = 12x^3 - 36x^2 + 10x$$

$$y'' = 36x^2 - 72x + 10$$

$$y''' = 72x - 72$$

$$y^{(4)} = 72$$

$$y^{(5)} = 0$$

$$y = e^{ax}$$

$$y' = a e^{ax}$$

$$y'' = a^2 e^{ax}$$

$$y''' = a^3 e^{ax}$$

$$y^{(n)} = a^n e^{ax}$$

$$y = \sin(ax+b)$$

$$y' = a \cos(ax+b)$$

$$= a \sin(ax + b + \frac{\pi}{2})$$

$$y'' = -a^2 \sin(ax + b + \frac{\pi}{2})$$

$$= -a^2 \cos(ax + b + \frac{\pi}{2})$$

$$y''' = a^3 \sin(ax + b + \frac{\pi}{2})$$

$$y^{(n)} = a^n \sin(ax + b + n \frac{\pi}{2})$$

$$y = \ln(ax+b)$$

$$y' = \frac{a}{ax+b} = a(ax+b)^{-1}$$

$$y'' = -a^2(ax+b)^{-2} = \frac{-a^2}{(ax+b)^2}$$

$$y''' = \frac{2a^3(-1)^2}{(ax+b)^3}$$

$$y^{(n)} = (-1)^{n-1} a^n (ax+b)^{-n}$$

Leibniz's theorem

$$y = uv$$

$$y' = u \cdot v' + v u'$$

$$y'' = u \cdot v'' + v' \cdot u' + v u'' + u' v'$$

$$= u \cdot v'' + 2u' v' + u'' \cdot v$$

$$y^{(n)} = (uv)^{(n)} = u^{(n)} v + n u^{(n-1)} v'$$

$$+ \frac{n(n-1)}{2!} u^{(n-2)} v''$$

$$2!$$

$$+ \frac{n(n-1)(n-2)}{3!} u^{(n-3)} v'''$$

$$3!$$

$$+ \dots + u v^{(n)}$$

$$y = x^3 e^{2x}$$

$$u = e^{2x}$$

$$v = x^3$$

$$u^{(n-3)} = \frac{2^{n-3}}{1} e^{2x}$$

$$v' = 3x^2$$

$$u^{(n-2)} = 2^{n-2} e^{2x}$$

$$v'' = 6x$$

$$u^{(n-1)} = 2^{n-1} e^{2x}$$

$$v''' = 6$$

$$u^{(n)} = 2^n e^{2x}$$

$$v^{(4)} = 0$$

$$y^{(n)} = (x^3 e^{2x})^{(n)}$$

$$= x^3 (2^n e^{2x}) + n (3x^2) 2^{n-1} e^{2x}$$

$$+ \frac{n(n-1)}{2!} (6x) (2^{n-2} e^{2x})$$

$$+ \frac{n(n-1)(n-2)}{3!} (6) (2^{n-3} e^{2x})$$

$$+ 0$$

$$= 2^n x^3 e^{2x} + 3n x^2 2^{n-1} e^{2x}$$

$$+ n(n-1)(n-2) 2^{n-3} e^{2x}$$

$$y = \sin(2x+3) \ln(3x+2)$$

$$u = \sin(2x+3)$$

$$u^{(n)} = 2^n \sin\left(2x+3 + n\frac{\pi}{2}\right)$$

$$v = \ln(3x+2)$$

$$v^{(n)} = \frac{(-1)^{n-1} (n-1)! \cdot 3^n}{(3x+2)^n}$$

$$\begin{aligned} y^{(n)} = [\sin(2x+3) \ln(3x+2)]^{(n)} &= 2^n \sin\left(2x+3 + n\frac{\pi}{2}\right) \cdot \ln(3x+2) \\ &+ n 2^{n-1} \sin\left(2x+3 + (n-1)\frac{\pi}{2}\right) \left(\frac{3}{(3x+2)}\right) \\ &+ \frac{n(n-1)}{2!} 2^{n-2} \sin\left(2x+3 + (n-2)\frac{\pi}{2}\right) \left(\frac{-3^2}{(3x+2)^2}\right) \\ &+ \dots + \sin(2x+3) \left(\frac{(-1)^{n-1} (n-1)! \cdot 3^n}{(3x+2)^n}\right) \end{aligned}$$

الاختبار

أسئلة

Q₁ Find the domain of the f_n $f(x) = ???$

Q₂ Prove that $e^{-1} = \ln \dots$

Q₃ Discuss the continuity of the f_n $f(x) = \{$

Q₄ Find y' $y = \dots \dots \dots$ $y = \dots \dots \dots$ $y = \dots \dots \dots$