

الهندسة التحليلية

نقل ودوران المحاور المتوازية
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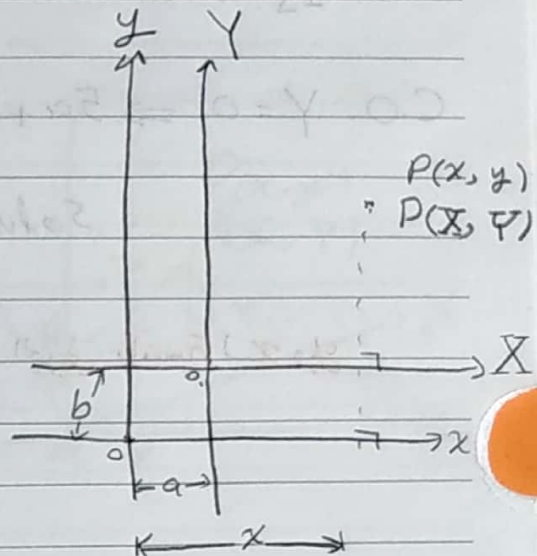
1st

Translation of axes

if 0 is translated

$$x = X + a$$

$$y = Y + b$$



Ex: find the new eqn. of the curve $x^2 + y^2 + 8x + 10y - 8 = 0$ $P(-4, -5)$

$$\text{let } x = X - 4, \quad y = Y - 5$$

$$(X - 4)^2 + (Y - 5)^2 + 8(X - 4) + 10(Y - 5) - 8 = 0$$

$$X^2 + Y^2 - 49 = 0$$

Notes:-

1) To eliminate x from the eqn then translate the axis to $(a, 0)$

2) " " " " " " " " " " " " $(0, b)$

3) " " " " " " " " " " " " (a, b)

Ex: eliminate both x and y from the eqn. $x^2 + 5xy - 19x - 22y + 26 = 0$

$$\text{let } x = X + a, \quad y = Y + b$$

$$(X + a)^2 + 5(X + a)(Y + b) - 19(X + a) - 22(Y + b) + 26 = 0$$

$$C.O. X = 2a + 5b = 19$$

$$C.O. Y = 0 \Rightarrow 5a + 2b = 22$$

Solve to get a, b

يمكن الحصول على المعادلتين عن طريق التفاضل الجزئي بالنسبة لـ x و y
ثم التحويل عن x بـ a و y بـ b

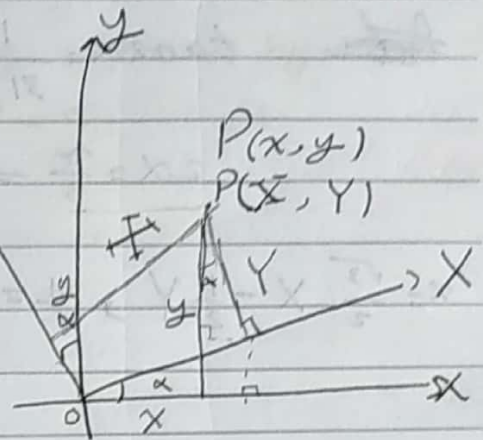
Rotation of axis

$$x = X \cos \alpha - Y \sin \alpha$$

$$y = X \sin \alpha + Y \cos \alpha$$

$$\begin{pmatrix} x \\ y \end{pmatrix} \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} = \begin{pmatrix} X \\ Y \end{pmatrix}$$

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To eliminate 2nd degree we rotate the axis with α

Ex: eliminate xy from the eqn. $ax^2 + 2hxy + ey^2 + 2fx + 2gy + e = 0$

let $x = X \cos \alpha - Y \sin \alpha$, $y = X \sin \alpha + Y \cos \alpha$

$$a(X \cos \alpha - Y \sin \alpha)^2 + 2h(X \cos \alpha - Y \sin \alpha)(X \sin \alpha + Y \cos \alpha) + b(X \sin \alpha + Y \cos \alpha)^2 + 2f(X \cos \alpha - Y \sin \alpha) + 2g(X \sin \alpha + Y \cos \alpha) + e = 0$$

$$\text{C.O. } X = 0 \Rightarrow -2a \sin \alpha + 2h(\cos^2 \alpha - \sin^2 \alpha) + 2b \sin \alpha \cos \alpha = 0$$

$$-2a \sin 2\alpha + 2h \cos 2\alpha + b \sin 2\alpha = 0$$

$$(b - a) \sin 2\alpha + 2h \cos 2\alpha = 0 \Rightarrow \tan 2\alpha = \frac{2h}{a - b}$$

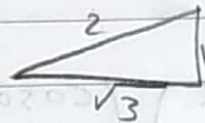
Ex: eliminate xy from the eqn. $31x^2 + 10\sqrt{3}xy + 21y^2 = 144$

~~Let~~ $\tan 2\alpha = \frac{10\sqrt{3}}{31-21} = \sqrt{3}$

$$2\alpha = \frac{\pi}{3} \Rightarrow \alpha = \frac{\pi}{6}$$

$$x = \frac{\sqrt{3}}{2} X - \frac{1}{2} Y, \quad y = \frac{1}{2} X + \frac{\sqrt{3}}{2} Y$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix}$$



Double eqn of Line pair

The Homogenous eqn of 2nd degree in x, y represents by two lines passing through the roots of $ax^2 + 2hxy + by^2 = 0$ --- (1)

$$b\left(\frac{y}{x}\right)^2 + 2h\left(\frac{y}{x}\right) + a = 0$$

$$\frac{y}{x} = \frac{-2h \pm \sqrt{4h^2 - 4ab}}{2b}$$

Ex: Find the lines represents by the eqn. $y^2 - 7xy + 10x^2 = 0$

$$(y - 2x)(y - 5x) = 0$$

$$y = 2x ; y = 5x$$