COLLEGE
Tanta Engineering

إعدادي 2020

التكامل

سنتر فيوتشر

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Subject:......إعدادي رياضة......

Chapter:......التكامل......

Mob: 0112 3333 122

0109 3508 204

integration   التكامل

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$\cot^2 x + 1 = \csc^2 x$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\sin x = \frac{1}{\csc x}$$

$$\cos x = \frac{1}{\sec x}$$

$$\tan x = \frac{1}{\cot x}$$

$$\sin 2x = 2 \sin x \cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$= 2\cos^2 x - 1$$

$$= 1 - 2\sin^2 x$$

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

$$\sinh x = \frac{e^x - e^{-x}}{2}, \quad \cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x} \qquad \operatorname{sech} x = \frac{1}{\cosh x}$$

$$\operatorname{cosech} x = \frac{1}{\sinh x} \qquad \tanh x = \frac{1}{\coth x}$$

$$\ln(xy) = \ln x + \ln y \qquad \ln(x/y) = \ln x - \ln y$$

$$\ln x^y = y \ln x$$

$$\ln e^{\square} = \square \qquad e^{\ln \square} = \square$$

$$\sin a \cos b = \frac{1}{2}\left[\sin(a+b) + \sin(a-b)\right]$$

$$\cos a \cos b = \frac{1}{2}\left[\cos(a+b) + \cos(a-b)\right]$$

$$\sin a \sin b = -\frac{1}{2}\left[\cos(a+b) - \cos(a-b)\right]$$

$$\sinh x + \cosh x = e^{x}, \qquad \cosh x - \sinh x = e^{-x}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\coth^2 x - 1 = \operatorname{csch}^2 x$$

$$* \int x^m \, dx = \frac{x^{m+1}}{m+1} + c \qquad m \neq -1$$

$$* \int \frac{u'}{u} \, du = \ln|u|$$

البسط تفاضل المقام ← المقام ln

$$* \int [f(x)]^n f'(x) \, dx = \frac{(f(x))^{n+1}}{n+1} + c$$

$$* \int e^u \cdot u' = e^u + c$$

$$\int a^u \cdot u' = \frac{a^u}{\ln a} + c$$

$$\int u' \sin u = -\cos u + c$$

$$\int u' \cos u = \sin u + c$$

$$\int u' \sec^2 u = \tan u + c$$

$$\int u' \csc^2 u = -\cot u$$

$$\int u' \sec u \cdot \tan u = \sec u$$

$$\int u' \csc u \cdot \cot u = -\csc u$$

$$\int u' \tan u = \ln \sec u$$

$$\int u' \cot u = \ln|\sin u|$$

$$\int u' \sec u \, du = \ln|\sec u + \tan u|$$

$$\int u' \csc u = \ln|\csc u - \cot u|$$

$$-\ln|\csc u + \cot u|$$

Find

$$\int \left(x^3 + \frac{1}{\sqrt[3]{x}} + 7\right) dx$$

$$= \int (x^3 + x^{-1/3} + 7)\, dx = \frac{x^4}{4} + \frac{3x^{2/3}}{2} + 7x + c$$

$$* \int \left(x + \frac{1}{x}\right)^2 dx$$

$$\int \left(x^2 + \frac{1}{x^2} + 2\right) dx = \int x^2 + 2 + x^{-2}\, dx$$

$$= \frac{x^3}{3} + 2x + \frac{x^{-1}}{-1} + c$$

$$* \int_0^2 \frac{3}{\sqrt{x}}\, dx = \int_0^2 3x^{-1/2}\, dx = \left.\frac{3\sqrt{x}}{1/2}\right|_0^2$$

$$= 6\left[\sqrt{2} - \sqrt{0}\right]$$ نعوض فوق عن X

بعدين - نعوض عن X تحت بالمثل

$$* \int \left(\frac{1}{x \ln x}\right) dx = \int \frac{1/x}{\ln x}\, dx$$

$$= \ln\left|\ln|x|\right| + c$$

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$$*\int \frac{1}{x+\sqrt{x}}\,dx$$

$$\int \frac{1}{\sqrt{x}\,(1+\sqrt{x})}\,dx = 2\int \frac{\frac{1}{2\sqrt{x}}}{1+\sqrt{x}}\,dx$$

$$= 2\ln|1+\sqrt{x}|+c$$

<u>OR</u> $\int \frac{1}{x+\sqrt{x}}\,dx$ let $x = u^2$ نعوض

تفاضل $dx = 2u\,du$

$$I = \int \frac{1}{u^2+u}\cdot 2u\,du = 2\int \frac{1}{u+1}\,du$$

$$= 2\ln|1+u| \Rightarrow 2\ln|1+\sqrt{x}|+c$$

$$*\int \frac{1}{x\ln x + \ln\ln x \cdot x\ln x}\,dx$$

تفاضل $\ln\ln x$

$= \frac{1}{\ln x}\cdot\frac{1}{x}$

$$\int \frac{\frac{1}{x\ln x}}{1+\ln\ln x}\,dx$$

البسط تفاضل المقام

$$= \ln|1+\ln\ln x|+c$$

$$* \int \left(x^{2} + \frac{1}{x}\right)^{7} \cdot x^{9}\, dx$$

$$\int \left(x^{2} + \frac{1}{x}\right)^{7} x^{7} \cdot x^{2}\, dx$$

$$\int (x^{3}+1)^{7} \cdot x^{2}\, dx \Rightarrow \frac{1}{3}\int 3x^{2}(x^{3}+1)^{7} dx$$

$$= \frac{1}{3}\frac{(x^{3}+1)^{8}}{8} + c$$

$$* \int \frac{\cos x}{4+\sin x}\, dx = \ln|4+\sin x| + c$$

$$* \int \frac{\cot x}{2+\ln \sin x}\, dx$$

$$\because \frac{d}{dx}(2+\ln \sin x) = \frac{1}{\sin x}\cos x = \cot x$$

$$I = \ln|2+\ln \sin x| + c$$

$$\int e^{4x}\, dx = \frac{1}{4}\int 4 e^{4x} dx = \frac{1}{4} e^{4x} + c$$

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$$*\int \left(e^{5x} + \frac{3}{e^{2x}}\right) dx$$

$$= \int \left(e^{5x} + 3e^{-2x}\right) dx = \left[\frac{e^{5x}}{5} - \frac{3e^{-2x}}{2}\right] + c$$

$$\int e^{4x} \cdot \sinh(2x)\, dx$$

$$I = \int \frac{e^{4x}}{2}\left(e^{2x} - e^{-2x}\right) dx = \frac{1}{2}\int e^{6x} - e^{2x}\, dx$$

$$= \frac{1}{2}\left[\frac{e^{6x}}{6} - \frac{e^{2x}}{2}\right] + c$$

$$*\int \frac{e^{\sin x}}{\sqrt{1+\tan^2 x}}\, dx$$

حل

$$\int \frac{e^{\sin x}}{\sec x}\, dx = \int e^{\sin x} \cdot \cos x\, dx$$

$$= e^{\sin x} + c$$

$$\int e^{\sin^2 x} \cdot \sin 2x\, dx = e^{\sin^2 x} + c$$

$$\because \frac{d}{dx} \sin^2 x = 2\sin x \cos x = \sin 2x$$

$$\int 4^{x} + e^{-2x} + \frac{1}{x+1}\, dx$$

$$= \frac{4^{x}}{\ln 4} - \frac{e^{-2x}}{2} + \ln|x+1| + c$$

$$* \int e^{x} \cdot a^{x}\, dx$$

$$\int (ae)^{x}\, dx = \frac{(ae)^{x}}{\ln(ae)} + c$$

$$= \frac{a^{x} e^{x}}{\ln a + \ln e} = \frac{a^{x} e^{x}}{1 + \ln a} + c$$

$$\int \frac{\sin 2x}{4 + \sin^2 x}\, dx \; *$$

$$\ln|4 + \sin^2 x| + c$$

$$* \int \frac{\sin 2x}{\sqrt{4 + \sin^2 x}}\, dx = 2\sqrt{4 + \sin^2 x}$$

البسط تفاضل ماتحت الجذر

$$2\sqrt{} =$$

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$$\int \frac{x+\sin^{-1}x}{\sqrt{1-x^2}}\,dx$$

$$-\frac{1}{2}\int \frac{-2x}{\sqrt{1-x^2}}\,dx + \int \left[\sin^{-1}x\right]\frac{1}{\sqrt{1-x^2}}\,dx$$

$$= -\frac{1}{2}\cdot 2\sqrt{1-x^2} + \frac{(\sin^{-1}x)^2}{2} + c$$

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* $$\int \frac{x+e^{\tan^{-1}x}}{1+x^2}\,dx$$

$$\int e^{\tan^{-1}x}\frac{1}{1+x^2}\,dx + \int \frac{x}{1+x^2}\,dx$$

$$= e^{\tan^{-1}x} + \frac{1}{2}\ln|1+x^2| + c$$

$$\int \frac{\cos x+\sin x}{\cos x-\sin x}\,dx =$$

$$-\int \frac{-\cos x-\sin x}{\cos x-\sin x}\,dx = -\ln|\cos x-\sin x| + c$$

$$I = \int \frac{1-\tan x}{1+\tan x}\,dx$$

$$\int \frac{1-\frac{\sin x}{\cos x}}{1+\frac{\sin x}{\cos x}}\,dx = \int \frac{\cos x - \sin x}{\cos x + \sin x}\,dx$$

$$= \ln|\cos x + \sin x| + c$$

$$* \int \frac{e^{x-1} + x^{e-1}}{e^x + x^e}\,dx$$

$$\frac{1}{e}\int \frac{e \cdot e^{x-1} + e\,x^{e-1}}{e^x + x^e}\,dx = \frac{1}{e}\int \frac{e^x + e\,x^{e-1}}{e^x + x^e}\,dx$$

$$= \frac{1}{e}\ln|e^x + x^e| + c$$

$$* \int \frac{10^{x+1}}{\sqrt{2+10^x}}\,dx$$

$$\frac{10}{\ln 10}\int \frac{10^x \ln 10}{\sqrt{2+10^x}}\,dx = \frac{20}{\ln 10}\sqrt{2+10^x}$$

$$\int e^{x}(1+e^{x})^{7}\,dx = \frac{(1+e^{x})^{8}}{8}+c$$

$$*\int \frac{e^{x}}{1+e^{x}}\,dx = \ln|1+e^{x}|+c$$

مهم

$$*\int \frac{1}{1+e^{x}}\,dx$$

$$I = \int \frac{1+e^{x}}{1+e^{x}} - \frac{e^{x}}{1+e^{x}}\,dx$$

$$= x - \ln|1+e^{x}| + c$$

:نضربه e^{-x}

OR $$\int \frac{1}{1+e^{x}}\,dx$$

$$-\int \frac{-e^{-x}}{e^{-x}+1}\,dx = -\ln|1+e^{-x}|+c$$

OR $$\int \frac{1}{1+e^{x}}\,dx$$

$$1+e^{x} = u$$

$$e^{x}\,dx = du$$

$$dx = \frac{du}{u-1}$$

$$\int \frac{1}{u(u-1)}\,du$$

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$$\int \frac{1}{u(u-1)}\,du = \int \frac{A}{u} + \frac{B}{u-1}\,du$$

$\underline{\underline{u=0}}$ $\qquad A=-1$

$u=1$ $\qquad B=1$

$$I = \int \frac{-1}{u} + \frac{1}{u-1}\,du$$

$$= -\ln u + \ln|u-1| + c$$

$$= -\ln|1+e^x| + \ln|e^x \cancel{+1} \cancel{-1}| + c$$

$$= x - \ln|1+e^x| + c$$

$$\int \frac{e^x - e^{-x}}{e^x + e^{-x}}\,dx = \ln|e^x + e^{-x}| + c$$

$$* \int \sin 3x\,dx = \frac{1}{3}\int 3\sin 3x\,dx$$

$$= -\frac{1}{3}\cos 3x + c$$

$$* \int \cos(5x)\,dx = \frac{\sin(5x)}{5} + c$$

$$* \int \sec^2(3x)\,dx = \frac{1}{3}\tan(3x) + c$$

$$* \int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx$$

$$2\int \frac{1}{2\sqrt{x}}\cos\sqrt{x}\,dx = 2\sin\sqrt{x} + c$$

$$* \int \frac{\sec^2(\ln x)}{x}\,dx = \int \frac{1}{x}\sec^2(\ln x)\,dx$$

$$= \tan(\ln x) + c$$

$$* \int \frac{\sec\left(\frac{1}{x}\right)\tan\left(\frac{1}{x}\right)}{x^2}\,dx$$

$$\frac{d}{dx}\left(\frac{1}{x}\right) = -\frac{1}{x^2}$$

$$-\int \frac{-1}{x^2}\sec\left(\frac{1}{x}\right)\tan\left(\frac{1}{x}\right)dx$$

$$= -\sec\left(\frac{1}{x}\right)$$

$$* \int \frac{e^x}{\sqrt{\cot^2(e^x)+1}}\,dx \rightarrow \int \frac{e^x}{\csc(e^x)}\,dx$$

$$\int e^x \sin(e^x)\,dx = -\cos(e^x) + c$$

$$\int \frac{\sin(\ln x^2)}{x}\, dX$$

$$\frac{1}{2}\int \frac{2}{x} \sin(2\ln x)\, dX = -\frac{1}{2}\ln \cos(\ln x^2)$$

$$* \int (X^2+1) \cos(x^3+3X+7)\, dX$$

$$= \frac{1}{3}\int (3X^2+3) \cos(x^3+3X+7)\, dX$$

$$\frac{1}{3}\sin(x^3+3X+7)+c$$

$$* \int x \sqrt{X^2+4}\, dX$$

$$\frac{1}{2}\int 2x (x^2+4)^{1/2} dX = \frac{1}{2}\frac{(x^2+4)^{3/2}}{3/2}+c$$

$$* \int \sin^3 x \cdot \cos x\, dX \Rightarrow \int (\sin x)^3 \cos x\, dX$$

$$= \frac{\sin^4 x}{4}+c$$

$$* \int \frac{\cos X}{\sqrt{4-\sin x}}\, dX = 2\sqrt{4-\sin x}+c$$

$$\int \sin(3x)\cdot\cos(2x)\,dx$$

$$= \frac{1}{2}\int \sin(5x)+\sin(x)\,dx$$

$$\frac{1}{2}\left[-\frac{\cos(5x)}{5}-\cos x\right]+c$$

$$* \int \cos(3x)\cos x\,dx = \frac{1}{2}\int \cos(4x)+\cos(2x)\,dx$$

$$\frac{1}{2}\left[\frac{\sin(4x)}{4}+\frac{\sin(2x)}{2}\right]+c$$

$* \int$ تكاملات $\sin x$ أو $\cos x$ زوجي
نقلل بقوانين ضعف الزاوية ←

$$\sin^2 x = \frac{1}{2}(1-\cos 2x) \qquad \cos^2 x = \frac{1}{2}(1+\cos 2x)$$

$$* \int \cos^2 x\,dx \Rightarrow \frac{1}{2}\int (1+\cos 2x)\,dx$$

$$= \frac{1}{2}\left[x+\frac{\sin 2x}{2}\right]+c$$

$$* \int \sin^4 x\,dx = \int \left(\frac{1}{2}(1-\cos 2x)\right)^2 dx$$

$$= \frac{1}{4}\int 1-2\cos 2x+\cos^2(2x)\,dx$$

$$I = \frac{1}{4}\int 1 - 2\cos 2x + \frac{1}{2}(1+\cos 4x)\,dx$$

$$\frac{1}{4}\int \frac{3}{2} - 2\cos 2x + \frac{1}{2}\cos 4x)\,dx$$

$$\frac{1}{4}\left[\frac{3}{2}x - \sin 2x + \frac{1}{2}\frac{\sin 4x}{4}\right) + c$$

$$*\int \sin^2 2x \cdot \cos x \, dx$$

$$\frac{1}{2}\int (1 - \cos 4x) \cdot \cos x \, dx$$

$$\frac{1}{2}\int \cos x - \cos x \cos 4x)\,dx$$

$$\frac{1}{2}\left[\int \cos x - \frac{1}{2}(\cos 5x + \cos 3x)\,dx\right.$$

$$\frac{1}{2}\left(\sin x - \frac{1}{2}\frac{\sin 5x}{5} - \frac{1}{2}\frac{\sin 3x}{3}\right) + c$$

<u>OR</u> $\int (2\sin x \cos x)^2 \cos x \, dx$

$$4\int \sin^2 x \, \underline{\cos^2 x} \cdot \cos x \, dx$$

$$4\int (1 - \sin^2 x)\sin^2 x \cdot \cos x \, dx$$

$$4\int \sin^2 x \cos x - \sin^4 x \cos x \, dx = \frac{1}{4}\left(\frac{\sin^3 x}{3} - \frac{\sin^5 x}{5}\right) + c$$

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تكاملات $\sin x$ ، $\cos x$ أس فردي

نفرد واحدة ونحول الباقي الى الثاني $\sin^2 x+\cos^2 x=1$

$$\int \sin^3 x\, dx = \int \sin x\,(1-\cos^2 x)\, dx$$

$$\int \sin x + \int -\sin x(\cos x)^2\, dx$$

$$-\cos x + \frac{\cos^3 x}{3} + c$$

$$* \int \cos^5 x\, dx \longrightarrow \int \cos x\,(\cos^2 x)^2\, dx$$

$$\int \cos x(1-\sin^2 x)^2\, dx$$

$$\int \cos x - 2\cos x(\sin x)^2 + \cos x(\sin x)^4\, dx$$

$$= \sin x - 2\frac{\sin^3 x}{3} + \frac{\sin^5 x}{5} + c$$

$$\int \frac{\sin^3 x}{\sqrt{\cos x}}\, dx \longrightarrow \int \sin x\left(\frac{1-\cos^2 x}{\sqrt{\cos x}}\right) dx$$

$$-\int \frac{-\sin x}{\sqrt{\cos x}} + \int -\sin x\,(\cos x)^{1.5}\, dx$$

$$= -2\sqrt{\cos x} + \frac{(\cos x)^{2.5}}{2.5}$$

$$\int \frac{1}{1+\cos x}\,dx$$

دائماً أن

$$\frac{1}{1 \pm \cancel{\sin x}}$$

$\cos x$

اضرب في المرافق

$$\int \frac{1-\cos x}{1-\cos^2 x}\,dx = \int \frac{1-\cos x}{\sin^2 x}\,dx$$

$$\int \frac{1}{\sin^2 x} - \frac{\cos x}{\sin x \cdot \sin x}\,dx$$

$$\int \csc^2 x - \cot x \cdot \csc x\,dx$$

$$= -\cot x + \csc x + c$$

$$\int \frac{1}{\cos x \cdot \cot x}\,dx$$

$$\rightarrow \int \sec x \tan x\,dx = \sec x + c$$

$$* \int \frac{1}{\sin^{-1} x \sqrt{1-x^2}}\,dx$$

$$\int \frac{\frac{1}{\sqrt{1-x^2}}}{\sin^{-1} x}\,dx = \ln|\sin^{-1} x| + c$$

$$\int \frac{\sec x \tan x}{\sqrt{\tan^4 x + 2\tan^2 x + 1}}\, dx$$

$$\int \frac{\sec x \tan x}{\sqrt{(1+\tan^2 x)^2}}\, dx = \int \frac{\cancel{\sec x} \tan x}{\cancel{\sec^2 x}}\, dx$$

$$\int \frac{\sin x}{\cos x \sec x}\, dx = \int \sin x\, dx = -\cos x +$$

$$\int \frac{\tan x}{\sqrt[3]{\sec x}}\, dx$$

$$\int \frac{\sec x \tan x}{(\sec x)^{4/3}}\, dx = \int \sec x \tan x (\sec x)^{-4/3}\, dx$$

$$= -3(\sec x)^{-1/3} = \frac{-3}{\sqrt[3]{\sec x}} + c$$

$$\int \frac{\sin 2x}{\cos^3 x}\, dx = \int \frac{2 \sin x \cancel{\cos x}}{\cancel{\cos^3 x}}\, dx$$

$$2 \int \frac{\sin x}{\cos x} \frac{1}{\cos x}\, dx = 2\int \sec x \tan x\, dx$$

$$= 2\sec x + c$$