

# Lecture 1

رياضة



Subject: .....

Date: the first lecture

# Linear Partial Differential equation :

$$\frac{\partial u}{\partial x} \rightarrow u_x$$

$$u = f(x, y)$$

فانقسم الى اعتبار  $x$  مع اعتبار  $y$  ثابت

$$\frac{\partial^2 u}{\partial x^2} \rightarrow u_{xx}$$

$$\text{ثابت } x \rightarrow \frac{\partial u}{\partial y} = u_y$$

$$\frac{\partial^2 u}{\partial x \partial y} = u_{xy}$$

\* 2<sup>nd</sup> order P.D.E

$$a u_{xx} + 2h u_{xy} + b u_{yy} + 2p u_x + 2q u_y = G(x, y)$$

\* Type of P.D.E

- 1. Laplace eq<sup>n</sup> :  $\Delta u = 0$

$$\Delta = \nabla^2 = \nabla \cdot \nabla$$

$$\nabla = \left( \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right)$$

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

ترتيب في  $x$  و  $y$  = متساوية

- 2. wave eq<sup>n</sup> :  $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$

- 3. Heat eq<sup>n</sup> :  $\frac{\partial^2 u}{\partial x^2} = \frac{1}{k} \frac{\partial u}{\partial t}$

- 4. Euler eq<sup>n</sup> :  $a u_{xx} + 2h u_{xy} + b u_{yy} = f(x, y)$

\* Classification of P.D.E

(Pg 13)

1. Linear and non linear P.D.E

1<sup>st</sup> degree :  $1 =$  linear

2<sup>nd</sup> degree :  $2 =$  non linear

انها لا تحتوي على  $u$  او مشتقاتها

non linear



Subject:

وتكون المعادلات؟

Date:

\* يجب سؤال تصنيف مع order, degree

1] Homogeneous and non Homo

1- دى تسمى معادلات Left H.S فقط

0 = right H.S

وذلك ان احد الطرفين ليس صفر معادلات "x, y, 1, xy"

non Homo يصبح 0 ≠ right

الرجوع

2] Parabolic, Hyperbolic and Elliptic P.D.E

∴  $h^2 - ab = 0$  ∴ The eq is Parabolic

∴  $h^2 - ab = +ve$  ∴ " " is Hyperbolic

∴  $h^2 - ab = -ve$  ∴ " " " Elliptic

\* write the order and degree of:

$$(u_{xxxy})^2 + (u_{xxy})^5 - 2xy[u_{xy}]^{10} + u^{15} = x^{20}$$

order → 3

degree → 5

Third D.E

Homogeneous, linear

$$[u_{xy}]^{1/2} + u_{xxy} = u_{yy}$$

order = 3

degree =  $(u_{xy})^{1/2} = u_{yy} - u_{xxy}$  بتوزيع المعادلات

$$u_{xy} = (u_{yy})^2 + (u_{xxy})^2 - 2u_{yy}u_{xxy}$$

degree = 2

Homo

nonlinear

Ex 1.  $xu_x + u_{yy} = x^3$

Linear

u معادلات  
معادلات 3 دى تسمى  
واسمها

2.  $x^2y u_{vxy} + 5u_{yyx} = 0$

Linear D.E, nonhomo

3.  $(u_{xx})^2 - 5u_{xy} = xy$

nonlinear D.E

non Homo

4)  $3xy u_{xx} + 5y u_{yy} + \sqrt{u} = 0$

non linear D.E

← R.H.S لايكون يبقى  
← L.H.S و  
مشتقات u

What you write today can make history tomorrow

2020/2/8 14:38

$$5 - u_{yy} + u_{xy}^2 + 5 u_x u_y = \sinh(x y)$$

1.  $\sinh$  - دالة

2.  $u_{xy}^2$  - الترتيب

3.  $u_x u_y$  - مشتقات

Ex 3: Pg(14)

① Laplace  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

$$u_{xx} + u_{yy} = 0$$

$$a = 1$$

$$h = 0 \rightarrow u_{xy}$$

$$\therefore h^2 - ab = 0 - 1 = -1 < 0$$

$$b = 1$$

Elliptic eq<sup>n</sup>

② Wave eq<sup>n</sup>  $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$

$\vec{t}$  direction

$$u_{xx} - \frac{1}{c^2} u_{tt} = 0$$

$$u_{tt}$$

$$a = 1$$

$$h = 0$$

$$b = -\frac{1}{c^2}$$

$$\therefore h^2 - ab = 0 - \frac{1}{c^2} > 0$$

$\therefore$  wave eq<sup>n</sup> is Hyperbolic

③ heat eq<sup>n</sup>  $\frac{\partial^2 u}{\partial x^2} = \frac{1}{\kappa} \frac{\partial u}{\partial t}$

$$u_{xx} - \frac{1}{\kappa} u_t = 0$$

$$a = 1$$

$$h = 0$$

$$h^2 - ab = 0 - 1 \neq 0 = 0$$

$$h = 0$$

$\therefore$  Parabolic D.E

iv) Euler eq<sup>n</sup>  $8 \frac{\partial^2 u}{\partial x^2} + 6 \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$

Constant C.P.E

$$8 u_{xx} + 6 u_{xy} + u_{yy} = 0$$

$$a = 8$$

$$h^2 - ab = 9 - 8 = 1$$

$$2h = 6$$

$$b = 1$$

$\therefore$  Hyperbolic D.E



$$5] \quad y \frac{\partial^2 u}{\partial x^2} + 2x \frac{\partial^2 u}{\partial x \partial y} + y \frac{\partial^2 u}{\partial y^2} = 0$$

$$y u_{xx} + 2x u_{xy} + y u_{yy} = 0$$

$$\therefore h^2 - ab = x^2 - y^2$$

$$a = y$$

$$2h = 2x$$

$$b = y$$

المعادلة التفاضلية  
 $= 0, < 0, > 0$

$$① \quad h^2 - ab = 0$$

Parabolic eq<sup>n</sup>

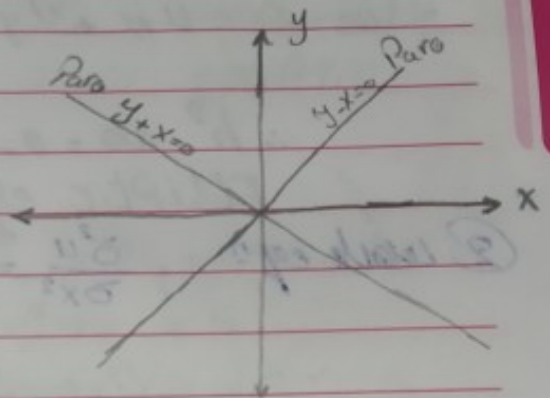
$$x^2 - y^2 = 0$$

$$y^2 = x^2$$

$$y = \pm x$$

$$y + x = 0$$

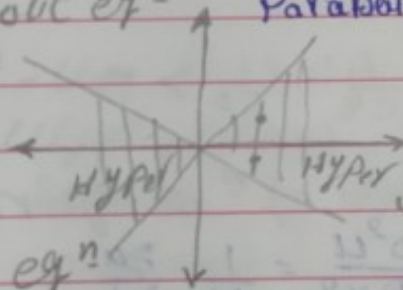
$$y - x = 0$$



$$2. \quad h^2 - ab > 0$$

$$x^2 - y^2 > 0$$

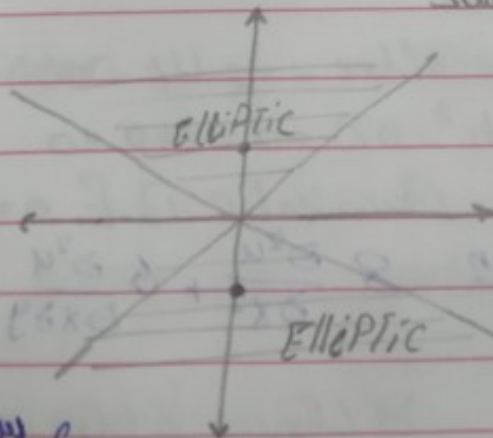
Hyperbolic eq<sup>n</sup>



$$3. \quad h^2 - ab < 0$$

$$x^2 - y^2 < 0$$

Elliptic eq<sup>n</sup>



المعادلة التفاضلية  
 Parabolic المعادلات التفاضلية

المعادلة التفاضلية

$$y = x \text{ و } y = -x$$

المعادلة التفاضلية  
 المعادلات التفاضلية

$$\left. \begin{array}{l} a \rightarrow G \text{ و } a = 2 \text{ و } a \\ a \rightarrow E \text{ و } a = 3 \text{ و } a \end{array} \right\}$$