COLLEGE
Tanta Engineering

إعدادي 2020

تكامل

التكامل بإزالة الجذور والتجزئة

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Integration by removing the roots

1] $\sqrt{a^2 - x^2}$ 2] $\sqrt{a^2 + x^2}$ 3] $\sqrt{x^2 - a^2}$

$\sin^2 x + \cos^2 x = 1$, $\tan^2 x + 1 = \sec^2 x$, $\cosh^2 x - \sinh^2 x = 1$

① $x = a\sin\theta$
$= a\cos\theta$

② $x = a\tan\theta$
$= a\sinh\theta$

③ $x = a\sec\theta$
$= a\cosh\theta$

Ex:- $I = \int \sqrt{a^2 - x^2}\, dx$ let $x = a\sin\theta$
$dx = a\cos\theta\, d\theta$

$$I = \int \sqrt{a^2 - a^2\sin^2\theta}\, a\cos\theta\, d\theta$$
$$= \int a^2 \sqrt{1 - \sin^2}\, \cos\theta\, d\theta$$
$$= a^2 \int \sqrt{\cos^2\theta}\, \cos\theta\, d\theta$$
$$= a^2 \int \cos^2\theta\, d\theta$$
$$= \frac{a^2}{2} \int (1 + \cos 2\theta)\, d\theta$$
$$= \frac{a^2}{2} \left(\theta + \frac{1}{2}\sin 2\theta\right) + C \quad \#$$

Ex: $I = \int \frac{dx}{x^2\sqrt{x^2+4}}$ let $x = 2\tan\theta$

$dx = 2\sec^2\theta \, d\theta$

$$I = \int \frac{2\sec^2\theta \, d\theta}{8\tan^2\theta \sec\theta}$$

$$= \frac{1}{4}\int \frac{\sec\theta}{\tan^2\theta} d\theta$$

$$= \frac{1}{4}\int \sin^{-2}\theta \cos\theta \, d\theta$$

$$= \frac{-1}{4}\left[\sin^{-1}\theta\right] + C$$

$$= \frac{-1}{4\sin\theta} + C \quad \#$$

Ex: $I = \int \frac{dx}{\sqrt{x^2+3}}$ let $x = \sqrt{3}\sec\theta$

$dx = \sqrt{3}\sec\theta\tan\theta \, d\theta$

$$I = \int \frac{\cancel{\sqrt{3}}\sec\theta \cancel{\tan\theta} \, d\theta}{\cancel{\tan\theta} \cancel{\sqrt{3}}}$$

$$= \int \sec\theta \, d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C \quad \#$$

$$\int \frac{1}{f(\theta)} , \frac{1}{f(\sin x, \cos x)}$$

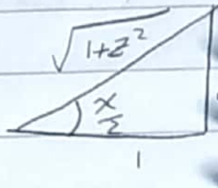
Ex: $\int \frac{1}{2+\cos x}\,dx$

let $z = \tan\frac{x}{2}$

$\sin x = \frac{2z}{1+z^2}$

$\cos x = \frac{1-z}{1+z^2}$

$dx = \frac{2\,dz}{1+z^2}$



$$= \int \frac{2\,dz}{1+z^2\left[2+\frac{1-z}{1+z^2}\right]}$$

$$= \int \frac{2\,dz}{1+z^2}$$

$$= 2\tan^{-1} z + C$$

Integration by Parts.

$$\int u\,dv = uv - \int v\,du$$

u قابلة للتفاضل

Ex: $I = \int \ln x \, dx$

$u = \ln x \qquad dv = dx$

$du = \frac{dx}{x} \qquad v = x$

$I = x \ln x - \int x \frac{dx}{x}$

$= x \ln x - x + C$

$= x(\ln x - 1) + C$

Ex: $\int x e^x dx$

u دالة تفاضلها ينتهي

$u = x \qquad dv = e^x dx$

$du = dx \qquad v = e^x$

$= x e^x - \int e^x dx$

$= x e^x - e^x + C$

$= e^x (x - 1) + C$

Ex: $\int x e^{x^2} dx$

$u = x^2 \qquad \cancel{dv = x\,dx}$

$du = 2x\,dx \qquad \cancel{v = \frac{x^2}{2}}$

$= \int e^u \frac{du}{2}$

$= \frac{1}{2} e^u + C$

$= \frac{1}{2} e^{x^2} + C$

Ex: $\int x^2 \sin 2x\,dx$

$u = x^2 \quad dv = \sin 2x\,dx$

$du = 2x\,dx \quad v = -\frac{1}{2}\cos 2x$

$= -\frac{1}{2}x^2\cos 2x + \int x\cos 2x\,dx$

$u = x \quad dv = \cos 2x\,dx$

$du = dx \quad v = \frac{1}{2}\sin 2x$

$= -\frac{1}{2}x^2\cos 2x + \frac{1}{2}\sin 2x$
$-\frac{1}{2}\int \sin 2x\,dx$

$= -\frac{1}{2}x^2\cos 2x + \frac{1}{2}\sin 2x - \frac{1}{2}\left(-\frac{1}{2}\cos 2x\right) + C$

Ex: $\int e^{2x}\cos 3x\,dx$

$u = e^{2x} \quad dv = \cos 3x\,dx$

$du = 2e^{2x} \quad v = \frac{1}{3}\sin 3x$

$= e^{2x}\cdot\frac{1}{3}\sin 3x - \frac{2}{3}\int e^{2x}\sin 3x\,dx$

$u = e^{2x} \quad dv = \sin 3x\,dx$

$du = 2e^{2x} \quad v = -\frac{1}{3}\cos 3x$

$= \frac{1}{3}e^{2x}\sin 3x - \frac{2}{3}\left(-\frac{1}{3}\cos 3x\, e^{2x}\right.$
$+ \frac{2}{3}\int e^{2x}\cos 3x$

$\int e^{2x}\cos 3x\,dx = \frac{9}{13}\cdot\left[\frac{1}{3}e^{2x}\sin 3x - \frac{1}{3}e^{2x}\cos 3x\right]$

اتاكد منها

Ex: $\int x \ln(1+x)\, dx$ $\quad u = \ln(1+x) \quad dv = x\,dx$

$du = \frac{dx}{1+x} \quad v = \frac{1}{2}x^2$

$I = \frac{1}{2}x^2 \ln(1+x) - \frac{1}{2}\int \frac{x^2}{1+x}\, dx$

لأن درجة البسط أكبر من درجة المقام بدرجة اقسم مرتين

$I = \int \frac{dx}{ax^2+bx+c}$

$I = \int \frac{ex+f}{ax^2+bx+c}\, dx$

$I = \int \frac{ex+f}{\sqrt{ax^2+bx+c}}\, dx$

Completing of squares

ax^2+bx+c

$a\left[x^2 + \frac{b}{a}x + \frac{c}{a}\right]$

$a\left[\left(x+\frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a}\right]$

Ex: $\int \frac{dx}{x^2-6x+7} = \int \frac{dx}{(x-3)^2-9+7} = \int \frac{dx}{(x-3)^2-2}$

$= \frac{1}{\sqrt{2}} \tanh^{-1}\left(\frac{x-3}{\sqrt{2}}\right) + C$

$\int \frac{dx}{x^2-a^2} = \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C$

Ex: $\int \frac{dx}{\sqrt{2x^2+10x+3}} = \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{x^2+5x+\frac{3}{2}}}$

$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{(x-\frac{5}{2})^2-\frac{25}{4}+\frac{3}{2}}}$

$= \frac{1}{\sqrt{2}} \int \frac{dx}{\sqrt{(x+\frac{5}{2})^2-\frac{13}{4}}}$

$= \frac{1}{\sqrt{2}} \cosh^{-1} \frac{(x+\frac{5}{2})}{\frac{\sqrt{13}}{2}} + c$

Ex: $I = \int \frac{5x+3}{2x^2+4x+3}$ $\qquad 5x+3 = \lambda(4x+4)+\mu$

$\lambda = \frac{5}{4}, \quad \mu = -2$

$= \int \frac{\frac{5}{4}(4x+4)-2}{2x^2+4x+3}$

$= \frac{5}{4} \int \frac{(4x+4)\,dx}{2x^2+4x+3} - 2\int \frac{dx}{2x^2+4x+3}$

$$I = \int \frac{3x-4}{\sqrt{2x^2-7x+5}}$$

$$3x-4 = \lambda(4x-7)+\mu$$

$$\lambda = \frac{3}{4}, \quad \mu = \frac{5}{4}$$

$$I = \frac{3}{4}\int \frac{4x-7}{\sqrt{2x^2-7x-5}}\,dx + \frac{5}{4}\int \frac{dx}{\sqrt{2x^2-7x+5}}$$