

سنتر فیوتشر

Subject:..... اعدادی ریاضی

Chapter:..... الدوال الزائدية

hyperbolic Functions

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Hyperbolic function الدوال الزائدية

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{1}{\tanh x}$$

$$\operatorname{sech} x = \frac{1}{\cosh x}, \quad \operatorname{cosech} x = \frac{1}{\sinh x}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\coth^2 x - 1 = \operatorname{cosech}^2 x$$

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$

$$= 2\cosh^2 x - 1$$
$$= 1 + 2\sinh^2 x \quad (1)$$

تذکرہ خواص اللوغاریتمیہ

$$\ln x + \ln y = \ln xy$$

$$\ln (x/y) = \ln x - \ln y$$

$$\ln x^y = y \ln x$$

$$e^{\ln x} = x$$

$$\ln e^x = x$$

$$\ln (x-y) = \text{تقدیر}$$

$$\ln (x+y) = \text{تقدیر}$$

$$\frac{\ln x}{\ln y} = \log_y x$$

$$\log_b x = y$$

$$x = b^y$$

$$\log_x x = 1$$

$$\log x = y$$

$$10^y = x$$

$$\log_e x = \ln x$$

$$\log_y x = \frac{\ln x}{\ln y}$$

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Prove that

$$\cosh^2 x - \sinh^2 x = 1$$

$$\therefore \text{L.H.S.} = \cosh^2 x - \sinh^2 x$$

$$= \left(\frac{e^x + e^{-x}}{2} \right)^2 - \left(\frac{e^x - e^{-x}}{2} \right)^2$$

$$= \frac{e^{2x} + e^{-2x} + 2}{4} - \frac{e^{2x} + e^{-2x} - 2}{4} = \frac{4}{4} = 1$$

$$e^x \cdot e^{-x} = e^0 = 1$$

Prove that $\cosh x + \sinh x = e^x$

$$\therefore \cosh x + \sinh x = \frac{e^x + e^{-x}}{2} + \frac{e^x - e^{-x}}{2}$$

$$= \frac{e^x + e^{-x} + e^x - e^{-x}}{2} = \frac{2e^x}{2}$$

$$= \underline{e^x}$$

Prove $\cosh x - \sinh x = e^{-x}$

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$$\begin{aligned}\cosh x - \sinh x &= \frac{e^x + e^{-x}}{2} - \left(\frac{e^x - e^{-x}}{2} \right) \\ &= \frac{\cancel{e^x} + e^{-x} - \cancel{e^x} + e^{-x}}{2} = \frac{2e^{-x}}{2} \\ &= e^{-x}\end{aligned}$$

$$* \cosh^2 x - \sinh^2 x = 1$$

$$\therefore \cosh x + \sinh x = e^x \quad \text{نثبت من قبل}$$

$$\cosh x - \sinh x = e^{-x}$$

$$\cosh^2 x - \sinh^2 x = e^x e^{-x} = 1 \quad \text{مضروب}$$

Prove that

$$1 - \tanh^2 x = \operatorname{sech}^2 x$$

$$\begin{aligned}\therefore 1 - \tanh^2 x &= 1 - \left[\frac{e^x - e^{-x}}{e^x + e^{-x}} \right]^2 \\ &= \frac{(e^x + e^{-x})^2 - (e^x - e^{-x})^2}{(e^x + e^{-x})^2} \quad \text{مربع الكسور}$$

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$$\begin{aligned}
 1 - \tanh^2 x &= \frac{e^{2x} + e^{-2x} + 2 - [e^{2x} + e^{-2x} - 2]}{(e^x + e^{-x})^2} \\
 &= \frac{4}{(e^x + e^{-x})^2} = \frac{2^2}{(e^x + e^{-x})^2} \\
 &= \left(\frac{1}{\cosh x} \right)^2 = \operatorname{sech}^2 x
 \end{aligned}$$

Prove that

$$\sinh 2x = 2 \sinh x \cosh x$$

$$\begin{aligned}
 2 \sinh x \cosh x &= 2 \frac{(e^x - e^{-x})(e^x + e^{-x})}{2 \cdot 2} \\
 &= \frac{e^{2x} - e^{-2x}}{2} = \sinh 2x
 \end{aligned}$$

$$\sinh \pi = \frac{e^\pi - e^{-\pi}}{2}$$



ملاحظة

$$\text{Prove } \sinh^{-1} x = \ln [x + \sqrt{1+x^2}]$$

Let $\sinh^{-1} x = y$

$$\therefore x = \sinh y$$

$$x = \frac{e^y - e^{-y}}{2}$$

بالمضروب $2e^y$ الطرفين

$$2xe^y = e^{2y} - 1$$

$$e^{2y} - 2xe^y - 1 = 0$$

$$(e^y)^2 - 2x(e^y) - 1 = 0$$

$$a = 1$$

$$b = -2x$$

$$c = -1$$

$$e^y = \frac{2x \pm \sqrt{4x^2 - 4(-1)}}{2}$$

$$e^y = \frac{2x \pm 2\sqrt{x^2 + 1}}{2}$$

$$y = \ln [x \pm \sqrt{x^2 + 1}]$$

نأخذ الإشارة الموجبة

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$$\therefore \sinh^{-1} x = \ln [x + \sqrt{x^2 + 1}]$$

Prove that

$$\cosh^{-1} x = \ln [x \pm \sqrt{x^2 - 1}]$$

let $y = \cosh^{-1} x$

$$\cosh y = x$$

$$\frac{e^y + e^{-y}}{2} = x$$

بالضرب $2e^y$

$$e^{2y} + 1 = 2xe^y$$

$$(e^y)^2 - 2xe^y + 1 = 0$$

مصادمتي
الدرجتين الثاني

$$e^y = \frac{2x \pm \sqrt{4x^2 - 4}}{2} = \frac{2x \pm 2\sqrt{x^2 - 1}}{2}$$

$$e^y = x \pm \sqrt{x^2 - 1}$$

لذا نأخذ هنا اللوغاريتم

$$y = \ln [x \pm \sqrt{x^2 - 1}]$$

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Show that

$$\tanh^{-1} x = \frac{1}{2} \ln \left[\frac{1+x}{1-x} \right]$$

$$y = \tanh^{-1} x$$

$$\tanh y = x$$

$$\frac{e^y - e^{-y}}{e^y + e^{-y}} = \frac{x}{1}$$

$$\frac{e^y}{e^y} \text{ multiply}$$

$$\frac{e^{2y} - 1}{e^{2y} + 1} = \frac{x}{1}$$

$$\therefore x e^{2y} + 1 = e^{2y} - 1$$

$$1 + x = e^{2y} - x e^{2y}$$

$$1 + x = e^{2y} [1 - x]$$

$$e^{2y} = \frac{1+x}{1-x}$$

$$2y = \ln \left[\frac{1+x}{1-x} \right] \xrightarrow{\text{divide by 2}}$$

$$y = \frac{1}{2} \ln \left[\frac{1+x}{1-x} \right]$$

$$\therefore \tanh^{-1} x$$

$$\textcircled{1} = \frac{1}{2} \ln \left[\frac{1+x}{1-x} \right]$$

Find logarithm form for $\cosh^{-1}(x)$

او هذا هو اللوغاريتم

$$y = \cosh^{-1} x$$

$$\cosh y = x$$

$$\frac{e^y + e^{-y}}{e^y - e^{-y}} = \frac{x}{1}$$

$$\frac{e^y + e^{-y}}{e^y - e^{-y}} = \frac{x}{1}$$

$$\frac{e^{2y} + 1}{e^{2y} - 1} = \frac{x}{1}$$

$$\therefore x e^{2y} - x = e^{2y} + 1$$

$$x e^{2y} - e^{2y} = x + 1$$

$$e^{2y} [x - 1] = x + 1$$

$$e^{2y} = \frac{x + 1}{x - 1}$$

$$2y = \ln \left[\frac{x + 1}{x - 1} \right]$$

$$\therefore y = \frac{1}{2} \ln \left(\frac{x + 1}{x - 1} \right) = \frac{1}{2} \ln \left(\frac{1 + x}{1 - x} \right)$$

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Prove that

$$\operatorname{sech}^{-1} x = \ln \left[\frac{1 \pm \sqrt{1-x^2}}{x} \right]$$

$$y = \operatorname{sech}^{-1} x$$

$$\operatorname{sech} y = x$$

$$\therefore \cosh y = \frac{1}{x}$$

$$\frac{e^y + e^{-y}}{2} = \frac{1}{x}$$

$$2e^y = \frac{2}{x} - 1$$

$$2e^y + 1 = \frac{2}{x} e^y$$

$$2e^y - \frac{2}{x} e^y + 1 = 0$$

$$x(2e^y - 2e^y) + x = 0$$

$$e^y = \frac{2 \pm \sqrt{4 - 4x^2}}{2x} =$$

$$\therefore e^y = \frac{1 \pm \sqrt{1-x^2}}{x}$$

هذا هو المطلوب

$$y = \ln \left[\frac{1 \pm \sqrt{1-x^2}}{x} \right]$$

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Prove that

$$\operatorname{Cosech}^{-1} x = \ln \left[\frac{1 + \sqrt{1+x^2}}{x} \right]$$

let $y = \operatorname{Cosech}^{-1} x$

$$\operatorname{Cosech} y = x$$

$$\frac{1}{\sinh y} = x$$

$$\sinh y = \frac{1}{x}$$

$$\therefore x \left[\frac{e^y - e^{-y}}{2} \right] = 1$$

$$2e^y \text{ and } 2e^{-y}$$

$$x [e^{2y} - 1] = 2e^y$$

$$x e^{2y} - 2e^y - x = 0$$

$$e^y = \frac{2 \pm \sqrt{4 + 4x^2}}{2}$$

$$e^y = \frac{2 \pm 2\sqrt{1+x^2}}{2}$$

$$e^y = 1 \pm \sqrt{1+x^2}$$

$$y = \ln [1 + \sqrt{1+x^2}]$$

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Prove that $\sinh^{-1} x = \cosh^{-1}(\frac{1}{x})$

$$y = \sinh^{-1} x$$

$$\sinh y = x$$

$$\cosh y = \frac{1}{x}$$

$$y = \cosh^{-1}(\frac{1}{x})$$

$$\therefore \sinh^{-1} x = \cosh^{-1}(\frac{1}{x})$$

Prove that $\tanh^{-1}(x) = \coth^{-1}(\frac{1}{x})$

$$\text{let } \tanh^{-1} x = y$$

$$\tanh y = x$$

$$\coth y = \frac{1}{x}$$

$$y = \coth^{-1}(\frac{1}{x})$$

$$\therefore \tanh^{-1} x = \coth^{-1}(\frac{1}{x})$$

Prove that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

$$y = \sin^{-1} x \quad \text{---} \quad \sin y = x$$

$$\cos\left(\frac{\pi}{2} - y\right) = x$$

$$\frac{\pi}{2} - y = \cos^{-1} x$$

$$\frac{\pi}{2} = y + \cos^{-1} x$$

$$\therefore \frac{\pi}{2} = \sin^{-1} x + \cos^{-1} x$$

$$\sin(90^\circ - x) = \cos x$$

$$\cos(90^\circ - x) = \sin x$$

Show that

$$\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$$

$$y = \tan^{-1} x \quad \text{---} \quad \tan y = x$$

$$\cot\left(\frac{\pi}{2} - y\right) = x$$

$$\frac{\pi}{2} - y = \cot^{-1} x$$

$$\frac{\pi}{2} = y + \cot^{-1} x$$

$$\frac{\pi}{2} = \tan^{-1} x + \cot^{-1} x$$

$$\tan(90^\circ - x) = \cot x$$

$$\cot(90^\circ - x) = \tan x$$

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Prove that

$$\tanh^{-1} \left(\frac{x^2 - 1}{x^2 + 1} \right) = \ln x$$

$$\text{let } \tanh^{-1} \left(\frac{x^2 - 1}{x^2 + 1} \right) = y$$

$$\frac{x^2 - 1}{x^2 + 1} = \tanh y = \frac{e^y - e^{-y}}{e^y + e^{-y}}$$

$$\frac{x^2 - 1}{x^2 + 1} = \frac{e^{2y} - 1}{e^{2y} + 1} \quad \frac{e^y}{e^y} \text{ cancel}$$

$$(x^2 + 1)(e^{2y} - 1) = (x^2 - 1)(e^{2y} + 1)$$

$$\cancel{x^2 e^{2y}} + e^{2y} - \cancel{x^2} - 1 = \cancel{x^2 e^{2y}} - e^{2y} + \cancel{x^2} + 1$$

$$\cancel{2x^2} = \cancel{2e^{2y}}$$

$$x^2 = e^{2y}$$

$$2y = \ln x^2$$

$$\cancel{2y} = \cancel{2} \ln x$$

لقد هالضرة

$$y = \ln x$$

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