The School of Mathematics

Analysis of the impact of time granularity change in modelling TV attribution

by

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Own Work Declaration

I confirm that the work contained in this report is my own except where otherwise indicated.

Executive summary

Background

Proving the return on marketing investment becomes more and more important in nowadays due to the digitisation of marketing. To measure the impact of marketing expenditure of the marketing mix is crucial. However, to do so a valid data quantity is needed in order to confidently estimate relations between marketing variables. Although transforming from weekly to daily data granularity increases the data quantity, it implies changes that need to be understood and considered when implemented in a commercial environment.

Research question

This work analyses the changes in the transformation from weekly to daily data granularity in the framework of TV advertisement attribution practised by TVSquared Ltd and aims to fill some research gaps encountered.

Data

Provided by 4 different clients, the data used is comprised of 2 groups of data sets. The first group includes time series of daily web response data, keyword specific Google Trends as well as initial TV attributed web response data. The second group consists of web conversions data including time series of daily web conversions and initial TV attributed web conversions.

Methods

The approach taken is based on the fact that the data transformation to a more granular level leads to potential multiple seasonal patterns which play a major role in this framework of TV advertisement attribution. Here, 2 different methods to detect daily seasonality, utilising indicator variables, and multiple seasonal patterns, utilising Fourier Transformation, are described and detection performance evaluated before they are compared in the framework environment.

Results

The results show that, although the more complex seasonal detection approach is leading to a better model fit when it comes to seasonal pattern detection, it is outperformed by the simple indicator approach in the framework model. In conclusion, 2 major results can be shown. First, transforming from weekly to daily data granularity increases the amount of data that can be used for modelling. However, the resulting outcomes need to be compared to the previous methodology in order to increase the explainability to the clients. Second, further research needs to be undertaken in order to analyse the models stability for different clients before implementing it in a production environment.

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1 Introduction

According to the 'State of inbound 2018 - Global Report' published by HubSpot Research giving insights on the current state of inbound marketing, more then 40% of the marketers surveyed set proving their return on marketing investment as the top priority for the next 12 months (see An 2018).
In order to do so, measuring the effect of marketing activity becomes crucial. Dating back to 1969, McCarthy minted the term 'Marketing Mix' by defining the 4 P's of the marketing mix: Product, Price, Place and Promotion (see McCarthy 1969). Since then, modelling the marketing mix variables became a topic of interest for researchers. While evaluating model types from basic linear models to complex hierarchical models, 7 different patterns of marketing effects were identified (see Tellis 2006). Among them, current effects describing the immediate change of the response measures due to the simultaneous exposure to advertising and carryover effects denoting the impact of advertising following in the time period after initial advertisement pulse. As the carryover effect, referred to as 'advertising adstock', became more and more popular for measuring the long term impact of advertisement, several adstock transformation models were analysed concluding that they are able to account for nonlinear and dynamic effects of advertising (see Joy 2006).

With the increasing digitisation, marketing research is focused on modelling the 4th 'P' in the marketing mix - Promotion through digital media. David Chan and Michael Perry see difficulties to overcome in the selection bias through ad targeting, seasonality and funnel effects arising when different ad channels influence each other, as well as in data specific areas like correlation between input variables, limited data ranges and low data quantity (see Chan and Perry 2017).
Estimating time series models on weekly data for commercial marketing purposes can therefore be challenging. One reason is, that the average number of weeks per year is 52.18. However, most of the methods used to model the underlying seasonality in the provided time series require integer seasonal periods, as pointed out in Hyndman and Athanasopoulos 2018, Section 12.1. Another important fact to consider is that the low number of data points for one year leads to a long on-boarding process for clients wanting to utilise the model results in a commercial environment.

2 Research question

Based on the mentioned foundation, this work analyses the modelling changes when transforming from weekly to daily data granularity in the framework of TV advertisement attribution practised at TVSquared Ltd, a worldwide leader in TV attribution, by implementing a new approach to capture multiple seasonality and evaluating the model performance on different data sets.

In Section 3, the methodology of TV attribution for web response data based on weekly granularity is reviewed. This method is divided into the initial web response attribution and the long term web response attribution. The data sets used for this analysis are described in Section 4. Following this, the methodology proposed in this analysis is explained in Section 5. The new approach implemented, accounting for multiple seasonality patterns is described in Section 6.2.2. Building on, the models and results are evaluated in Section 6 and the best performing model selected. In Section 7 the selected model is applied to different data set, which utilises web conversions instead of web responses.
Finally, the analysis is critically concluded in Section 8.

3 Background

Alternative to the marketing attribution approaches described in Section 1 the following method is built on the idea of measuring the impact of TV advertisement by response driven attribution. Here, the method builds onto the use of a specified response measure and TV spot characteristics. In order to understand the following modelling approaches, the most important terms used are defined in the following table

Term	Definition
response	reaction of a customer to a specified call-to-action (i.e. open a web site)
web session	a group of user interactions with a web site (i.e. page views, e-commerce transaction)
adstock	the decay (carry-over effect) of the long term impact of TV advertisement in %
uplift factor	multiplier that expresses the ratio of long term response to the initial response
conversion	completed activity, online or offline, that is important to the business success (i.e. login, sale)

Table 1: Most important marketing terms and their corresponding definitions from [reference]

When modelling the impact of TV advertisement on customer response in a commercial setting several business constraints can restrict the choice and use of the chosen models. Such constraints involve costs including expensive computational power for more complex and efficient models as well as the expensive acquisition of external data each time the models are fitted to the data. Another major constraint that is often encountered is the low quantity and poor quality of data provided by the client which are used in statistical modelling. The proposed methods in this analysis are designed to account for the above mentioned constraints. They are to provide consistent results using a minimum of data while being flexible to adapt to data sets provided by different clients.

3.1 Modelling initial response

The method is described here in a generic way through a probabilistic approach described in "TVSquared - Attribution Methodology" and practised by TVSquared Ltd (see TVSquared 2019b). The primary emphasis is placed on modelling the relation between initial web responses and TV advertisement. Input data used are the number of web sessions started on a clients specific web site for every minute of the day, referred to as 'web responses' and the TV spot log files containing information about spot time, spot length and the medium the spot was aired on.

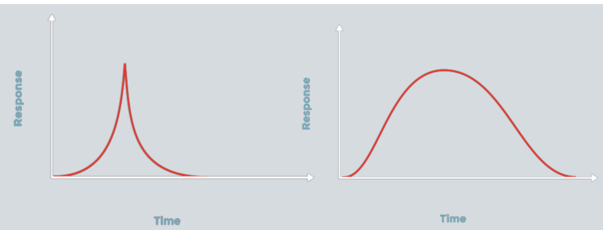

Figure 1: Example of reference response profiles for two different brands

A 3 step process is utilised. In the calibration step TV spots are identified with clear response profiles shown in Figure 1. They differ depending on the brand, the spot length and medium as customers respond differently to different brands. In the second step the baseline is computed. By filtering the web sessions based on the IP address locations, it is possible to only use web sessions in regions where the TV spot was aired. Web sessions that are unlikely to be directly TV-driven, for example when originated from affiliate web sites or digital-only campaigns, are removed from the data set. Using the filtered time series, the baseline is computed utilising the signal-processing method called frequency decomposition. Advantages of this approach are the automatic adjustments to changes in behaviour for specific days of the week or holidays. The calculated range of frequencies is mapped against the identified reference response profile creating a baseline consisting of web sessions that are not part of the mapped set. This set of web sessions could be attributed to TV.

In the final attribution step a probability set is linked to each web session of the attributable set. This probability set denotes the likelihood of being a response that corresponds to a particular TV spot. In the calculation of the probability sets factors such as the level of the baseline, the

time difference between spot airings and web sessions, the size of the audience, and the brand specific reference response profiles are taken into account. In a further step the resulting probability score for each web session can then be used to attribute any action, i.e. sale, associated with that session to the linked TV spot.
This probabilistic approach allows to take situations into account when multiple TV spots air within a close time period in the same market. The number of web responses attributed to any TV spot per day is used in the subsequent modelling approach described in the following Section.

3.2 Modelling long term response

The approach to model the long term effect of TV advertisement described here builds on the process described in Section 3.1 (see TVSquared 2019a). It is explained in more detail in "TVSquared - ADeffect Methodology". Several important characteristics need to be mentioned. The data used is in weekly granularity. With utilising the outcomes of the initial response attribution approach it focuses only on response to TV and no other marketing channel. In contrast to the previous method described this one is based around multiple linear regression in time series. It is structured in multiple phases. In the phase of data capture the time series for the covariates of the multiple regression model are extracted. Here is to be noted that the already attributed response is derived from the previous described method and together with a chosen adstock value used in an exponential decay model to compute the adstock influenced TV attributed response time series. Motivated by a study from Choi and Varian, indicating that the

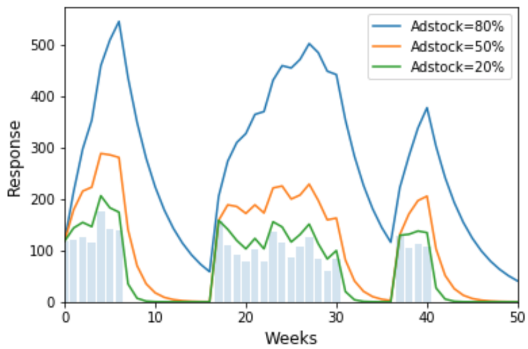

Figure 2: Example of TV attributed response variable transformed with different adstock values

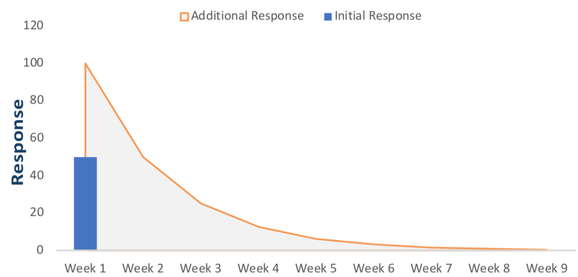

Figure 3: Example of computed long term response for one week of TV advertisement activity

use of Google Trends data improves model performance for predicting present trends, the weekly seasonal index is derived from an indicator variable model based on indicators representing each week of the year. The time series used here is extracted from 5 years of weekly Google Trends data. Google Trends is an index representing the volume of the search queries by geographic location and keyword category. Here, each data point is the the ratio of the total volume for the specified search term in the given geographic region to the total number of searches in this region at specific time point. In order to capture the generic business growth, the global trend time series is derived from the use of the Savitzky-Golay filter. This method is first described in 1964 in "Smoothing and differentiation of data by simplified least squares procedures." and can be reviewed in more detail in *Numerical Recipes in C.* (see Savitzky and Golay 1964; W. Press et al. 2002). It gives the advantage of detecting micro changes in a dynamical way. In the next phase the adstock is identified. Doing this, several multiple regression models using different adstock values are fitted to the data and evaluated based on the following criteria

- R^2 denoting a measure of model fit
- t-value significance adstock transformed TV attributed response variable
- positive coefficients

- overall contribution of variables

The adstock of the model with the best criteria fit is accepted. An important measure of advertising impact is the uplift factor which takes additional in-week attributed response and the ongoing weekly adstock into account. Figure 3 shows an example of the calculated total response after applying the computed uplift factor. In the last phase a confidence score is derived based on input data quantity and quality, model statistics and stability. Built on the confidence score the final uplift factor is computed which consists of a weighted Uplift Factor with the confidence score and a pooled Uplift Factor derived from the industry average. This analysis is based on data from 4 clients, therefore the pooled Uplift Factor is not considered. The model results of the weekly model are displayed in the Appendix Section C Table A.1. A more detailed description of the process on daily granularity follows in Section 5.1.

4 Data

Two groups of data sets are used to analyse the long term effect of TV advertisement on two specified response measures. Each group consists of 4 data sets provided by different clients.

4.1 Web response

As a direct response measure of TV impact the measure of web response is chosen which is represented by the daily number of web sessions started after a user opened a client specific website. A filtering process is used applying the following steps

- filter web sessions by IP address locations where the TV spot was aired
- filter web sessions by source of origin; from search engines or direct web browser input
- filter out time periods with low web response in the beginning and end of the available clients data

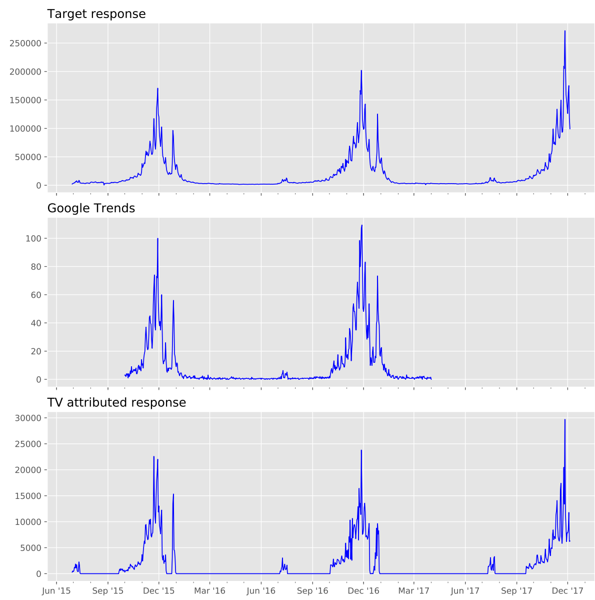

Figure 4: Client 1 - time series of target response, TV attributed response with number of responses and Google Trends with search activity level on the y-axis

This process yields a time series which will denote the dependent variable in this analysis referred to as 'target response'. Two additional time series are considered as covariates in the model analysed. The daily number of web sessions attributed to TV advertisement derived from the process described in Section 3.1 which represents a measure of immediate response to TV advertisement and is further referred to as 'TV attributed response'. In order to capture seasonal patterns Google Trends time series are used denoting the daily number of keyword searches relative to the highest number in the specified time range. Constraint by the longest possible Google Trends search period for daily data which is 90 days, the extracted 18 months time series are re-scaled in relation to the first data point in the series. Based on the time series pattern of the 'target response', 4 different clients are chosen out of the provided client pool.

For Client 1, operating in the United States of America, the above described extracted time series are shown in Figure 4. This time series experiences a clearly visible seasonal pattern over the year, with increasing variance, but no clear daily seasonal pattern over the week. This behaviour can be explained by the characteristics of the clients product which is highly related to the celebration of Christmas. Due to confidentiality this cannot be specified in more detail. In Figure A.1 the target response for Client 2, operating in Sweden, shows a strong daily pattern over the week but no seasonal pattern over the year. A clear seasonal pattern over the year and over the week is visible in Figure A.2 for Client 3, who operates in the United States of America. Additional to strong seasonal patterns over the year and week, a clear increasing trend is visible in the target response in Figure A.3 provided by Client 4, who operates in Spain. The above described data sets are used to apply the proposed models and methods on, in order to evaluate their behaviour for different data sets with different time series patterns.

4.2 Web conversions

For evaluating the modelling approach in a different setting, a second data set is provided by the above mentioned clients. This data includes several daily time series. The first denotes the number of total conversions linked to started web sessions on each day of the series, referred to as 'target conversions'. The second time series represents the aggregated number of conversions that were attributed to web sessions from customers estimated to being exposed to a TV spot at each day of the series. This time series is used to derive the long term impact of TV advertisement from and referred to as 'TV attributed conversions'. As described in the previous section, seasonality index is derived from 'Google Trends' under the assumption that it accounts for the impact of other business activities besides TV advertisement.

5 Methodology

The transformation of data granularity for a model which is based on time series data in weekly granularity to a model based on time series data on daily level implies several changes. The major change affects the capture of multiple seasonality patterns. In this chapter the framework model of the analysis will be introduced in Section 5.1. A major factor driving digital response to advertisement is seasonality which depends on the characteristics of the business of interest (e.g. seasonal product), its market (e.g. regional differences) and the consumer behaviour. Different approaches to capture multiple seasonality patterns are described in Section 5.2 in which a time domain based model is specified in Section 5.2.1 and a frequency domain based model is constructed in Section 5.2.2.

5.1 Framework Model: Multiple Linear Regression

To derive the uplift factor and adstock explained in Section 3.2 a multiple regression model is used reflecting the business constraints described in Section 3.

Let y_t denote the response measurement at time point t, with $t = 1, ..., T$ and X_{it} denote the i-th covariate at time point t with $i = 1, ..., p$. Assuming linearity in the conditional mean function yields the multiple linear regression model,

$$y_t = \alpha + \sum_{i=1}^{p} \beta_i X_{it} + \varepsilon_t \tag{5.1}$$

Under the following assumptions for the random error component,

- $E(\varepsilon_t) = 0$
- $Var(\varepsilon_t) = \sigma^2$
- $Corr(\varepsilon_i, \varepsilon_j) = 0$, where i and j denote different points in time

the method of ordinary least squares (OLS) is used to estimate the regression coefficients. The least-squares function is given by

$$\mathcal{Q}(\beta_0, ..., \beta_k) = \sum_{t=1}^{T} \left(y_t - \alpha \sum_{i=1}^{k} \beta_i X_{it} \right)^2 = \sum_{t=1}^{T} \varepsilon_t^2 = (\mathbf{y} - \mathbf{X}\beta)^\top (\mathbf{y} - \mathbf{X}\beta) \tag{5.2}$$

where $\mathbf{y}$ denotes the response vector of size $T \times 1$, $\mathbf{X}$ denotes the design matrix of size $T \times p$ and β denoting the $(p+1)$-dimensional vector of unknown coefficients. Minimising $\mathcal{Q}(\beta_0, ..., \beta_k)$ with respect to β yields

$$\hat{\beta} = (\mathbf{X}^\top \mathbf{X})^{-1} \mathbf{X}^\top \mathbf{y}$$

A more detailed description can be found in *Introduction to linear regression analysis* (see Montgomery, Peck, and Vining 2012, p. 67-73). The given framework model used in this analysis is described by

$$\text{Target}_t = \text{constant} + \beta_1 \text{TV}_t + \beta_2 \text{Season}_t + \beta_3 \text{ Trend}_t + \varepsilon_t \tag{5.3}$$

Hereby, the dependent variable Target_t, with $t = 1, ..., n$, is derived from the process described in Section 4.1. The independent variable Adstock_t is derived from an exponential decay function given by

$$\text{TV}(t, v) = \sum_{d=0}^{m} v^d \cdot \text{TV}_{t-d}^{(\text{initial})} \tag{5.4}$$

where $\text{TV}_t^{(\text{initial})}$ denotes the TV attributed response at time t, derived from the process explained in Section 3.1 and the factor $v \in V$ with $V = [0, 0.05, ..., 0.95, 1]$ denotes the adstock value, describing the uplift percentage of the initial response with d representing the index of day that the adstock lasts. Season(t) describes the seasonality index derived from Google Trends and Trend(t) represents the global trend of the target response at time t to capture the general business growth or public interest for the product utilising the Savitzky-Golay filter.

For each $v \in V$ a multiple regression model is fitted. To select the best model its statistics are evaluated under the following company given criteria

- $\beta_1 > 1$ - to ensure positive uplift from TV advertisement
- $R^2 > 0.4$ - to ensure a flexible but relevant model fit for different clients
- $\text{t-value}^{(\text{Adstock})} > 2$ - to ensure significant impact of TV on the target response

Derived from the methodology of TVSquared, the coefficient for the TV attributed response variable of the selected best model β^* and its corresponding adstock value v^* are used to calculate the uplift factor F, given by

$$\text{F} = \frac{\beta_1^*}{1 - v^*} \tag{5.5}$$

In order to evaluate the resulting uplift factor F and its corresponding adstock v^*, a confidence score is computed taking the following factors into account

- Multicollinearity
- Quantity of the data
- Stability in the uplift factor
- Contribution of $\text{TV}^{(\text{initial})}$

Two major factors, collinearity and the quantity of the data can influence the model results of this approach. Multicollinearity describes the phenomenon of correlation among the explanatory variables which can lead to unreliable estimates for the coefficients due to large variances in the distributions of the coefficients (see Montgomery, Peck, and Vining 2012, p. 117). Another consequence of high multicollinearity is the possibility of over-fitting which leads to unstable estimates when using different input samples.

Although collinearity between the Target and Adstock is expected, very high collinearity leads to over-attribution of web response to TV advertisement. It is important to notice that this issue is highly related to the settings in the process deriving the initial response $\mathrm{TV}^{(\mathrm{initial})}$ described in Section 3.1. Collinearity between the Target and Season may lead to under-attribution of web response to TV advertisement, as a high proportion of the Target can be explained by the seasonality. If high collinearity is detected between the explanatory variables, Adstock and Season the model will give one of these variables a much higher relevance compared to the other while in fact both variables have relevant influence on the Target. Due to the importance of data quantity in an industrial setting, the number of input data points for the model is considered. With an increase of input data the model estimates are not only stabilising but less frequent seasonal patterns can be detected and accounted for. A similar mechanism is applicable for the initial response measure $\mathrm{TV}^{(\mathrm{initial})}$. The higher the contribution of initial response $\mathrm{TV}^{(\mathrm{initial})}$ to the Target, the more relevant it is in explaining the dependent variable. However, very high contribution is not preferred either, as this might lead to over-attributing web responses to the TV advertisement. As it is important for the client to base decisions on reliable measures, the stability of the uplift factor over time is evaluated and taken into consideration in the confidence score.

5.2 Seasonality analysis

In this chapter seasonality modelling is approached in two different ways - the time domain and the frequency domain.
A common simple method to model seasonality in the time domain is the use of seasonal indicator variables (see Hyndman and Athanasopoulos 2018, Section 5.4). This approach is explained in Section 5.2.1.
One distinguishable characteristic of seasonality in time series is its periodic occurrence. Methods that transform time series from the time domain into the frequency domain utilise this feature. To extract periodic seasonality from a given time series, one of the most popular approaches is the use of Fourier Transformation which is explained in Section 5.2.2.

5.2.1 Time domain: Seasonal indicator model

In the time domain seasonality can be modelled with seasonal indicator variables.

Let $Y = y_1, ..., y_t$ be an arbitrary detrended time series. The seasonal indicator model

$$Y_t = \alpha_1 D_{1t} + ... + \alpha_k D_{kt} + \varepsilon_t = \sum_{s=1}^{k} \alpha_s D_{st} + \varepsilon_t \tag{5.6}$$

is based on a multiple linear regression model explained in Section 5.1 where D_{st}, with $s = 1, ..., k$, denote the s-th seasonal indicator variable at time point t, α_s denote the corresponding regression coefficients and ε_t a random error component at time point t.

Minimising the corresponding least-squares function yields the regression coefficient estimates for each seasonal indicator. It is important to note that an intercept is excluded in the model as the regression coefficients are estimated to reflect a seasonality index for all seasonal indicator variables and therefore all coefficients are needed. If an intercept and all needed seasonal indicator variables are included, perfect multicollinearity will be an issue when fitting the model (see Hyndman and Athanasopoulos 2018, Section 5.4). As the seasonal indicator model is fitted to detrended data, a possible trend or median level does not need to be considered.

One advantage of this approach to model seasonality in an industrial setting is that once the seasonal coefficients are computed they can be applied to future data points under the assumption that the underlying seasonality pattern is not changing over time for this specific client pattern. Furthermore, this approach convinces with its simple implementation in a commercial environment which can be explained to potential clients in a simple understandable way. Nevertheless, it is limited by the inflexible identification of multiple existing seasonality pattern. All possible patterns should be included into the model as indicator variables which quickly leads to a high number of covariates and possible worse performance.

5.2.2 Frequency domain: Fourier transformation

To overcome the issues of a high number of indicator variables when using multiple regression to capture seasonality, a new approach is taken. This approach is based on the idea that any given signal in the time domain can be expressed as a periodic function in the frequency domain. To analyse seasonality in the frequency domain a common technique is the Fourier Transformation. The following scheme is introduced as a procedure to transform a given time series from the time domain into the frequency domain, select desired seasonal patterns and re-transform back into the time domain.

Let $y = y_0, ..., y_T$ be an arbitrary time series.

1. Step: Subtract mean
$$x_t = y_t - \bar{y}$$

2. Step: Discrete Fourier Transformation
Under the assumption that x_t is a realisation of a periodic function $g(t)$, this function can be expressed as a sum of sinusoids in the form of
$$g(t) = \sum_{0<j<\frac{T}{2}} A(f_j)cos(2\pi f_j t) + B(f_j)sin(2\pi f_j) \tag{5.7}$$
where $f_j = \frac{j}{T}$ with $j = 1, ..., \frac{T-1}{2}$ and
$$A(f) = \frac{2}{T}\sum_{t=0}^{T-1} x_t \cos(2\pi f t)$$
$$B(f) = \frac{2}{T}\sum_{t=0}^{T-1} x_t \sin(2\pi f t)$$
The Discrete Fourier Transform can be expressed as
$$X_f = \frac{A(f)}{2} - i\frac{B(f)}{2} = R(f)e^{i\phi(f)} \tag{5.8}$$
with $R(f)$ denoting the magnitude and $\phi(f)$ denoting the phase (see Bloomfield 2004, p. 38).

3. Step: Apply frequency threshold
A Band-Pass-Filter based on the variability of the signal is used as a threshold for frequency selection. The relationship between the variance of a zero-mean signal and the energy of its Fourier transform is based on Parseval's theorem given by
$$\sum_{t=0}^{T-1} |x_t|^2 = \frac{1}{T}\sum_{f=0}^{T-1} |X_f|^2 \tag{5.9}$$
where X_f is the Discrete Fourier Transform of x_t (see William H Press et al. 2007, p. 390).

   The frequency filter is chosen so that the power density spectrum of a frequency is individually exceeding a given proportion $m \in (0, 1)$ of the variance of the signal. The corresponding variance threshold is selected minimising the Root Mean Squared Error of the out-of-sample forward cross-validation. The theoretical foundation is provided in "A note on the validity of cross-validation for evaluating autoregressive time series prediction" (see Bergmeir, Hyndman, and Koo 2018).

4. Step: Fourier Inverse
   The altered signal can be computed after applying the filter to the frequencies chosen by calculating the Inverse Fourier Transform (see Bloomfield 2004, p. 45) given by

$$x_t^{(\text{inv})} = \sum_j X_{f_j} e^{2\pi i f_j t} \tag{5.10}$$

This approach of detecting and modelling seasonality in a given signal has several advantages. It is able to identify multiple seasonality patterns which can be easily detected by analysing the magnitude of the computed Discrete Fourier Transform. Furthermore, a wide range of frequency selection methods can be applied which controls the number of covariates in the resulting seasonality model.

However, the challenges in applying this approach in a commercial environment are its complex frequency selection process that needs advanced domain knowledge leading to a difficult interpretability and explainability to possible clients. As this work gives a first analysis using this method, further research on the implementation of it is recommended.

6 Models and results: web response

To transform the existing modelling approach from weekly time granularity to daily time granularity, 4 different time series derived from different client data which are to be used in this analysis are explored in Section 6.1 in order to identify seasonal patterns, trends and special characteristics to be considered in the further analysis. In Section 6.2 the different approaches to model seasonality are applied and evaluated. Following the evaluation, in Section 6.3 the different methods are implemented in the framework model described and compared in Section 5.1.

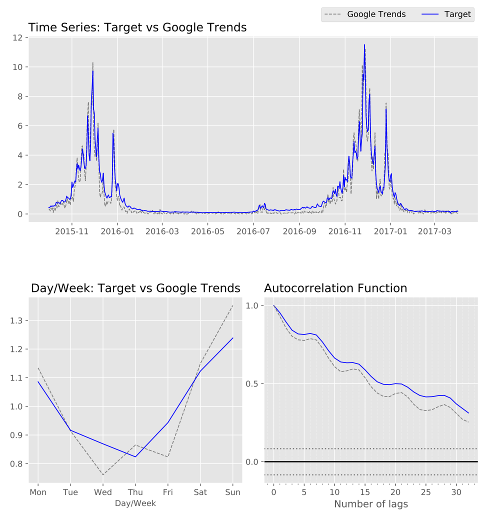

Figure 5: Visualisation of the target variable in number of responses relative to its mean and Google Trends over time for Client 1 (top) in search activity level relative to its mean; summed relative measures of target and Google Trends variable for each day of the week (bottom right); autocorrelation function for target and Google Trends over 32 lags (bottom right)

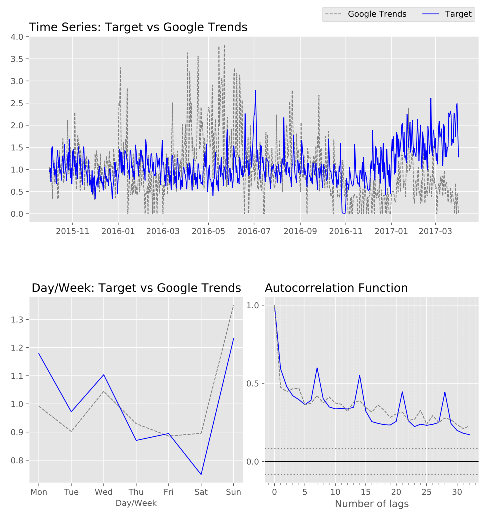

Figure 6: Visualisation of the target variable in number of responses relative to its mean and Google Trends over time for Client 2 (top) in search activity level relative to its mean; summed relative measures of target and Google Trends variable for each day of the week (bottom right); autocorrelation function for target and Google Trends over 32 lags (bottom right)

6.1 Exploratory data analysis

First, the time series of the target variable denoting the search and direct web response on the clients website is normalised by division of the mean for comparison of the target variable and Google Trends over the time range where both time series overlap.

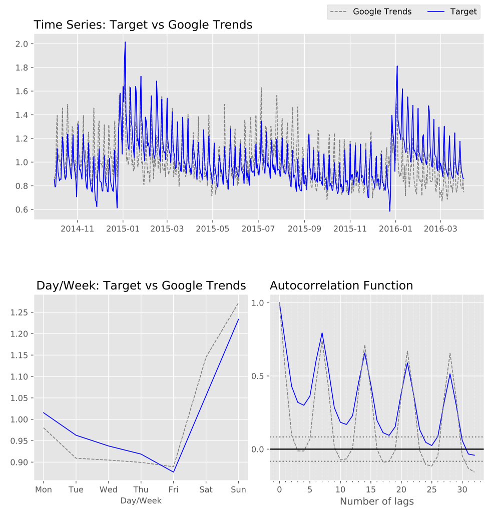

Figure 7: Visualisation of the target variable in number of responses relative to its mean and Google Trends over time for Client 3 (top) in search activity level relative to its mean; summed relative measures of target and Google Trends variable for each day of the week (bottom right); autocorrelation function for target and Google Trends over 32 lags (bottom right)

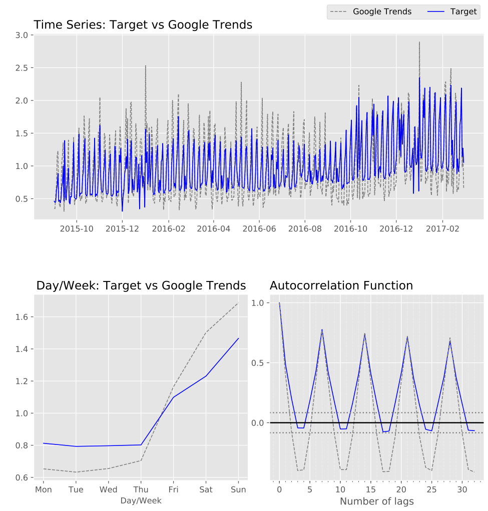

Figure 8: Visualisation of the target variable in number of responses relative to its mean and Google Trends over time for Client 4 (top) in search activity level relative to its mean; summed relative measures of target and Google Trends variable for each day of the week (bottom right); autocorrelation function for target and Google Trends over 32 lags (bottom right)

For Client 1 the full daily time series plot in Figure 5 shows high website activity around Christmas and before New Year's Eve, followed by very low activity till July where increased activity is visible. The Google Trends time series mostly follows the behaviour of the target variable. To identify potential daily seasonality the sum of website activity and Google Trends for each day of the week is plotted, shown in the bottom left graph in 5. For both variables high activity over the weekend followed by low activity over the middle of the week are visible. Here, the target variable shows a global minimum of activity of Thursday, whereas Google Trends shows two local minima on Wednesday and Friday. In the bottom right plot, the autocorrelation for 32 lags corresponding to a period of one month is plotted along 5% significance bounds around zero. The autocorrelation plot confirms the detected daily seasonal pattern.

In Figure 6 the plot of the full daily time series for Client 2 is shown. In this case the target variable indicates very different website traffic compared to the keyword searches which are reflected in the Google Trends time series. However, the target time series experiences zero activity around the end of October 2016 while Google Trends reveals low search activity from January till March 2016 and from November 2016 till January 2017. Despite Google Trends shows high volatility, the target is less volatile and does not show a specific trend or seasonal pattern over the year. For identification of daily seasonal patterns during the week the sum of web traffic and search traffic respectively is plotted for each day of the week in the bottom left graph of Figure 6. Google Trends as well as the target time series show a similar daily pattern, but differ in intensity. While Monday, Wednesday and Sunday record high activity, the rest of the days show a low activity with a global minimum on Saturday. The autocorrelation function plotted in the bottom right graph confirms a strong daily seasonal pattern combined with a slight trend indicated by the decreasing correlation with increasing number of lags. The distributions of weekend activity against weekday activity shown in A.4 overlap, which indicates no specific difference.

Figure 7 shows the time series plots of the target and Google Trends for Client 3. Several seasonal patterns can be identified. A clear yearly seasonal pattern and a daily seasonal pattern over the week are visible for both time series. With very high activity level in the beginning of the year a general downwards trend with a shift in summer is revealed in the target time series as well as the Google Trends time series. Evaluating the bottom left graph in Figure 7 the above mentioned daily seasonality can be seen, where both time series display a decreasing level of activity during the working days till Thursday but increasing activity from Friday up to a maximum on Sunday. The identified trend and seasonal patterns for both variables are confirmed by the autocorrelation plot in the bottom right graph in which every 7 lags a significant correlation is detected.

Displayed in Figure 8 the target and Google Trends time series for Client 4 in the top graph show characteristics of a strong daily pattern combined with an increasing trend. Outliers are visible in the target time series on the 24th and 31st of December where the activity level experiences sudden drops. The fluctuation of both variables are high and may indicate a strong daily seasonal pattern over the week. When evaluating the bottom left graph in which the daily data is accumulated for each day of the week, both variables show a similar pattern. Low activity from Monday to Thursday and rising activity till Sunday is visible. This very strong daily pattern over the week is confirmed in the autocorrelation graph on the bottom right graph in which a significant strong positive correlation every 7 lags is detected. The distribution of web traffic for the client grouped by weekend and weekday visualised in Figure **??** in the Appendix A show that the amount of web traffic and keyword searches via Google are higher during the weekend compared to during the working week.

6.2 Seasonality analysis

In the analysis of seasonality two modelling approaches are compared. The first follows a similar time domain based methodology used for the model on weekly time series granularity. Here,seasonal indicator models, described in Section 5.2.1, are used to capture daily seasonal patterns.The model and its results are outlined in Section 6.2.1. As the transformation from weekly to daily granularity expands the number of possible seasonal patterns, a more complex approach is taken, introduced in Section 5.2.2, and evaluated in Section 6.2.2. The model performance is analysed under the limitation of reduced input data quantity in 6.2.3 to take account for cost constraints often occurring in business environment.

6.2.1 Time domain: Seasonal indicator model

To built onto the existing procedure based in the time domain, two multiple regression models are estimated.

In the first regression model each day of the week is used as indicator variable,

$$\mathrm{GT}_t = \alpha_1 D_{1t} + ... + \alpha_k D_{kt} + \varepsilon_t = \sum_s \alpha_s D_{st} + \varepsilon_t \tag{6.1}$$

where $\mathrm{GT_t}$ denotes the activity level of keyword searches derived from Google Trends and D_{st} denotes the daily indicator variable at time $t = 1, ..., T$ with

$s \in$ (Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday) describing the index for each day of the week.

In order to take effects of special events into account, the second model follows a similar approach but includes client specific holiday indicator variables additionally to the daily indicator variables. The model can written as

$$\mathrm{GT}_t = \sum_s \alpha_s D_{st} + \sum_h \beta_h H_{ht} + \varepsilon_t \tag{6.2}$$

where additionally H_{ht} denoting the holiday indicator variable at time t with h indexing each client country specific holiday.

In order to detect potential seasonality patterns the input time series derived from Google Trends is detrended by using a one degree polynomial regression model described in the Appendix Section **??**. The model fit plotted against the detrended input time series of Model 1 is shown in Figure A.5 in the Appendix Section A. Evaluating the fitted result it is visible that Model 1 picks up the existing daily seasonal pattern over the week, where a high amplitude of the model fit indicates a strong daily seasonal pattern. However, despite of utilising a limited approach of focusing on daily seasonality this model is not flexible enough to adapt to different client input data. In Figure 9 the model fit of Model 2 indicates the significance of including holidays into the model as they are an additional source to explain the variability of the time series. Here it is important to mention that the impact of specific events depends not only on country specific holidays the client is operating in, but also depends on market specific events. A client advertising for a beauty related product might set Black Friday as an important date to consider.

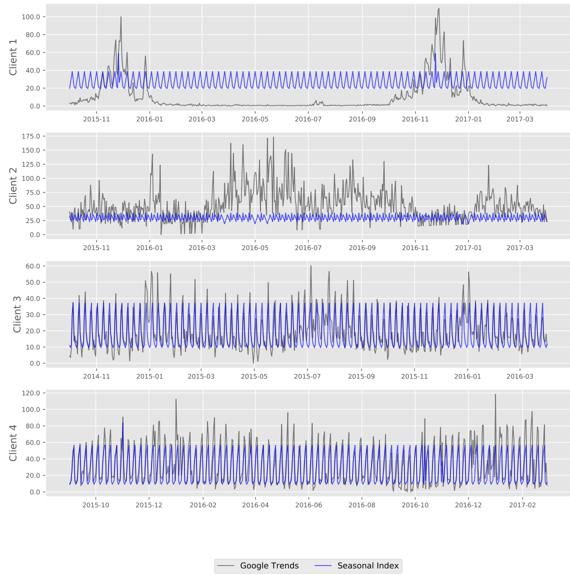

Figure 9: Model 2: multiple regression including indicators for each day of the week and client country specific holidays

6.2.2 Frequency domain: Fourier Transformation

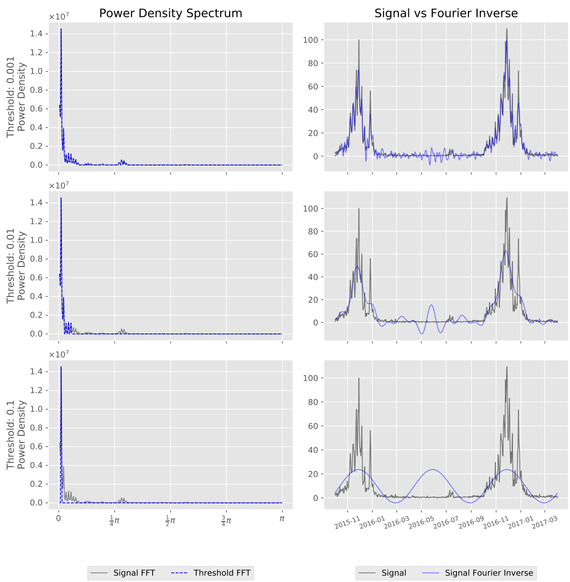

Figure 10: Client 1: Power density spectrum of pure signal and signal using variance thresholding with thresholds selecting frequencies that explain 0.1%, 1% and 10% of the signal variance (left side); signal and Fourier Inverse of the signal altered by the corresponding variance threshold

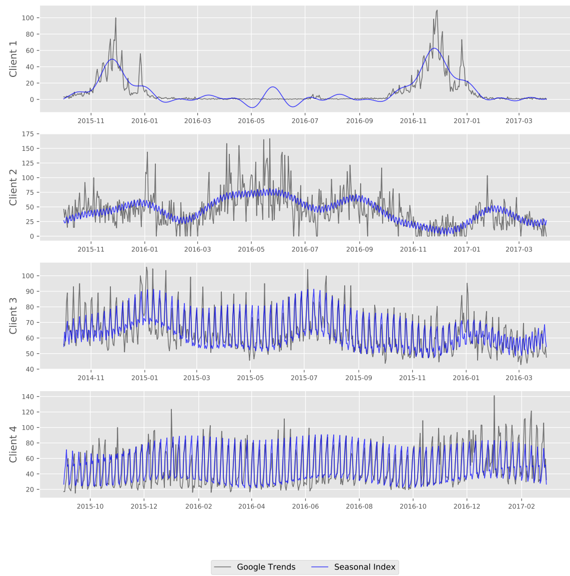

Figure 11: Model 3: Fourier Transformation with variance threshold 0.01 for all selected clients with response on the y-axis

The idea of utilising Fourier Transformation to detect multiple seasonality patterns is a very promising approach. However, in practice difficulties can be experienced when deciding on a feasible threshold for frequency selection. Here the Band-Pass-Filter using Equation 5.9 is applied.

For Client 1 Figure 10 shows on the left side plots of selected frequencies derived from the Discrete Fourier Transformation of the input signal (blue), explained in Equation 5.8, and the corresponding fitted time series derived from Equation 5.10. Several variance thresholds are applied including 0.001, 0.01 and 0.1. Doing so displays that choosing a low variance threshold drives the Band-Pass-Filter to select frequencies with a low amplitude (top left plot) leading the resulting Fourier Inverse to overfit the input signal (top right plot). Whereas, choosing a high variance threshold forces the Band-Pass-Filter to select few frequencies with very high amplitudes (bottom left plot) which leads to not detecting possibly important seasonal patterns that can bee seen in the bottom right plot. For the further analysis a variance threshold of 0.01 is selected as it minimises the root mean squared error using out-of-sample cross-validation.

The model fit of the seasonal pattern detection of Model 3 for all 4 clients is shown in Figure 11. For Client 1 the model detects the increase in search traffic from November to January including the increase in the variance from 2015 to 2016. However, the very weak daily pattern over the week occurring during the high traffic periods is not picked up by the model due to its low relevance compared over the whole time range of the data. The weak daily pattern over the week as well as a weekly trend over the year is picked up by the model for Client 2. In this case the noise around these patterns was successfully filtered by the Band-Pass-Filter. The very strong daily pattern over the week combined with a clear weekly pattern over the year is modelled for Client 3, in which not only the high daily peaks but also weak valleys are picked up by the Fourier Transformation model. A similar result is visible for Client 4.

	Client 1	Client 2	Client 3	Client 4
Model 1	0.02	0.05	0.60	0.73
Model 2	0.11	0.10	0.62	0.83
Model 3	0.78	0.41	0.66	0.65

Table 2: R^2 values from all models for all clients

The model fit of all 3 models is compared in Table 2. Here, a clear improvement from Model 1 to Model 3 can be seen for clients 1,2 and 3. However, Model 3 seems to worsen the model fit for Client 4. In this case using Model 2, which includes indicator variables for each day of the week and client country specific holiday indicator, results in a better model fit compared to the more complex Fourier Transformation method implemented in Model 3.

To summarise, it can be said that including client specific events in the model improves the model fit as it contributes to explaining the variance of the given time series. The implementation of seasonal pattern detection in the frequency domain results in a more flexible model picking up the most relevant seasonal patterns. However, this approach highly depends on the choice of the variance threshold.

6.2.3 Stability evaluation

One major constraint of any kind of modelling in an industrial or commercial environment is low data availability. In order to analyse the influence of changing amounts of input data the 3 proposed season detection models are evaluated on 3 different time ranges.

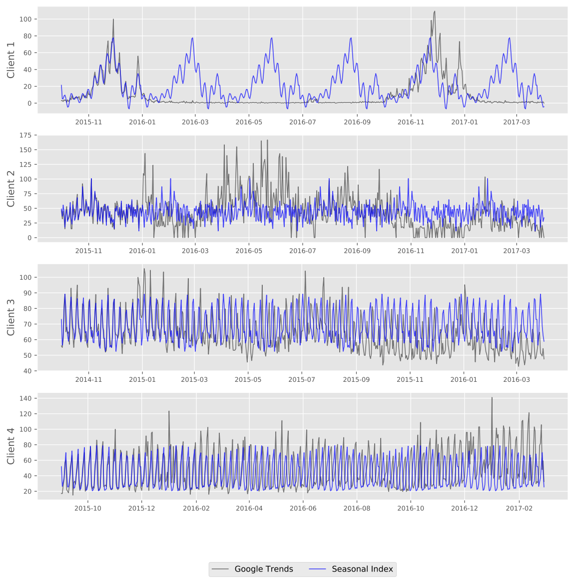

Figure 12: Model 3: Fourier Transformation with variance threshold 0.01 for all clients with 90 days of input data

Concluding from Table A.2 in Appendix A each model shows different characteristics of model fit when comparing the R^2. Model 1 and Model 2 perform best with 30 days of training data over all 4 clients. This can be explained be their simplicity and limitation to daily seasonal pattern detection. Whereas, Model 3 due to its capability of detecting multiple seasonal patterns show its best performance with the highest number days of training data. In this case 365 days. Additionally, the special characteristics of client data need to be taken into consideration. While Client 1 shows high search traffic over few months in winter in which it is easy to capture daily seasonal patterns for Model 1 and Model 2, this pattern is difficult to capture by Model 3 due to its low frequent behaviour. This phenomenon can be observed in Figures A.10 - A.19 in Appendix A. In Figure 12 the model fit of Model 3 is displayed with a variance threshold of 0.01 for all 4 clients using a training data period of 90 days. In this setting Model 3 is able to capture the daily and monthly seasonal patterns for Client 3 and Client 4.

However, due to the infrequent search traffic of Client 2 the model fit shows low accuracy in the the period following the one it is trained on. Nevertheless, for Client 1 this model layout is able to capture the high activity during the winter months in 2015 and it can be seen that this estimated pattern can be predicted for the same time range in 2016. This setting will be used amongst others for the implementation into the framework model in the upcoming Section 6.3.

6.3 Framework model comparison

Based on the framework model given in Equation 5.3 the following 7 models are estimated. The models are described by

$$\text{Model 1:}\quad \text{Target}_t^{365} = \alpha + \beta_1 \text{TV}(t, v^*) + \beta_2 \text{S}^{\text{Fourier}}(t) + \beta_3\ \text{T}^{\text{Linear}}(t) + \varepsilon_t \tag{6.3}$$

$$\text{Model 2:}\quad \text{Target}_t^{365} = \alpha + \beta_1 \text{TV}(\text{t}, \text{v}^*) + \beta_2 \text{S}^{\text{Fourier}}(\text{t}) + \beta_3\ \text{T}^{\text{Linear}}(\text{t}) + \beta_4 \text{H}(\text{t}) + \varepsilon_{\text{t}} \tag{6.4}$$

$$\text{Model 3:}\quad \text{Target}_t^{90} = \alpha + \beta_1 \text{TV}(\text{t}, \text{v}^*) + \beta_2 \text{S}^{\text{Fourier}}(\text{t}) + \beta_3\ \text{T}^{\text{SG}}(\text{t}) + \varepsilon_{\text{t}} \tag{6.5}$$

$$\text{Model 4:}\quad \text{Target}_t^{90} = \alpha + \beta_1 \text{TV}(\text{t}, \text{v}^*) + \beta_2 \text{S}^{\text{Fourier}}(\text{t}) + \beta_3\ \text{T}^{\text{SG}}(\text{t}) + \beta_4 \text{H}(\text{t}) + \varepsilon_{\text{t}} \tag{6.6}$$

$$\text{Model 5:}\quad \text{Target}_t^{365} = \alpha + \beta_1 \text{TV}(\text{t}, \text{v}^*) + \beta_2 \text{S}^{\text{Indicator-Day}}(\text{t}) + \beta_3\ \text{T}^{\text{SG}}(\text{t}) + \varepsilon_{\text{t}} \tag{6.7}$$

$$\text{Model 6:}\quad \text{Target}_t^{365} = \alpha + \beta_1 \text{TV}(\text{t}, \text{v}^*) + \beta_2 \text{S}^{\text{Indicator-Day-Holiday}}(\text{t}) + \beta_3\ \text{T}^{\text{SG}}(\text{t}) + \varepsilon_{\text{t}} \tag{6.8}$$

$$\text{Model 7:}\quad \text{Target}_t^{365} = \alpha + \beta_1 \text{TV}(\text{t}, \text{v}^*) + \beta_2 \text{S}^{\text{Indicator-Day}}(\text{t}) + \beta_3\ \text{T}^{\text{SG}}(\text{t}) + \beta_4 \text{H}(\text{t}) + \varepsilon_{\text{t}} \tag{6.9}$$

where Target^{365} denotes the web response time series with the seasonal index derived from 365 days of Google Trends data and Target^{90} denotes the web response time series with the seasonal index derived from 90 days of Google Trends data. The positive constant is here denoted as α. $\mathrm{TV}(t, v^*)$ represents a function to derive adstock time series described in Equation 5.4 with v^* being the adstock value chosen based on selection criteria described in Section 5.1. The seasonal index derived by the function $\mathrm{S}^{\text{Fourier}}(t)$ which is based on the Fourier Transformation described in Section 5.2.2 uses a variance threshold of 0.01. Whereas, the seasonal index $\mathrm{S}^{\text{Indicator-Day}}(t)$ is derived from the indicator model described in Equation 6.1. $\mathrm{S}^{\text{Indicator-Day-Holiday}}(t)$ represents a function to derive the seasonal index based on indicator model described in Equation 6.2. The linear trend index is calculated with the function $\mathrm{T}^{\text{Linear}}(t)$ which is described in Section 5.1 and the function $\mathrm{T}^{\text{SG}}(t)$ computes the trend index based in Savitzky-Golay filter described in the Appendix Section **??**. $\mathrm{H}(t)$ describes a function to derive holiday indicator index derived from method based on Equation 5.6.

The proposed models differ in 4 characteristics. First, two different amounts of data to derive the seasonal index from are used in order to account for the business constraint of low data availability.

Second, the methods used to compute the seasonal index vary in combination with the trend derivation method. Here, the limited seasonal indicator approach is combined with the flexible Savitzky-Golay filter deriving the trend index. Whereas, the flexible Fourier Transformation approach to obtain the seasonal index is combined with a linear first degree polynomial fit to calculate the trend index capturing the overall business growth.

Third, the above mentioned combinations are tested with and without including a holiday index in the framework model in order to capture its impact on the comparison metrics, described in the following.

For each client the above described models are estimated and comparison metrics computed including R^2 as a measure of model fit, Adstock as an important measure to evaluate the ling term effect of TV advertisement on customer response, TV-Factor as an important measure to evaluate the long term intensity of the initial response leveraged by TV advertisement, the coefficients of the corresponding independent variables to evaluate the influence of the input data, the total contribution of the independent variable towards the model fit and the corresponding t-value as a measure of its significance.

The resulting metrics for Client 1 are shown in the Appendix C Table A.3. Here, all estimated models show a high model fit. The Adstock chosen is either 0.5 for the models using a linear trend index and 0.6 for models using the Savitzky-Golay filter for trend derivation which can be interpreted as toady's TV spot leads to a 60% decrease of toady's initial web response. The estimated TV-Factor is with 3.91 higher for models using the Savitzky-Golay filter meaning that the total long term impact of toady's TV spot measured in numbers of web responses is 3.91 times higher compared to the initial level of web response. As already seen in the exploratory data analysis in Section 6.1, the TV attributed response time series of Client 1 matches closely the pattern of the target response time series. Therefore, it is not surprising that the transformed TV attributed response variable using the above mentioned Adstock still leads to a high contribution for all models ranging from 59.93% to 65.18%. Another important result should be mentioned here. For models utilising the Savitzky-Golay filter for obtaining the trend, the corresponding seasonal index is insignificant and does not play a role in explaining the model fit. Although, all other included variables play a significant role with t-values$\gg 2.0$. When comparing the models under these metrics, Model 7 is selected as the best performing one for this client.

The exploratory analysis of the data from Client 2 already suggests difficulties when modelling the seasonal index as the pattern of Google Trends does not match with the web response time series. This phenomenon is confirmed when evaluating the resulting model metrics displayed in Table A.4 in the Appendix Section C, where Model 1, Model 2 and Model 3 cannot be considered further due the very low R^2 values of 0.02, 0.1 and 0.37 respectively. For the remaining models it can be seen that the contribution of the TV variable is very low and therefore does not support the model fit to a high degree. However, this detail influences the important measures which leads to a low Adstock of 0.1 and low TV significance. Most of the model fit can be explained by the daily indicator seasonal index in combination with the Savitzky-Golay trend index. The overall best performing model under such circumstances is Model 7, which includes a significant holiday index leading to an Adstock of 0.1.

When estimating the proposed models on data showing clear daily seasonality over the week combined with weekly seasonality over the year, as shown for Client 3, the resulting metrics can be evaluated in the Appendix Section C Table A.5. Here, Model 1 and Model 2, both implementing the Fourier Transformation to derive the seasonal index in combination with a linear trend index, show a better model fit. Nevertheless, the corresponding daily Adstock with 0.15 and 0.2 respectively, is very low. When evaluating the model metrics it is important to notice, that Model 3 and Model 4 which include the Fourier Transformation seasonality approach combined with the Savitzky-Golay trend index trained on 90 days of input data, do not result in significant seasonality indices. Whereas, the opposite result can be found for Model 5, Model 6 and Model 7. Looking at the overall best performance of the proposed models, Model 7 is selected.

Metrics	**Client 1**	**Client 2**	**Client 3**	**Client 4**
R^2	0.94	0.93	0.74	0.84
v^*	0.50	0.10	0.40	0.30
TV-factor	7.83	1.91	1.66	3.86
TV-coefficient	3.91	1.72	0.99	2.70
TV-contribution	64.97	1.37	26.70	4.55
TV t-value	81.55	3.70	27.34	6.26
S-coefficient	0.00	75.33	131.16	835.90
S-contribution	0.00	72.33	19.86	34.75
S t-value	0.00	43.99	30.96	33.63
T-coefficient	4 022.12	8 008.87	14 796.46	74 579.30
T-contribution	38.37	25.43	11.62	54.11
T t-value	20.42	24.15	23.36	49.44
H-coefficient	0.05	0.06	0.21	0.36
H-contribution	0.19	1.16	0.47	0.92
H t-value	1.35	2.67	13.07	12.64
Confidence	0.90	0.80	0.75	0.60

Table 3: Model 7: model metrics of selected model for all 4 clients

For Client 4, the results of the model show a very different situation which lead to the same conclusion. Although, obtaining a high model fit for all estimated models the results for the other metrics are quite different. Notably is that the models which implement the Fourier Transformation approach to compute the seasonal index lead to an Adstock of 0.00. This makes them difficult to recommend for the future implementation into the product. For the other 3 models it is important to notice that despite the fact that the adstock transformed TV time series does not contribute to the model fit on a high level, it still leads to Adstock values and TV-Factors up to 0.30 as well as 4.74 respectively. When looking at the overall performance, Model 7 is selected, although Model 5 and Model 6 show similar results.
The model best performing under the given circumstances of different client data is Model 7. The model metrics computed for all 4 clients are displayed in Table 3. Several important differences are noticeable. Looking at the two important measures of interest, Adstock and TV-Factor, the models obtain very different estimates ranging from Adstock values from 0.10 to 0.50 as well

as TV-Factor values from 1.91 to 7.83. These very different outcomes can be attributed to the very different patterns in the input data the models very estimated on. The results show, that although the same model specification were used, the outcome is highly influenced by the input data, which needs to be taken into consideration when using the results for decision making in business context.

6.4 Translation: weekly to daily granularity

In the previous step an appropriate model obtaining valid estimates for the daily Adstock value as well as daily TV-Factor was found. When transforming from weekly data granularity to daily granularity it is crucial to compare both results in order to understand their relation so that the changes can be explained to the client.

In theory it is expected that the TV-Factor stays approximately the same when estimated on weekly compared to daily level. In Equation 6.10, the relation between the TV-Factors based on different data granularity is displayed.

$$\text{Factor}_{weekly}^{TV} = \frac{\text{coefficient}_{\text{weekly}}^{\text{TV}}}{1 - \text{Adstock}_{\text{weekly}}} \approx \frac{\text{coefficient}_{\text{daily}}^{\text{TV}}}{1 - \text{Adstock}_{\text{daily}}} = \text{Factor}_{\text{daily}}^{\text{TV}} \tag{6.10}$$

Despite the expected change in the selected Adstock as well as the estimated TV-coefficient, the Uplift Factor of the corresponding TV spots should remain the same, assuming that TV spot specific characteristics like spot length, creative, medium on which the spot was aired do not alter. Under this assumption the relation between both Adstock value is straight forward. Given the definition of weekly Adstock from Table 1, the daily Adstock needs to be multiplied 7 times by the initial web response amount corresponding to the each day of the week, in order to reach the same level of web response achieved by the weekly Adstock. In Equation 6.11 this theoretical translation is shown. It is clearly visible that the daily Adstock values are expected to be higher than its weekly pedant.

$$\text{Adstock}_{\text{daily}} = (\text{Adstock}_{\text{weekly}})^{\frac{1}{7}} \tag{6.11}$$

When applying this translation to the estimated values, $\text{Adstock}_{\text{weekly}}^{\text{model}}$, the derived theoretical values, $\text{Adstock}_{\text{daily}}^{\text{theory}}$, show deviations. Presented in Table 4, the estimated Adstock values derived from the selected model based on daily granularity are greater than the weekly Adstock values. However, the expected level derived from the theoretical translation is not met. This can be explained by differences in the model settings applied when selecting the model with the best Adstock value.

Client	$\text{Adstock}_{\text{weekly}}^{\text{model}}$	$\text{Adstock}_{\text{daily}}^{\text{theory}}$	$\text{Adstock}_{\text{daily}}^{\text{model}}$	$\text{Factor}_{\text{weekly}}^{\text{model}}$	$\text{Factor}_{\text{daily}}^{\text{model}}$
Client 1	0.20	0.79	0.50	5.94	7.83
Client 2	0.80	0.97	0.90	3.51	3.13
Client 3	0.00	0.00	0.35	1.02	1.63
Client 4	0.05	0.65	0.05	2.97	4.05

Table 4: Comparison: daily against weekly Adstock and TV-Factor

The underlying assumption, stating that despite different time granularity the TV-Factors remain approximately the same, cannot be confirmed by the results. In this case, further research on testing this hypothesis needs to be carried out on a greater number of client data sets, in order to obtain a confident result.

7 Models and results: web conversions

The method based on web responses proposed in the previous sections, is an approach to measure the general relation between TV advertisement and customer response. However, the interest of businesses in justifying their return on marketing investment is increasing. [reference] Therefore, trying to not just understand the relation between TV advertisement and web response, but furthermore to analyse the relation between TV advertisement and web conversions such as sales, log ins or downloads, is the next step. The exploratory data analysis in Section 7.1 summarises the main characteristics of the data provided by the previous 4 clients and the previously derived model framework is applied in Section 7.2.

7.1 Exploratory data analysis

Plotting the conversions time series for each client reveals very similar patterns to the ones seen in Section 4.

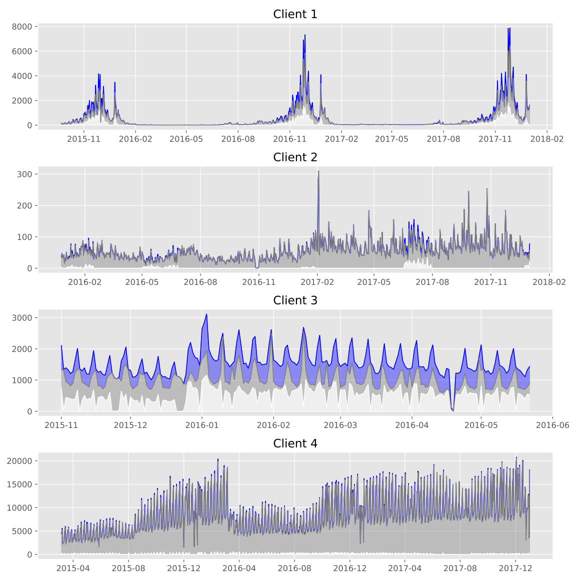

Figure 13: Conversions: target conversions (blue), TV attributed conversions (white) and non TV attributed conversions (grey) time series for all clients with the number of conversions on the y-axis

Due to the characteristics of the product, the conversions of Client 1 experience high activity level from October to February with a small peak in July and very low to zero activity during the other months, shown in Figure 13. Important to notice here is that the TV attributed initial conversions time series in white matches the pattern of the target conversions time series almost exactly. Learned from Section 6, this characteristics of the data seems to have a high influence on the performance and resulting metrics of the framework model. A different behaviour can be seen for the conversions time series provided by Client 2. Compared to the web response time series, the target conversions (blue) seem not to follow a specific seasonal pattern. However, it needs to be noticed that the mean level of conversions increases after experiencing a big spike in February 2017. When looking at the TV attributed conversions (white) only in the periods of January to February 2016 and May to June in 2016 and 2017, a very low number of initial conversions were attributed to TV advertisement. Having such low quantity of data to derive the impact of TV advertisement from, may lead to unreliable model results when applying the framework model. In contrary to Client 2, the provided time series from Client 3 show a very clear daily seasonal pattern over the week in combination with a decreasing yearly trend. Here, all 3 time series experience a big drop of conversion numbers in the second half of April 2016. Although the TV attributed conversions (white) mirror the daily seasonal pattern, the the repeating steep drops on Mondays recover quickly over the rest of the week. A clear upwards trend is visible for the target conversions (blue) time series for Client 3. However, a strong shift change takes place from April 2016 which could be explained by the acquisition of a national operating competitor in this period. The strong daily seasonality in the target conversions is mirrored in from the web response data from this client. The most important insight from this plot is, that the TV attributed conversions are very low to zero in this case which will make it difficult for the model to attributed any long term effect to TV advertisement.

7.2 Framework model results

In order to analyse its behaviour on a different set of data, the framework Model 7, described in Equation 6.9, is applied. In Table 5 the corresponding model metrics are shown. Overall, the result that stands out is the Adstock value of 0.00 selected for Client 1, Client 2 and Client 4. This outcome is closely linked to the low quantity of TV attributed conversions data the model had to deal with, which was already visible in the exploratory data analysis in Section 7.1. As a very low number of conversions are attributed to TV, the few existing conversions transformed with the Adstock values do not have an impact on a better model fit. As an Adstock value of 0.00 is selected, the TV-Factor equals the TV-coefficient due to their relation described in 5.5. In the case of Client 3 the quantity for the TV attributed conversions data is higher. Therefore, the model was able to relate the long term effect of the initial attributed conversions to the target conversions.

Metrics	**Client 1**	**Client 2**	**Client 3**	**Client 4**
R^2	0.94	0.85	0.79	0.96
Adstock	0.00	0.00	0.60	0.00
TV-Factor	3.55	0.98	1.13	1.78
TV-coefficient	3.55	0.98	0.45	1.78
TV-contribution	72.38	2.92	34.96	3.07
TV t-value	80.48	4.24	8.67	3.46
S-coefficient	1.89	0.48	28.02	112.74
S-contribution	7.72	53.80	36.42	40.52
S t-value	2.40	17.02	13.28	34.37
T-coefficient	108.94	58.53	4 440.98	8 986.59
T-contribution	20.23	40.17	14.48	55.62
T t-value	7.42	16.85	1.56	60.59
H-coefficient	0.14	0.17	0.00	0.31
H-contribution	0.77	3.36	0.00	0.89
H t-value	3.97	4.55	0.00	9.96
Confidence	0.85	0.00	0.46	0.60

Table 5: Conversions: comparison metrics of the selected model for all clients

Looking at the contribution of the TV attributed conversions towards the model fit, the results confirm the above described strong impact of the quantity of the input data. However, it shows that care needs to be taken, as a high contribution of TV attributed conversions indicates a possible over attribution of TV impact. It is intuitive and visible in Figure 13. When the TV attributed conversions time series closely matches the shape of the target conversions time series, there is a risk of over-attributing conversions to the TV advertisement. When evaluating the significance metrics, all variables are significant with $|t - value| > 1.96$ but for Client 3. In this case the trend index indicates to be insignificant with $|t - value| = 1.56 < 1.96$. In addition, with having a negative coefficient the holiday indicator variable is not included in the model.

To summarise, applying the model derived from web responses onto a different data set including conversions, 3 major conclusions can be drawn. First, as expected the low input data quantity leads to the selection of an Adstock value of 0.00 as the model has not enough data for estimation. Second, already discovered in Section 6, the shape of the TV attributed data highly impacts the contribution of this variable towards the model fit. Here, a high contribution indicates a possible over attribution of conversions towards the TV advertisement. Third, with this modelling approach it is possible to account for significant daily seasonality, a flexible general business trend and client specific holidays.

8 Conclusion

The research question this work set to answer is about the changes that are encountered when moving from a weekly to a daily data granularity in the framework of TV advertisement attribution practised by TVSquared Ltd.
The approach taken is focused on comparing two different methods for modelling seasonality patterns using 4 given sets of client data . The first method is based in the time domain utilising a set of seasonal indicator variables for each day of the week and a set indicator variables for each client country specific holiday. The second method is based in the frequency domain. Here, the signal processing method of Fourier Transformation is used in combination with a Band-Pass filter in order to select frequencies that drive the seasonal pattern. Fitting the 3 models reveals that the time domain based model which includes holiday indicator variables outperforms the model without them. However, the model based in the frequency domain is able to capture multiple seasonal patterns which results in a better model fit as the indicator variable models are limited to daily seasonality. Although, when testing on different input data quantity, the time domain based models pick up the daily seasonality better with lower data quantity compared to the frequency domain model. Following the seasonality analysis, 7 different framework models that aim to model the relation between TV advertisement and web responses are fitted to the 4 given data sets. Each model incorporates one of the proposed seasonal methods to derive a seasonality index, a related trend method to derive a trend index and the TV attributed web response time series transformed with a selected adstock value. In order to analyse the influence of holidays in the framework model, for some models a client country specific holiday index is added. Based on predefined model metrics, Model 7, described in Equation 6.9, performs best compared over all 4 client data sets. This result is surprising, as the favoured frequency domain based seasonal method did not outperform the limited time domain based methods when implemented in the framework model. Due to the difference in granularity, the measure of interest, namely the adstock value, is analysed further. Here, a theoretical relation is introduced, which suggests a greater daily adstock derived from the new proposed methods in comparison to the weekly adstock value derived from the current approach. The results of this transformation show that this theory is confirmed. However, the theoretical value is not fulled reached. This can be explained with slightly different model settings in the weekly model compared to the daily. In order to evaluate the behaviour of the new selected model, it is applied to different client provided data sets incorporating web conversions instead of web responses. Here, a major difference is that the quantity of TV attributed web conversions is much lower compared to TV attributed web responses. Therefore it is expected that the selected framework model experiences difficulties in capturing a significant long term effect of TV advertisement. The resulting model metrics confirm this hypothesis, as an adstock value of 0.00 is chosen. Taking the given framework of modelling the relation between TV advertisement and web responses into consideration, it can be seen that changing the data granularity from weekly to daily level increases the data quantity but comes with several important aspects to consider. Although this change leads to multiple seasonality patterns in the data, the proposed simple approach outperforms methods of higher complexity. This brings the advantages of easier explainability to existing and potential clients in a commercial environment. Another aspect to consider is, that the preferred outcome measures need to be translated which additionally needs to be explained to existing and potential clients.
In conclusion, this analysis is a first step towards getting a better understanding of the differences being encountered when changing from weekly to daily data granularity in a commercial modelling environment. However, further research needs to be done in testing the new proposed method in different settings and on different clients in order to verify its stability and

consistency.

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Appendices

A Models and results

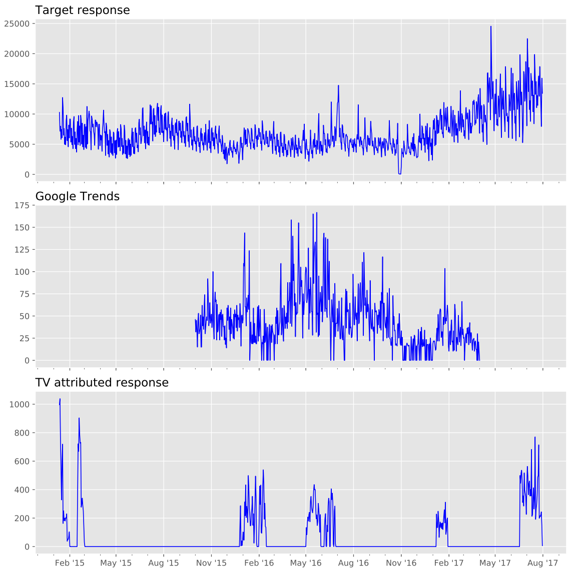

Figure A.1: Client 2 - time series of target response, Google Trends and TV attributed response

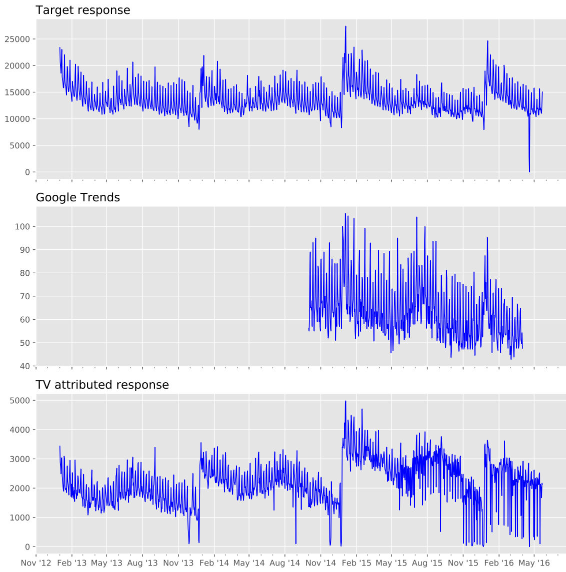

Figure A.2: Client 3 - time series of target response, Google Trends and TV attributed response

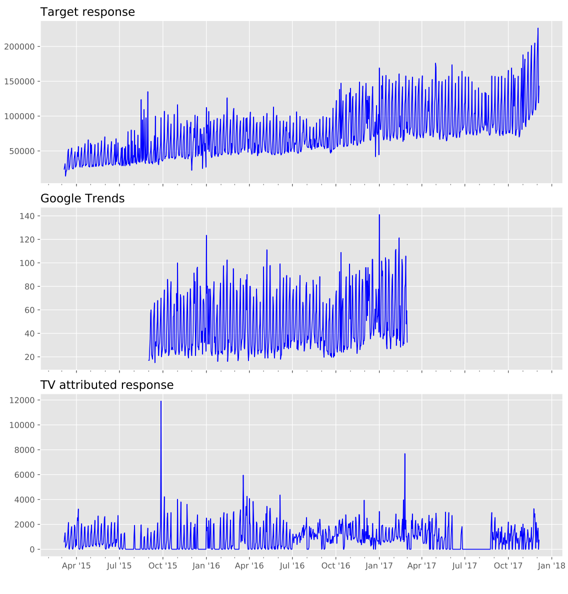

Figure A.3: Client 4 - time series of target response, Google Trends and TV attributed response

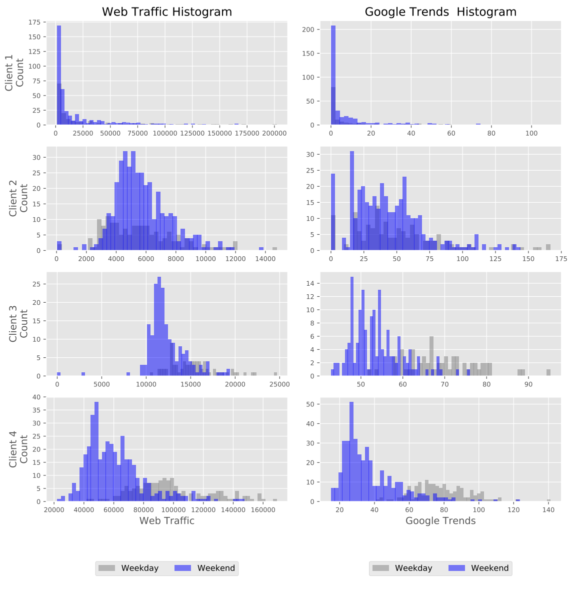

Figure A.4: Histograms of target variable and Google Trends separated by weekday and weekend

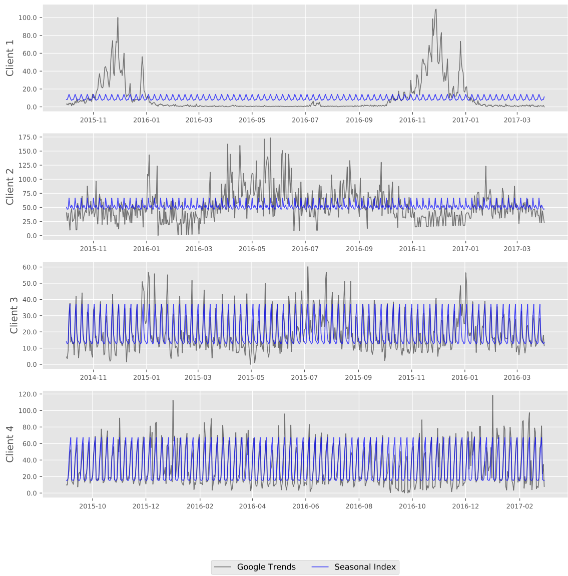

Figure A.5: Model 1: model fit of multiple regression including indicators for each day of the week plotted against the initial input time series

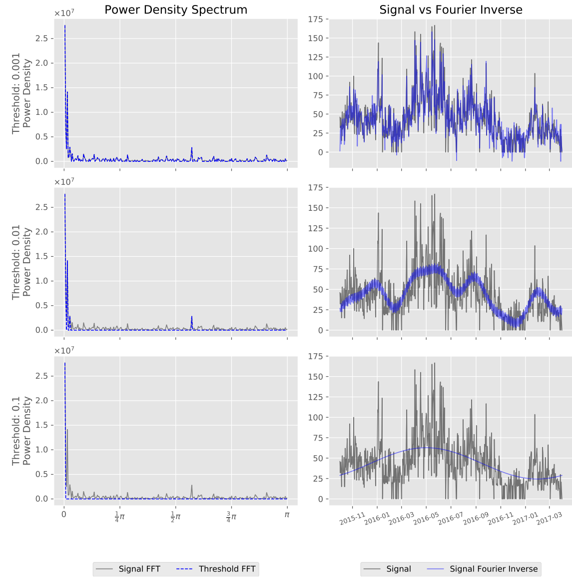

Figure A.6: Client 2 - Power density spectrum of pure signal and signal using variance thresholding with thresholds selecting frequencies that explain 0.1%, 1% and 10% of the signal variance (left side); signal and Fourier Inverse of the signal altered by the corresponding variance threshold described in Section 6.2.2

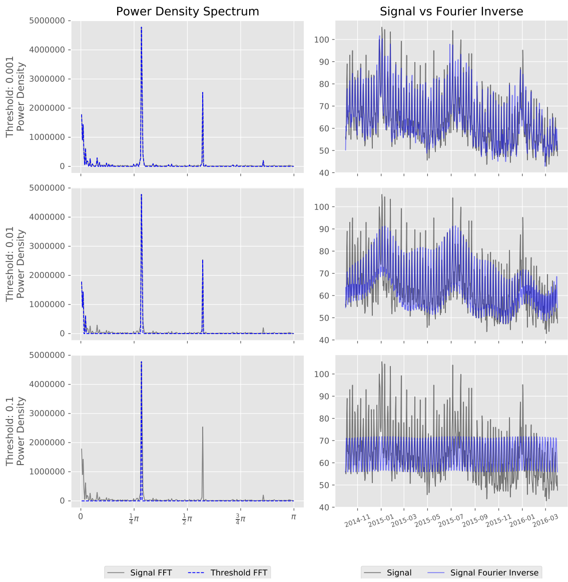

Figure A.7: Client 3 - Power density spectrum of pure signal and signal using variance thresholding with thresholds selecting frequencies that explain 0.1%, 1% and 10% of the signal variance (left side); signal and Fourier Inverse of the signal altered by the corresponding variance threshold described in Section 6.2.2

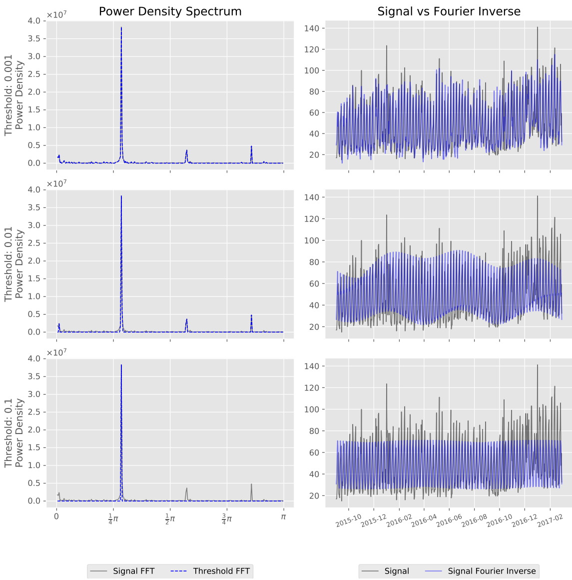

Figure A.8: Client 4 - Power density spectrum of pure signal and signal using variance thresholding with thresholds selecting frequencies that explain 0.1%, 1% and 10% of the signal variance (left side); signal and Fourier Inverse of the signal altered by the corresponding variance threshold described in Section 6.2.2

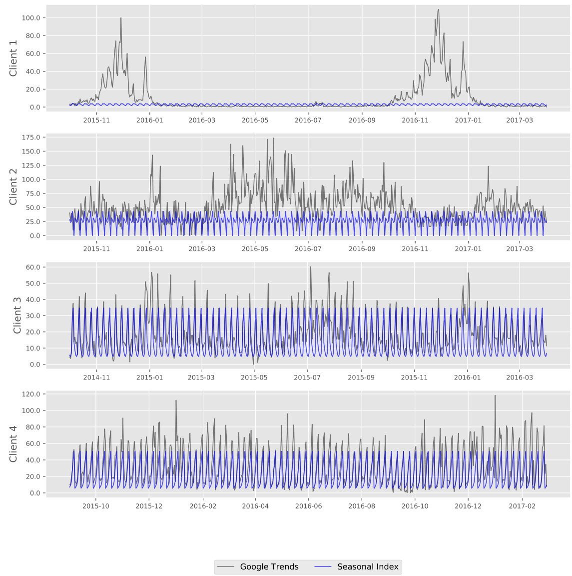

Figure A.9: Model 1: Fourier Transformation with variance threshold 0.01 for all clients with 30 days of input data

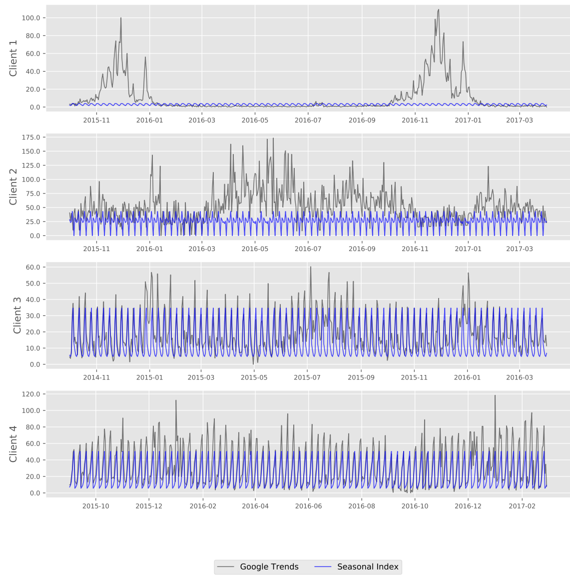

Figure A.10: Model 2: Fourier Transformation with variance threshold 0.01 for all clients with 30 days of input data

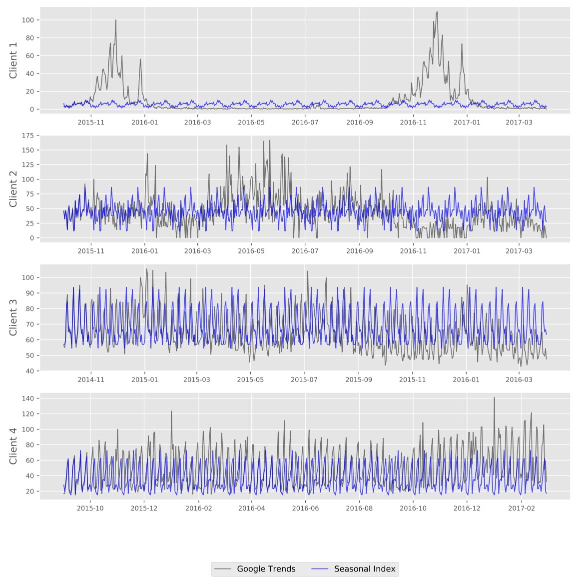

Figure A.11: Model 3: Fourier Transformation with variance threshold 0.01 for all clients with 30 days of input data

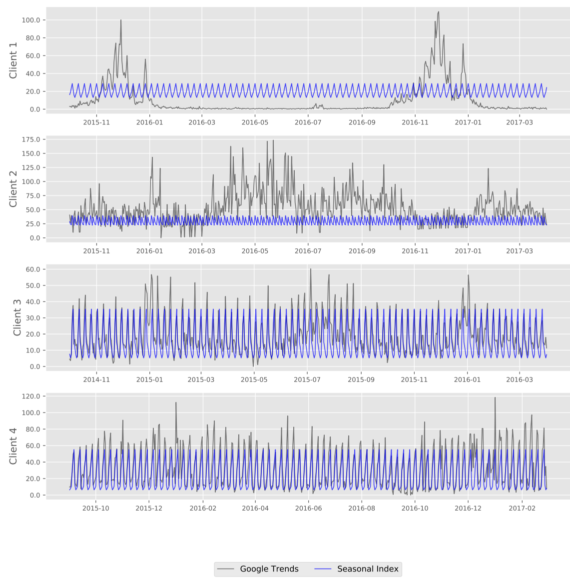

Figure A.12: Model 1: Fourier Transformation with variance threshold 0.01 for all clients with 60 days of input data

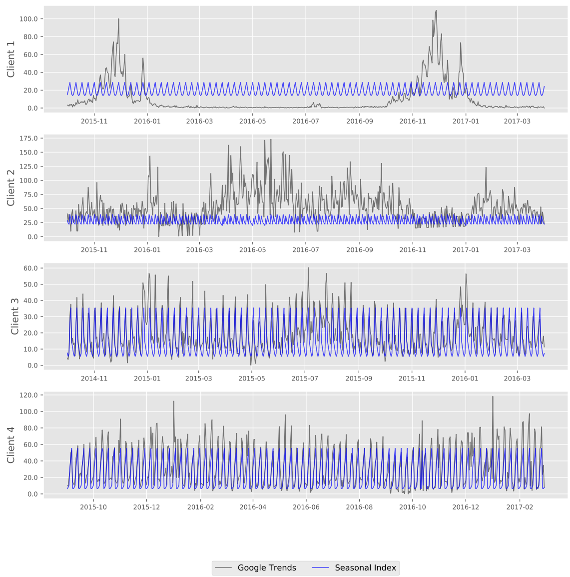

Figure A.13: Model 2: Fourier Transformation with variance threshold 0.01 for all clients with 60 days of input data

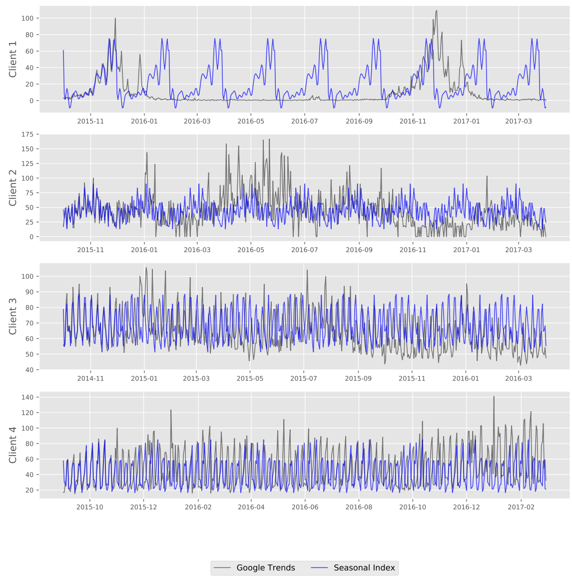

Figure A.14: Model 3: Fourier Transformation with variance threshold 0.01 for all clients with 60 days of input data

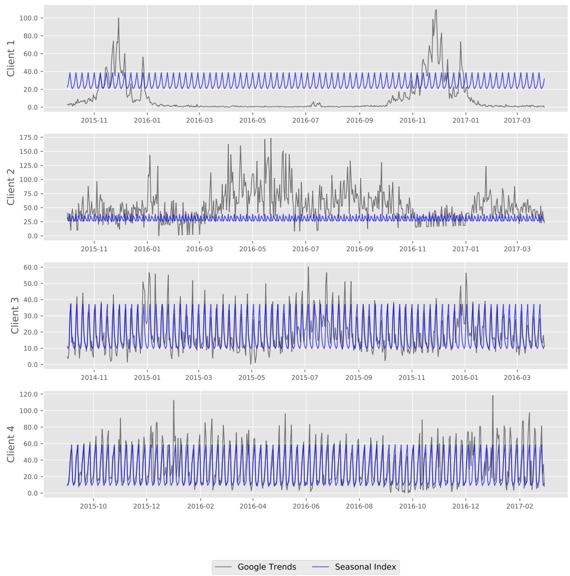

Figure A.15: Model 1: Fourier Transformation with variance threshold 0.01 for all clients with 90 days of input data

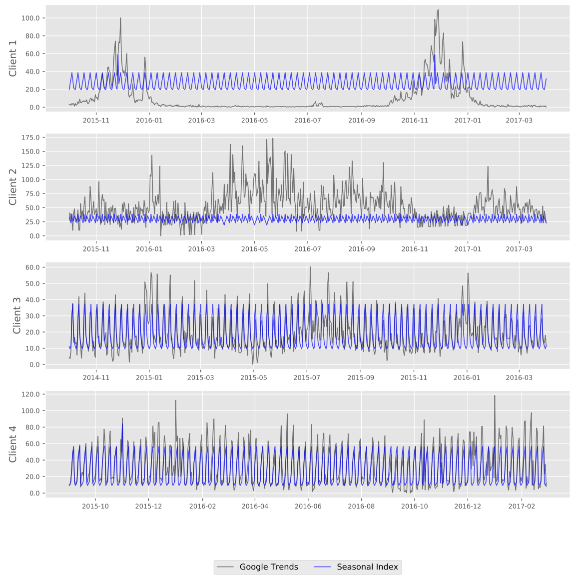

Figure A.16: Model 2: Fourier Transformation with variance threshold 0.01 for all clients with 90 days of input data

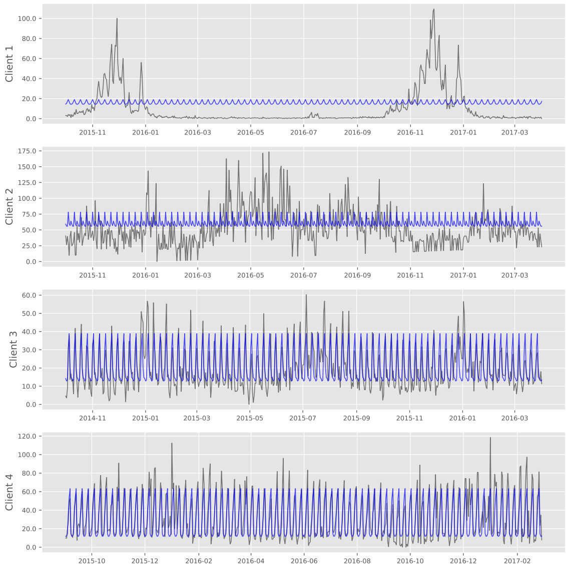

Figure A.17: Model 1: Fourier Transformation with variance threshold 0.01 for all clients with 365 days of input data

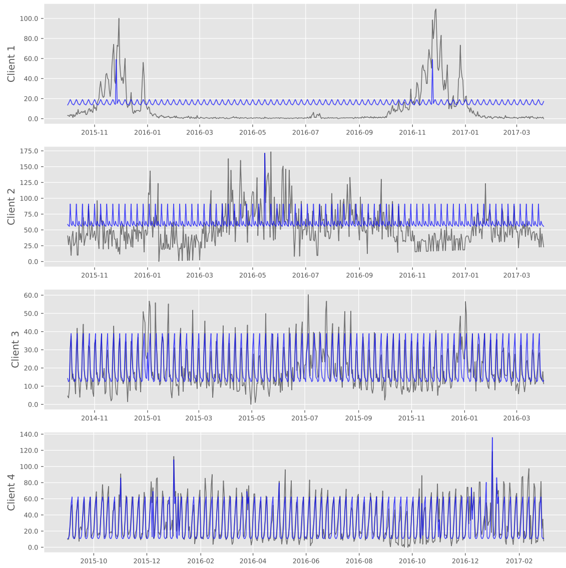

Figure A.18: Model 2: Fourier Transformation with variance threshold 0.01 for all clients with 365 days of input data

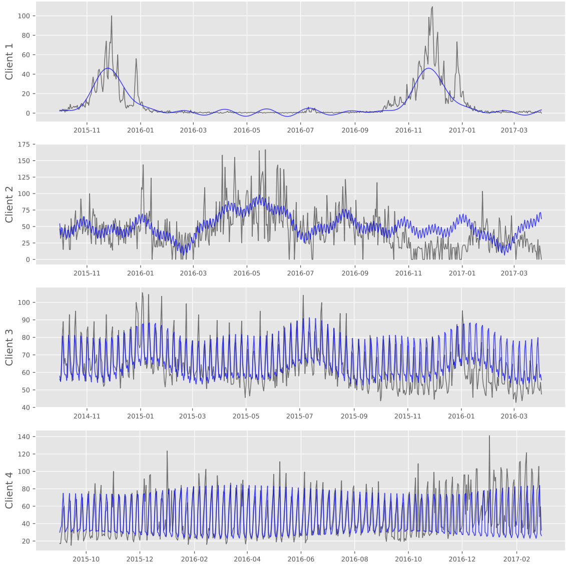

Figure A.19: Model 3: Fourier Transformation with variance threshold 0.01 for all clients with 365 days of input data

Metrics	Client 1	Client 2	Client 3	Client 4
R^2	0.95	0.84	0.74	0.95
v^*	0.20	0.80	0.00	0.05
TV-Factor	5.94	3.51	1.02	2.97
TV-coefficient	4.75	0.70	1.02	2.82
TV-contribution	50.09	2.63	16.48	3.34
TV t-value	9.30	2.69	7.69	2.62
S-coefficient	48 684.80	6 176.59	20 548.30	77 039.40
S-contribution	37.34	12.60	21.22	14.27
S t-value	4.78	3.63	9.31	5.40
T-coefficient	0.21	0.98	1.16	0.99
T-contribution	19.29	28.38	11.68	55.89
T t-value	5.06	23.81	13.11	48.33

Table A.1: Current weekly model: model metrics for all clients

Training data: number of days	**Model**	**Client 1**	**Client 2**	**Client 3**	**Client 4**
30	Model 1	0.28	0.26	0.89	0.94
	Model 2	0.31	0.26	0.89	0.94
	Model 3	-0.08	-0.35	-0.89	-0.64
60	Model 1	0.17	0.13	0.86	0.83
	Model 2	0.19	0.13	0.87	0.91
	Model 3	-1.59	-0.25	-0.76	-0.65
90	Model 1	0.10	0.08	0.73	0.84
	Model 2	0.16	0.11	0.73	0.90
	Model 3	-1.00	-0.17	-0.85	-0.71
365	Model 1	0.02	0.06	0.62	0.78
	Model 2	0.07	0.11	0.65	0.87
	Model 3	0.72	0.21	0.40	0.43

Table A.2: R^2 values for model training data periods of 30,60,90 and 365 days

Metrics	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
R^2	0.93	0.93	0.94	0.94	0.94	0.94	0.94
v^*	0.60	0.60	0.50	0.50	0.50	0.50	0.50
TV-factor	7.24	7.20	7.85	7.83	7.85	7.85	7.83
TV-coefficient	2.90	2.88	3.93	3.91	3.93	3.93	3.91
TV-contribution	60.27	59.93	65.18	64.97	65.18	65.18	64.97
TV t-value	31.66	31.38	83.40	81.55	83.40	83.40	81.55
S-coefficient	346.89	348.30	0.00	0.00	0.00	0.00	0.00
S-contribution	13.29	13.34	0.00	0.00	0.00	0.00	0.00
S t-value	5.83	5.86	0.00	0.00	0.00	0.00	0.00
T-coefficient	0.22	0.22	4 020.39	4 022.12	4 020.39	4 020.39	4 022.12
T-contribution	26.49	26.49	38.35	38.37	38.35	38.35	38.37
T t-value	13.50	13.52	20.40	20.42	20.40	20.40	20.42
H-coefficient	0.00	0.07	0.00	0.05	0.00	0.00	0.05
H-contribution	0.00	0.27	0.00	0.19	0.00	0.00	0.19
H t-value	0.00	1.82	0.00	1.35	0.00	0.00	1.35
Confidence	0.70	0.80	0.80	0.80	0.90	0.90	0.90

Table A.3: Client 1: Model comparison for target response based on selected metrics

Metrics	**Model 1**	**Model 2**	**Model 3**	**Model 4**	**Model 5**	**Model 6**	**Model 7**
R^2	0.02	0.10	0.37	0.45	0.93	0.45	0.93
v^*	0.00	0.00	0.00	0.10	0.05	0.10	0.10
TV-factor	0.00	0.00	0.00	2.03	1.83	1.94	1.91
TV-coefficient	0.00	0.00	0.00	1.83	1.74	1.74	1.72
TV-contribution	0.00	0.00	0.00	1.46	1.32	1.39	1.37
TV t-value	0.00	0.00	0.00	3.82	3.57	3.67	3.70
S-coefficient	0.00	0.00	0.00	1.28	77.19	57.59	75.33
S-contribution	0.00	0.00	0.00	0.90	74.12	57.08	72.33
S t-value	0.00	0.00	0.00	0.29	49.15	11.01	43.99
T-coefficient	0.00	0.00	0.00	8 088.66	7 918.28	8 136.46	8 008.87
T-contribution	0.00	0.00	0.00	25.69	25.14	25.84	25.43
T t-value	0.00	0.00	0.00	23.55	23.91	23.91	24.15
H-coefficient	0.00	0.00	0.00	0.22	0.00	0.00	0.06
H-contribution	0.00	0.00	0.00	4.38	0.00	0.00	1.16
H t-value	0.00	0.00	0.00	10.32	0.00	0.00	2.67
Confidence	0.00	0.00	0.00	0.80	0.80	0.80	0.80

Table A.4: Client 2: Model comparison for target response based on selected metrics

Metrics	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
R^2	0.98	0.98	0.54	0.56	0.71	0.72	0.74
v^*	0.15	0.20	0.10	0.10	0.35	0.35	0.40
TV-factor	2.17	2.09	2.15	2.07	1.76	1.71	1.66
TV-coefficient	1.84	1.67	1.94	1.86	1.15	1.11	0.99
TV-contribution	34.94	33.70	34.70	33.34	28.42	27.54	26.70
TV t-value	28.46	27.57	30.56	29.76	27.80	27.60	27.34
S-coefficient	49.18	52.80	0.00	0.00	124.80	128.58	131.16
S-contribution	24.01	25.78	0.00	0.00	18.90	19.38	19.86
S t-value	8.59	9.52	0.00	0.00	27.42	29.43	30.96
T-coefficient	0.41	0.40	14 723.56	14 409.70	15 189.67	15 054.82	14 796.46
T-contribution	41.13	40.17	11.56	11.31	11.92	11.82	11.62
T t-value	14.58	14.65	17.33	17.38	22.43	22.89	23.36
H-coefficient	0.00	0.20	0.00	0.17	0.00	0.00	0.21
H-contribution	0.00	0.43	0.00	0.37	0.00	0.00	0.47
H t-value	0.00	8.68	0.00	7.80	0.00	0.00	13.07
Confidence	0.85	0.85	0.85	0.85	0.75	0.75	0.75

Table A.5: Client 3: Model comparison for target response based on selected metrics

Metrics	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
R^2	0.94	0.94	0.64	0.66	0.81	0.83	0.84
v^*	0.00	0.00	0.00	0.00	0.10	0.30	0.30
TV-factor	11.19	10.38	13.33	12.69	4.74	4.23	3.86
TV-coefficient	11.19	10.38	13.33	12.69	4.26	2.96	2.70
TV-contribution	13.20	12.24	15.72	14.97	5.59	4.98	4.55
TV t-value	16.49	15.45	21.31	20.42	8.00	6.73	6.26
S-coefficient	230.72	239.40	103.02	101.09	792.38	817.15	835.90
S-contribution	14.82	15.38	6.07	5.95	32.94	33.81	34.75
S t-value	7.88	8.38	3.01	3.01	28.45	33.70	33.63
T-coefficient	0.73	0.73	72 307.18	72 317.45	74 552.04	74 371.12	74 579.30
T-contribution	73.76	73.48	52.46	52.47	54.09	53.96	54.11
T t-value	41.75	42.65	32.61	33.27	45.71	48.25	49.44
H-coefficient	0.00	0.29	0.00	0.25	0.00	0.00	0.36
H-contribution	0.00	0.73	0.00	0.65	0.00	0.00	0.92
H t-value	0.00	6.97	0.00	6.15	0.00	0.00	12.64
Confidence	0.70	0.70	0.80	0.80	0.60	0.45	0.60

Table A.6: Client 4: Model comparison for target response based on selected metrics

Metrics	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
R^2	0.91	0.92	0.92	0.92	0.94	0.94	0.94
v^*	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TV-Factor	3.65	3.61	3.58	3.55	3.58	3.58	3.55
TV-coefficient	3.65	3.61	3.58	3.55	3.58	3.58	3.55
TV-contribution	74.30	73.56	72.93	72.34	72.97	72.91	72.38
TV t-value	43.32	43.05	81.21	80.20	81.48	81.46	80.48
S-coefficient	1.76	1.85	0.00	0.00	1.86	1.61	1.89
S-contribution	2.78	2.92	0.00	0.00	7.62	6.61	7.72
S t-value	1.18	1.25	0.00	0.00	2.34	2.07	2.40
T-coefficient	0.09	0.09	119.03	117.82	110.32	113.79	108.94
T-contribution	12.23	12.26	22.11	21.88	20.49	21.13	20.23
T t-value	1.43	1.45	7.94	7.94	7.45	7.77	7.42
H-coefficient	0.00	0.15	0.00	0.14	0.00	0.00	0.14
H-contribution	0.00	0.80	0.00	0.76	0.00	0.00	0.77
H t-value	0.00	3.97	0.00	3.94	0.00	0.00	3.97
Confidence	0.65	0.65	0.75	0.75	0.85	0.85	0.85

Table A.7: Client 1: model comparison for conversions based on selected metrics

Metrics	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
R^2	0.80	0.82	0.33	0.39	0.84	0.84	0.85
v^*	0.00	0.15	0.00	0.00	0.00	0.00	0.00
TV-Factor	1.46	1.43	0.00	0.00	1.02	1.00	0.98
TV-coefficient	1.46	1.21	0.00	0.00	1.02	1.00	0.98
TV-contribution	4.36	4.27	0.00	0.00	3.04	3.00	2.92
TV t-value	5.69	5.56	0.00	0.00	4.35	4.31	4.24
S-coefficient	0.00	0.00	0.00	0.00	0.53	0.50	0.48
S-contribution	0.00	0.00	0.00	0.00	59.41	58.01	53.80
S t-value	0.00	0.00	0.00	0.00	20.10	20.45	17.02
T-coefficient	0.98	0.92	0.00	0.00	56.30	57.62	58.53
T-contribution	96.22	91.11	0.00	0.00	38.64	39.55	40.17
T t-value	45.60	42.29	0.00	0.00	16.12	16.95	16.85
H-coefficient	0.00	0.28	0.00	0.00	0.00	0.00	0.17
H-contribution	0.00	5.48	0.00	0.00	0.00	0.00	3.36
H t-value	0.00	7.25	0.00	0.00	0.00	0.00	4.55
Confidence	0.90	0.90	0.00	0.00	0.00	0.00	0.00

Table A.8: Client 2: model comparison for conversions based on selected metrics

Metrics	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
R^2	0.97	0.97	0.47	0.47	0.79	0.82	0.79
v^*	0.25	0.25	0.40	0.40	0.60	0.65	0.60
TV-Factor	1.08	1.08	1.33	1.33	1.13	1.13	1.13
TV-coefficient	0.81	0.81	0.80	0.80	0.45	0.39	0.45
TV-contribution	33.57	33.57	41.22	41.22	34.96	34.65	34.96
TV t-value	10.09	10.09	7.88	7.88	8.67	9.01	8.67
S-coefficient	15.60	15.60	0.00	0.00	28.02	29.43	28.02
S-contribution	66.77	66.77	0.00	0.00	36.42	38.25	36.42
S t-value	18.21	18.21	0.00	0.00	13.28	15.59	13.28
T-coefficient	0.00	0.00	1 230.98	1 230.98	4 440.98	4 329.04	4 440.98
T-contribution	0.00	0.00	4.01	4.01	14.48	14.11	14.48
T t-value	0.00	0.00	0.29	0.29	1.56	1.65	1.56
H-coefficient	0.00	0.00	0.00	0.00	0.00	0.00	0.00
H-contribution	0.00	0.00	0.00	0.00	0.00	0.00	0.00
H t-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Confidence	0.56	0.56	0.56	0.56	0.46	0.46	0.46

Table A.9: Client 3: model comparison for conversions based on selected metrics

Metrics	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7
R^2	0.90	0.90	0.58	0.59	0.96	0.00	0.96
v^*	0.00	0.00	0.00	0.00	0.00	0.00	0.00
TV-Factor	9.21	8.34	12.18	11.76	3.08	0.00	1.78
TV-coefficient	9.21	8.34	12.18	11.76	3.08	0.00	1.78
TV-contribution	15.92	14.41	21.04	20.31	5.32	0.00	3.07
TV t-value	12.70	11.37	21.57	20.51	5.90	0.00	3.46
S-coefficient	34.30	36.30	17.66	17.40	107.27	0.00	112.74
S-contribution	19.04	20.15	8.98	8.85	38.55	0.00	40.52
S t-value	7.14	7.64	4.04	4.00	31.56	0.00	34.37
T-coefficient	0.66	0.66	9 118.40	9 061.76	9 086.90	0.00	8 986.59
T-contribution	66.37	66.04	56.43	56.08	56.24	0.00	55.62
T t-value	26.53	26.77	28.14	28.11	58.43	0.00	60.59
H-coefficient	0.00	0.27	0.00	0.16	0.00	0.00	0.31
H-contribution	0.00	0.77	0.00	0.45	0.00	0.00	0.89
H t-value	0.00	5.35	0.00	3.60	0.00	0.00	9.96
Confidence	0.70	0.70	0.80	0.80	0.60	0.00	0.60

Table A.10: Client 4: model comparison for conversions based on selected metrics

B Code

The corresponding code can be found here:

https://drive.google.com/open?id=1YKba5aTkMso58XmDnzDh785g7QAfvguV