DT

November 17, 2019

[1]:
```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
```

0.1 Dataset

0.1.1 Load dataset

[2]:
```
column_names = ['Attr.1', 'Attr.2', 'Attr.3', 'Attr.4', 'label']
data = pd.read_csv('iris.data',names=column_names, header=None)
data.head(5)
```

[2]:

	Attr.1	Attr.2	Attr.3	Attr.4	label
0	5.1	3.5	1.4	0.2	Iris-setosa
1	4.9	3.0	1.4	0.2	Iris-setosa
2	4.7	3.2	1.3	0.2	Iris-setosa
3	4.6	3.1	1.5	0.2	Iris-setosa
4	5.0	3.6	1.4	0.2	Iris-setosa

0.1.2 Dataset info.

[3]:
```
data.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 150 entries, 0 to 149
Data columns (total 5 columns):
Attr.1    150 non-null float64
Attr.2    150 non-null float64
Attr.3    150 non-null float64
Attr.4    150 non-null float64
label     150 non-null object
dtypes: float64(4), object(1)
memory usage: 5.9+ KB
```

[4]:
```
data.describe()
```

[4]:

```
            Attr.1      Attr.2      Attr.3      Attr.4
count   150.000000  150.000000  150.000000  150.000000
mean      5.843333    3.054000    3.758667    1.198667
std       0.828066    0.433594    1.764420    0.763161
min       4.300000    2.000000    1.000000    0.100000
25%       5.100000    2.800000    1.600000    0.300000
50%       5.800000    3.000000    4.350000    1.300000
75%       6.400000    3.300000    5.100000    1.800000
max       7.900000    4.400000    6.900000    2.500000
```

[5]:

```
data.groupby('label').size()
```

[5]:

```
label
Iris-setosa        50
Iris-versicolor    50
Iris-virginica     50
dtype: int64
```

0.2 Several Functions

0.2.1 Calculate entropy

$$H(D) = -\sum_{k=1}^{K} p_i \log p_i = -\sum_{k=1}^{K} \frac{|C_k|}{|D|} \log \frac{|C_k|}{|D|}$$

Function: cal_entropy - Description: calculate entropy using above formula. - Patameters: - data: Dataframe type with label and attrbutes as columns. - Return: - entropy: float type

[6]:

```
def cal_entropy(data):
    p = data.groupby('label').size() / len(data)
    h = -sum(p * np.log(p))
    return h
```

0.2.2 Calculate entropy of the splited set

given split point s of attribute a, split D into two disjoint sets D_1, D_2, satisfy

$$D_1 = \{x|x.a <= s\} D_2 = \{x|x.a > s\}$$

then calculate:

$$H(D|a,s) = \frac{|D_1|}{|D|} H(D_1) + \frac{|D_2|}{|D|} H(D_2)$$

Function: cal_entropy_after_split - Description: calculate entropy with the splited data by split_point of attr_name. - Patameters: - data: Dataframe type with label and attrbutes as columns. - attr_name: which is the split_point? - split_point: thredhold to split data in attr_name column. - Return: - entropy: float type

[7]:

```
def cal_entropy_after_split(data, attr_name, split_point):
    d_1 = data[data[attr_name] <= split_point]
    d_2 = data[data[attr_name] >  split_point]
```

```python
    h_1 = cal_entropy(d_1)
    h_2 = cal_entropy(d_2)
    h = len(d_1) / len(data) * h_1 + len(d_2) / len(data) * h_2
    return h
```

0.2.3 Find best split point

using gold section search algorithm

given $a, b, a < b$, let

$$\mu = a + (1 - \phi)(b - a)\lambda = a + \phi(b - a)$$

repeat

if $f(\mu) > f(\lambda)$, then $a = \mu, \mu = \lambda, \lambda = a + \phi(b - a)$

if $f(\mu) < f(\lambda)$, then $b = \lambda, \lambda = \mu, \mu = a + (1 - \phi)(b - a)$

until satisfy the end condition.

$$s_{best} = \arg\min_{s} Criterita(D|a, s)$$

criterita may be min_entropy, min_error, min_GINI, etc.

Function: find_best_split_point - Description: find best split point in attr_name column to minimize criterion function - Patameters: - data: Dataframe type with label and attrbutes as columns. - attr_name: in which column? - criterion_function: the smaller criteria, the better. - Return: - split_point - Note: Using gold section search algorithm, maybe fall into minimum if criterion function is not convex function.

[8]:
```python
def find_best_split_point(data, attr_name, criterion_function):
    fhi = 0.618
    f = criterion_function
    a = min(data[attr_name])
    b = max(data[attr_name])
    mu = a + (1 - fhi) * (b - a)
    lamb = a + fhi * (b - a)
    f_mu = f(data, attr_name, mu)
    f_lambda = f(data, attr_name, lamb)

    while(f_mu != f_lambda):
        if f_mu > f_lambda:
            a = mu
            mu = lamb
            lamb = a + fhi * (b - a)
            f_mu = f_lambda
            f_lambda = f(data, attr_name, lamb)
        else:
            b = lamb
            lamb = mu
            mu = a + (1 - fhi) * (b - a)
            f_lambda = f_mu
            f_mu = f(data, attr_name, mu)
    split_point = (mu + lamb) / 2
```

```
    return split_point
```

[9]:
```
split_point = find_best_split_point(data, 'Attr.1', cal_entropy_after_split)
split_point
```

[9]: 5.574850320834822

0.2.4 Can gold section search find optimal split point?

[10]:
```
def get_entropy_by_splitpoint(data, attr_name):
    attr_max = max(data[attr_name])
    attr_min = min(data[attr_name])
    entropy = []
    split_point = []
    for i in np.arange(0, attr_max - attr_min, 0.01):
        split_point.append(attr_min + i)
        entropy.append(cal_entropy_after_split(data, attr_name, attr_min + i))
    return entropy, split_point
```

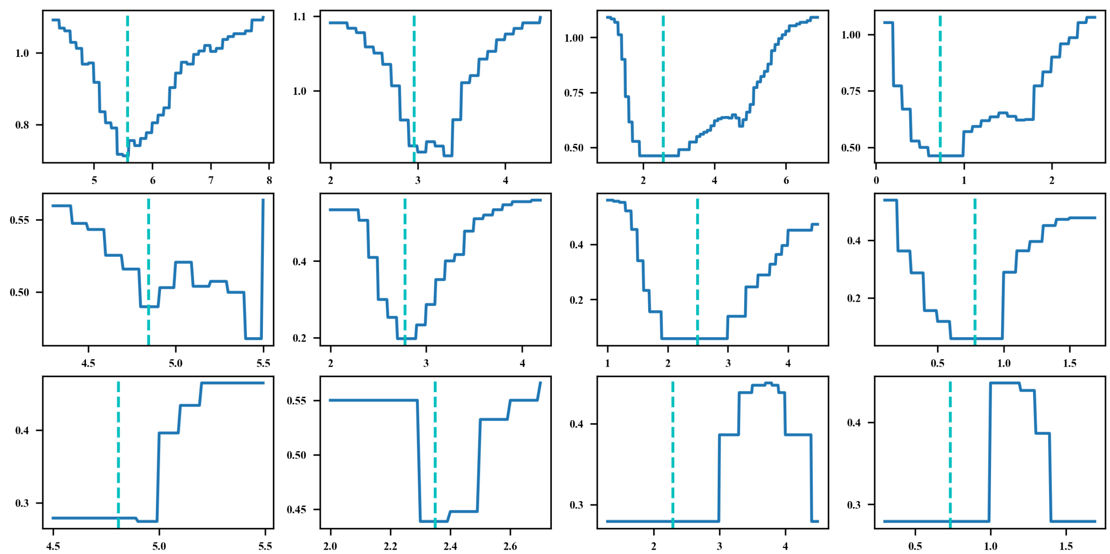
[11]:
```
entropy = []
split_point = []
best_split_point = []
for i in range(1, 5):
    e, s = get_entropy_by_splitpoint(data, 'Attr.%d' % i)
    b_s = find_best_split_point(data, 'Attr.%d' % i, cal_entropy_after_split)
    entropy.append(e)
    split_point.append(s)
    best_split_point.append(b_s)
```

[12]:
```
data_1 = data[data['Attr.1'] <= best_split_point[0]]
entropy_1 = []
split_point_1 = []
best_split_point_1 = []
for i in range(1, 5):
    e, s = get_entropy_by_splitpoint(data_1, 'Attr.%d' % i)
    b_s = find_best_split_point(data_1, 'Attr.%d' % i, cal_entropy_after_split)
    entropy_1.append(e)
    split_point_1.append(s)
    best_split_point_1.append(b_s)
```

[13]:
```
data_2 = data_1[data_1['Attr.2'] <= best_split_point_1[1]]
entropy_2 = []
split_point_2 = []
best_split_point_2 = []
for i in range(1, 5):
    e, s = get_entropy_by_splitpoint(data_2, 'Attr.%d' % i)
    b_s = find_best_split_point(data_2, 'Attr.%d' % i, cal_entropy_after_split)
    entropy_2.append(e)
    split_point_2.append(s)
```

```
    best_split_point_2.append(b_s)
```

[14]:
```
plt.rcParams['figure.dpi'] = 300
plt.rcParams['savefig.dpi'] = 300

fig = plt.figure(figsize=(10, 5), dpi=300)
for i in range(4):
    ax = fig.add_subplot(3, 4, i + 1)
    plt.plot(split_point[i], entropy[i])
    #plt.xlabel('split point')
    #plt.ylabel('entropy')
    plt.yticks(fontproperties = 'Times New Roman', size = 6)
    plt.xticks(fontproperties = 'Times New Roman', size = 6)
    plt.axvline(x=best_split_point[i], c = "c", ls = "dashed")
    ax = fig.add_subplot(3, 4, i + 5)
    plt.plot(split_point_1[i], entropy_1[i])
    plt.yticks(fontproperties = 'Times New Roman', size = 6)
    plt.xticks(fontproperties = 'Times New Roman', size = 6)
    plt.axvline(x=best_split_point_1[i], c = "c", ls = "dashed")
    ax = fig.add_subplot(3, 4, i + 9)
    plt.plot(split_point_2[i], entropy_2[i])
    plt.yticks(fontproperties = 'Times New Roman', size = 6)
    plt.xticks(fontproperties = 'Times New Roman', size = 6)
    plt.axvline(x=best_split_point_2[i], c = "c", ls = "dashed")

#plt.savefig('./gss_split_point.jpg')
#plt.savefig('./gss_split_point.eps', dpi=300, format='eps')
plt.show()
```



0.2.5 Find best attribute

$$a^* = \arg\max_{a \in A} Gain(D|a)$$

Function: find_best_attribute - Description: find best attribute for next branch to maximize gain_function. - Patameters: - data: Dataframe type with label and attributes as columns. - gain_function: the bigger gain, the better. - Return: - best_attr: the best attribute

[15]:
```python
def find_best_attribute(data, gain_function):
    gain = gain_function
    attr_name_list = data.columns[:-1]

    best_attr = attr_name_list[0];
    max_gain = gain(data, best_attr)
    for attr in attr_name_list:
        g = gain(data, attr)
        if max_gain < g:
            best_attr = attr
            max_gain = g
    return best_attr
```

Entropy gain:

$$Gain(D|a) = H(D) - H(D|a, s)$$

Function: entropy_gain - Description: calculate entropy gain if using attr as next branch attribute. - Patameters: - data: Dataframe type with label and attributes as columns. - attr: attribute - Return: - gain

[16]:
```python
def entropy_gain(data, attr):
    old_entropy = cal_entropy(data);
    split_point = find_best_split_point(data, attr, cal_entropy_after_split)
    new_entropy = cal_entropy_after_split(data, attr, split_point)
    gain = old_entropy - new_entropy
    return gain
```

Minimum Error Criteria

predict all samples in D to the label l, then error

$$E(D|l) = \frac{|D(x.label \neq l)|}{|D|}$$

[17]:
```python
def cal_error(data):
    label, precise = get_max_precise(data)
    error = 1 - precise
    return error
```

[18]:
```python
def cal_error_after_split(data, attr_name, split_point):
    d_1 = data[data[attr_name] <= split_point]
    d_2 = data[data[attr_name] >  split_point]
    error_1 = cal_error(d_1)
    error_2 = cal_error(d_2)
    error = len(d_1) / len(data) * error_1 + len(d_2) / len(data) * error_2
```

```
    return error
```

[19]:
```
def error_gain(data, attr):
    old_error = cal_error(data);
    split_point = find_best_split_point(data, attr, cal_error_after_split)
    new_error = cal_error_after_split(data, attr, split_point)
    gain = old_error - new_error
    return gain
```

Maximun Entropy Ratio Gain

$$Gain\_ratio = \frac{Gain(D|a)}{H(D|a)}$$

[20]:
```
def entropy_ratio_gain(daata, attr):
    pass # need to continue.
```

Minimum GINI index criteria

$$Gini(D) = 1 - \sum_{k=1}^{|A|} p_k^2 = 1 - \sum_{k=1}^{K} \left(\frac{|C_k|}{|D|}\right)^2$$

[21]:
```
def cal_gini(data):
    p = data.groupby('label').size() / len(data)
    gini = 1 - np.sum(np.power(p, 2))
    return gini
```

[22]:
```
def cal_gini_after_split(data, attr_name, split_point):
    d_1 = data[data[attr_name] <= split_point]
    d_2 = data[data[attr_name] >  split_point]
    error_1 = cal_gini(d_1)
    error_2 = cal_gini(d_2)
    error = len(d_1) / len(data) * error_1 + len(d_2) / len(data) * error_2
    return error
```

[23]:
```
def gini_gain(data, attr):
    old_error = cal_gini(data);
    split_point = find_best_split_point(data, attr, cal_gini_after_split)
    new_error = cal_gini_after_split(data, attr, split_point)
    gain = old_error - new_error
    return gain
```

0.2.6 make decision tree

- Algorithm:
  - (1) find the best attribute for next branch.
  - (2) find the best split point of the best brunch attribute.
  - (3) split the data into disjoint subset.

- (4) then repeat above procedure on these two subset recurrently until santify the end condition.

- End condition:
  - every sample in data has the same label.
- Note: This function may contain some repeated calculations. For example, when we find the best attribute, we have computed the best split point of every attribute, but after find the best attribute we compute the split point of the best attribute again. Although exists some repeated computation, this can maintain the atomic function of each method and easy unstanding of code.

Function: make_tree - Description: make decision tree on data - Patameters: - data: Dataframe type with label and attributes as columns. - Return: - decision tree: - data structure: python build-in dictory. - for example, {'branch_condition1': subtree1,'branch_condition2': subtree2} - tree type: binary tree. - interal node: branch condition with the form: - "attribute_name<=split_point" or "attribute_name>split_point". - leaf node: label, string type, - for example: 'Iris-virginica'.

[24]:
```
def make_tree(data):
    attr_name_list = data.columns[:-1]
    label_name = data.groupby('label').size().index

    if len(label_name) == 1:
        return label_name[0]
    best_attr = find_best_attribute(data, entropy_gain)
    split_point = find_best_split_point(data, best_attr,␣
 ↪cal_entropy_after_split)
    tree = {}
    tree['%s<=%f' % (best_attr, split_point)] = make_tree(data[data[best_attr]␣
 ↪<= split_point])
    tree['%s>%f' % (best_attr, split_point)] = make_tree(data[data[best_attr] >␣
 ↪split_point])
    return tree
```

[25]:
```
tree = make_tree(data)
tree
```

[25]:
```
{'Attr.3<=2.557290': 'Iris-setosa',
 'Attr.3>2.557290': {'Attr.4<=1.749965': {'Attr.3<=4.964518':
{'Attr.4<=1.566300': 'Iris-versicolor',
    'Attr.4>1.566300': {'Attr.1<=5.600000': 'Iris-virginica',
     'Attr.1>5.600000': 'Iris-versicolor'}},
   'Attr.3>4.964518': {'Attr.4<=1.550000': 'Iris-virginica',
    'Attr.4>1.550000': {'Attr.1<=6.829154': 'Iris-versicolor',
     'Attr.1>6.829154': 'Iris-virginica'}}},
  'Attr.4>1.749965': {'Attr.3<=4.953220': {'Attr.2<=3.045481': 'Iris-virginica',
    'Attr.2>3.045481': 'Iris-versicolor'},
   'Attr.3>4.953220': 'Iris-virginica'}}}
```

Function: predict - Description: predict sample's label - Patameters: - sample: a series with attributes. - tree: the tree trained before. - Return: - label

[26]:
```python
def predict(sample, tree):
    if not isinstance(tree, dict):
        return tree

    attr, split_point = list(tree.keys())[0].split('<=')
    split_point = (float)(split_point)
    if sample[attr] <= split_point:
        return predict(sample, tree[list(tree.keys())[0]])
    else:
        return predict(sample, tree[list(tree.keys())[1]])
```

[27]:
```python
data.loc[53, :]
```

[27]:
```
Attr.1                 5.5
Attr.2                 2.3
Attr.3                   4
Attr.4                 1.3
label      Iris-versicolor
Name: 53, dtype: object
```

[28]:
```python
predict(data.loc[53, :], tree)
```

[28]:
```
'Iris-versicolor'
```

Function: evaluate - Description: evaluate the performance of the tree on data - Patameters: - data: Dataframe type with label and attributes as columns. - tree: the tree trained before. - Return: - precise: the number of right predicted samples divided by the size of data.

[29]:
```python
def evaluate(data, tree):
    right_count = 0
    for _, sample in data.T.iteritems():
        label = sample[-1]
        if label == predict(sample, tree):
            right_count += 1
    return right_count / len(data)
```

[30]:
```python
evaluate(data, tree)
```

[30]:
```
1.0
```

Function: get_max_precise - Description: compute the max precise when predict every samples in data to the same label. - Patameters: - data: Dataframe type with label and attributes as columns. - Return: - label: the label which have max percentage over data. - precise: the max precise when see all samples as the same label.

[31]:
```python
def get_max_precise(data):
    precise_dict = dict(data.groupby('label').size() / len(data))
    if len(precise_dict) == 0:
        return '', 0
    else:
        label = max(precise_dict, key = precise_dict.get)
        precise = precise_dict[label]
```

```
        return label, precise
```

Function: get_max_precise_after_split - Description: compute the precise after split data according to the given split_point of attr. - Patameters: - data: Dataframe type with label and attributes as columns. - attr: attribute name. - split_point: split point. - Return: - precise

[32]:
```
def get_max_precise_after_split(data, attr, split_point):
    data_1 = data[data[attr] <= split_point]
    data_2 = data[data[attr] >  split_point]
    _, precise_1 = get_max_precise(data_1)
    _, precise_2 = get_max_precise(data_2)
    precise = len(data_1) / len(data) * precise_1 + len(data_2) / len(data) *␣
 ↪precise_2
    return precise
```

0.2.7 Pre. pruning

- Algorithm:
  - (1) find the best attribute for next branch.
  - (2) compute the best split point of the best attribute.
  - (3) calculate the current precision and the precise on evaluate dataset when we add the next brunch.
  - (4) compare these two precision, if current precision higher, then,
    * stop to brunch, and return the label with the max precision on current dataset.
  - (5) if addition the next brunch is better, then,
    * split the data into disjoint subset according to the split point of the best attribute.
    * then repeat above procedure on these two subset recurrently until santify the end condition.
- End condition:
  - every sample in data has the same label.
  - Over fitting.

Function: make_tree_with_pre_pruning - Description: make decision tree with previous pruning. - Patameters: - train_set. - evaluate_set. - Return: - the pre-pruned tree.

[33]:
```
def make_tree_with_pre_pruning(train_set, evaluate_set):
    attr_name_list = train_set.columns[:-1]
    label_name = train_set.groupby('label').size().index

    if len(label_name) == 1:
        return label_name[0]
    best_attr = find_best_attribute(train_set, entropy_gain)
    split_point = find_best_split_point(train_set, best_attr,␣
 ↪cal_entropy_after_split)

    label, precise = get_max_precise(train_set)
```

```
    precise_after_split = get_max_precise_after_split(evaluate_set, best_attr,␣
↪split_point)

    if precise >= precise_after_split:
        return label
    else:
        tree = {}
        tree['%s<=%f' % (best_attr, split_point)] = \
                                        ␣
↪make_tree_with_pre_pruning(train_set[train_set[best_attr] <= split_point],
                                                                        ␣
↪evaluate_set[evaluate_set[best_attr] <= split_point])
        tree['%s>%f' % (best_attr, split_point)] = \
                                        ␣
↪make_tree_with_pre_pruning(train_set[train_set[best_attr] > split_point],
                                                                       ␣
↪evaluate_set[evaluate_set[best_attr] > split_point])
        return tree
```

[34]:
```
data_shuffled = data.sample(frac=1)
data_shuffled.head(5)
```

[34]:
```
    Attr.1  Attr.2  Attr.3  Attr.4            label
42     4.4     3.2     1.3     0.2      Iris-setosa
19     5.1     3.8     1.5     0.3      Iris-setosa
56     6.3     3.3     4.7     1.6  Iris-versicolor
47     4.6     3.2     1.4     0.2      Iris-setosa
27     5.2     3.5     1.5     0.2      Iris-setosa
```

[35]:
```
frac = 0.2
train_set = data_shuffled.iloc[0: int(frac * len(data_shuffled)), : ]
evaluate_set = data_shuffled.iloc[int(frac * len(data_shuffled)) : , :]
```

[36]:
```
tree_pre_pruned = make_tree_with_pre_pruning(train_set, evaluate_set)
```

[37]:
```
tree_pre_pruned
```

[37]:
```
{'Attr.1<=5.504006': 'Iris-setosa',
 'Attr.1>5.504006': {'Attr.4<=1.650031': 'Iris-versicolor',
  'Attr.4>1.650031': 'Iris-virginica'}}
```

[38]:
```
evaluate(data, tree_pre_pruned)
```

[38]:
```
0.86
```

0.2.8 Post. pruning

- Algorithm:
  - (1) split current evaluate_set into two disjoint set according to the brunch condition.

- (2) prune the two subtree first with corresponding evaluate dataset(using post-order traversal walk through the tree).
- (3) After prune subtree, calculate the precision on evaluate dataset of current tree and the pruned tree.
- (4) compare the two precision, if current tree is better, then don't change, return current tree.
- (5) otherwise, prune current tree by returning the label with maximum percentage.

Function: post_pruning - Description: post-prune tree. - Patameters: - tree. - evaluate_set. - Return: - the post-pruned tree. - Warning: the original tree will be changed.

[39]:
```
def post_pruning(tree, evaluate_set):
    if not isinstance(tree, dict):
        return tree

    attr, split_point = list(tree.keys())[0].split('<=')
    split_point = (float)(split_point)

    tree[list(tree.keys())[0]] = post_pruning(tree[list(tree.keys())[0]],
                                              evaluate_set[evaluate_set[attr]␣
 ↪<= split_point])
    tree[list(tree.keys())[1]] = post_pruning(tree[list(tree.keys())[1]],
                                              evaluate_set[evaluate_set[attr] >␣
 ↪ split_point])

    precise_before_pruning = evaluate(evaluate_set, tree)
    label, precise_after_pruning = get_max_precise(evaluate_set)
    if precise_after_pruning > precise_before_pruning:
        return label
    else:
        return tree
```

[40]:
```
tree = make_tree(data.sample(frac=0.2))
tree
```

[40]:
```
{'Attr.3<=4.820511': {'Attr.3<=2.350729': 'Iris-setosa',
  'Attr.3>2.350729': 'Iris-versicolor'},
 'Attr.3>4.820511': 'Iris-virginica'}
```

[41]:
```
evaluate(data, tree)
```

[41]:
```
0.9533333333333334
```

[42]:
```
tree_pruned = post_pruning(tree, data.sample(frac=0.6))
tree_pruned
```

[42]:
```
{'Attr.3<=4.820511': {'Attr.3<=2.350729': 'Iris-setosa',
  'Attr.3>2.350729': 'Iris-versicolor'},
 'Attr.3>4.820511': 'Iris-virginica'}
```

[43]:
```
evaluate(data, tree)
```

[43]: 0.9533333333333334

0.3 Wrap into class

[44]:
```
class DecisionTree:

    def __init__(self, criteria='Entropy'):
        self.criteria = criteria
        self.tree = None
        if criteria == 'Entropy':
            self.gain_function = self._entropy_gain
            self.cal_criterion_after_split = self._cal_entropy_after_split
        elif criteria == 'minError':
            self.gain_function = self._error_gain
            self.cal_criterion_after_split = self._cal_error_after_split
        elif criteria == 'Gini':
            self.gain_function = self._gini_gain
            self.cal_criterion_after_split = self._cal_gini_after_split


    def make_tree(self, data, ratio = 0.1, pruning='no pruning',␣
↪max_depth=1024):
        if pruning == 'pre pruning':
            #print('Using pre-pruning.')
            train_set, evaluate_set = self._split_data(data, ratio)
            self.tree = self._make_tree(train_set, max_depth, True,␣
↪evaluate_set)
        elif pruning == 'post pruning':
            #print('Using post-pruning.')
            train_set, evaluate_set = self._split_data(data, ratio)
            tree = self._make_tree(train_set, max_depth=max_depth)
            self.tree = self._post_pruning(tree, evaluate_set)
        else:
            #print('No pruning.')
            self.tree = self._make_tree(data, max_depth=max_depth)


    def predict(self, sample):
        return self._predict(sample, self.tree)


    def evaluate(self, data):
        return self._evaluate(data, self.tree);


    def _split_data(self, data, frac):
```

```
        data = data.sample(frac=1)
        train_set = data.iloc[(int)(len(data) * frac) : , : ]
        evaluate_set = data.iloc[0 : (int)(len(data) * frac), : ]
        return train_set, evaluate_set


    def _predict(self, sample, tree):
        if not isinstance(tree, dict):
            return tree

        attr, split_point = list(tree.keys())[0].split('<=')
        split_point = (float)(split_point)
        if sample[attr] <= split_point:
            return self._predict(sample, tree[list(tree.keys())[0]])
        else:
            return self._predict(sample, tree[list(tree.keys())[1]])


    def _evaluate(self, data, tree):
        if len(data) == 0:
            return 1

        right_count = 0
        for _, sample in data.T.iteritems():
            label = sample[-1]
            if label == self._predict(sample, tree):
                right_count += 1
        return right_count / len(data)


    def _make_tree(self, train_set, max_depth=1024, pre_pruning=False,␣
↪evaluate_set=None):
        attr_name_list = train_set.columns[:-1]
        label_name = train_set.groupby('label').size().index
        if max_depth == 0:
            label, precise = self._get_max_precise(train_set)
            return label

        if len(label_name) == 0:
            return ''

        if len(label_name) == 1:
            return label_name[0]

        best_attr = self._find_best_attribute(train_set, self.gain_function)
        split_point = self._find_best_split_point(train_set, best_attr, self.
↪cal_criterion_after_split)
```

```
        if len(train_set.groupby(best_attr).size()) == 1:
            label, precise = self._get_max_precise(train_set)
            return label
        tree = {}
        if pre_pruning:
            label, precise = self._get_max_precise(train_set)
            precise_after_split = self.
↪_get_max_precise_after_split(evaluate_set, best_attr, split_point)

            if precise >= precise_after_split:
                return label
            tree['%s<=%f' % (best_attr, split_point)] = \
                                self._make_tree(train_set[train_set[best_attr] <=␣
↪split_point], max_depth - 1, True,
                                                                            ␣
↪evaluate_set[evaluate_set[best_attr] <= split_point])
            tree['%s>%f' % (best_attr, split_point)] = \
                                self._make_tree(train_set[train_set[best_attr] >␣
↪split_point], max_depth - 1, True,
                                                                           ␣
↪evaluate_set[evaluate_set[best_attr] >  split_point])
        else:
            tree['%s<=%f' % (best_attr, split_point)] = \
                                self._make_tree(train_set[train_set[best_attr] <=␣
↪split_point], max_depth - 1)
            tree['%s>%f' % (best_attr, split_point)] = \
                                self._make_tree(train_set[train_set[best_attr] > ␣
↪split_point], max_depth - 1)
        return tree


    def _post_pruning(self, tree, evaluate_set):
        if not isinstance(tree, dict):
            return tree

        attr, split_point = list(tree.keys())[0].split('<=')
        split_point = (float)(split_point)

        tree[list(tree.keys())[0]] = self._post_pruning(tree[list(tree.
↪keys())[0]],
                                                                      ␣
↪evaluate_set[evaluate_set[attr] <= split_point])
        tree[list(tree.keys())[1]] = self._post_pruning(tree[list(tree.
↪keys())[1]],
```

```
↪evaluate_set[evaluate_set[attr] >␣ split_point])

        precise_before_pruning = self._evaluate(evaluate_set, tree)
        label, precise_after_pruning = self._get_max_precise(evaluate_set)
        if precise_after_pruning > precise_before_pruning:
            return label
        else:
            return tree


    def _find_best_split_point(self, data, attr_name, criterion_function):
        fhi = 0.618
        f = criterion_function
        a = min(data[attr_name])
        b = max(data[attr_name])
        mu = a + (1 - fhi) * (b - a)
        lamb = a + fhi * (b - a)
        f_mu = f(data, attr_name, mu)
        f_lambda = f(data, attr_name, lamb)

        while(f_mu != f_lambda):
            if f_mu > f_lambda:
                a = mu
                mu = lamb
                lamb = a + fhi * (b - a)
                f_mu = f_lambda
                f_lambda = f(data, attr_name, lamb)
            else:
                b = lamb
                lamb = mu
                mu = a + (1 - fhi) * (b - a)
                f_lambda = f_mu
                f_mu = f(data, attr_name, mu)
        split_point = (mu + lamb) / 2
        return split_point


    def _find_best_attribute(self, data, gain_function):
        gain = gain_function
        attr_name_list = data.columns[:-1]

        best_attr = attr_name_list[0];
        max_gain = gain(data, best_attr)
        for attr in attr_name_list:
            g = gain(data, attr)
            if max_gain < g:
```

```
                best_attr = attr
                max_gain = g
        return best_attr


    def _get_max_precise(self, data):
        precise_dict = dict(data.groupby('label').size() / len(data))
        if len(precise_dict) == 0:
            return '', 0
        else:
            label = max(precise_dict, key = precise_dict.get)
            precise = precise_dict[label]
            return label, precise


    def _get_max_precise_after_split(self, data, attr, split_point):
        if len(data) == 0:
            return 0
        data_1 = data[data[attr] <= split_point]
        data_2 = data[data[attr] >  split_point]
        _, precise_1 = self._get_max_precise(data_1)
        _, precise_2 = self._get_max_precise(data_2)
        precise = len(data_1) / len(data) * precise_1 + len(data_2) / len(data)␣
↪* precise_2
        return precise


    def _cal_entropy(self, data):
        p = data.groupby('label').size() / len(data)
        h = -sum(p * np.log(p))
        return h


    def _cal_entropy_after_split(self, data, attr_name, split_point):
        d_1 = data[data[attr_name] <= split_point]
        d_2 = data[data[attr_name] >  split_point]
        h_1 = self._cal_entropy(d_1)
        h_2 = self._cal_entropy(d_2)
        h = len(d_1) / len(data) * h_1 + len(d_2) / len(data) * h_2
        return h


    def _entropy_gain(self, data, attr):
        old_entropy = self._cal_entropy(data);
        split_point = self._find_best_split_point(data, attr,
                                                  self._cal_entropy_after_split)
        new_entropy = self._cal_entropy_after_split(data, attr, split_point)
```

```
        gain = old_entropy - new_entropy
        return gain


    def _cal_error(self, data):
        _, precise = self._get_max_precise(data)
        error = 1 - precise
        return error


    def _cal_error_after_split(self, data, attr_name, split_point):
        d_1 = data[data[attr_name] <= split_point]
        d_2 = data[data[attr_name] >  split_point]
        error_1 = self._cal_error(d_1)
        error_2 = self._cal_error(d_2)
        error = len(d_1) / len(data) * error_1 + len(d_2) / len(data) * error_2
        return error

    def _error_gain(self, data, attr):
        old_error = self._cal_error(data);
        split_point = self._find_best_split_point(data, attr,
                                                  self._cal_error_after_split)
        new_error = self._cal_error_after_split(data, attr, split_point)
        gain = old_error - new_error
        return gain


    def _cal_gini(self, data):
        p = data.groupby('label').size() / len(data)
        gini = 1 - np.sum(np.power(p, 2))
        return gini


    def _cal_gini_after_split(self, data, attr_name, split_point):
        d_1 = data[data[attr_name] <= split_point]
        d_2 = data[data[attr_name] >  split_point]
        error_1 = self._cal_error(d_1)
        error_2 = self._cal_error(d_2)
        error = len(d_1) / len(data) * error_1 + len(d_2) / len(data) * error_2
        return error


    def _gini_gain(self, data, attr):
        old_error = self._cal_gini(data);
        split_point = self._find_best_split_point(data, attr,
                                                  self._cal_gini_after_split)
        new_error = self._cal_gini_after_split(data, attr, split_point)
```

```
        gain = old_error - new_error
        return gain
```

[45]:
```
decision_tree = DecisionTree(criteria='minError')
```

[46]:
```
decision_tree.make_tree(data.sample(frac=0.6), data.sample(frac=0.4), 
 ↪pruning='no pruning', max_depth=6)
```

[47]:
```
decision_tree.evaluate(data)
```

[47]: 0.9666666666666667

0.4 Evaluate DT

0.4.1 cross evaluation

[48]:
```
def cross_validation(model, k, data, pruning):
    distance = int(len(data) / k)
    accuracy = []
    for begin in range(0, len(data), distance):
        evaluate_set = data.iloc[begin : begin + distance, :]
        train_set = pd.concat([data.iloc[0:begin, :], data.iloc[begin +
 ↪distance:, :]])
        model.make_tree(train_set, pruning=pruning)

        acc = model.evaluate(evaluate_set)
        accuracy.append(acc)
    avg_acc = sum(accuracy) / len(accuracy)

    return avg_acc, accuracy
```

[49]:
```
data_shuffled = data.sample(frac=1)
```

[50]:
```
model = DecisionTree('Gini')
```

[51]:
```
cross_validation(model, 5, data_shuffled, 'post pruning')
```

[51]: (0.9400000000000001, [0.9333333333333333, 0.9, 0.9, 0.9666666666666667, 1.0])

0.4.2 Overfitting

[52]:
```
data_shuffled = data.sample(frac=1)
```

[53]:
```
 def split_data(data, frac):
    data = data.sample(frac=1)
    train_set = data.iloc[(int)(len(data) * frac) : , : ]
    evaluate_set = data.iloc[0 : (int)(len(data) * frac), : ]
    return train_set, evaluate_set
```

[54]:
```
train_set, evaluate_set = split_data(data_shuffled, 0.3)
```

[55]:
```
len(train_set)
```

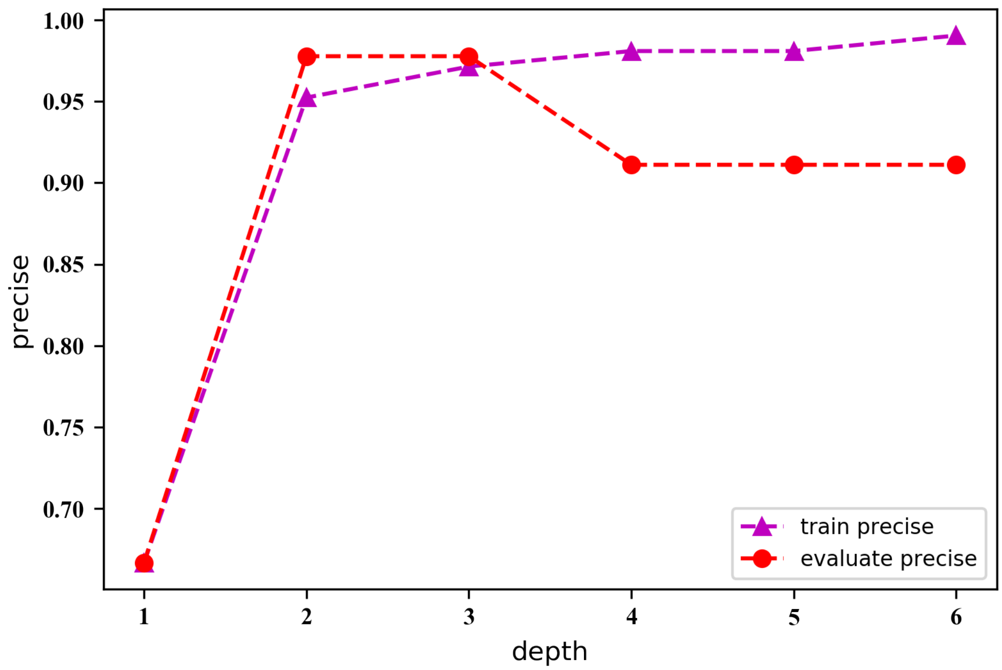
[55]: 105

[56]:
```
len(evaluate_set)
```

[56]: 45

[57]:
```
depth = []
train_precise = []
evaluate_precise = []
tree = DecisionTree('Entropy')
for d in range(1, 7):
    tree.make_tree(train_set, max_depth=d, pruning='no pruning')
    t_p = tree.evaluate(train_set)
    e_p = tree.evaluate(evaluate_set)
    depth.append(d)
    train_precise.append(t_p)
    evaluate_precise.append(e_p)
```

[58]:
```
plt.rcParams['figure.figsize'] = (6, 4)
plt.rcParams['figure.dpi'] = 300
plt.rcParams['savefig.dpi'] = 300

plt.plot(depth, train_precise, 'm^--', label='train precise')
plt.plot(depth, evaluate_precise, 'ro--', label='evaluate precise')
plt.xlabel('depth')
plt.ylabel('precise')
plt.yticks(fontproperties = 'Times New Roman', size = 9)
plt.xticks(fontproperties = 'Times New Roman', size = 9)
plt.legend(fontsize=8)
#plt.savefig('./overfitting.jpg')
#plt.savefig('./overfitting.eps', dpi=300, format='eps')
plt.show()
```



0.4.3 Can pruning prevent overfitting?

[59]:
```
tree = DecisionTree('Entropy')
pre_max_precise, _ = cross_validation(tree, 4, data_shuffled, 'pre pruning')
post_max_precise, _ = cross_validation(tree, 4, data_shuffled, 'post pruning')
no_max_precise, _ = cross_validation(tree, 4, data_shuffled, 'no pruning')
```

[60]:
```
pre_max_precise, post_max_precise, no_max_precise
```

[60]:
```
(0.9567567567567569, 0.9459459459459459, 0.972972972972973)
```

[61]:
```
tree.make_tree(train_set, pruning='no pruning')
no_precise = tree.evaluate(evaluate_set)
tree.make_tree(train_set, pruning='pre pruning')
pre_precise = tree.evaluate(evaluate_set)
tree.make_tree(train_set, pruning='post pruning')
post_precise = tree.evaluate(evaluate_set)
```

[62]:
```
no_precise, pre_precise, post_precise
```

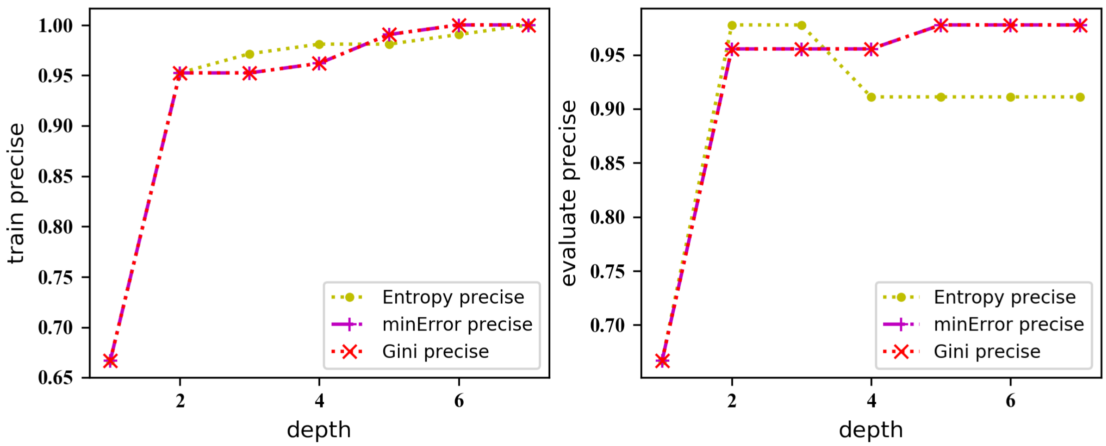
[62]:
```
(0.9111111111111111, 0.9111111111111111, 0.9777777777777777)
```

0.4.4 preformance by different criteria

[63]:
```
depth = []
entropy_train_precise = []
error_train_precise = []
gini_train_precise = []
entropy_evaluate_precise = []
error_evaluate_precise = []
gini_evaluate_precise = []
entropy_tree = DecisionTree('Entropy')
error_tree = DecisionTree('minError')
gini_tree = DecisionTree('Gini')
for d in range(1, 8):
    entropy_tree.make_tree(train_set, max_depth=d, pruning='no pruning')
    error_tree.make_tree(train_set, max_depth=d, pruning='no pruning')
    gini_tree.make_tree(train_set, max_depth=d, pruning='no pruning')
    en_t_p = entropy_tree.evaluate(train_set)
    en_e_p = entropy_tree.evaluate(evaluate_set)
    er_t_p = error_tree.evaluate(train_set)
    er_e_p = error_tree.evaluate(evaluate_set)
    gi_t_p = gini_tree.evaluate(train_set)
    gi_e_p = gini_tree.evaluate(evaluate_set)
    depth.append(d)
    entropy_train_precise.append(en_t_p)
    entropy_evaluate_precise.append(en_e_p)
    error_train_precise.append(er_t_p)
    error_evaluate_precise.append(er_e_p)
    gini_train_precise.append(gi_t_p)
    gini_evaluate_precise.append(gi_e_p)
```

[64]:
```
plt.rcParams['figure.dpi'] = 300
plt.rcParams['savefig.dpi'] = 300

fig = plt.figure(figsize=(8, 3), dpi=300)
ax0 = fig.add_subplot(1, 2, 1)
ax0.plot(depth, entropy_train_precise, 'y.:', label='Entropy precise')
ax0.plot(depth, error_train_precise, 'm+-.', label='minError precise')
ax0.plot(depth, gini_train_precise, 'rx:', label='Gini precise')
plt.xlabel('depth')
plt.ylabel('train precise')

plt.yticks(fontproperties = 'Times New Roman', size = 9)
plt.xticks(fontproperties = 'Times New Roman', size = 9)
plt.legend(fontsize=8)
ax1 = fig.add_subplot(1, 2, 2)
ax1.plot(depth, entropy_evaluate_precise, 'y.:', label='Entropy precise')
ax1.plot(depth, error_evaluate_precise, 'm+-.', label='minError precise')
ax1.plot(depth, gini_evaluate_precise, 'rx:', label='Gini precise')
```

```
plt.xlabel('depth')
plt.ylabel('evaluate precise')

plt.yticks(fontproperties = 'Times New Roman', size = 9)
plt.xticks(fontproperties = 'Times New Roman', size = 9)
plt.legend(fontsize=8)
#plt.savefig('./precise_vs_criteria.jpg')
#plt.savefig('./precise_vs_criteria.eps', dpi=300, format='eps')
plt.show()
```



[65]:
```
entropy_tree = DecisionTree('Entropy')
error_tree = DecisionTree('minError')
gini_tree = DecisionTree('Gini')
```

[66]:
```
entropy_precise, _ = cross_validation(entropy_tree, 5, data_shuffled, 'no␣
↪pruning')
error_precise, _ = cross_validation(error_tree, 5, data_shuffled, 'no pruning')
gini_precise, _ = cross_validation(gini_tree, 5, data_shuffled, 'no pruning')
```

[67]:
```
entropy_precise, error_precise, gini_precise
```

[67]:
```
(0.96, 0.9200000000000002, 0.9200000000000002)
```

0.4.5 Something went wrong!

[68]:
```
index = [72, 133, 56, 87]
```

[69]:
```
test_data = data_shuffled.loc[index]
test_data
```

[69]:

	Attr.1	Attr.2	Attr.3	Attr.4	label
72	6.3	2.5	4.9	1.5	Iris-versicolor
133	6.3	2.8	5.1	1.5	Iris-virginica

```
56      6.3     3.3     4.7     1.6  Iris-versicolor
87      6.3     2.3     4.4     1.3  Iris-versicolor
```

[70]:
```
find_best_attribute(test_data, error_gain)
```

[70]: 'Attr.1'

[71]:
```
find_best_split_point(test_data, 'Attr.3', cal_error_after_split)
```

[71]: 4.75

[72]:
```
cal_error_after_split(test_data, 'Attr.1', 6.3)
```

[72]: 0.25

0.4.6 Show how decision works

[73]:
```
label_name_list = list(data.groupby('label').size().index)
attr_name_list = list(data.columns[:-1])
```

[74]:
```
def split_data(data):
    data_ = []
    for i in range(len(label_name_list)):
        data_.append(data[data['label'] == label_name_list[i]])
    attr_data = []
    for i in range(len(data_)):
        temp_data = []
        for j in range(len(attr_name_list)):
            temp_data.append(data_[i].iloc[:,j])
        attr_data.append(temp_data)
    return attr_data
```

[75]:
```
def plot_sub_scatter(attr_data, i, j):
    plt.scatter(attr_data[0][i], attr_data[0][j], c='c', s=3,␣
 ↪label=label_name_list[0])
    plt.scatter(attr_data[1][i], attr_data[1][j], c='g', s=3,␣
 ↪label=label_name_list[1])
    plt.scatter(attr_data[2][i], attr_data[2][j], c='b', s=3,␣
 ↪label=label_name_list[2])
    plt.xlabel('$Attr.%d$' % (i + 1), fontsize=6)
    plt.ylabel('$Attr.%d$' % (j + 1), fontsize=6)
    plt.yticks(fontproperties = 'Times New Roman', size = 6)
    plt.xticks(fontproperties = 'Times New Roman', size = 6)
    plt.legend(loc=1, fontsize=6)
```

[76]:
```
def plot_scatter(attr_data, attr_name, split_point, save_name=None):
    plt.rcParams['figure.dpi'] = 300
    plt.rcParams['savefig.dpi'] = 300

    fig = plt.figure(figsize=(10, 6), dpi=300)
    fig.add_subplot(2, 3, 1)
```

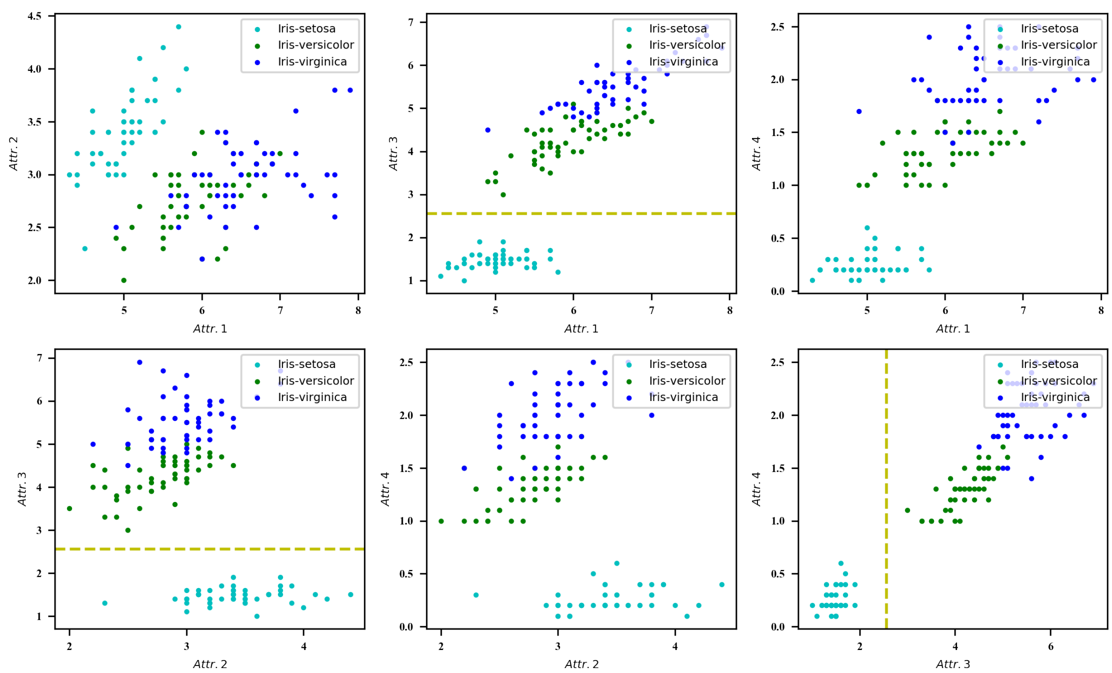
```python
    plot_sub_scatter(attr_data, 0, 1)
    if attr_name == 'Attr.1':
        plt.axvline(x=split_point, c = "y", ls = "dashed")
    elif attr_name == 'Attr.2':
        plt.axhline(y=split_point, c = "y", ls = "dashed")

    fig.add_subplot(2, 3, 2)
    plot_sub_scatter(attr_data, 0, 2)
    if attr_name == 'Attr.1':
        plt.axvline(x=split_point, c = "y", ls = "dashed")
    elif attr_name == 'Attr.3':
        plt.axhline(y=split_point, c = "y", ls = "dashed")

    fig.add_subplot(2, 3, 3)
    plot_sub_scatter(attr_data, 0, 3)
    if attr_name == 'Attr.1':
        plt.axvline(x=split_point, c = "y", ls = "dashed")
    elif attr_name == 'Attr.4':
        plt.axhline(y=split_point, c = "y", ls = "dashed")

    fig.add_subplot(2, 3, 4)
    plot_sub_scatter(attr_data, 1, 2)
    if attr_name == 'Attr.2':
        plt.axvline(x=split_point, c = "y", ls = "dashed")
    elif attr_name == 'Attr.3':
        plt.axhline(y=split_point, c = "y", ls = "dashed")

    fig.add_subplot(2, 3, 5)
    plot_sub_scatter(attr_data, 1, 3)
    if attr_name == 'Attr.2':
        plt.axvline(x=split_point, c = "y", ls = "dashed")
    elif attr_name == 'Attr.4':
        plt.axhline(y=split_point, c = "y", ls = "dashed")

    fig.add_subplot(2, 3, 6)
    plot_sub_scatter(attr_data, 2, 3)
    if attr_name == 'Attr.3':
        plt.axvline(x=split_point, c = "y", ls = "dashed")
    elif attr_name == 'Attr.4':
        plt.axhline(y=split_point, c = "y", ls = "dashed")

    if save_name is not None:
        plt.savefig(save_name + '.jpg')
        plt.savefig(save_name + '.eps', dpi=300, format='eps')
    plt.show()
```

[77]:
```python
attr_data = split_data(data)
```

[78]:
```python
best_attribute = find_best_attribute(data, entropy_gain)
split_point = find_best_split_point(data, best_attribute,␣
 ↪cal_entropy_after_split)
best_attribute, split_point
```

[78]:
```
('Attr.3', 2.5572901556)
```

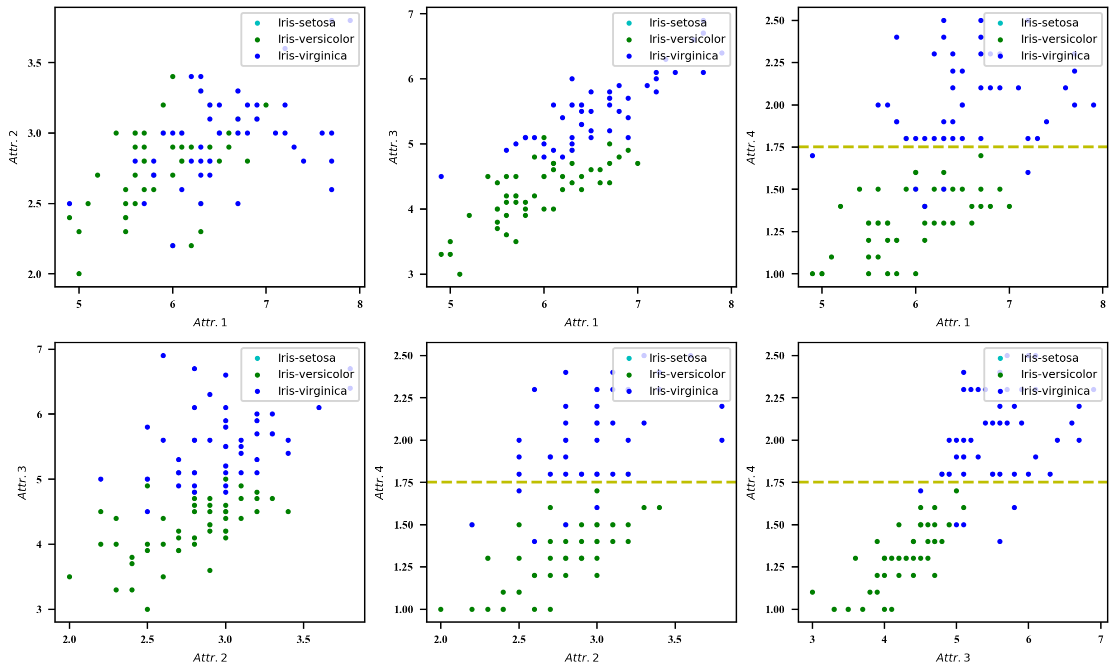
[79]:
```python
plot_scatter(attr_data, best_attribute, split_point)
```



[80]:
```python
data_1 = data[data[best_attribute] > split_point]
attr_data = split_data(data_1)
best_attribute = find_best_attribute(data_1, entropy_gain)
split_point = find_best_split_point(data_1, best_attribute,␣
 ↪cal_entropy_after_split)
best_attribute, split_point
```

[80]:
```
('Attr.4', 1.749964774)
```

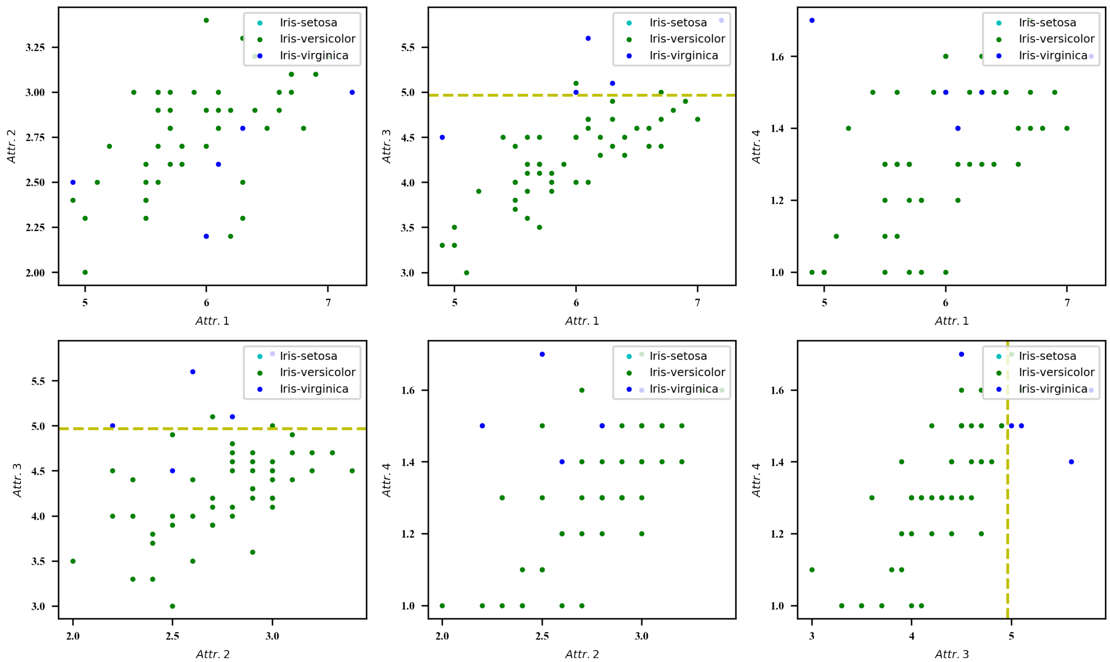
[81]:
```python
plot_scatter(attr_data, best_attribute, split_point)
```



[82]:
```
data_2 = data_1[data_1[best_attribute] <= split_point]
attr_data = split_data(data_2)
best_attribute = find_best_attribute(data_2, entropy_gain)
split_point = find_best_split_point(data_2, best_attribute,␣
 ↪cal_entropy_after_split)
best_attribute, split_point
```

[82]: ('Attr.3', 4.964518196849209)

[83]:
```
plot_scatter(attr_data, best_attribute, split_point)
```


Iris-setosa
Iris-versicolor
Iris-virginica
Attr. 1
Attr. 2
Attr. 3
Attr. 4

[ ]:

[ ]: