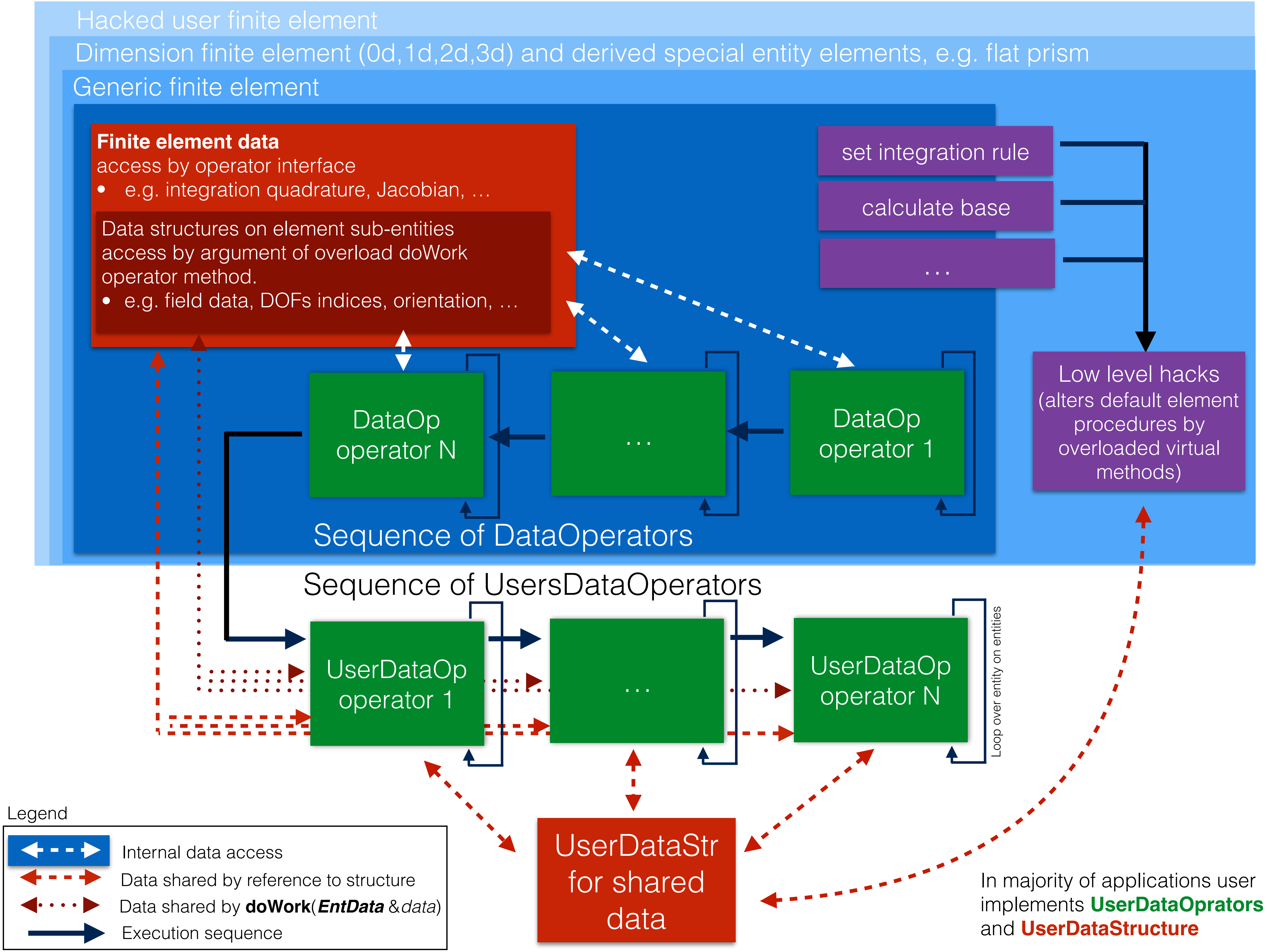
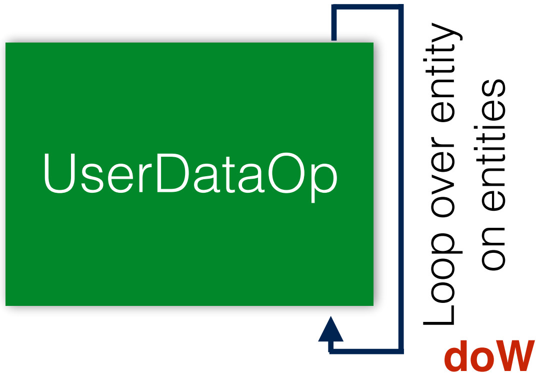

Hacked user finite element
Dimension finite element (0d,1d,2d,3d) and derived special entity elements, e.g. flat prism
Generic finite element
Finite element data
access by operator interface
- e.g. integration quadrature, Jacobian, …
Data structures on element sub-entities access by argument of overload doWork operator method.
- e.g. field data, DOFs indices, orientation, …
set integration rule
calculate base
…
Low level hacks (alters default element procedures by overloaded virtual methods)
DataOp operator N
…
DataOp operator 1
Sequence of DataOperators
Sequence of UsersDataOperators
UserDataOp operator 1
…
UserDataOp operator N
Loop over entity on entities
UserDataStr for shared data
Legend
Internal data access
Data shared by reference to structure
Data shared by **doWork**(***EntData*** &*data*)
Execution sequence

In majority of applications user implements **UserDataOprators** and **UserDataStructure**

Type: OPROW or OPCOL (indices on rows or on columns)

Constructor:

```
UserDataOperator(const std::string &field_name, const
char type)
```



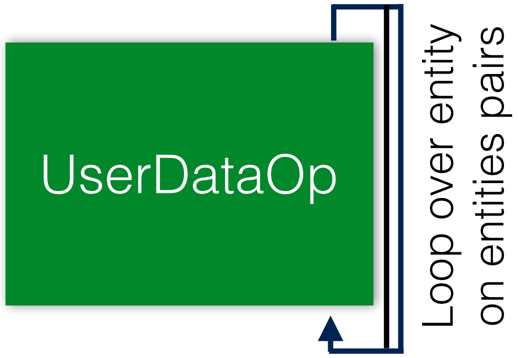
```
entities_set = { Vertices, Edge0, .., Edge5,
Face0, .., Face3, Volume }
for(o in operator_sequence)
  for(e in entities_set)
    o.doWork(side[e],type[e],ent_data[e])
```

doWork is overloaded method by user (loop is implicitly called by element)

Type: OPROWCOL (indices on rows & on columns)

Constructor:

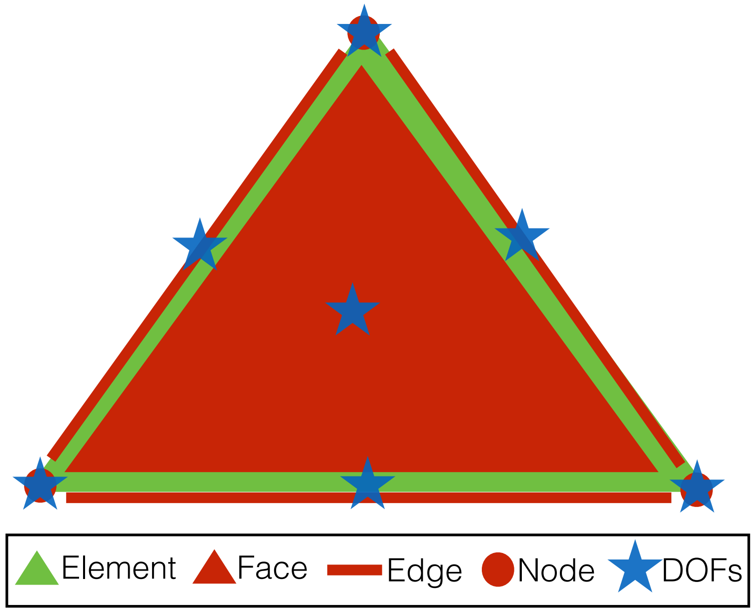
```
UserDataOperator(const std::string &row_field_name,
const std::string &col_field_name, const char type,
const bool symm=true)
```



```
entities_pair_set = { {Vertices, Vertices},
{Vertices,Edge0}, .., { Volume, Volume} }
for(o in operator_sequence)
 for(e in entities_pair_set)
  o.doWork(
   row_side[e.f],row_type[e.f],row_ent_data[e.f],
   col_side[e.s],col_type[e.s],col_ent_data[e.s]
  )
```

doWork is overloaded method by user (loop is implicitly called by element)

For square matrices & symmetric finite element *OPROW* & *OPCOL* are equivalent.
For *OPROWCOL*, when *symm = true*, only unique pairs are processed. It is third kind of operator, which not loop on entities of particular field, but entities of space, e.g. used to apply transformation to base functions. You can as well set *type = OPROW|OPROWCOL*.



Tetrahedral has base & DOFs on entities.

By space:

- **Space H1:** Vertices, 6 Edges, 4 Faces (Tri, Quad), 1 Volume
- **Space H-Curl:** 6 Edges,4 Faces, 1 Volume
- **Space H-Div:** 4 Faces, 1 Volume
- **Space L2:** 1 Volume (Tet, Prism, Hex, Wedge, …)

By order:

- **H1 order 1:** Only on Vertices
- **H1 order 2:** Vertices and Edges
- **H1 order 3:** Vertices, Edges & Faces
- **H1 order 4 and more:** Verices, Egdes, Faces and Volume

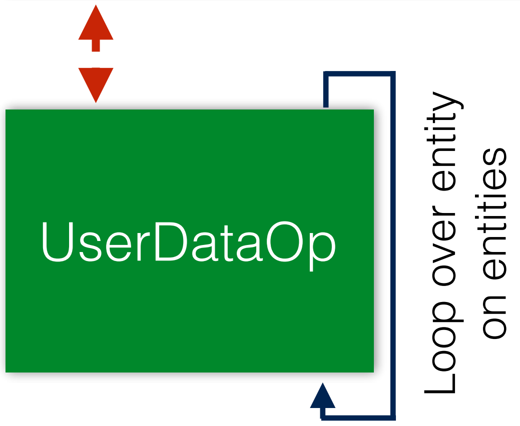
In similar way for other approximation spaces.

EntData:

- Values at DOFs
- Global/Local indices of DOFs
- Base/Space/Order/Sense
- Base functions & more

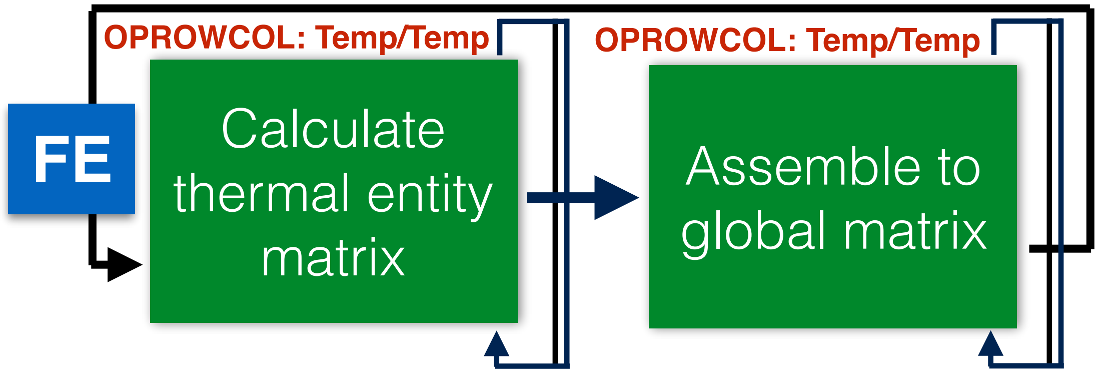
UserDataStr

- Temperature at Gauss pts.
- Stresses at Gauss pts.
- Strains at Gauss pts.
- Local entity matrix (Lhs)
- Local entity vector (Rhs)
- Global thermal matrix
- Global elastic matrix
- Global thermal vector
- Global elastic vector

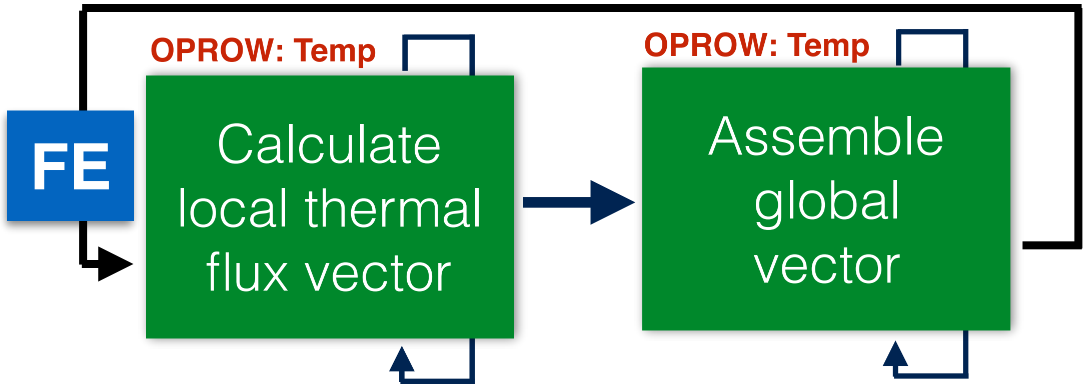


Thermal element

1) Loop body domain volume entities

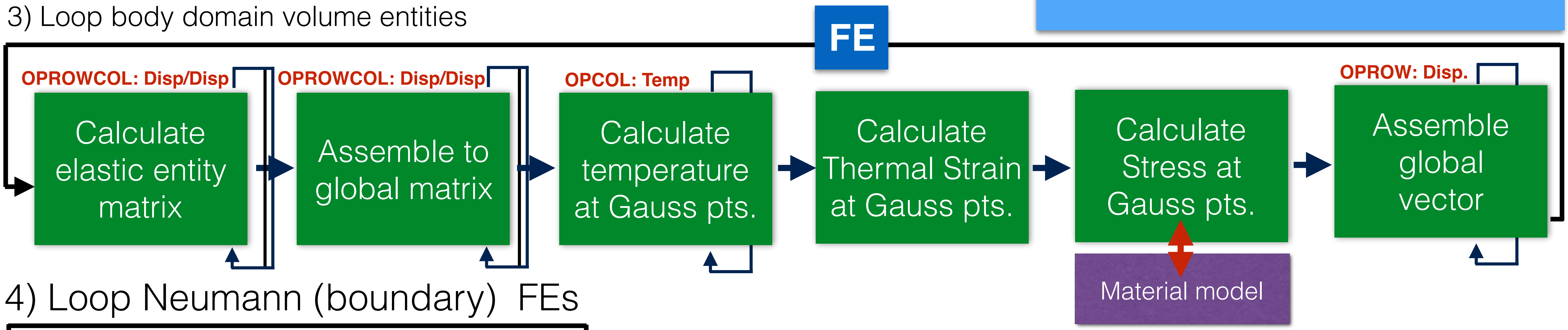


2) Loop Neumann (boundary) surface entities

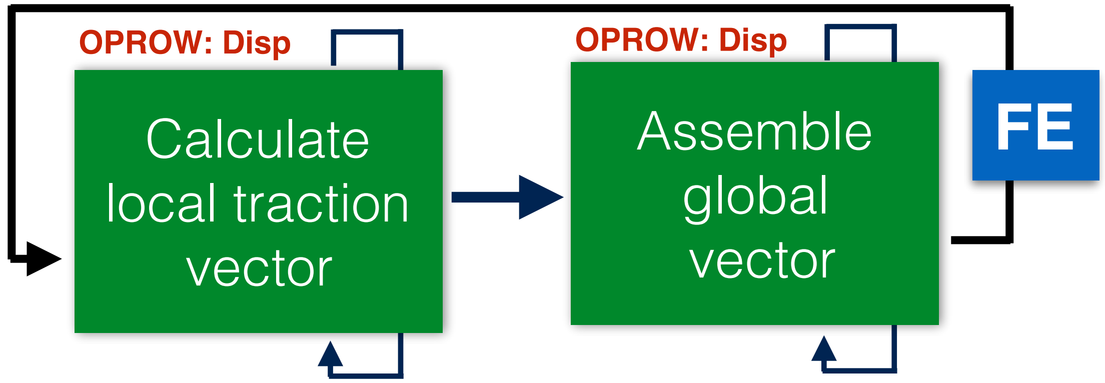


Elastic element

3) Loop body domain volume entities


FE
OPROWCOL: Disp/Disp
Calculate elastic entity matrix
OPROWCOL: Disp/Disp
Assemble to global matrix
OPCOL: Temp
Calculate temperature at Gauss pts.
Calculate Thermal Strain at Gauss pts.
Calculate Stress at Gauss pts.
Material model
OPROW: Disp.
Assemble global vector

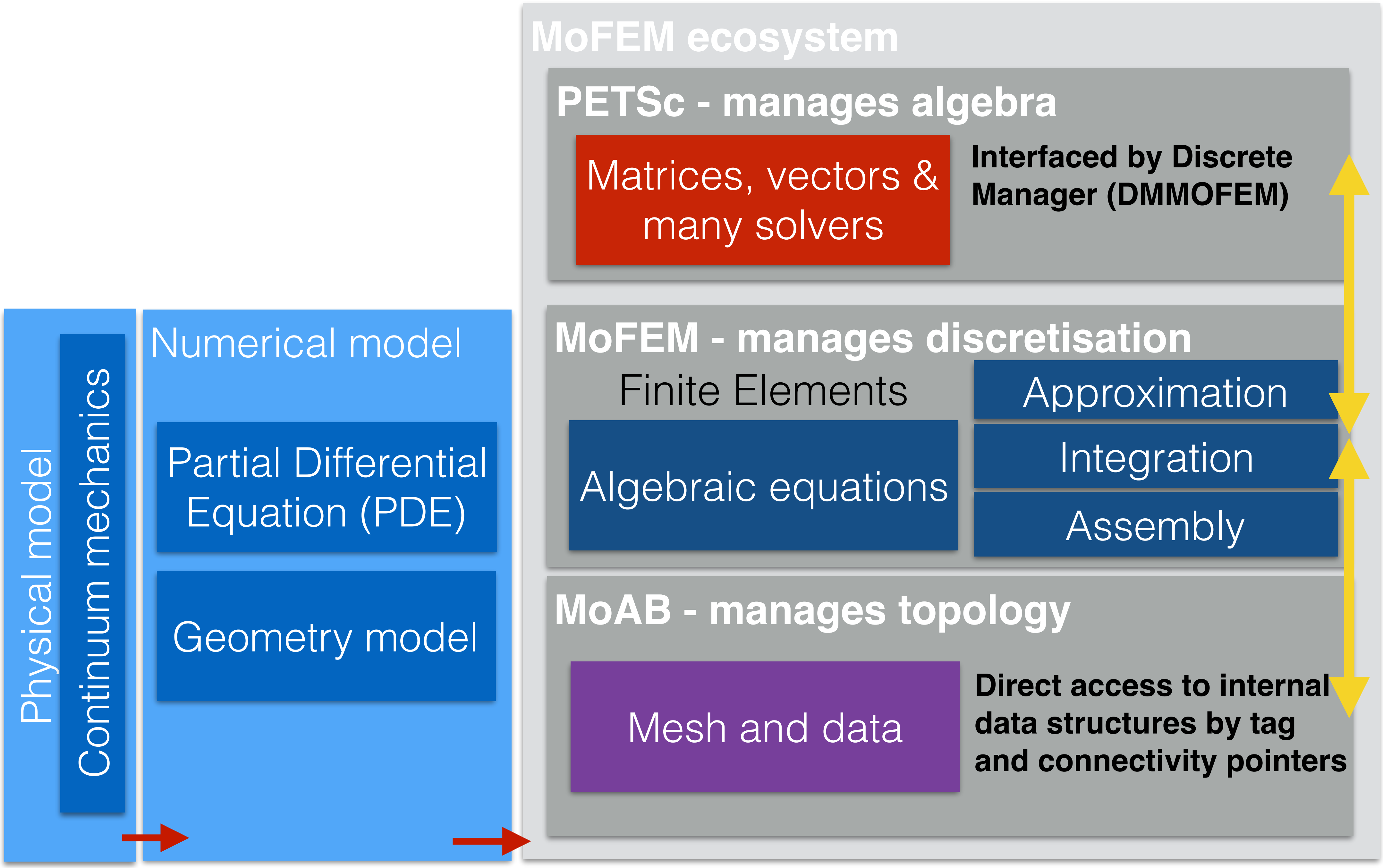
4) Loop Neumann (boundary) FEs



Notes:

- Operators can be implemented such that works for 2d/3d problems
- The same operator can be used in several different contexts, e.g. assembly of vector
- Unit (atom) tests can be written for each operator independently
- Each problem, thermal, elastic & thermo-elastic can be tested&run, by adding/removing operators
- Developer focus attention on operator implementation
- Loops, indexing, etc. are managed by MoFEM
- Each field is declared independently by space (H1, Hcurl,Hdiv, L2) & base (Legendre, Jacobi, …), on heterogenous and arbitrary approximation order
- Local and global indices on entity are passed by **EntData** as reference to overloaded **doWork** method in operator.

- Base functions and derivatives are evaluated by element and easy accessed from users operator from **EntData**
- **Note that implementation of UserOperator is for entities on element, not element itself.**



Physical model

Continuum mechanics

Numerical model

Partial Differential Equation (PDE)

Geometry model

MoFEM ecosystem

PETSc - manages algebra

Matrices, vectors & many solvers

Interfaced by Discrete Manager (DMMOFEM)

MoFEM - manages discretisation

Finite Elements

Algebraic equations

Approximation

Integration

Assembly

MoAB - manages topology

Mesh and data

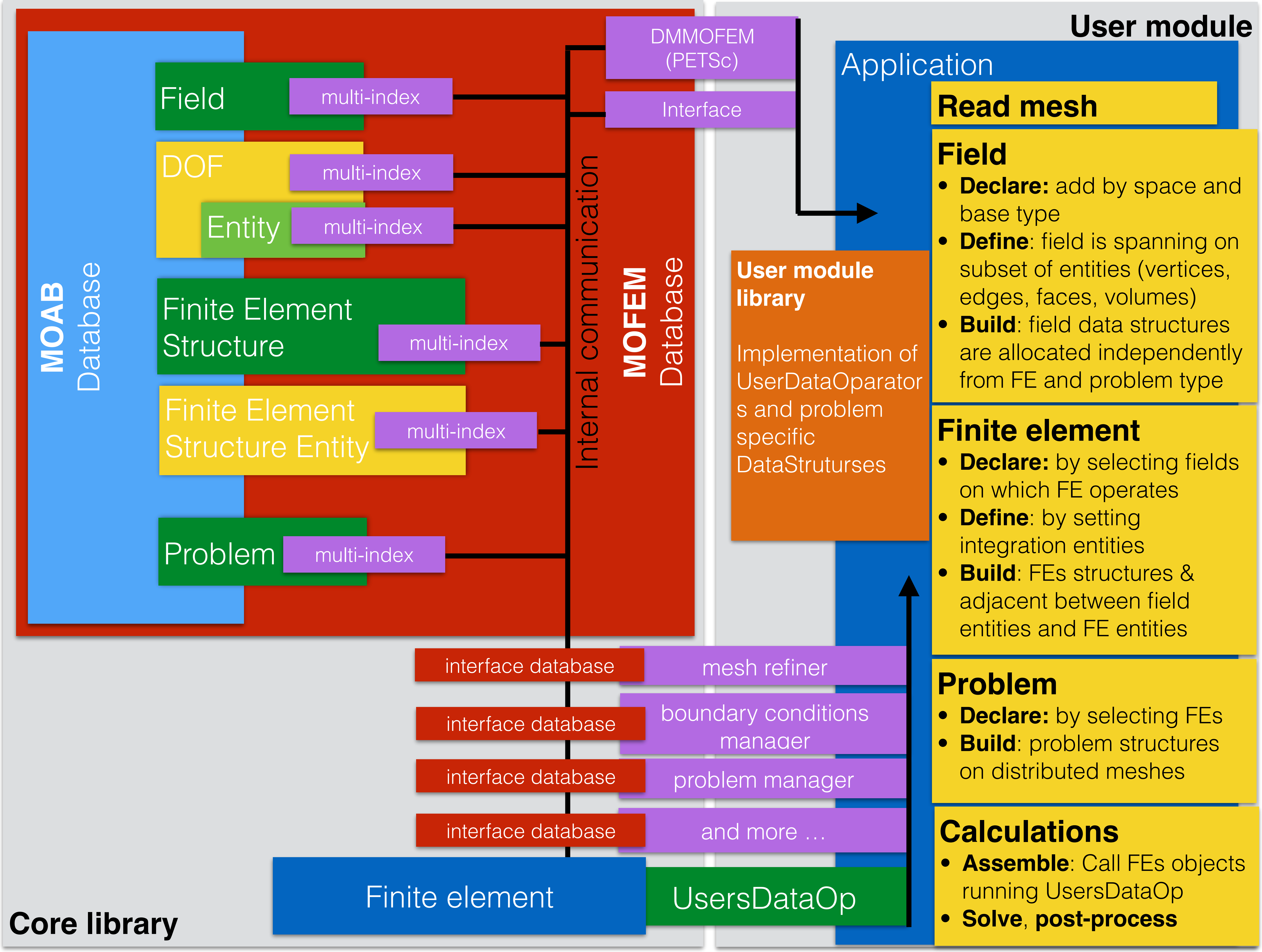
Direct access to internal data structures by tag and connectivity pointers

Design objectives

- scalability & runtime efficiency & memory efficiency (order is solver & problem size dependent)
- extendibility

- extendibility (hack ability)
- scalability
- runtime efficiency
- memory efficiency

- memory efficiency
- runtime efficiency
- scalability
- extendibility



User module library

Implementation of UserDataOparators and problem specific DataStrutures

Read mesh

Field

- **Declare:** add by space and base type
- **Define**: field is spanning on subset of entities (vertices, edges, faces, volumes)
- **Build**: field data structures are allocated independently from FE and problem type

Finite element

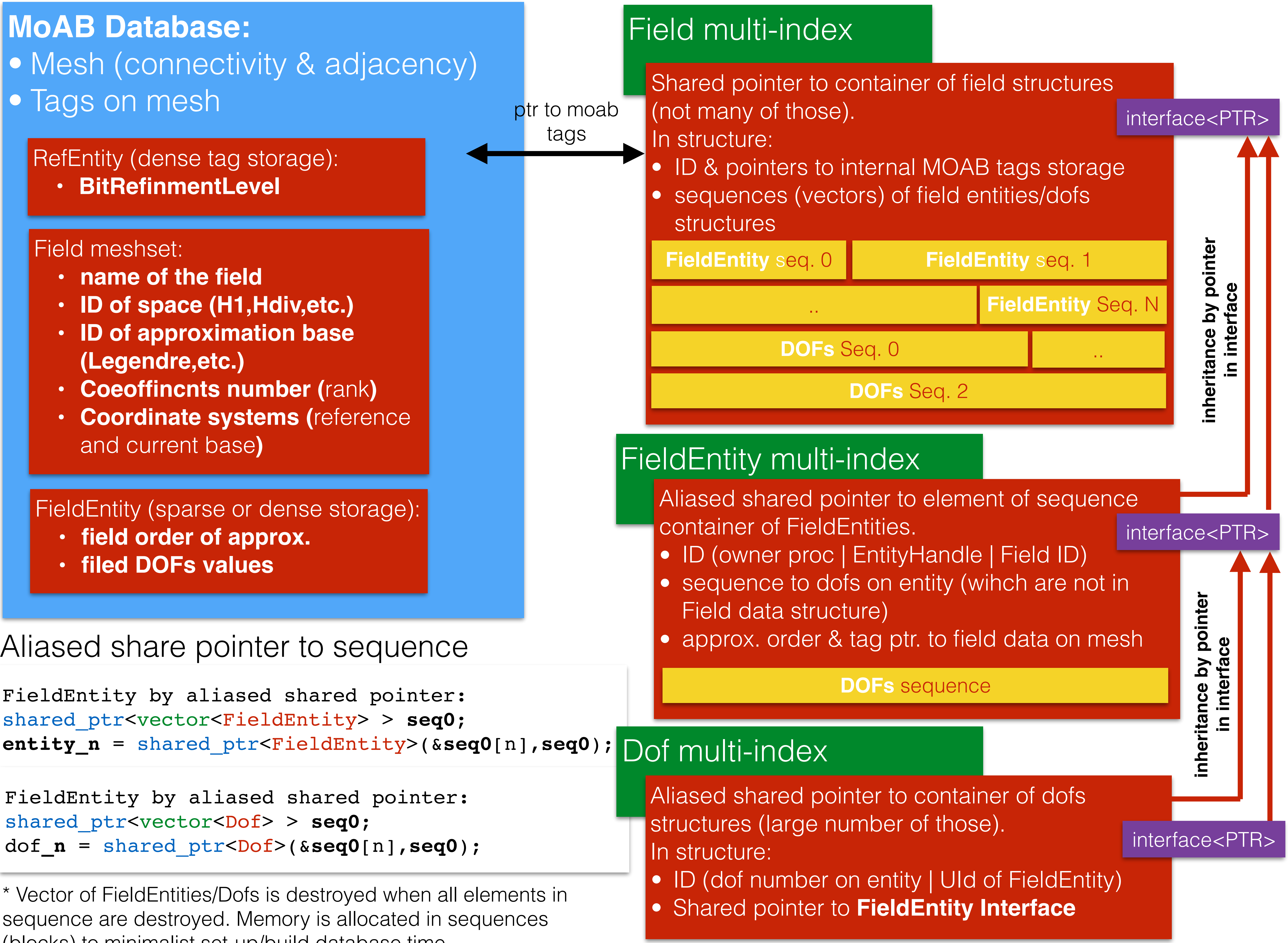
- **Declare:** by selecting fields on which FE operates
- **Define**: by setting integration entities
- **Build**: FEs structures & adjacent between field entities and FE entities

Problem

- **Declare:** by selecting FEs
- **Build**: problem structures on distributed meshes

Calculations

- **Assemble**: Call FEs objects running UsersDataOp
- **Solve**, **post-process**



MoAB Database:

- Mesh (connectivity & adjacency)
- Tags on mesh

RefEntity (dense tag storage):

- **BitRefinmentLevel**

Field meshset:

- **name of the field**
- **ID of space (H1,Hdiv,etc.)**
- **ID of approximation base (Legendre,etc.)**
- **Coeoffincnts number (**rank)
- **Coordinate systems (**reference and current base**)**

FieldEntity (sparse or dense storage):

- **field order of approx.**
- **filed DOFs values**

Field multi-index

Shared pointer to container of field structures (not many of those).
In structure:

- ID & pointers to internal MOAB tags storage
- sequences (vectors) of field entities/dofs structures

FieldEntity multi-index

Aliased shared pointer to element of sequence container of FieldEntities.

- ID (owner proc | EntityHandle | Field ID)
- sequence to dofs on entity (wihch are not in Field data structure)
- approx. order & tag ptr. to field data on mesh

Dof multi-index

Aliased shared pointer to container of dofs structures (large number of those).
In structure:

- ID (dof number on entity | UId of FieldEntity)
- Shared pointer to **FieldEntity Interface**

Aliased share pointer to sequence

```
FieldEntity by aliased shared pointer:
shared_ptr<vector<FieldEntity> > seq0;
entity_n = shared_ptr<FieldEntity>(&seq0[n],seq0);
```

```
FieldEntity by aliased shared pointer:
shared_ptr<vector<Dof> > seq0;
dof_n = shared_ptr<Dof>(&seq0[n],seq0);
```

* Vector of FieldEntities/Dofs is destroyed when all elements in sequence are destroyed. Memory is allocated in sequences (blocks) to minimalist set-up/build database time.