

A MODEL FOR DIURNAL VARIATION IN SOIL AND AIR TEMPERATURE*

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ABSTRACT

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A model for predicting diurnal changes in soil and air temperatures given the maximum and minimum temperatures has been developed. The model uses a truncated sine wave to predict daytime temperature changes and an exponential function to predict nighttime temperatures. The model is based upon hourly soil and air temperatures for 1977 at a shortgrass prairie site and is parameterized for 150-cm and 10-cm air temperatures and for soil-surface and 10-cm soil temperatures. The absolute mean error for the model ranged from a maximum of 2.64°C for the 10-cm air temperature to a minimum of 1.20°C for the 10-cm soil temperature. The model was also parameterized for hourly air temperature data for Denver, Colorado. Comparison of the model with other models showed that it did a superior job of fitting the data with a smaller number of parameters.

INTRODUCTION

The two most frequently used techniques for simulating diurnal variation in soil and air temperatures are empirical models and energy-budget models. The energy-budget models (Myrup, 1969; Lemon et al., 1971; Goudriaan and Waggoner, 1972) use different methods to calculate the energy budget at the plant canopy and soil surface and thus determine soil and air temperatures. Empirical models, such as those developed by Langbein (1949), Fluker (1958), and Parton (1978), use the Fourier heat conduction equation to predict soil temperature as a function of soil-surface temperature. Linear (Sanders, 1975) and nonlinear models (Heuer et al., 1978), Fourier series models (Walter, 1967; Watanabe, 1978), and sinusoidal models (Johnson and Fitzpatrick, 1977a, b) have been used to predict diurnal variation in soil and air temperatures. Energy-budget models are difficult to use because they require extensive computer time and data inputs (i.e., solar radiation, wind-speed, dew point, and air temperature); whereas the empirical model may require only the maximum and minimum temperatures.

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We have developed a model for predicting diurnal variations in soil and air temperatures given the maximum and minimum temperatures and the Julian date. The model uses a truncated sine wave to predict variation of daytime temperatures and an exponential function to predict nighttime temperatures. The use of a sine wave during the daytime is very similar to the approach used in the sinusoidal model proposed by Johnson and Fitzpatrick (1977a). This model is based on hourly soil and air temperature data observed during 1977 at a shortgrass prairie site (the Pawnee Site, field research facility of the Natural Resource Ecology Laboratory, Colorado State University, located on the USDA, SEA-AR Central Plains Experimental Range) in northeastern Colorado. Over 95% of the leaf area index (*LAI*) at the site is grass and forb ground cover, primarily *Bouteloua gracilis*, which is 3–6 cm tall. Leaf area index of live biomass at peak standing crop is 0.5 and occurs during June or July (Knight, 1973). Hourly values of soil temperature (0, –10, –20, and –50 cm) and air temperature (10 and 150 cm) were observed for one year and recorded on an automatic data-logger system. The air and soil temperatures were measured with thermistors. The air temperature thermistors were shielded from direct radiation with metal sheets above and below the sensor, while the soil surface temperature was measured with an unshielded thermistor placed directly on the soil surface. The data were recorded on cassette tapes that were dumped into Colorado State University computer system every two weeks. The data were screened by comparing them with manually observed maximum and minimum daily air and soil temperature data from the site.

Parameter values in the equations were determined by fitting the equations to the observed data (Powell, 1965). The model was also parameterized for hourly temperatures in Denver, Colorado. The model was compared with a Fourier series model and a curvilinear model (Heuer et al., 1978) and proved to be superior to both.

MODEL DESCRIPTION

Mathematical descriptions of the model for daytime and nighttime are shown in eqs. 1 and 2, respectively

$$T_i = (T_x - T_N) \sin \left(\frac{\pi m}{Y + 2a} \right) + T_N \quad (1)$$

$$T_i = T_N + (T_s - T_N) \exp - (bn/Z) \quad (2)$$

where Y is day length (h), Z is night length (h), T_i is temperature at the i th hour, T_x and T_N are the maximum and minimum temperatures, T_s is the temperature at sunset, m is the number of hours after the minimum temperature occurs until sunset, n is the number of hours after sunset until the time of the minimum temperature, a is the lag coefficient for the maximum tempera-

ture, and b is the nighttime temperature coefficient. Another parameter in the model is c , the lag of the minimum temperature from the time of sunrise. The model assumes that the maximum temperature will occur sometime during the daylight hours and that the minimum temperature will occur within a few hours before or after sunrise.

The input variables required by the model include maximum and minimum temperatures and Julian date. The day and night lengths (Y and Z) are determined as functions of Julian date and latitude, using the equation presented by Sellers (1965); the temperature at sunset (T_s) is calculated with eq. 1, while m and n are functions of time of day, day length, and the lag coefficient for the minimum temperature (c). The coefficients a , b , and c vary as functions of soil depth or height above the soil surface and could also vary as functions of soil type and soil water content. Values for these coefficients were determined for different soil depths and air temperatures, using temperature data from the Pawnee Site. A Fortran IV program of the model is presented in the Appendix and includes the equations for calculating day length as a function of latitude and Julian date.

The important assumptions in the model are (a) that maximum temperature will occur sometime during the daylight hours before sunset, (b) that the minimum temperature will occur during the early morning hours, and (c) that temperature variation during the day is described by a truncated sine wave, while the nighttime temperatures are described by an exponential function. The validity of these assumptions was tested with observed air and soil temperature data for 1977 (Figs. 1 and 2). Figure 1 shows the average time when the maximum and minimum temperatures occur and indicates that the minimum soil-surface and air temperatures occurred at about the same time, generally 2–3 hours before sunrise, and that significant increases in the soil temperature do not occur until after sunrise and are delayed with depth. A detailed analysis of the temperature data (see Fig. 3) shows that air temperature at sunrise and the minimum temperature differ

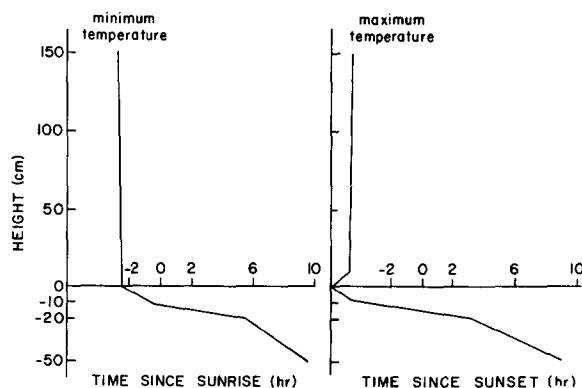


Fig. 1. Average time when the maximum and minimum soil and air temperatures occur at the Pawnee site for 1977. Time of sunrise and sunset is 6 a.m. and 6 p.m., respectively.

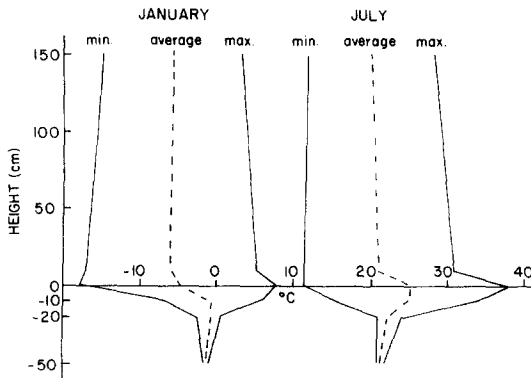


Fig. 2. Average daily maximum, minimum, and average soil and air temperature for January and July at the Pawnee Site for 1977.

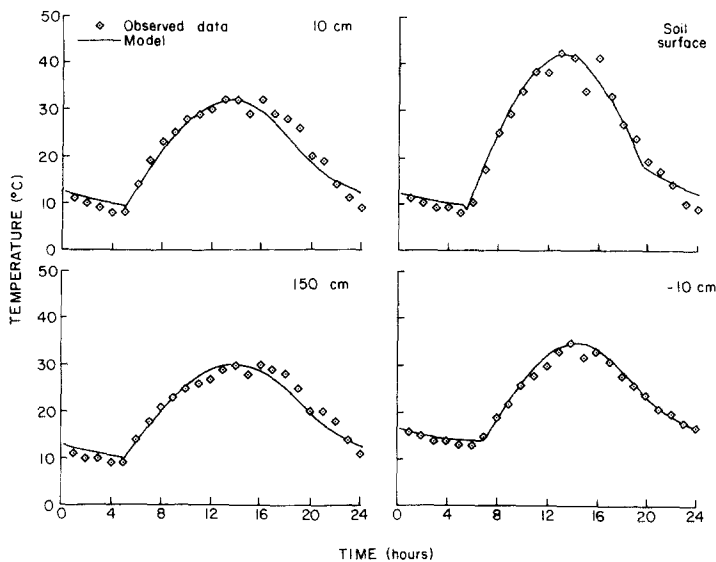


Fig. 3. Observed vs. simulated temperature for the 150- and 10-cm air temperature and the soil-surface and -10-cm soil temperature.

very little ($\sim 1^{\circ}\text{C}$) and that temperatures do not increase significantly until sunrise. Minimum temperature occurs later in the soil than in the air, with the time of the minimum temperature increasing steadily from 0312 at the soil surface to 1536 at the 50-cm soil depth.

The maximum temperature at the soil surface occurs at noon, and the time of the maximum temperature then increases with increasing height above the soil and also with increasing soil depth. From the soil surface to 20-cm soil depth, the time of maximum temperature was delayed from 1200 to 2100, while at the 150-cm height the maximum air temperature occurred

at 1315. The results suggest that the assumptions in the temperature model are consistent with the air temperature data but are verified by the soil data only from soil surface to the 15-cm depth. At depths greater than 15 cm the maximum temperature occurs after sunset, thus violating a critical model assumption.

Figure 2 shows the average maximum and minimum temperatures for January and July and indicates that the maximum diurnal variation occurred at the soil surface, then decreased with increasing height in the air and with increasing soil depth. The diurnal variation decreases much more rapidly with increasing soil depth than with increasing height above the soil surface. At the 20-cm soil depth the diurnal variation is less than 2°C , while at 150 cm the diurnal variation is greater than 15°C . The data for January and July show little seasonal variation in the magnitude of the diurnal temperature variation; however, the soil temperature variations are lower in January, while the air temperature variations are slightly lower in July. The major change between January and July is that the difference between the maximum soil-surface temperature and the 150-cm temperature is considerably greater in July than in January ($+11.5$ vs. $+2.5^{\circ}\text{C}$). The results (Figs. 1 and 2) suggest that the model can be used for the soil and air temperatures where the diurnal variations are greatest. The fact that the model does not work for soil depths greater than 15 cm is unimportant, since the diurnal variations are small ($< 2^{\circ}\text{C}$).

A comparison of the data for soil and air temperatures at the Pawnee Site with data at other grassland sites (Whitman, 1969; Old, 1969) and data from crop systems (Rosenberg, 1974) shows that different grassland sites and crop systems have the same basic types of temperature patterns, indicating that the model would be applicable for most locations.

Parameter estimation

Hourly values of soil and air temperatures for 1977 were used to estimate parameters for the 150- and 10-cm air temperatures and for the soil surface and 10-cm soil temperatures. Annual parameter values were estimated with Powell's (1965) function-minimizing algorithm. The soil and air temperatures were automatically recorded hourly with a data logger, then transferred to the Control Data Corporation CYBER computer system at Colorado State University.

Table I presents the value of the lag coefficient for the maximum temperature (a), the nighttime temperature coefficient (b), and the lag in minimum air temperature after sunrise (c) for the different soil and air temperature locations. The lag coefficient for the maximum temperature ranges from 0.45 h at the 10-cm soil depth to 1.86 h at the 150-cm height. The nighttime temperature coefficient ranges from 1.81 at soil surface to 2.28 at the 10-cm soil depth. The lag coefficient for the minimum temperature is -0.18 for the air temperatures and increases to 0.49 at the soil surface and to 1.83 at 10-cm

TABLE I

Annual values of the coefficients (a , b , and c) in eqs. 1 and 2 for different soil and air temperatures

Location	a (h)	b	c (h)	Absolute mean error* (°C)	Root mean square error* ² (°C)
150-cm air temperature	1.86	2.20	-0.17	2.35	3.14
10-cm air temperature	1.52	2.00	-0.18	2.64	3.62
Soil-surface temperature	0.50	1.81	0.49	2.61	3.66
-10-cm soil temperature	0.45	2.28	1.83	1.20	1.74

$$* u = \sum_{i=1}^n \text{ABS}(X_i^s - X_i^0)/N$$

$$*^2 R = [\sum_{i=1}^n (X_i^s - X_i^0)^2]^{1/2}/N$$

where u = absolute mean error, n = number of observations, X_i^0 = simulated temperature for the i th observation, X_i^s = observed temperature for the i th observation, and R = the root mean square error.

soil depth. The values for a and c are consistent with the observed times of maximum and minimum temperatures shown in Fig. 1.

The absolute mean error and root mean square error (Table I) are presented to indicate how well the model results compare with the observed data. The error terms are largest for the soil-surface and 10-cm air temperatures. The root mean square error ranged from 3.66 at the soil surface to 1.74°C for the 10-cm soil depth. The error terms are largest at the soil surface and 10-cm height because temperature at these levels responds most rapidly to short-term change in wind speed and cloud cover.

The hourly root mean squared error was also calculated (Fig. 4). The error term is fairly homogeneous throughout the day; however, the error tends to be largest during the afternoon. This pattern is most pronounced with the soil-surface temperatures, which have a root mean squared error of less than 3.0°C in the early morning and increase during the day to greater than 5°C in the late afternoon. With the other layers the diurnal range is generally less than 1°C. Analysis of hourly data shows that the large error term in late afternoon is caused by short-term variation in cloud cover.

The model was also parameterized for five years of hourly temperature data (1950–1954) for Denver. The estimated parameter values for a , b , and c were 1.80, 2.20, and 0.88, respectively. A comparison with the 150-cm parameter estimates of the Pawnee site shows that the values of a and b are very similar but that the value of c is one hour greater for Denver. The reason for the difference is unknown but could be related to the fact that

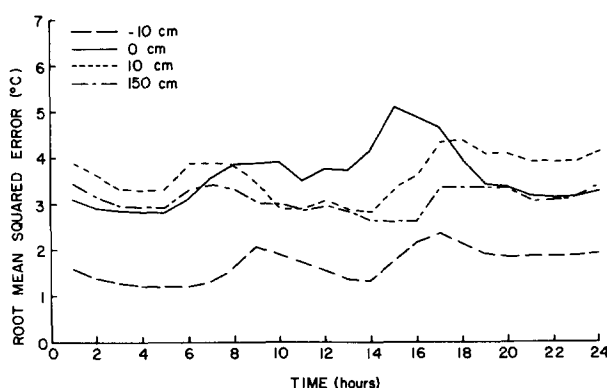


Fig. 4. Root mean squared error for the soil and air temperature layers as a function of time of day.

the Pawnee temperatures were measured with a thermistor shielded from direct radiation, while the Denver temperatures were observed in a standard meteorological shelter. The model did an excellent job of fitting the Denver data, since the absolute mean error was lower (1.62 vs. 2.35°C) for the Denver data.

The model was further verified by comparing observed data and model results for July 6, 1977 (Fig. 3). The results for the simulated air temperature and soil temperature layers compare very favorably with observed data. The major discrepancy is that the model does not predict the short-term variation in soil-surface temperature during the afternoon; however, most of the model results are within 1 to 2°C of the observed data.

Error analysis

A detailed error analysis shows that the model is least accurate on days when cloud cover is variable during the daytime and when the minimum temperature for the day occurs during the afternoon, as when a cold front passes through during the middle of the day. The error associated with this situation could be reduced if the minimum temperature from the following day is used to calculate the temperature decrease after sunset and if the early morning temperature only is used to calculate the minimum temperature for any day. These improvements were incorporated into the model, and the results showed that the root mean squared error could be reduced by 0.5°C for the 10-cm air temperature. Unfortunately, general use of the model with these improvements would be limited, since the maximum and minimum temperatures are recorded on a daily basis at most weather stations.

Sensitivity analysis

A sensitivity analysis of the model was performed by varying the model

coefficients (a , b , c) by $\pm 90\%$ about the optimum values. Figure 5 shows that changes of $\pm 50\%$ have relatively little impact on root mean squared error (increased $< 0.2^\circ\text{C}$). Results suggest that the root mean squared error for air temperatures are most sensitive to changes in a and b , that the error term for soil temperature is most sensitive to changes in c , and that the root mean squared error is relatively insensitive to fairly large changes (± 50) of the parameter values.

Model comparison

The sine-exponential model was compared with the Fourier series model and a curvilinear model, which Heuer et al. (1978) showed to be superior to the linear model of Sanders (1975). The Fourier series model is represented by the following equation

$$T_t = a_0 + \frac{R_x}{R_0} \sum_{i=1}^4 [a_i \cos(kit) + b_i \sin(kit)] \quad (3)$$

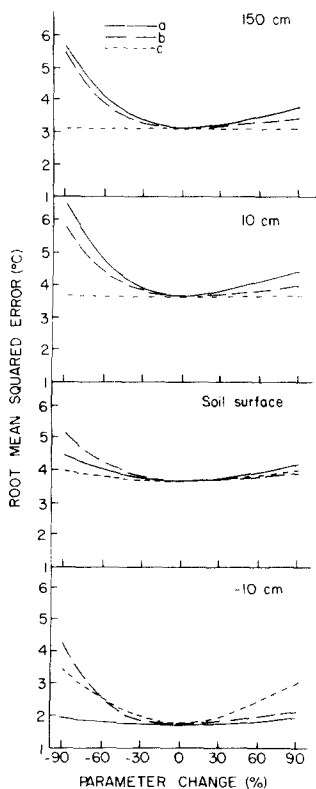


Fig. 5. Changes in the root mean squared error for the different temperature layers that result from changing the three parameters used in the model (a , b , and c).

where k is a Fourier constant describing the period of the temperature function, and a_1 and b_1 are the Fourier coefficients estimated with a standard algorithm (IBM-SSP program, FORIT, IBM, 1970). The average daily temperature (a_0) and the observed diurnal temperature range (R_0) are determined from the observed maximum and minimum temperatures. The average annual temperature range (R_x) was determined by the data-fitting process. The order of the fitted series (i) was increased until either the additional sum of squares was determined insignificant by a Fratis test or there was no further increase in the sum of squares (i never exceeded 4).

The mathematical description for the curvilinear model is shown by eqs. 4, 5, and 6

$$T_t = \frac{(T_x - T_m)\log(C_1 i + 1)}{\log(C_1 d + 1)}, \quad t_s < t \leq t_m \quad (4)$$

$$T_t = \frac{(T_m - T_x)\log(C_2 j + 1)}{\log(C_2(24 - d) + 1)}, \quad t_s > t > t_m \quad (5)$$

$$d = (\gamma C_3 - t_s) \quad (6)$$

where C_1 and C_2 are the shape coefficients for rising and falling parts of the temperature curve, γ is the day length, d is the time period for the rising part of the curve (h), C_3 is the coefficient that controls the value of d , t_s is the time of sunrise, t_m is the time when the maximum temperature occurs, i is number of hours after sunrise (rising part of curve), and j is number of hours since the maximum temperature occurred (falling part of the curve). i and j are determined as a function of the time of day, γ and t_s are calculated with equations presented by Sellers (1965), and t_m is equal to d plus t_s . C_1 , C_2 , and C_3 were determined by fitting equations to the observed data, using Powell's (1965) function-minimizing algorithm.

The sine-exponential model was compared with the other models by parameterizing these models for the 10-cm air temperature on an annual basis. The root mean square error was 4.39°C for the Fourier series fit and 4.33°C for the curvilinear model, compared with 3.62°C for the sine-exponential model. This is particularly impressive, since the sine-exponential model required only three parameters, compared with eight parameters for the Fourier series model. Our model is superior because it considers seasonal variation in day length and because the combination of a truncated sine wave and exponential function does a good job of describing diurnal temperature changes.

The three models were also parameterized monthly, which reduced the root mean square error for the sine-exponential model, the Fourier series model, and the curvilinear model by 0.06, 0.71, and 0.51°C, respectively. The more substantial decrease in error term of the Fourier series model resulted from the incorporation of the seasonal variation of day length. The sine-exponential model considers the variation in day length in both the monthly and annual versions of the model. For the monthly root mean

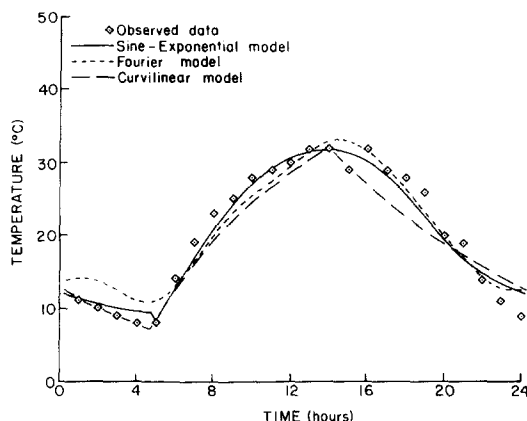


Fig. 6. Comparison of observed 10-cm air temperature data (July 6, 1977) with the simulated output for the Fourier, curvilinear, and sine-exponential models.

square error, the Fourier series model does a slightly better job of fitting the data model ($+3.50^{\circ}\text{C}$, compared with $+3.53^{\circ}\text{C}$ for the sine-exponential model and $+3.83^{\circ}\text{C}$ for the curvilinear model); however, the Fourier series model also requires many more parameters.

Figure 6 compares observed temperature data for 10 cm (July 6, 1977) with simulated model results for the monthly versions of the sine-exponential, Fourier series, and curvilinear models. All of the models adequately represent the data. The major discrepancy for the Fourier series model is an overestimate (2 to 3°C) of temperature in early morning hours. The curvilinear model best predicts temperature during the early morning hours but substantially underestimates temperature ($3\text{--}4^{\circ}\text{C}$) during the late afternoon hours. A summary of the results (Fig. 6) and the comparison of root mean square error for the different temperature models indicates that the sine-exponential model does the best job of simulating diurnal temperature changes and uses fewer parameters.

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APPENDIX

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SUBROUTINE TEMP(T,TMX,TMN,HR,A,B,C,NDAY,APHI)
C      THIS SUBROUTINE CALCULATES THE TEMPERATURE FOR A
C      SPECIFIC HOUR
C      GIVEN THE MAXIMUM AND MINIMUM AIR OR SOIL TEMPERATURE.
C      TMX=MAXIMUM TEMPERATURE
C      TMN=MINIMUM TEMPERATURE

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C      T=TEMPERATURE AT THE SPECIFIED HOUR
C      HR=HOUR FOR WHICH THE TEMPERATURE IS CALCULATED (0-24)
C      A=TIME LAG IN MAXIMUM TEMPERATURE AFTER NOON (HR)
C      B=COEFFICIENT THAT CONTROLS TEMPERATURE DECREASE AT
      NIGHT
C      C=TIME LAG FOR THE MINIMUM TEMPERATURE AFTER SUNRISE
      (HR)
C      NDAY=THE JULIAN DATE (1-365)
C      APhi=LATITUDE (RADIAN)
C      CALCULATE DAY LENGTH (ADY-HR) AND NIGHT LENGTH (ANI-HR)
      ADELTA=.4014*SIN(6.28*(NDAY-77.)/365.)
      TEM1=1.-(-TAN(APhi)*(ADELTA))**2
      TEM1=SQRT(TEM1)
      TEM2=(-TAN(APhi)*TAN(ADELTA))
      AHOU=ATAN2(TEM1,TEM2)
      ADY=(AHOU/3.14)*24.
      ANI=(24.-ADY)
C  DETERMINE IF THE HOUR IS DURING THE DAY OR NIGHT
      BB=12.-ADY/2.+C
      BE=12.+ADY/2.
      BT=HR
      IF (BT.GE.BB.AND.BT.LE.BE)GOTO 3
C  CALCULATE TEMPERATURE FOR A NIGHT TIME HOUR
      IF (BT.GT.BE)BBE=BT-BE
      IF (BT.LT.BB)BBD=(24.+BE)+BT
      DDY=ADY-C
      TSN=(TMX-TMN)*SIN((3.14*DDY)/(ADY+2*A))+TMN
      T=TMN+(TSN-TMN)*EXP(-B*BBD/ANI)
      GOTO 4
C  CALCULATE TEMPERATURE FOR A DAY TIME HOUR
      3  BBD=BT-BB
      T=(TMX-TMN)*SIN((3.14*BBD)/(ADY+2*A))+TMN
      4  CONTINUE
      RETURN
      END

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