Quant

Dainish Jabeen

August 16, 2023

1 Maths used in Quantitative Finance?

The main areas are differential equations and probability theory.

Useful mathematical tools:

- Asymptotic analysis
- Series solution
- Discretisation methods
- Green functions

2 Risk

Simplest method is using the std of portfolio returns, high standard deviation shows high risk. The central limit theory suggest the average of returns will be normally distributed allowing the use of confidence interval.

Uncertainty is when the probabilities of risk are unknown. Risk is based on known probabilities.

Operational Risk: Company and technology infrastructure risk.

2.1 Kelly Criterion

Most funds are trying to balance the growth rate via leverage with the risk via drawdown. A leverage balancing tool is the Kelly Criterion.

$$f_i = \frac{\mu_i}{\sigma_i^2}$$

- i:Strategy
- f:Vector of allocation of length N
- μ Average excess return
- σ Variation of excess return

Should be updated periodically using a trailing period over a window (3-6 months). The value can be considered as an upper bound for the leverage used.

2.1.1 Assumptions

- Gaussian return
- Focus on profit
- All profits are reinvested
- All strategies are independent

2.2 Value at Risk (VaR)

VaR is the amount that could be lost from a position, portfolio etc. Generally the maximum loss for a chosen confidence interval over a time period.

$$P(L \leq VaR) = 1 - c$$

- c: confidence level

The method used is simulate many realisations of the portfolio using monte carlo method. Using the distribution of values to define a confidence interval. Or can just use a historical data for the distribution. However, assumes normal market activity.

Time period for VaR used should have a minimum market impact; if applicable.

2.2.1 Methods to calculate

- Variance Covariance (Normal Dist)
- Monte Carlo (Non Normal Dist)
- Historic boot strapping

2.3 Crash Metrics

Portfolio assessment in extreme market conditions.

2.4 Properties of coherent risk measurement

2.4.1 Sub-additivity

$$\rho(X+Y) \leq \rho(X) + \rho(Y)$$

The risk of two portfolios combined cant be worse than the individual risks added.

2.4.2 Monotonicity

$$if\, X \leq Y\ then\, \rho(X) \leq \rho(Y)$$

2.4.3 Positive Homogeneity

$$\lambda > 0; \rho(\lambda X) \leq \lambda \rho(X)$$

2.4.4 Translation Invariance

$$constant\, c; \rho(X+c) = \rho(X) - c$$

3 Performance Measures

3.1 General

Important to compare the performance of strategies to limit opportunity cost, as capital to be allocated is limited.

General measures:

- Return
- Drawdown
- Risk
- Risk/Reward ratio

3.1.1 Returns

$$Total\ Return = \frac{P_f - P_i}{P_i} * 100$$

- P_f: Final portfolio value
- P_i: Inital portfolio value

Portfolio value over time is an equity curve.

3.1.2 Drawdown

Drops from high water marks, this being the top of an equity curve.

Maximum Drawdown: Largest percentage drop from previous peak to current/previous trough.

Drawdown duration: Period of this max drawdown event.

Can drawdown plot this over a time series.

3.2 Trade Analysis

- Number of wins and losses
- Mean profit
- Win/Loss Ratio

Trend Following trades: Uses found trends to make large trades to overcome small losses.

Pair trading mean reverting: Many small wins that can be wiped by big loss.

3.3 Risk Adjusted

Usually adjusted for risk, as the high return with low risk is better than high return with high risk.

Most popular measurement is the sharpe ratio.

3.3.1 Sharpe ratio

$$Sharpe = \frac{\mu - r}{\sigma}$$

- μ: Return over period (Must use profits)
- r: Risk free rate over period (I.e Bond interest rate)
- σ: std of returns

Poor at characterising tail risk, as based on normal dist.

Generally S>1 is bad.

3.3.2 Annualised Sharpe

$$S_A = \sqrt{N} * Sharpe$$

- N: Periods (used in Sharpe) over a year

3.3.3 Sortino Ratio

Same as sharpe but uses semi-variance which only has down side risk of to calculate the std.

3.3.4 Treynor Ratio

$$Treynor = \frac{\mu - r}{\beta}$$

- β: Systematic risk

3.3.5 Information ratio

$$Info = \frac{\mu - r}{Tracking\ error}$$

3.3.6 Calmar

$$CALMAR = \frac{E(R_a - R_b)}{max\ drawdown}$$

4 Volatility

Generally measured using annualised std of the returns. But this is not true volatility is in truth an instantaneous measurement of randomness not historic.

5 Back testing

5.1 Bias

There a few types of bias that can cause inaccuracy in back testing:

5.1.1 Optimisation bias

Overfitting of model parameters via back testing, can reduce impact via sensitivity analysis.

5.1.2 Look ahead bias

- Using data which at the time during back testing should not be available.
- Data used to train is then used to test.
- Using aggregate values ie average/max/min should be period lagged, as wont be available at the most recent time point.

5.1.3 Survivorship bias

Using only assets that have survived through a period of time. Avoid using data sets with stock delisiting.

5.1.4 Cognotive bias

Awareness of the psychological difficulty you can endure a large period of drawdown.

5.2 Exchange Issues

5.2.1 Order type

Market or limit.

5.2.2 Price Consolidation

Careful of composite feeds combining data from exchanges. As a single exchange will be used to execute the trade.

Foreign exchanges may not require the storage of the trade size and price.

5.2.3 Transaction Cost

- Commission
- Stamp Duty (UK Shares 0.5%)
- Slippage
- Market Impact (For larger orders on illiquid markets)
- Spread (bid - ask price)