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Determine the effect of distortion of Mercator chart, Gnomonic chart on maritime safety and method of navigation when using Gnomonic chart

T D Nguyen

Faculty of Navigation, Vietnam Maritime University, Hai Phong, Vietnam

E-mail: nguyenthaiduong@vimaru.edu.vn

Abstract. Nautical charts are an important tool to support safe navigation at sea. Several projections have been studied to develop charts for various maritime applications. Among them, the only Mercator projection satisfies the requirements of a nautical chart. However, the disadvantage of Mercator projection is the distortion that increases gradually to both poles. Therefore, Gnomonic charts are used to replace Mercator charts for navigation in some areas. However, the Gnomonic chart does not satisfy the basic requirements of a nautical chart, not built by an equiangular projection and the rhumb line is the curve on the chart. For building the nautical charts, the article will study the theoretical and practical basis of using Gnomonic projection instead of Mercator projection in determining the limit that the distortion of the Mercator projection affects the safe navigation, the conditions which the distortion of a Gnomonic projection has an influence on the safety of navigation in the allowable limits and finally determining the conditions and providing a method of navigation when using the Gnomonic chart instead of the Mercator chart.

1. Introduction

Nautical charts are used to manipulate estimated maritime routes and safely navigate. Although there is no official regulation from the International Maritime Organization on their specific scale, in reality, they used to navigate often have a small ratio more than 1: 500 000, including the following three basic types: the port charts, used to proceed in and out of the ports, in and out of the bridges, the ratio more than 1:50 000; the approach charts, used onshore, in anchoring areas, stern runs, narrow canals, the scale from 1: 50,000 ÷ 1: 150 000; the sailing charts, used along planned routes, with a scale of 1: 150 000 ÷ 1: 500 000 [1-5]. Nautical charts commonly used on ships are built by Mercator projection. Due to the limitations of this projection, in some areas of the sea, the Gnomonic projection is used to construct nautical charts. However, Gnomonic projection does not satisfy the requirements of the sea chart [6]. In this regard, there are a number of related published studies.

There are several studies on the distortion of the Mercator projection published in a number of basic documents on chart projection such as those of Peter Osborne [6], Daniel Daner [7]; depth study of Bertici, M. Herbei, Silvica Oncia, Laura Smuleac, Buasmv [8]; Karen Veize [9]. The above studies have published results on the distortion of the Mercator projection, the closer it is to the pole, the more the distortion is. However, there is no study determining how level of distortion to the limit will affect the safety of navigation at sea.

In addition to the aforementioned studies, there are a few other studies showing the distortion of Gnomonic projection such as those of Lysandros Tsoulos, Athanasios, Andriani Skopeliti [10]; In-depth study of L. Barazzetti, M. previtali, M. Scaioni. The results of these studies indicate that the farther the contact point of the projection, the more the distortion is. However, there is also no study showing how level of distortion will affect safe navigation.

In fact, Gnomonic projections are used to replace Mercator projections for building nautical charts in some areas (Figure 8). The issue is how to use Gnomonic chart to navigate safely and how to manipulate a ship's route when the rhumb line is a curve on the Gnomonic chart and the projection is not equiangular. This is an applied problem in maritime practice, but no research has been published on this issue.

Based on the analysis of outstanding issues on the theoretical and practical basis of using Gnomonic projection to build nautical charts, the author focused on studying three specific contents as follows:
First, determining the limited latitude that the distortion of the Mercator chart will affect the safe navigation.

Secondly, determining the conditions for the distortion of the Gnomonic projection to have maritime safety influence within the allowable limits.

Thirdly, determining the binding conditions and providing a safe method of navigation when using Gnomonic charts.

2. Mercator projection

2.1. Basic requirements of nautical charts
In planning and implementating a voyage, the navigation officer should perform on the charts some operations such as plan an established route; measure, draw azimuth and distance to the target; manipulate position lines to determine the position of the ship; ... Therefore, chart projection needs to meet the following basic requirements [9, 10]:

The rhumb line on the chart is a straight line. It cuts the meridians at equal angles and it is the ship path. The ship is always steered in a certain compass direction, then the center of gravity will shift on that direction. Therefore, the rhumb line on the chart must be a straight line so that the seafarers can manipulate the expected route, measure the direction and steaming distance on the prescribed route

The projection must be equiangular. If the projection is not equilateral, the actual angle on the earth's surface is not equal to the corresponding angle on the chart, which means that the seafarer cannot manipulate the exact direction of the ship or the bearing to the target on the chart.

2.2. Mercator projection characteristics
The Mercator projection is an equiangular cylindrical projection allowing the cylinder to contact the earth's surface at the equator, the cylinder axis to coincide with the earth axis and the projection center to be the center of the earth. After projecting all the points on the surface of the earth onto the cylinder surface, spreading the cylinder surface along the central meridian, then the results that is a projection plane (chart) reflect the entire surface of the earth

The Mercator projection is the unique one that satisfies the requirements of the nautical chart. The characteristics of the basic lines for navigation on Mercator charts are as follows [11]:

- Meridian lines are parallel lines equally spaced;
- Latitude lines are parallel lines, spaced according to Meridional Parts and perpendicular with meridians;
- The rhumb line is a straight line;
- The great circle is a curve having the concave surface returning to the equator.

2.2.1. The increase in the scale along the meridian in the Mercator projection. Satisfying the basic requirements of nautical charts, the Mercator projection has a certain distortion, the closer to the earth's pole, the more the distortion is. For equiangular conditions, the meridian arc on the earth's surface is extended according to Meridional Parts on the chart (figure 1).

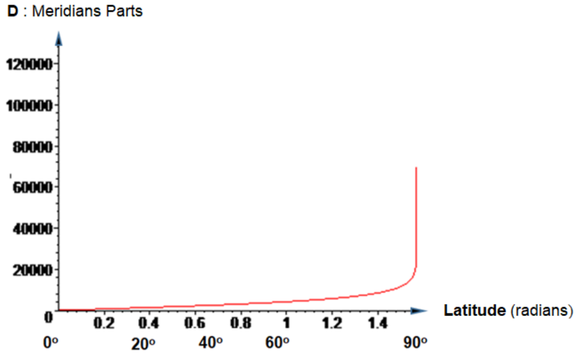

Figure 1. Graph of changes in Meridional Parts.

The meridional Part is determined by the formular:

$$D = 7929{,}915.\log_{10}\left\{\left[\tan\left(\frac{\pi}{4}+\frac{\varphi}{2}\right]\mathrm{x}\left(\frac{1-e\sin\varphi}{1+e\sin\varphi}\right)^{\frac{e}{2}}\right\} \qquad (1)$$

where: D - Meridional Parts; φ – Latitude; e - The eccentricity of ellipsoid.

The increase in the scale along the meridian, shown in graph form (Figure1).

In case of application of the earth model is the international ellipsoid WGS - 84 [12], with the parameters: semi-major axis a = 3,483,918 nautical miles, small semi-axial b = 3,432,372 nautical miles, the result of calculating the value and the increase in Meridional Parts (IF = MP / LAT) at different latitudes as the following: (table 1).

Table 1. The Increase in Meridional Part at different latitudes

Latitude	Meridional Parts	Increasing Factor
0°	0	-
10°	600,15	1,00
20°	1219,45	1,02
28°	**1743,48**	**1,04**
30°	1880,23	1,05
40°	2612,57	1,09
50°	3463,03	1,15
60°	1415,50	1,25
70°	5954,92	1,42
80°	8367,48	1,74
85°	**10760,93**	**2,11**
89°	16305,71	3,05
89°5	18692,92	3,48
90°0	**128543,35**	**23,80**

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2.2.2. The Increase in the scale in a random direction on the Mercator chart. The Meridional part represents the distortion along the meridian. In order for the projection to satisfy the equiangular condition, the parallels also extend along the equator, which increases the proportion of the Mercator chart.

Considering an extremely small region adjacent to the M (φ, λ) on the earth's surface as the MNPQ, the projection plane becomes as M'N'P'Q '(figure 2).

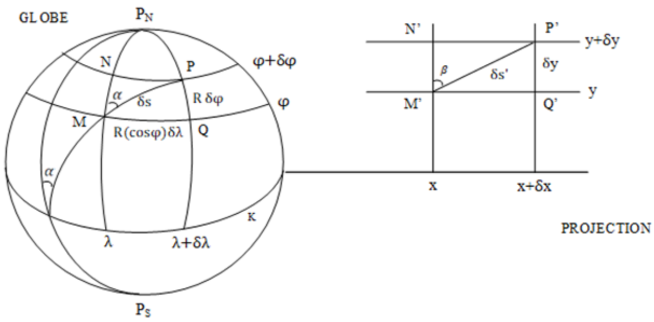

Figure 2. The increase in the scale on Mercator chart

On the Mercator chart, the meridians are parallel and perpendicular to the equator, so the distances along the line of latitude are found according to the formula x = R(λ – λo) and δx = R δλ, with:

The scale factor along the line of meridians is found as follows:

$$m_{(\varphi)} = \frac{M\prime N\prime}{MN} = \frac{\delta y}{R\,\delta\varphi} = \frac{y\prime_{(\varphi)}}{R} \tag{2}$$

The scale factor along the line of latitude is found as follows:

$$p_{(\varphi)} = \frac{M\prime Q\prime}{MQ} = \frac{\delta x}{R\cos\delta\varphi} = \sec\varphi \tag{3}$$

Satisfying the equiangular condition, the scale factor (SF) in a random direction is the same:

$$SF = \frac{M\prime P\prime}{MP} = \frac{\delta\prime s}{\delta s} = m_{(\varphi)} = \frac{M\prime N\prime}{MN} = p_{(\varphi)} = \frac{M\prime Q\prime}{MQ} = \sec\varphi \tag{4}$$

where SF: The scale factor; R: The earth radius; λo: The prime meridian in the geographic coordinate system; δφ, δλ: Latitude and longitude increments of an extremely small region adjacent to the M (φ, λ) on the earth's surface; δx , δy: corresponding values with δφ and δy on Mercator chart.

Where the earth model is ellipsoid, the scale factor is found as following:

$$SF = \sec\varphi\sqrt{1 - e^2\sin^2\varphi} \tag{5}$$

The increase in the scale on the Mercator chart corresponding to the latitude is shown in the graph below (figure 3):

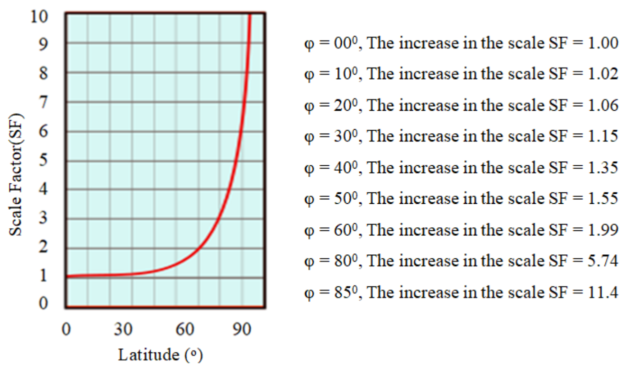

Figure 3. The increase in the scale on the Mercator chart corresponding to the latitude.

2.3. Effect of distortion on the Mercator chart on maritime safety

The distortion on the Mercator chart includes distortion along the meridian, horizontal distortion along the latitude and distortion in any direction. The increase in the scale along the meridian (table 1) indicates the distortion (error) of the distance on the Mercator chart. In accordance with resolution A.529 (13) [13] of the International Maritime Organization, the accuracy required for a given ship position shall not exceed 4% of the distance to the nearest danger point or 4 nautical miles, whichever is the bigger. Base on comparing with Table 1 and referring to Figure 4 to calculate the increasing scale factor (IF), the distance error is evaluated when determining the area probably containing the ship's position if it meets the conditions of the resolution as following

The condition which is latitude φ ≤ 28o, IF<1.04 (4%) meets the requirements of resolution A.529 (13) [12] when determining the area probably containing ship location

The another which is latitude φ ≥ 28o, IF >1.04 (4%) does not meets the requirements of resolution A.529 (13) [10] when determining the area probably containing ship location

High latitude areas (φ ≥ 60o, IF> 1.25), especially from latitude 85o and above, is an extreme distortion (IF = 2.11 ÷ 23.80). Therefore, this area was identified as a polar maritime area where often use azimuth perspective projection to build nautical charts. The navigation in this area is guided in Resolution A1024 (26) of the International Maritime Organization [14, 15].

The others area (φ = 28o ÷ 60o) is large distortion (IF = 1.04 (4%) ÷ 1.26 (26%)). This distortion may affect maritime safety depending on the ship operating area. Therefore, in some cases, the maritime safety agency developed a nautical chart using Gnomonic projection.

3. Gnomonic projection

Gnomonic projection is a perspective azimuth projection obtained by projecting all points on the surface of sphere from a sphere's center O in a plane that is tangent to a point K (figure 4).

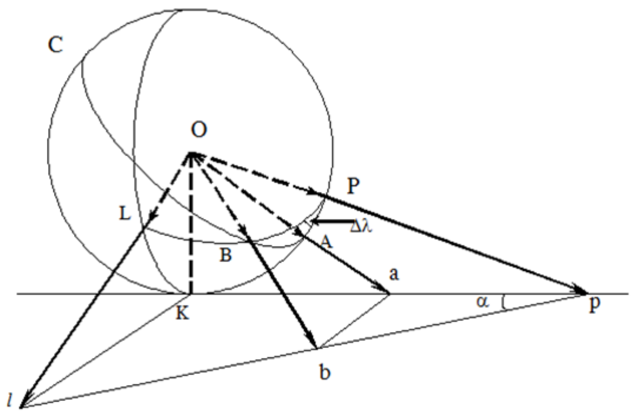

Figure 4. Gnomonic projection

Gnomonic projection does not meet the requirements of the nautical chart. The basic characteristics for navigation on Gnomonic chart are as follows:

- Meridians are the straight lines converging at a point which is the intersection of the earth axis with the projection plane
- Lines of latitude are curved,
- The rhumb line are curved,
- The great circle is a straight line.

3.1. Vertical distortion of Gnomonic projection $m_{(Z)}$

Vertical distortion of Gnomonic projection is the increase in the scale along the vertical arc, which is the intersection of the plane containing the center of Earth O and the point of contact K with the projection plane, symbolized as $m_{(Z)}$. The point M is on the vertical arc KMH, the projection of that on the projection plane is m (figure 5).

$$KM = R\,Z \Rightarrow d(KM) = R\,dZ$$

$$\widehat{Km} = R\tan\widehat{KOM} = R\tan Z \quad \Rightarrow \quad d(Km) = \frac{R\,dZ}{\cos^2 Z}$$

$$m_{(Z)} = \lim_{KM\to 0}\frac{Km}{KM} = \frac{d(Km)}{d(KM)}$$

$$m_{(Z)} = \frac{R\,dZ}{\cos^2 Z} : R\,dZ = \frac{1}{\cos^2 Z} = \sec^2 Z \tag{6}$$

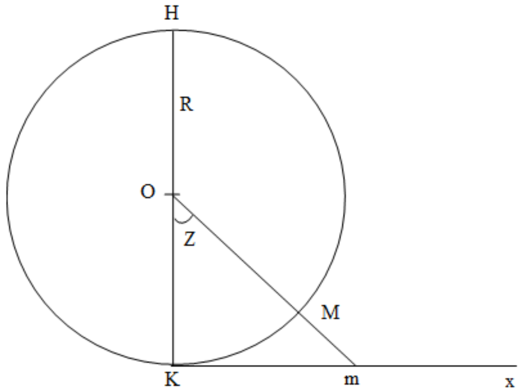

Figure 5. Vertical distortion of Gnomonic projection

3.2. Horizontal distortion of Gnomonic projection

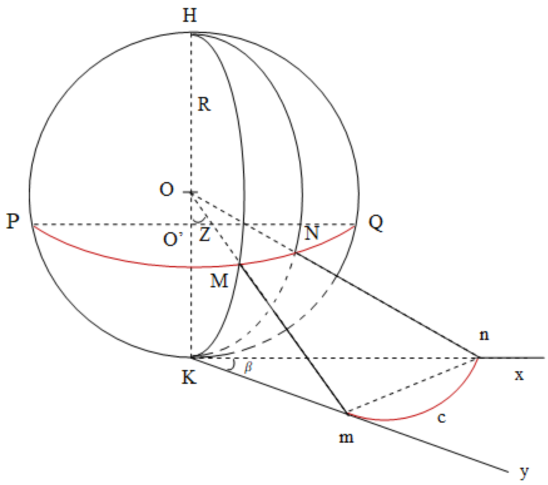

Figure 6. Horizontal distortion of Gnomonic projection

The horizontal distortion is the increase in the scale along the horizontal arc, the intersection of the plane perpendicular to the vertical axis OK at O and the earth's surface, symbolized as $p_{(Z)}$. If considering the extremely small arc MN ∈ PMQ, the projection of the arc MN on the projection plane is a curve mcn since MN is not a large arc. MN is found as below formula:

$$\widehat{MN} = O'M * \angle MO'N = R\sin Z * \beta \qquad d_{(\widehat{MN})} = R\sin Z_{.}\, d\beta \tag{7}$$

$\widehat{mcn}$ is considered as an arc, center K, radius Km ≈ Kn, and found as follows:

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$$\widehat{mcn} \approx Km * \beta \Rightarrow d_{\widehat{mcn}} = Km * d\beta = R\tan Z_{.} d\beta \quad (8)$$

So, horizontal distortion of Gnomonic projection:

$$p_{(Z)} = lim_{NM\to 0} \frac{\widehat{mcn}}{\widehat{MN}} = \frac{R\tan Z_{.}d\beta}{R\sin Z_{.}d\beta} = \frac{1}{\cos Z} = \sec Z \quad (9)$$

The distortion of a Gnomonic projection does not depend on the geographical location but on the distance to the point of contact. From (1) and (2), it shows that the further distance is away from the contact point, the larger the vertical angle Z is, the greater the vertical and horizontal distortions are. The vertical distortion is greater than the horizontal one. Table 2 is shown the distortion according to vertical angle Z:

Table 2. The increase in the scale on Gnomonic chart

Latitudes Z	**m (z) = sec^2 z**	**p(z) = sec z**
0^o	1	**1**
5^o	1.00765	1.00382
10^o	1.03109	1.01543
11^{o}3	**1.03993**	**1.01977**
20^o	1.13247	1,064
30^o	1.33333	1.15470
40^o	1.70409	1.30541
50^o	2,42028	1.55572
60^o	4.00000	2.00000
70^o	8.54863	2.92380
80^o	33.16334	5.75777
85^o	131.64610	11.47371
89^{o}9	328280.96833	**572.95809**
90^o	∞	∞

3.3. Use Gnomonic charts to navigate

Gnomonic projection does not satisfy the basic requirements of nautical charts. The rhumb line is a curve, so manipulating the courses and azimuths on the chart is impossible. Therefore, it is necessary to approximately manipulate the rhumb line which becomes the straight line on the Gnomonic chart. In essence, the rhumb line (the curve) is considered as the great circle (straight line) on the Gnomonic chart. The angle between the rhumb line and the great circle is called the Octo correction number ψ.

For caculating Octo correction number ψ (Conversion Angle) (figure 7).

The great circle cuts meridians at different angles called convergency angle C.

$$C = C_B - C_A \quad (10)$$

In the celestial triangle APNB, applying the 4-element formula in succession:

$$cotC_A = \cos\varphi_1 \tan\varphi_2 \cos e\, c\Delta\lambda - \sin\varphi_1 \cos\Delta\lambda \cos e\, c\Delta\lambda \quad (11)$$

$$cot_{.}(180^o - C_B) = \cos\varphi_{2.} \tan\varphi_{1.} \cos e\, c\Delta\lambda - \sin\varphi_{2.} \cos\Delta\lambda_{.} \cos e\, c\Delta\lambda \quad (12)$$

$$\tan C_B = \sin\varphi_2 \cos\Delta\lambda \mathrm{cosec}\Delta\lambda \text{-} \cos\varphi_2 \tan\varphi_1 \cos e\, c\Delta\lambda \quad (13)$$

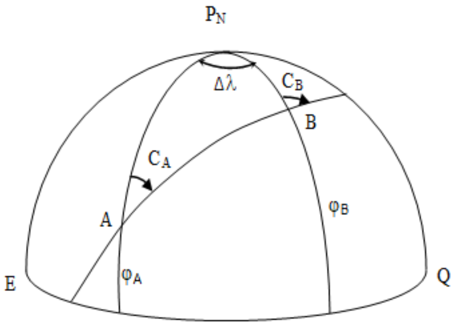

Figure 7. Convergency angle.

Thus, allowed angle error can be considered as 0^05 in maritime practice. When Octo correction number ψ is less than 0^o5, it considered that the rhumb line coincides with the great circle. Gnomonic chart can be used to safe navigate in that case. The latitude limit $\varphi = 28^0 \div 60^0$ is calculated in section

4. Conclusion

In this article, the author has studied the theoretical and practical basis of the use of Gnomonic projection to build nautical charts. Specific results are summarized as follows:

Firstly, the distortion of Mercator projection is analyzed and the limit that this distortion affects maritime safety is determined in the latitude range more than 280. However, the polar area often uses azimuthal projection to build nautical charts. Therefore, that area used Gnomonic projection to replace Mercator projection in latitude range from 280 to 600

Secondly, the distortion of Gnomonic projection is analyzed and the limit that this distortion affects maritime safety at the allowed level is determined at the vertical angle (Z) less than 1103, corresponding to the distance to the contact point of the projection shall not exceed 600 nautical miles

Thirdly, based on the regulations of the International Maritime Organization and the maritime practice, the error of direction and azimuth operation on the chart is determined as 005. Then, conditions that allow to consider the rhumb line coincide the the great circle on Gnomonic chart is also determined. That is the projection area with the longitude difference less than 201.

In short, the conditions for using Gnomonic projection in place of Mercator projection are latitude in the range from 280 to 600, longitude difference less than 201 and the furthest distance to the contact point not more than 600 nautical miles (vertical angle Z ≤ 1103). If satisfying these conditions, the navigation practice on the Gnomonic chart is exactly the same as the Mercator chart.

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