RC Filter; Numerical Simulation & Experiment Comparison Write Up

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Abstract

Application of filters to pass either low or high ranges of frequencies is ubiquitous in electronics for controlling signal information. Resistor-capacitor (RC) circuits provide common first-order filter input frequencies to select a desired range. In order to validate the real life RC circuit design, numerically modeling through differential equation methods derives equations to find the voltage at any instantaneous time of the circuit. This write-up models and experimentally tests low-pass and high-pass RC filters on breadboard with control input by a DDS signal generator and high quality data collected by an oscilloscope and its support software. The low-pass model showed agreement with experimental results, with a reduced χ^2 of 0.198 over 3198 degrees of freedom, confirming thorough data matching. The high-pass configuration was well-predicted by simulations, achieving reduced χ^2 of 0.42. Close alignment between simulations and measured responses supports using tractable RC equations to aid and understand the design optimization when building and targeting the necessary range of frequencies.

I. Introduction

Resistor-capacitor(RC) circuits or RC filters utilizing a resistor and capacitor are fundamental circuits used for filtering input signals by cutting off a certain range of frequencies. Determining the behavior of these filters for real-world applications requires numerical modeling to predict performance along with physical experimentation to verify models. In this work using the same input voltage and frequencies, an RC filter was numerically simulated on Google Sheet. The model was then compared to the experimental frequency captured by an oscilloscope from a physical breadboard RC circuit.

The two common RC filters are low-pass filters and high-pass filters. The location of the resistor and capacitor determines the type of filter - whether it allows ranges of low or high frequencies to pass through. Low-pass and high-pass RC filters were built on two separate breadboards. A square wave signal was input into each circuit and the output wave-forms were recorded using an oscilloscope.

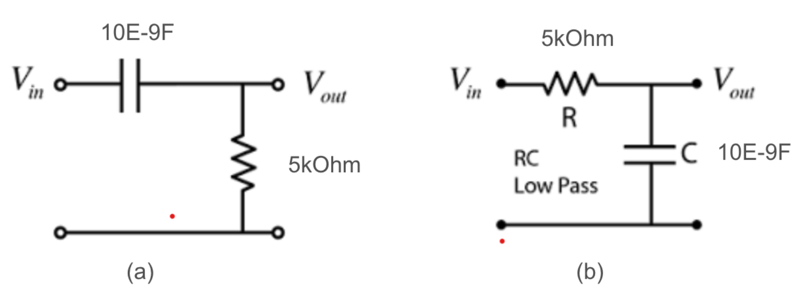

Fig. 1: RC filters with 10 nF capacitor and 5 kΩ resistor, (a) a high-pass filter with positive bridged by the resistor to ground, (b) a low-pass filter with positive bridged by the capacitor to ground.

The circuit contains a capacitor and the "application of the capacitor rules give"[1] the equation (1) and (2) where at time t = 0 of the current. Transforming the equation with finite differential methods from estimate the derivative of $I(t)$ at Δt [2] solved for any instantaneous voltage of capacitor at time t, $V_C(t)$ where $\alpha = \frac{RC}{\Delta t}$. Vice versa for finding the equation for any instantaneous voltage of resistor at time t $V_R(t)$ where $\beta = \frac{RC}{\Delta t}$

$$I = C\frac{dV}{dt} = \frac{V_{in} - V_C}{R} \Leftrightarrow V_C(t) = \frac{V_{in}(t) + \alpha V_C(t-1)}{\alpha - 1} \tag{1}$$

$$I = C\frac{d(V_{in} - V_R)}{dt} = \frac{V_R}{R} \Leftrightarrow V_R(t) = \frac{\beta(V_{in}(t) - V_{in}(t-1) + V_R(t-1))}{\beta - 1} \tag{2}$$

The voltages across the capacitor ($V_C(t)$) and resistor ($V_R(t)$) in the RC filter circuit are named to describe the output voltage ($V_{out}(t)$) in waveform. (1) and (2) equations provides the basis for numerically modeling the output voltage waveform over time for the RC filters.

The write up provides both numerical and physical experimental results for two types of RC filter. The goal is to compare the simulated frequency response to lab measurements to assess model accuracy. Differences will be discussed and potential explanations provided to inform future modeling enhancements. The analysis demonstrates the iterative process between simulation and experimentation in circuit design and provides context for refining models to get closer agreement with measured data. By improving modeling fidelity, development time for RC filter circuits can be shortened, benefiting numerous applications relying on these fundamental filters.

II. Experimental Setup

Our source (V_{in}) generated from a Waveform DDS TTL Signal generator where the signal was set with the following settings.

- Amplitude (AMPL): 05.00V
- Waveform (WAVE): Square
- Offset (OFF): 0.00V
- Duty cycle (DUTY): 50.0%
- Frequency (FREQ): 1591Hz (temporarily set)

Resistors of 5 $k\Omega$ and capacitors of 10 nF were used to build the RC circuits on breadboards. Breadboards were chosen to minimize potential reflections from long wires that would otherwise be attached to each circuit element. With the circuit components and topology defined from Fig. 2 and Fig. 3, the cutoff frequency f_{3dB} for both RC filters can be calculated analytically based on the chosen resistor and capacitor values.

$$f_{3dB} = \frac{1}{2\pi RC} = \frac{1}{2\pi * (5 \times 10^3) * (1 \times 10^{-8})} = 1591Hz \tag{3}$$

Make sure that channel 1 (CH1) is recording the input voltage (V_{in}) of the signal and channel 2 (CH2) is recording the output signals on the oscilloscope. There is no need to wire the ground to CH2 when the ground is already connected to CH1. To record higher quality data, the Hantek oscilloscope this experiment used has a compatible software called DigitalScope[3] (for Windows devices) that can connect with the physical device and record the waveform directly rather than graphically record from the scope.

Before inputting a signal into the circuits, the oscilloscope data was recorded without any input to estimate the variance and standard deviation of the background noise level. This provided a baseline measurement of the oscilloscope noise that could later be used in analyzing the experimental RC circuit data. The standard deviation of the oscilloscope noise will allow the significance of the measured signal

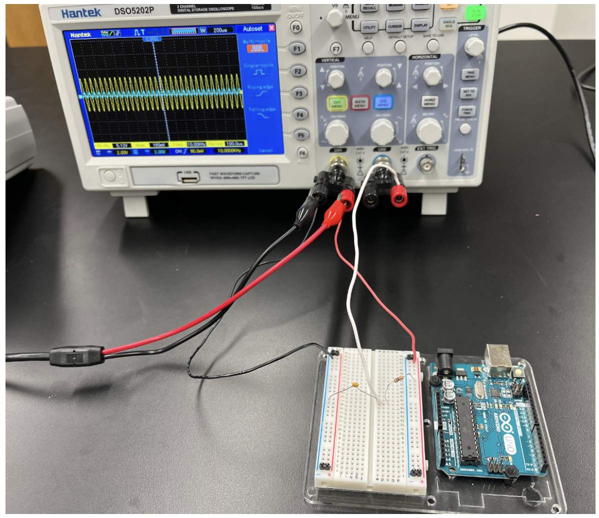

Fig. 2: Low-pass RC filter setup with DDS Signal generator (off the image to the left) and an oscilloscope.

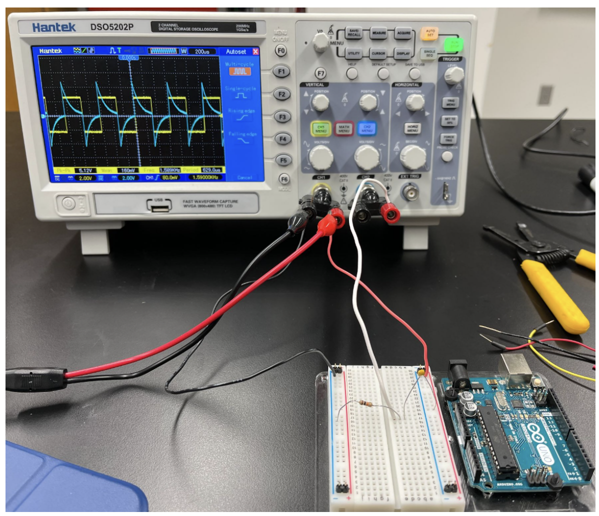

Fig. 3: High-pass RC filter setup with DDS Signal generator (off the image to the left) and an oscilloscope.

data to be quantified by comparison to the underlying noise floor. By accounting for the inherent noise, more meaningful comparisons can be made between the modeled and experimentally obtained RC circuit wave-forms. The oscilloscope variance and noise characterization thus facilitates better analysis when validating the circuit simulations against physical measurements.

III. Results

III.I. Low-pass RC filter

Top of Fig. 4 Voltage vs. Time shows the changes between the input at frequency f_{3dB} and the output of the signal. Frequencies that are lower than the cut-off frequency will allow to pass through the circuit where there are less cut-off from happening. The cut-off happens when the frequency is larger than the f_{3dB} because of the help of an RC producing a delayed digital waveform[1]. Fig. 4 has a time delay that is larger than half of the period of the square wave causing the output to the right by more than half of the wave length. The shift happens depends on the level of the frequencies fitting with the low-pass RC filters.

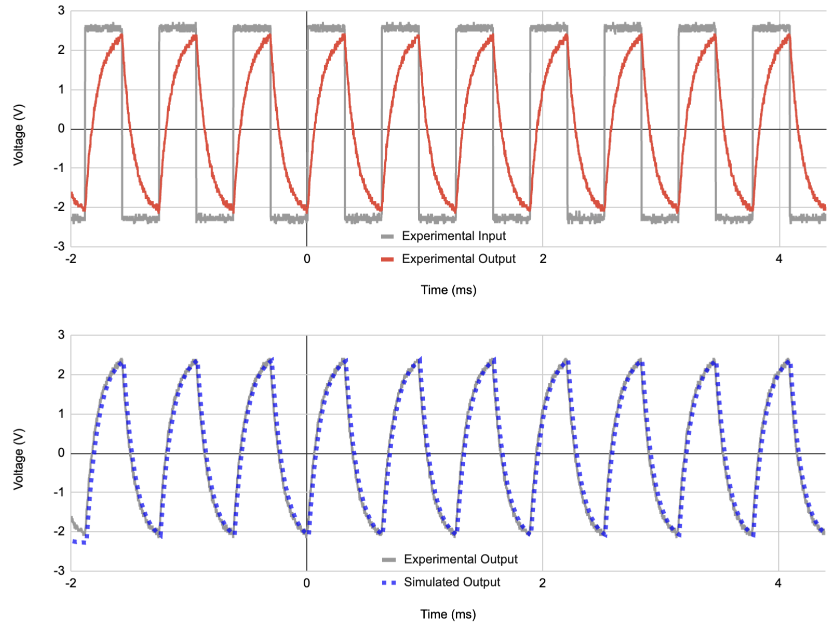

Fig. 4: Top Solid Grey: Experimental input signal wave frequency of 1591 Hz generated by DDS with a resolution of time interval of 2 μs. Solid Red: Experimental output signal after passing through the low-pass RC circuit. Dashed Blue: Simulated output.

Time (ms)	Experimental Input (V)	Experimental Output (V)	Simulated Output (V)
-2	-2.24	-1.6	-2.24
-1.998	-2.32	-1.68	-2.241568627
-1.996	-2.24	-1.68	-2.24153787
-1.994	-2.32	-1.6	-2.243076343
-1.992	-2.32	-1.76	-2.24458465
...	...	...	...

Table 1: The time negative here is because it is relative to the cursor that had set on the oscilloscope. Partial table for experimental data and simulated output for voltage and time of the low-pass RC circuit.

Table 1 contains partial data collected from the experiment along with the simulated output voltage. The simulation uses only the experimental input voltage data and a time interval of $\Delta t \approx 0.002ms$ and $\alpha = 50$. Using the equation (1) and initializing $V_C(0) = V_{in}(0) = -2.24V$, the output voltage over time is calculated by the model. As observed, the simulated output closely matches the experimentally measured waveform.

III.II. High-pass RC filter

In Fig. 5 where the top graph displays the input and output signal of the experiment with the 1591 Hz (cut-off frequency). Unlike low-pass, the circuit of high-pass only allow high frequency signal to pass through. With a lower frequency than f_{3dB}, the signal output will not disappear and output to be a much lower amplitude compared to the input. If the frequency is significantly higher than cut-off frequency then the whole signal will result to be similar at the output compared to the input. Instead of the time delay in low-pass circuit, the high-pass has the characteristic of "leading edges and trailing edges in pulse signals[1]" which can be observed with the spike between the transition of the square wave in Fig. 4.

Table 2 similar to Table 1 also contains partial data collected from the experiment along with the simulated output voltage. The simulation uses only the experimental input voltage data and a time

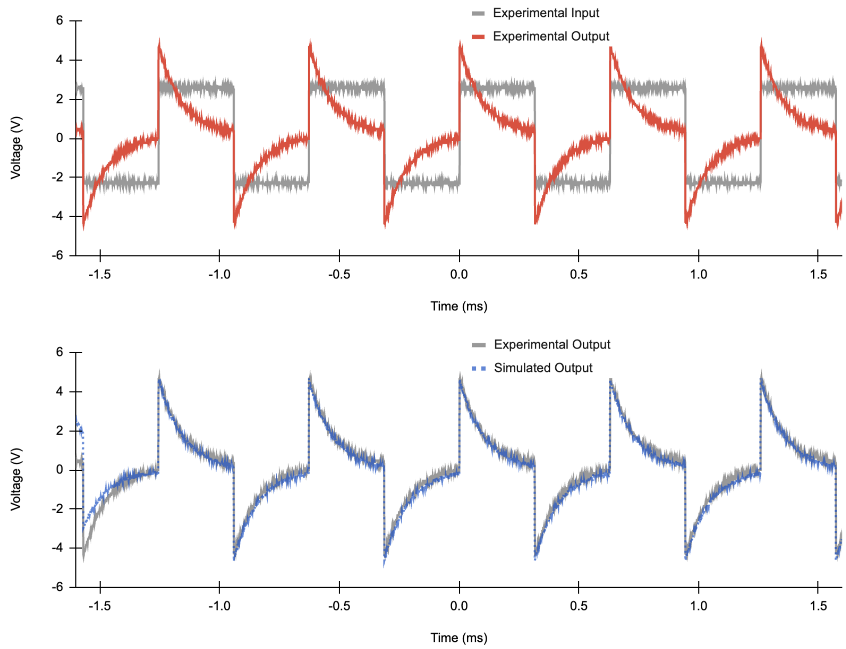

Fig. 5: Top Solid Grey: Experimental input signal wave frequency of 1591 Hz generated by DDS with a resolution of time interval of 4 μs. Solid Red: Experimental output signal after passing through the low-pass RC circuit. Dashed Blue: Simulated output.

Time (ms)	Experimental Input (V)	Experimental Output (V)	Simulated Output (V)
-1.6	2.48	0.48	2.48
-1.599	2.48	0.4	2.455445545
-1.598	2.56	0.48	2.510342123
-1.597	2.48	0.4	2.40627933
-1.596	2.56	0.48	2.461662703
...	...	...	...

Table 2: The time negative here is because it is relative to the cursor that had set on the oscilloscope. Partial table for experimental data and simulated output for voltage and time of the high-pass RC circuit.

interval found to be $\Delta t \approx 0.001ms$. The founded Δt used to find $\beta = 100$. Using the equation (2) and initializing $V_R(0) = V_{in}(0) = 2.48V$. As observed, the simulated output also matches the experimentally measured waveform.

IV. Discussion

IV.I. Low-pass RC filter

A standard deviation σ found from measuring the background noise from the scope $\sigma \approx 0.5644V$ and we use χ^2 to check for the goodness of fit with number of significant to be 5%. Even though Table 1 does not reflect the full size of the data set, the data set has the size of $N = 3198$ which is substantially large and this allow us to use the χ^2 test[4].

$$\chi^2 = \sum_{i=1}^{N} \frac{(V_{Experimental\ ouput}(i) - V_{Simulated\ output}(i))^2}{\sigma^2} \tag{4}$$

For low-pass RC experimental and simulated output the $\chi^2 \approx 63.374$, degree of freedom to account for is 3198 where $\chi^2_\nu = \frac{69.443}{3198} \approx 0.0217$. A reduced chi-squared that is much less than 1 typically means that the uncertainties in the data measurements are overestimated. So it suggests the actual measurement errors are smaller than what was founded from the background noise above . Since our χ^2_ν is less than 1, we can conclude immediately that agreement between our observations (experimental data and the simulated data) and the expected Poisson distribution is satisfactory[4]. The data were good and the error would be expected to be from device error and the expectation to see the simulated model to be a perfect fit with our experiment is guarantee. The probability of our χ^2_ν is found to be a solid 1. This can be explained by our large number for degree of freedom. A probability of 1 means that there is 100% chance of repeating the experiment and getting the same result, and justifying the numerical equation we found in equation (1) for any low-pass RC circuit.

IV.II. High-pass RC filter

With the same standard deviation found from the noise background mentioned in low-pass RC and the same data set size, the chi square for high-pass RC circuit is $\chi^2 \approx 1341.779$ found from equation (5). The degree of freedom for high-pass is the same with low-pass and $\chi^2_\nu \approx 0.42$. Again, we immediate satisfy our agreement for this case. The goodness-of-fit for a reduced chi square that close to 0.5 tells us that the variance in the data is well approximated. It means that the true variance σ^2 is very close to s^2[5]. The data amount are the same with the previous case and since we also derived the numerical equation (2) for the high-pass RC circuit, the simulation is expected to be a good fit with our experiment. The probability of our χ^2_ν is also found to be a solid 1 but we have more confidence for this to be true.

References

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