Medical Statistics

Homework 4

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1. Exercise 1

1.1 The t Test for Comparing Two Means

The cholesterol (mmol/L) is a continuous value with a normal distribution in population. 24 volunteers recruited were completely randomly divided into two groups with 12 individuals for each. If we want to judge whether the pre-study cholesterol level for the two treatments are equal on average, the t test for data of two pre-study cholesterol level under completely randomized design can be used.

First, we perform an F test to see if two cholesterol level variances are equal. The null hypothesis is $H_0 : \sigma_1^2 = \sigma_2^2$, the alternative hypothesis is $H_1 : \sigma_1^2 \neq \sigma_2^2$. The value $F \approx \frac{S_1^2}{S_2^2} = 0.344$, $P = 0.564 > 0.1$. As a result, we cannot reject null hypothesis. That is to say, the difference between two cholesterol level variances is not statistically significant.

It is reasonable to assume that $\sigma_1^2 = \sigma_2^2$, since the difference between two cholesterol level variancess is not statistically significant and S_1^2 and S_2^2 are close to each other. ($S_1^2 = 0.74$, $S_2^2 = 0.61$)

Then we perform the t test for the data of two pre-study cholesterol level. The null hypothesis and alternative hypothesis are:

$$H_0 : \mu_1 = \mu_2,\ H_1 : \mu_1 \neq \mu_2 \tag{1.1}$$

$$S_c^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 1} \tag{1.2}$$

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{S_c^2 \left(\frac{1}{n_1} + \frac{1}{n_2} \right)}} = 0.219 \tag{1.3}$$

$$\nu = n_1 + n_2 - 1 = 23 \tag{1.4}$$

Check up the table of t distribution, $P = 0.829 > 0.05$. Do not reject the null hypothesis. The difference between the pre-study cholesterol level means of two treatments is not statistically significant.

1.2 The t Test for Data under Randomized Paired Design

Whether the two treatments are effective can be analysed using the cholesterol level differences of pre-study and post-study, set as d. Assuming d follows a normal distribution, a zero mean will indicate that the treatment is not effective. Denote the population mean of d with μ_d.

$$H_0 : \mu_d = 0,\ H_1 : \mu_d \neq 0 \tag{1.5}$$

*Github repo: https://github.com/MoRunbing/Medical_Statistics

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$$S_d = \frac{\left[\Sigma d_i^2 - \frac{1}{n}\left(\Sigma d_i\right)^2\right]}{n-1} \tag{1.6}$$

$$t = \frac{\bar{d} - 0}{S_d/\sqrt{n}} \tag{1.7}$$

$$\nu = n - 1 \tag{1.8}$$

For Group A who received a special diet, $t = 3.150$, $P = 0.009 < 0.05$. Reject the null hypothesis. The difference before and after receiving a special diet is statistically significant. That is to say, a special diet is effective in reducing cholesterol level.

For Group B who received a medical therapy, $t = 2.411$, $P = 0.035 < 0.05$. Reject the null hypothesis. The difference before and after receiving a medical therapy is statistically significant. That is to say, a medical therapy is also effective in reducing cholesterol level.

1.3 The t Test for Comparing Two Means of Two Differences

To analyse whether the effects on reducing cholesterol are equal on average, we perform a t test for d of two groups under completely randomized design.

First, we perform an F test to see if d variances of two groups are equal. The null hypothesis is $H_0 : \sigma_1^2 = \sigma_2^2$, the alternative hypothesis is $H_1 : \sigma_1^2 \neq \sigma_2^2$. The value $F \approx \frac{S_1^2}{S_2^2} = 1.118$, $P = 0.302 > 0.1$. As a result, we cannot reject null hypothesis. That is to say, the difference between d variances is not statistically significant.

Then we perform the t test for d under completely randomized design. The null hypothesis and alternative hypothesis are:

$$H_0 : \mu_{d1} = \mu_{d2},\ H_1 : \mu_{d1} \neq \mu_{d2} \tag{1.9}$$

$$S_d^2 = \frac{(n_1 - 1)S_{d1}^2 + (n_2 - 1)S_{d2}^2}{n_1 + n_2 - 1} \tag{1.10}$$

$$t = \frac{\bar{X}_{d1} - \bar{X}_{d2}}{\sqrt{S_d^2\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}} = -1.773 \tag{1.11}$$

$$\nu = n_1 + n_2 - 1 = 23 \tag{1.12}$$

Check up the table of t distribution, $P = 0.090 > 0.05$. The difference between the differences before and after two treatments is not statistically significant. As a result, there is no significant difference between effects on reducing cholesterol of a special diet and a medical therapy.