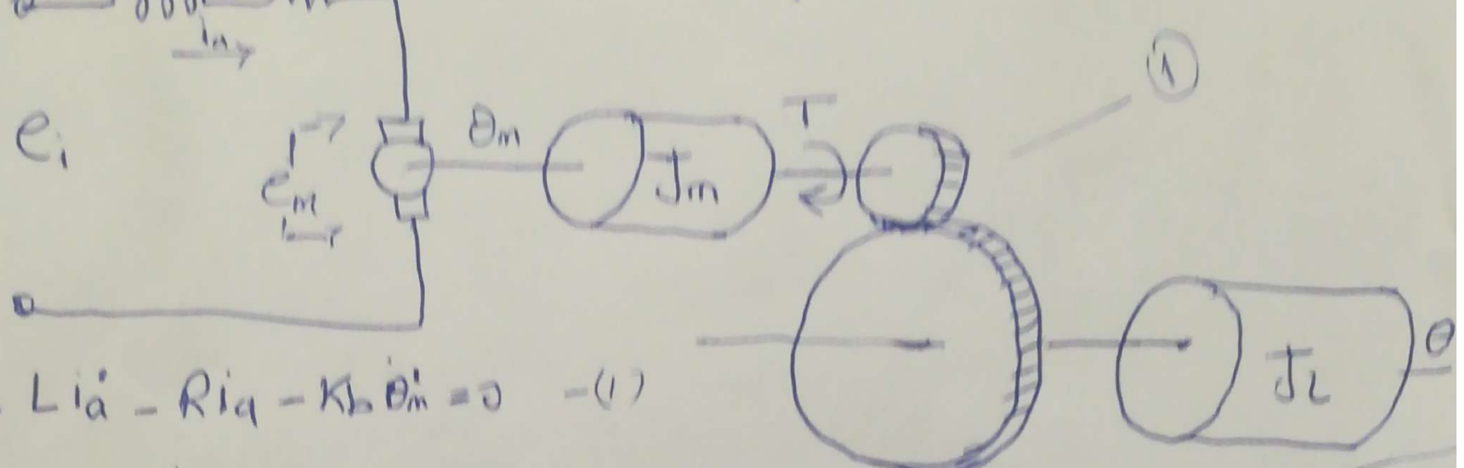
Example 1C



$$e_i - L\dot{i}_a - Ri_a - K_b\dot{\theta}_m = 0 \quad (1)$$

$$J_m\ddot{\theta}_m = Ki_a - T \quad (2)$$

$$T = \frac{\theta}{\theta_m}T_L = nT_L \quad (3)$$

K_b = back emf constant ($e_m = K_b\dot{\theta}_m$)
K = Torque constant ($T_m = Ki_a$)

* The back emf of the motor is directly proportional to the speed. (e_m) = $K_b\dot{\theta}_m$

* The torque generated by the motor is directly proportional to the current. → Ki_a

load eqn.

$$T_L = J_L\ddot{\theta} \quad (4)$$

sub 4,3 → $J_L\ddot{\theta} = \frac{T}{n}$ (5)

sub 5 in 2

$$J_m\ddot{\theta}_m = Ki_a - nJ_L\ddot{\theta}$$

$$Ki_a = nJ_L\ddot{\theta} + J_m\left(\frac{1}{n}\ddot{\theta}\right)$$

$$nKi_a = (n^2J_L + J_m)(\ddot{\theta}) \quad (6)$$

* for shaft 1

$$\Sigma M = J_m\ddot{\theta}_m = \overset{\text{motor}}{Ki_a} - \overset{\text{load}}{T}$$

* for shaft 2

$$\Sigma m = J_L\ddot{\theta} = \overset{\text{from 1st shaft}}{T_L}$$

1A TF

Example 5 : Obtain TF of 1A

$$m_1 x_1'' + b x_1' + (K_1+K_2)x_1 = b x_2' + K_2 x_2 + u$$

$$m_2 x_2'' + b x_2' + (K_2+K_3)x_2 = b x_1' + K_2 x_1$$

$$s^2 m_1 X_1 + sbX_1 + (K_1+K_2)X_1 = bs X_2 + K_2 X_2 + u$$

$$(m_1 s^2 + sb + K_1+K_2) X_1 = (bs+K_2)X_2 + U(s) \quad \text{— (1)}$$

$$(m_2 s^2 + bs + (K_2+K_3)) X_2(s) = (bs+K_2) X_1(s) \quad \cancel{+U(s)} \quad \text{(2)}$$

$$X_1 = \left(\frac{(bs+K_2)X_2 + U(s)}{m_1 s^2 + sb + K_1 + K_2}\right) \quad \text{— (3)}$$

Sub by 3 in (2)

$$(m_2 s^2 + bs + K_2 + K_3) X_2 = (bs+K_2)\left(\frac{(bs+K_2)X_2 + U(s)}{m_1 s^2 + sb + K_1 + K_2}\right)$$

$$\left(m_2 s^2 + bs + K_2 + K_3 - \frac{(bs+K_2)^2}{(m_1 s^2 + sb + K_1 + K_2)}\right) X_2 = \frac{(bs+K_2)}{m_1 s^2 + sb + K_1 + K_2} U(s)$$

$$\frac{X_2}{U(s)} = \frac{\dfrac{(bs+K_2)}{m_1 s^2 + sb + K_1 + K_2}}{m_2 s^2 + bs + K_2 + K_3 - \dfrac{(bs+K_2)^2}{m_1 s^2 + sb + K_1 + K_2}} \times \frac{m_1 s^2 + sb + K_1 + K_2}{''}$$

$$= \frac{bs + K_2}{(m_2 s^2 + bs + K_2 + K_3)(m_2 s^2 + bs + K_2 + K_3) - (bs+K_2)^2} \quad \text{— 7}$$

From (2)

$$X_2 = \frac{bs + K_2}{m_2 s^2 + bs + K_2 + K_3} X_1 \rightarrow \text{Sub in (7)}$$

$$\frac{X_1}{U(s)} = \frac{m_2 s^2 + bs + K_2 + K_3}{(m_2 s^2 + bs + K_2 + K_3)(m_2 s^2 + bs + K_2 + K_3) - (bs+K_2)^2}$$

Tutorial 2

Example 5: TF of 1B

$$CR_2 \dot{e}_i + CR_2 \dot{e}_o + e_o - e_i = 0$$

$$sCR_2 E_i(s) + sCR_2 E_o(s) + E_o(s) + E_i(s) = 0$$

$$E_o(s) \cdot (sCR_2 + 1) = E_i(s)(1 - sCR_2)$$

$$\frac{E_o(s)}{E_i(s)} = \frac{1 - sCR_2}{1 + sCR_2} = \frac{\frac{1}{CR_2} - s}{\frac{1}{CR_2} + s} = -\frac{s - \frac{1}{CR_2}}{s + \frac{1}{CR_2}}$$

Example 5 : Obtain TF of 1C

1C 7+

$J_m + n^2 J_L \ddot{\theta} = nK i_a$

$L \dot{i}_a + R i_a + K_b \dot{\theta}_m = e_i$

$E_i(s) = (R + L s) I_a(s) + K_b s \theta_m(s)$

$J_m + n^2 J_L s^2 \theta(s) = nK I_a(s)$

$$n = \frac{\theta}{\theta_m}$$

$$E_i(s) = (Ls + R) \frac{J_m + n^2 J_L s^2}{nK} \theta(s) + \frac{K_b s}{n} \theta(s)$$

$$E_i(s) = \left((Ls + R) \frac{J_m + n^2 J_L s^2}{nK} + \frac{K_b s}{n} \right) (\theta(s))$$

$$\frac{\theta(s)}{E_i(s)} = \frac{1}{(Ls + R)(J_m + n^2 J_L s^2) + K K_b s}$$